The use of mixed integer linear programming for long-term scheduling in block caving mines

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Caving is a mass mining method that is normally applied to large, low grade orebodies because of its low production cost and high capacity. However, equipment breakdown and poor draw management can cause mining costs to exceed that of all other mining methods. One way to minimize the effect of disruptions in the mining cycle is to use Mixed Integer Linear Programming (MILP) for long-term scheduling.

This paper describes a MILP goal program which has the dual objectives of minimizing deviation from the ideal draw profile while achieving a production target. Schedule optimization is performed using a life of mine approach in which all production periods are optimized simultaneously. The underlying assumption being that material mixing in the short-term (between monthly optimization cycles) has a minimal effect on the long-term state of the panel.

Schedule optimization is carried out using six general constraint types to model and control production. The constraint types define: deviation from ideal practice, panel state, material flow conservation, production quality, material flow capacity and production control. These constraints define production targets and drive production toward the ideal profile while maintaining correct mining practice.

Application of the model using production scheduling data for a De Beers kimberlite mine. It is shown how different production control constraints regulate production from individual drawpoints as well as recovery of the ideal panel profile by implementing an optimized draw schedule.

Introduction

The math programming model and results described herein are part of a larger research project concerned with draw control in mass mining operations. The International Caving Study (ICS II) is now in its second three-year phase of funding by a consortium of major mining companies involved with block and sublevel caving (SLC) operations. In the context of this paper, draw control refers to the long-term scheduling of production from block and panel cave drawpoints. The requirements for sublevel caving are significantly different and are not discussed herein. This paper reports specifically on results arising from ICS II R&D associated with a prototype production scheduling MILP for block caving originating from a model developed by De Beers (Guest, et al.) that was made available to researchers at the Julius Kruttschnitt Mineral Research Centre (JKMRC) as part of De Beers’ involvement as an ICS II sponsor.

There have been few applications of Math Programming technologies applied to underground mining in general (cf. Smith, et al.) and even fewer applied to mass mining: including an application in block cave drawpoint scheduling aimed at production control by Chandra and a SLC application for Kiruna developed by Almgren. Both applications used binary variables in an MILP formulation to schedule and sequence production in mass mining operations, but in both applications the emphasis was on production rather than on draw control as in the case for De Beers’ MILP and the subsequent ICS II MILP.

Draw control

Cave mining started at De Beers in the early 1950s with the first installation at the Bulfontein Mine in Kimberley. Its success has been such that it continues to be used in a number of De Beers’ mines (Figure 1). De Beers recognized in the early days of block caving that block cave operations needed to strictly adhere to correct draw control (Gallagher and Loftus, 1960). This realization is acknowledged in the operation’s slogan, ‘Draw Control is King’. Failure to adhere to strict draw control practice has resulted in serious ground control and grade problems. As a result, mine management has expended a lot of effort and capital in the development and implementation draw control systems over the years. Guest et al. describe the evolution of the draw control systems within De Beers Consolidated Mines. De Beers currently uses Mixed Integer Linear Programming (MILP) based long-term scheduling tools for a number of its caving operations. The more recent short-term scheduling systems can handle daily reconciliations and shift production planning in addition to logging production data.

Current practice is to monitor drawpoints on a daily basis with the following information collected into the mine database:

- Daily tonnage (planned versus actual as recorded by the vehicle monitoring system)
The critical draw control parameters constraining production and their purpose are:

- **Relative draw rate between adjacent drawpoints**—Ensure interactive draw and limit the depletion difference between adjacent drawpoints. Interactive draw aims to pull neighbouring drawpoints simultaneously to promote equal draw down over the entire footprint. Minimizing depletion level deviation between adjacent drawpoints avoids early waste ingress by maintaining a smooth ore/waste interface.

- **Minimum draw rate**—Eliminate point loading by avoiding ore recompaction in the drawpoints.

- **Maximum draw rate** (Maturity rule)—Limit the maximum draw rate to ensure that the cave advance rate does not exceed the undercut rate. Also, the draw rate is chosen so that the secondary fragmentation process within the cave (induced by mechanical interaction of the caved material) reduces secondary breakage by blasting.

- **Waste limit**—Waste content in the drawpoint cannot exceed 15% for more than 10 working days. This constraint is imposed because processing plant efficiency drops dramatically with increasing waste percentage and consistent waste in a drawpoint indicates that ore-waste interface has reached the production level. Pulling the drawpoint further could promote waste ingress in neighbouring drawpoints.

The implementation of these constraints in the MILP and their effect on panel production are described in the remainder of this paper.

### Cave scheduling model

The goal of a long-term scheduling model is to find a solution to the draw control scheduling problem in terms of the tonnage to be drawn from individual drawpoints for a sequence of planning periods spanning the life of the geologic resource. As such, the model must define: a function for optimization, the materials handling system and flow through that system, as well as the constraints which force production to adhere to allowable draw practice as described above.

The objective of the goal programming model is to reduce deviations in both the level of ideal depletion and the deviation from a target production rate. If both the ideal profile and production rate are maintained the objective function evaluates to zero.

This type of formulation makes the constraints defining the deviation from the ideal depletion and production fundamental to determining an optimized schedule. The definition of these variables for ideal depletion and production are shown in Figure 2 and Figure 3 respectively.

The variables (blockDev+, blockDev-, Prodev+, and Prodev-) are defined such that if the '+' variable is non-zero then the '-' variable is zero. This relationship holds in reverse. The model then minimizes the sum of the ideal depletion and production deviation variables for all time periods simultaneously in order to stay as close to the ideal draw plan as possible. Although the primary purpose of the scheduling model is to maintain the shape of the ore waste contact within the panel while achieving a target production rate. There are a large number of additional constraints required to represent the mine production system.

There are often more than one source of ore material in a mining operation. This is especially true of mature operations that are in transition between different ore deposits as well as operations that are changing mining method. This requires constraints to handle the contents of alternate ore sources such as dumps, stockpiles and other mining operations in addition to the blocks within the active cave. These constraints ensure that the material remaining in the source at the end of a production period equals the amount at the beginning of the period less the amount transferred out of the source during the period. Likewise, there must be continuity between periods by ensuring that the amount in a source at the beginning of a period is equal to the closing amount of the previous period.

The total production from an individual source during a given period is not the only material flow that must be...
accounted for in optimizing a production schedule. The movement of individual types of material must be tracked if constraints on grade, waste, and/or rock types are to be implemented. The mass transfer of a component material is its fraction of the total material moved. This material flow is conserved as it moves from its source to its final destination (the plant). These constraints treat each material source and the parts of the materials handling system as nodes within the production network.

The network flow constraints state that the material flow through a node must equal the sum of its flow to all nodes which it feeds. They also ensure that the flow through a node equals the total flow from all nodes which feed it. An example of these constraints is the total flow through a tunnel must equal the total amount transported from that tunnel into orepasses and it must equal the flow from all drawpoints in that tunnel.

The node constraints dealing with flow through the production system (rather than specific sources of material) is the quality and quantity constraint set. This type of constraint has a dramatic effect on production. It is used to regulate flow through the mine as well as to limit production based on processing plant criteria. Tunnel capacity can be constrained to implement a specific LHD utilization level by setting the minimum flow rate through a tunnel at some acceptable level. The maximum flow rate through the tunnel should reflect the maximum productivity of the LHD.

In a similar way, the quality and quantity constraints can also be used to blend material from different sources at the processing plant. For example, if production from the cave is well below the processing plant’s acceptable waste limit, additional high waste material can be fed into the plant.

The previously described constraints define the mine production system and ensure production continuity. However, they do nothing to maintain the ideal panel profile or to ensure correct draw control practice. That is the domain of a number of production control constraints. There are four types of constraints governing cave production. They are production precedence, relative draw rates between adjacent drawpoints, the lower bound on production from a drawpoint, and the upper bound on drawpoint production. The upper limit on drawpoint production is also known as the ‘maturity rules’ production limit. The purpose and form of each constraint follows.

Production precedence in the context of cave scheduling means that production cannot begin from a drawpoint until a certain amount of material has been extracted from a predecessor drawpoint. In general terms the constraint says that the amount of material mined from the predecessor must be greater than a defined depletion limit for mining to begin from the dependent drawpoint. An example of defined precedence relationships can be seen in Figure 4. The relationships shown are:

![Figure 4. Diagram of production precedence definition](image_url)
• Production from B2 depends on the depletion of reference block B3
• Production from B4 depends on the depletion of reference block B3
• Production from B1 depends on the depletion of reference block B2
• Production from B5 depends on the depletion of reference block B4
• Production from B1 and B5 are tied indirectly to B3 through B2 and B4.

The effect of these precedences on drawpoint production is described in the next section.

The relative draw rate constraint is similar to the precedence constraints in that they limit production based on drawpoint to drawpoint relationships. However, they differ in that they ensure that ore is extracted across the panel in a uniform manner rather than limiting production based on the state of a drawpoint. It ensures that drawpoints are not drawn in isolation, thereby reducing the chance of premature dilution entry.

The relative draw rate (RDR) constraints state that the production from a drawpoint must be greater than and less than a percentage of all related (adjacent) drawpoints. A graphical example of drawpoint relationships is given in Figure 5. The arrows in the diagram indicate relationships between drawpoint (blocks). The relationships are defined as a set of independent/dependent relationships (i.e. B5 depends on production from B1) but in fact the model constraints implement a two-way relationship.

The reciprocal relationship between the production from adjacent drawpoints has a profound effect on panel production. This arises because production in any given block will be controlled indirectly by the production of all other drawpoints through the declaration of these two-way relationships. This effect will be discussed in more detail later.

The last set of production control constraints is the lower and upper bounds on production. The first of these requires a minimum draw rate from all drawpoints that have been drawn previously to ensure that the ore column above the drawpoint does not recompact and transfer stress onto the production level. The second limits production to below an appropriate level. The maximum draw rate should prevent overdraw and the subsequent development of an air gap above the caved material as well as maintaining a balance between the extraction advance and rock caving rates.

For example, the upper bound on production can be regulated by a set of disjunctive constraints which control production based on the amount of material that has been mined from the drawpoint in previous periods. This constraint regulates production according to the schematic shown in Figure 6. It can be seen that the upper bound on production increases in a stepwise fashion according to the depletion level. Production is limited to 1000 tpm until depletion exceeds 1%, 1500 tpm until 2.5%, 2000 tpm until 6.5%, and 4000 tpm until 93.5%. It then decreases in a manner similar to its initial ramp up.

The effect of the lower and upper production limit constraints are discussed further at the end of the next section.

Cave scheduling
A series of scheduling trials were conducted using the iterative approach shown in Figure 7. The current state of the panel was imported from the mine database and a life of mine optimization was performed. The number of production periods in the life of mine optimization can be found in Table 1 along with the length of each period (in months).

There were three sets of trials conducted to illustrate the effect of each of the production control constraints. The first was conducted to show the effect of production precedence on the commissioning of five drawpoints. The draw scheduling cycle was repeated twenty-four times with the production panel beginning in a virgin state. The
relative draw rate constraints were relaxed in this first trial. The second trial mirrored the first in both duration and starting state. However, the relative draw rate constraints were activated by tightening the lower and upper bounds. The lower bound on the relative draw rate was set at (25%) while the upper bound was calculated for each block pair as the ratio of the reserves at the start of the scheduling cycle.

The last trial was a panel recovery exercise in which the scheduling cycle was repeated 52 times. The panel state at the end of each cycle was used as the opening state for the next trial. The panel recovery trial provided the data for evaluating the effect of the draw maturity rules as well as showing how the panel profile can be improved if the optimized schedule is used to plan production.

Production control
There are four types of production control constraints in the MILP that regulate production in the panel. The effect of production precedence, relative draw rates, production lower bound, and production upper bound constraints are given here.

Production precedence
The precedence relationships between the blocks in this trial were as shown in Figure 4. Production could not begin from drawpoints B2 and B4 until B3 was 5% depleted. The commissioning of B1 and B5 depended in the same way on the depletion of B2 and B4. The resulting production profile of the five drawpoints can be seen in Figure 8.

It can be seen that B3 is the dominant drawpoint in this example. It is the only drawpoint without a predecessor and no other drawpoint produces until it reaches 5% depletion. It took 7 months of production before its dependents, B2 and B3, began to produce. As production proceeds, drawpoints, B1 and B5 begin to produce. Note the one month lag between the start of B1 and B5. The predecessor of B5 (B4) takes an additional month to achieve 5% depletion because it contains more ore than B2.

It is interesting to note that all five drawpoints obey the draw maturity rules. They increase their production stepwise through 1000, 1500, 2000, and 4000 tpm as their depletion levels increase. This trend changes dramatically once the relative draw rate constraints are activated.

Relative draw rates
The constraints on the relative draw rates (RDR) are designed to ensure that production is uniform across the cave. The goal being to eliminate isolated draw and reduce the likelihood of premature dilution entry. The effect of the RDR constraints are best described by comparison to the results of the production precedence trials. The set up of the RDR trial was identical to that described in the previous section with the exception that the lower bound was increased from 0 to 0.25 and the upper bound was decreased from 1000 to a ratio of the reserves remaining above the related drawpoints. The upper bound ranged between 1.01350 and 1.14488 for this trial.

It was stated previously that the RDR constraint set has a dramatic effect on production. Figure 9 highlights this fact. Drawpoint activation is delayed for three of the five drawpoints and production is decreased throughout most of the trial. Production from drawpoints B1, B2, and B5 is delayed by a month because their opening in months 15, 8, and 15 respectively would have lowered the total production in those periods by restricting production from their neighbours. This clamping effect on their neighbour's production is shown in months 9 and 16. Production in B3 drops from 2000 to 1013 tpm when B2 activates according to their reserve ratio of 1.013. In the sixteenth month B1 and B5 affect the production from B2 and B4 in the same manner. Production from both drops from 2000 to 1066 and 1231 tpm for B2 and B4 respectively.

As production proceeds past the sixteenth month, all the drawpoints produce at similar rates due to the tight upper bound on their relative draw rates. This means that mature drawpoints do not achieve their full production capacity until their neighbours mature. This ensures a smooth draw down but it significantly reduced the total production. This decrease is shown in Figure 10. Total production during the trial decreased from 198500 to 129702 tons when the RDR constraints were tightened. Production has been sacrificed in the short-term but this ensures that the panel is drawn down more evenly. Production under tighter RDR constraints does not catch up to the unconstrained case until month 25. In practice, this means that more drawpoints will be needed early on to achieve a given production target.

It is heartening to note that as the group of drawpoints
mature they begin to produce at the production rated defined by the draw maturity rules.

Production lower and upper bounds
The data used to show the effect of the lower and upper bounds on production come from the panel recovery trial described in the next section. At this point it is sufficient to recognize that the minimum draw rate was defined as 5 tpm and that the upper bound followed that shown in Figure 6. The lower bound on production from an individual drawpoint ensures that once extraction has been initiated it continues until the block is fully depleted. The upper bound on production is determined from geotechnical knowledge and desired production. The extraction rate must correlate
to the rate of caving to ensure that a large air gap does not form.

The two bounds on production will be described concurrently. Their effect on drawpoint production can be seen in Figure 11 which shows the production profiles for four drawpoints. The first profile (circles) shows production from a drawpoint that lags behind its ideal depletion level throughout most of the trial. Production levels are at the maximum production allowed at each depletion level. Once the drawpoint achieves its ideal depletion level, production drops to just below 1000 tpm to maintain panel shape until the block is fully depleted.

The production profile for the second drawpoint (inverted triangle) is typical of a drawpoint that has been mined previously yet still lags behind its ideal depletion. The maximum production rate for a mature drawpoint is scheduled for the first three months of the trial. Production drops sharply thereafter to the minimum draw rate (5 tpm) for six months as other drawpoints lag farther behind in production and are more critical to recovering the panel shape. Production then resumes at 4000 tpm for most of the remainder of the drawpoint’s life. Note that production ramps down to 2000, 1500 and 1000 tpm as the drawpoint nears full depletion.

The next profile (squares) shows production from a drawpoint that is nearly at its ideal depletion level at the beginning of the trial (Actual: 0%, Ideal 14.7%). Production ramps up according the draw maturity for the first six months. It then decreases to slightly below 1000 tpm to maintain the level of ideal depletion throughout the remainder of the trial.

The final profile, indicated by diamonds, shows a drawpoint that has over produced previously and must be maintained at the minimum draw rate until the actual and ideal depletions coincide. The minimum production level is maintained for the first 39 months of the trial after which time production increases to nearly 1000 tpm to maintain the level of ideal depletion. (There are 39 diamonds stacked on top of each other at the 60% depletion mark.)

**Panel recovery**

The set up of the panel recovery trial was similar to that for the previously described precedence and relative draw rate trials. The main exceptions being the number of times the planning cycle was conducted and the selection of constraint parameters. The panel recovery trial was carried out for 52 scheduling cycles (months). The production precedence parameter (required depletion level) was set at 5% for one drawpoint pair. That is, one drawpoint could not produce until its predecessor had been depleted by more than 5%. Production precedence is not implemented at the caving operation so it was not a major part of this trial. The constraint was included and activated during the trial to ensure that the constraint did not conflict with other production control constraints.

The lower and upper bounds on the RDR constraint was relaxed to 0 and 1000 respectively. This deactivated the RDR constraint because a drawpoint could now draw between 0 and 100 times the tonnage of its neighbours. (Production was still bound by the draw maturity rules described earlier.) The current state of the panel required relaxation of the RDR constraints because historically poor mining practice resulted in huge differences between the depletion of adjacent drawpoints (see Figure 12). The panel recovery trial relied on the production lower bound to maintain draw from active drawpoints. The lower bound on production was set at 5 tpm and the upper bound followed the previously described maturity rules.

The results of the panel recovery trial can be seen in Figure 12 to Figure 15 (The graphs show the panel profiles for months 1, 12, 24 and 52 sequentially). As production progressed the panel profile began to smooth towards the...
ideal chevron shape. This improvement was achieved while maintaining the target production rate of 150,000 tpm and limiting production from individual drawpoints according to the draw maturity rules. The improvement in the profile shape is best shown by comparison to the actual mine profile after 52 months of production (Figure 16). The total production in the panel recovery trial was equal to mine production over the same 52 month period.

A numerical comparison of the two profiles is facilitated by calculating their deviation from the ideal profile for each month. The deviation was calculated as the sum of the squared difference from the level of ideal depletion. A plot comparing the deviations for the trial and actual profiles is shown in Figure 17. It can be seen that there is a steady improvement in the profile if the optimized schedule is followed. The final recovered profile has a deviation that is 13% of that achieved by actual practice at the mine.

It is unlikely that the mine would be able to achieve the optimized schedule each month, breakdowns, hang-ups and repairs would cause unforeseen deviations. However, this trial has shown that production plans developed using the model will drive production towards the ideal draw profile if followed to the best of the mine’s ability.

Conclusions

The MILP goal program described in this paper shows the major facets of a caving operation that must be included when scheduling mine production. The goal objective in this case was to minimize the deviation from both the ideal depletion level of each drawpoint while maintaining a production target of 150,000 tpm.

The objective function defined in this model is not the only type available for use. Maximizing production or net present value could also have been selected. Maximizing NPV was not used in this case as the dynamic nature of the ore reserve as material flows in the cave prohibits accurate estimation of the material contained above a drawpoint with time. Drawpoint (block) content depends on mining practice throughout the life of the operation. It was therefore felt that the current depletion level was a more realistic variable for life of mine optimization.

It has been shown that the production control constraints defining draw control have a significant impact on mine production. The precedence constraints delay the activation of drawpoints depending on the current state of their predecessors. The relative draw rate constraints ensure even draw down across the panel and the production lower and upper bound constraints regulate the minimum and maximum amount of material that can be drawn from each drawpoint during a month.

Finally, the model is suitable for recovering a panel profile if poor mining practice and other disruptions in the mining cycle have caused a distortion of the shape of the ore/waste contact.

Ongoing R&D aims at delivery of an Integrated Draw Control System (IDCS) that includes both mixing algorithms of particle flow in a cave and draw control optimization using a variety of optimization technologies aimed at improved solution times. IDCS is wrapped in a Windows GUI and include an Access database for linking to site installation specific mine databases.

Acknowledgements

Funding for this project was provided by the sponsors of the International Caving Study (ICS II) a three-year project (2001–2004) coordinated and run by the Julius Kruttschnitt Mineral Research Centre (JKMRC). The sponsoring companies are De Beers, Anglo American Technical Services, Rio Tinto Technical Services, CODELCO Chile, Sandvic Tamrock, LKAB, Northparkes Mines, Newcrest Mining and WMC. Special thanks are due to De Beers for the contribution of their MILP formulation to ICS II and the JKMRC Draw Control Team including Alan Cocker and Kai Riihioja.

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Figure 16. Actual Panel Profile After 52 Months

Figure 17. Profile deviation, sum of square difference between ideal and current


