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Determining the value of additional drilling

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The spatial distribution of copper grades at the Mansa Mina project is highly variable due to the presence of high grade copper/silver/arsenic breccia bodies. Copper grades within the breccia bodies tend to average about 4.5% copper while grades in the surrounding stockwork average about 1.0%. The breccia is known to occur in the vicinity of significant controlling structures, however, the outline and therefore the thickness of the bodies are far from regular. Due to the high grade of these bodies a significant proportion of the value of the project is derived from the breccia tonnages. The irregularity in the outlines/thickness translates into an uncertainty in the breccia tonnage and therefore in the value of the project.

Acknowledging the economic importance of the breccia bodies and the risk associated with the breccia tonnage uncertainty, significant additional drilling has been proposed to both understand the spatial distribution of the breccias and reduce project risk. The costs associated with this additional drilling are not inconsequential; thus, a reasonable question is 'what value is derived from this additional drilling'?

To obtain an answer to this important question, a simulation approach is applied. First the deposit is simulated using the existing drilling and the uncertainty in the resource is defined. Next the proposed drilling program is simulated and the simulated data are added to the existing data. Using this expanded data set a second simulation of the deposit is created and once again the uncertainty in the resource is defined. Using this approach, the reduction in the uncertainty in the resource can be determined. To convert this reduction in uncertainty (or reduction in risk) in the stated resource into a dollar value, a method of defining the increase in the project rate of return necessary to account for risk is presented. Using the two different rates of return, the difference in project NPV and hence the value of additional drilling is determined.

Study area and statistics of the data

The Mansa Mina project is located in northern Chile south of the Chuquicamata mine and north of the town of Calama. The primary owner of the project is the Corporacion Nacional del Cobre de Chile (Codelco). The deposit is associated with the West Fissure structure that truncates the Chuquicamata orebody. Near surface the deposit is seen as a series of high grade structurally controlled breccias that contain significant amounts of copper, silver, and arsenic. Deep drilling (to 900 m) has shown that there is a significant porphry deposit underlying the breccia system with copper grades in the vicinity of 1%.

The volume selected for this study is within the most completely studied portion of the deposit. The dimensions of the study area are 125 m, 475 m, and 100 m in the EW, NS and vertical directions. The box is centred on the 2200 m level which is the most intensively investigated of the two subsurface levels. The study volume contains 3558 samples of 1.5 m length. Samples are obtained from surface drilling, underground drilling and channel sampling.

The samples were logged and coded into 5 major unidades geologicas (UGs) following a statistical and geological evaluation of the grade, alteration, lithology, and mineralogy data. The breccia units are economically the most important. Surrounding the breccia a stockwork unit is found. The transition between these units is clear. As the distance from the breccia bodies increases grade decreases. The contact between the stockwork and the low grade C1 unit is based on the presence or absence of alunite. In the eastern portion of the study area a separate low arsenic unit containing primarily chalcopyrite is found. This unit which has copper grades similar to the stockwork is separated on the basis of arsenic. Finally there is a mapable, continuous barren zone in the centre of the deposit. The basic total copper statistics by unit are presented in Table I.

Figure 1 shows the distribution of copper grades through a lognormal probability plot. As shown, individual sample grades in the breccia units can exceed 20% total copper. These high values do not appear to be outliers as they fit the tail of the distribution fairly well; furthermore, the observed maximum values for this study area are less than the capping values used in the resource estimation of the entire deposit. For these reasons, plus the fact that the simulation is created to reproduce variability not average properties, no high grade cutting is performed. All of the data shown in Figure 1 are used in the simulation without modification.

Initial simulation of the study area

The first simulation of the study area uses the existing data to obtain 30 simulations of UG and total copper on a 2 × 2 × 5 m grid. The simulations are performed using a modification of the GSLIB routines SGSIM (sequential simulation of grade) and SISIM (sequential simulation of indicators). The modified program allows simultaneous simulation of both grade and UG. The steps in the simulation are:

Table I
Total copper statistics by UG

Unidad Geologica	Number of samples	Average (%)	Median (%)	Std.Dev. (%)	Coef.of variation
Breccia	547	5.05	4.30	3.09	0.61
Cero1	629	0.28	0.25	0.20	0.71
Stockwork	1,667	1.29	1.08	0.81	0.63
Chalcopyrite	380	0.91	0.77	0.56	0.61
Waste	209	0.05	0.02	0.12	2.34

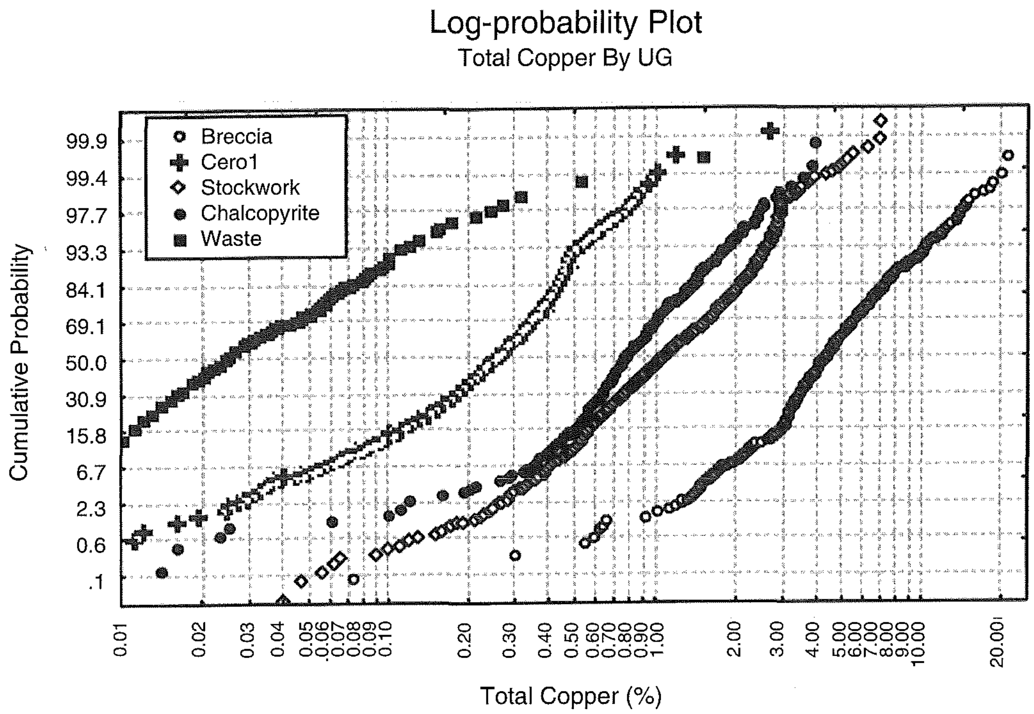


Figure 1. Copper log probability plot by UG

- Transform grades (by UG) to follow standard normal variables
- Model the variogram of each transform of copper grades
- Model the UG indicator variograms
- Simulate the grades for each UG at each node (5 simulations of grade in total)
- Simulate the UG at each node
- Select the simulated grade appropriate for the simulated UG. Each node now has one simulated UG and one simulated grade
- Average the simulated grades at the nodes to produce simulated grades for 10 x 10 x 10 m blocks
- Assign the most common simulated UG within the block to the block.

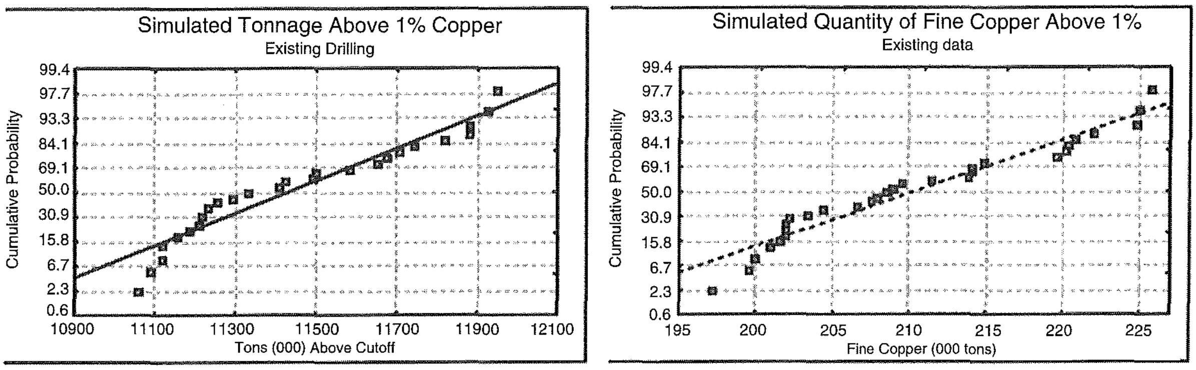
To perform this simulation, variograms of transformed copper grade and indicators were modelled for each UG. That is a total of 10 variogram models were required. Most of these experimental variograms were erratic and therefore difficult to model. The simulations were checked by comparing the distributions of simulated and data grades, by comparing the variograms (both grade and indicator) of the actual and simulated data, and by comparing the simulated spatial distributions of UG and grade with the data. In all cases the simulation appeared to be a reasonable representation of the data.

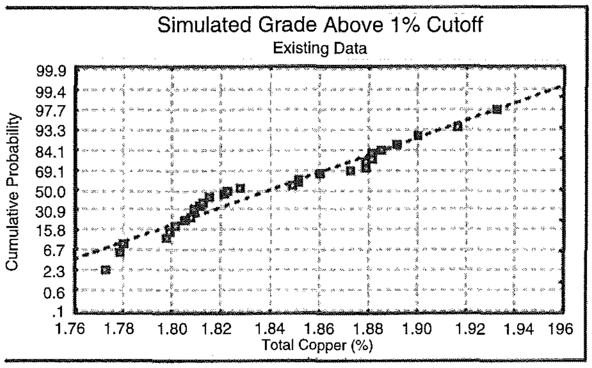
In each of the 30 simulations, the same general features were seen. The details, however, differed significantly between simulations. In particular the location and particularly the size of the high grade breccia pods (>4%) changed significantly between simulations. To quantify the uncertainty or variability exhibited by the multiple simulations, the grade and tonnage of 10 m blocks exceeding a cutoff grade will be examined. As an example, the uncertainty in the grade, tonnage, and quantity of metal exceeding a 1% cutoff is determined. The shape of the individual statistical distributions for tonnage, grade, and quantity of metal above cutoff are shown in Figure 2. For each of these quantities, the distribution is more or less normal although there are some deviations from normality in the tails of the distributions. The summary statistics show that, as expected, uncertainty is largest for the quantity of metal (product of tonnage and grade).

Simulation of additional data

To obtain additional information concerning the form of the

Uncertain in Estimated Resource





Summary Statistics

Statistics	Tons	Grade	Metal
Average	11430	1.84	210
Std. Dev.	283	0.043	8.75
RSD	2.5%	2.4%	4.2%

Figure 2. Uncertainty in resource based on initial data

high grade breccia bodies and reduce the uncertainty in the resource estimate, a program of additional drilling was designed. Within the study area considered here, this drill program consisted of 3561 additional 1.5 m samples. The majority of these samples were collected from underground drilling. To show the approximate location of the additional drillholes and the final data density, two plots of the sample locations are provided in Figure 3. Both of the images shown are plan projections of all sample points to the 2200 m level.

For the proposed drilling only the sample locations are known. To evaluate the reduction in uncertainty that should be expected following completion of the drilling both UG and grade information are required. Since actual drilling results are not available, these data values were obtained using a single detailed simulation (1 x 1 x 1 m) of the study area. At each node both UG and grade are simulated. The drillholes are then passed through the simulation and the simulated value at the node closest to the expected sample location was assigned to the sample. Through this process, 3561 sample grades and UGs are generated. The univariate statistics and variograms for the simulated data were then compared with those for the actual data to confirm that the new simulated data provided a fair representation of the deposit.

Second simulation of the study area

The second simulation of the study area uses the actual data plus the simulated data to obtain 30 simulations of UG and total copper on a 2 x 2 x 5 m grid. The simulations are performed using the same simulation routine used in the first simulation step. This modification of existing GSLIB routines allows simultaneous simulation of both grade and UG. The steps in the simulation are:

- Transform simulated and actual grades (by UG) to follow standard normal variables
- Use the variogram models previously defined for the UG indicator and transformed copper grade
- Simulate the grades for each UG at each node
- Simulate the UG at each node
- Select the simulated grade appropriate for the simulated UG so that each node has only one simulated grade
- Average the simulated grades at the nodes to produce simulated grades for 10 x 10 x 10 m blocks.

As for the first simulation, the uncertainty in the estimated resource for the 10 m blocks above a 1% copper cutoff can be defined. As previously, normal probability plots are used to show the distribution of the estimated tonnage, grade, and metal above cutoff (Figure 4). To show the reduction in the variability of the simulated resource, the results for the first simulation are also shown. For each of the tonnage, grade and metal above cutoff the averages for the two data sets are nearly identical. This indicates that adding simulated data does not impact the expected resource. In other words, the conditional simulation procedure is nearly conditionally unbiased.

For grade above cutoff, the average and median grades are the same for both data sets indicating that both distributions are strongly centred (or close to normal) and there is no conditional bias. The only significant difference in the distributions of grade above cutoff is the variance of the two distributions. This is shown by the difference in slope of the two curves with a steeper slope indicating less variance.

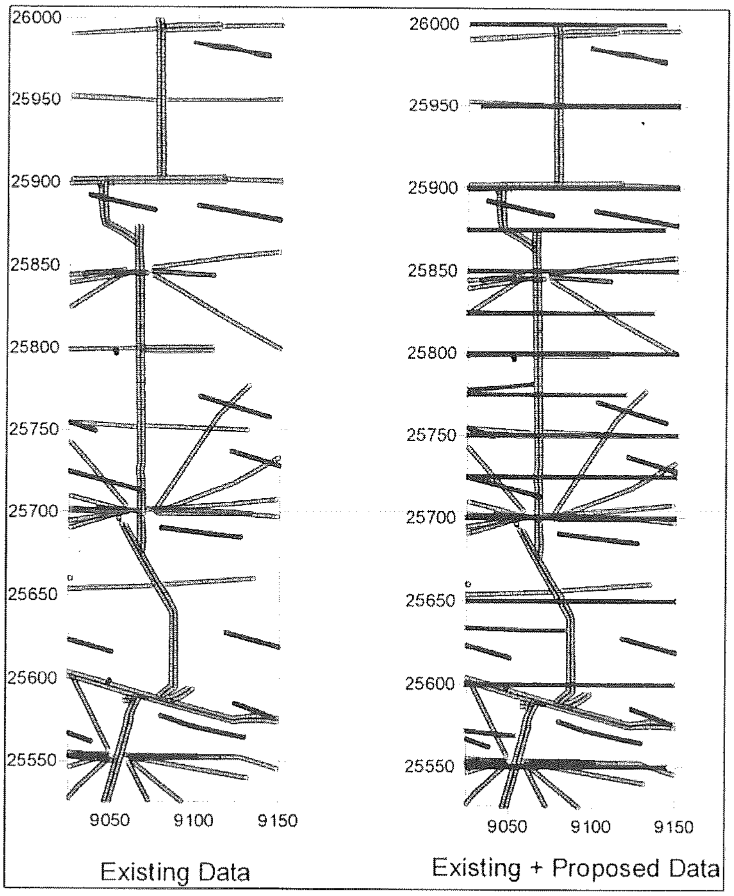
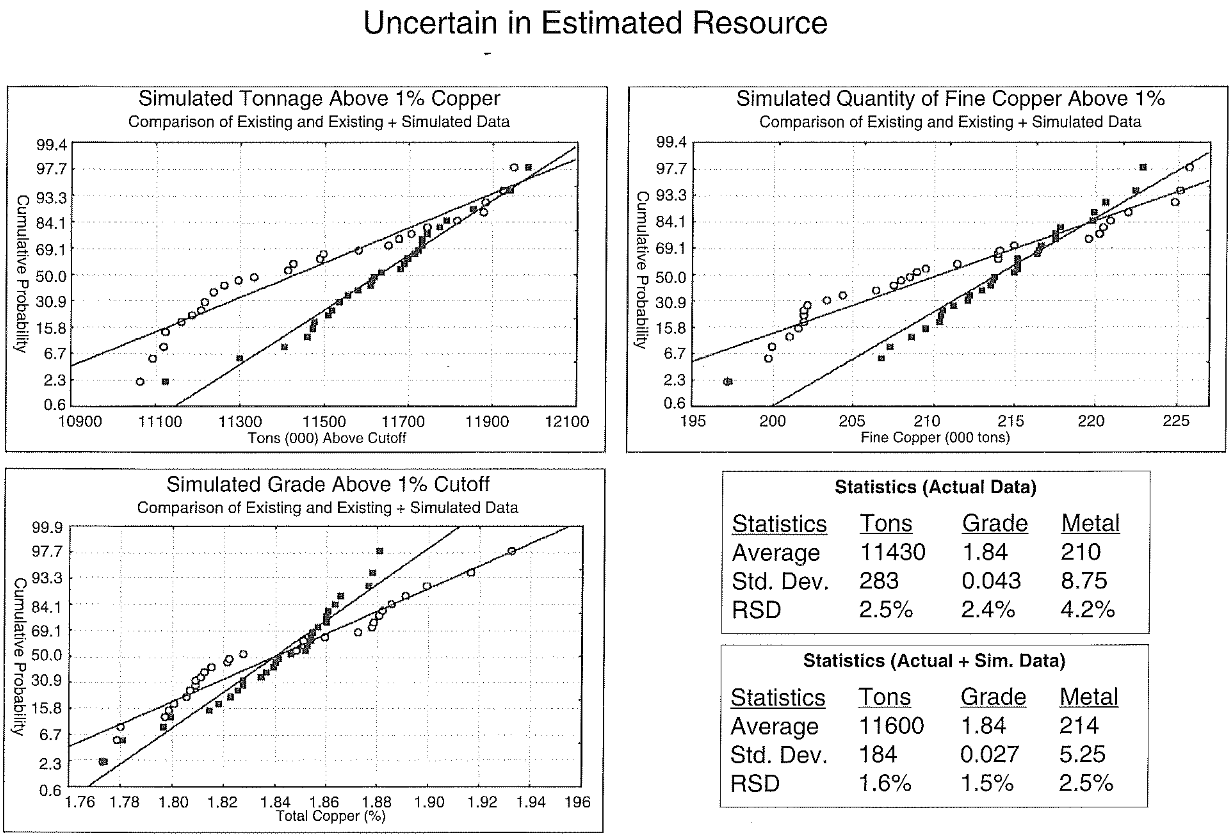


Figure 3. Location of existing and proposed data (plan view)



Statistics (Actual Data)

Statistics	Tons	Grade	Metal
Average	11430	1.84	210
Std. Dev.	283	0.043	8.75
RSD	2.5%	2.4%	4.2%

Statistics (Actual + Sim. Data)

Statistics	Tons	Grade	Metal
Average	11600	1.84	214
Std. Dev.	184	0.027	5.25
RSD	1.6%	1.5%	2.5%

Figure 4. Uncertainty in the resource estimate after incorporation additional data

For tonnage above cutoff, there is a small difference in the expected quantity above cutoff (11.6 vs. 11.4 mt or 1.5%). This difference is not sufficiently large to conclude that a conditional bias is present but it is large enough to produce an offset in the two curves. Despite the offset it is clear that both distributions of tonnage are very similar to normal distributions and the uncertainty in the tonnage above cutoff is less when both existing and simulated data are used. The reduction in tonnage and grade uncertainty with the increase in data translates into a reduction in the uncertainty in the quantity of metal above cutoff. Since the quantity of metal, is the product of two uncertain quantities it is not surprising that the uncertainty in the quantity of metal is the largest. This is shown in the table of statistics. The relative standard deviation (RSD) is largest for this quantity. With the additional data the uncertainty in the quantity of metal shows the greatest reduction. The reduction would have been larger if not for an outlier simulation showing a very low amount of metal. Some thought was given to eliminating this outlier from consideration but in the end all simulations were considered.

The reduction of uncertainty due to an increase in drilling can be examined across the range of cutoffs. The change in grade uncertainty can be seen in Figure 5. Several variables are graphed here. The points represent the expected grades above cutoff. As with all the variables graphed here, the points are offset to the left and right of the line representing the cutoff grade for clarity. The information shown on the left side is for the original drilling while that shown on the right side is for the original plus additional drilling. As a function of cutoff, the two expected grades above a particular cutoff are very similar across the range of cutoffs and there are no systematic differences. Based on the similarity of these values, it is concluded that the simulation does not contain a conditional bias. Around the expected values, box and whisker plots show the central 50% (25th and 75th percentiles) and the minimum and maximum of the 30 simulated values. As shown, up to a cutoff of 3% there is an important reduction in the variability of grade above cutoff. At higher cutoffs the reduction in uncertainty is much less and at the 4% cutoff there is no apparent reduction in uncertainty. A similar result is shown by the bar graphs showing the relative standard deviation of the grade above cutoff. These results indicate the difficulty of estimating the grade above cutoff at high cutoffs.

As for the grade above cutoff, the uncertainty in the tonnage above cutoff can also be examined as a function of cutoff grade. Using the same format as described for the uncertainty in grade, both the relative and absolute uncertainty are shown in Figure 6.

As shown by the box and whisker plots, the absolute uncertainty is small at low and high cutoffs and largest at cutoffs where approximately half the total tonnage is above cutoff (the 1.5% cutoff in this case). This is sensible since there is little possibility of error at extreme cutoffs since nearly all of the deposit is either above or below cutoff. Near the median cutoff, however, there is ample room for both positive and negative errors. On a relative basis, uncertainty increases as a function of cutoff. Once again this trend is more a function of the denominator (rapid reduction in the tonnage above cutoff) than the numerator. That is at high cutoffs there is little tonnage above cutoff so that on a relative basis even a small error appears large.

It is important to note that for mid-range cutoffs (say 1.0 and 1.5%) the relative uncertainty in the tonnage is equal to or larger than the relative uncertainty in the grade. Approaches that classify a resource or define resource quality solely on the basis of grade uncertainty (kriging variance approaches for example) thus disregard a very significant source of error and are highly optimistic. Furthermore, an approach that does not consider the impact of cutoff grade on uncertainty disregards another fundamental control on uncertainty and therefore is also in error. Based on the results shown here, any determinations of resource quality that do not consider all of tonnage uncertainty, grade uncertainty, and the impact of cutoff grade should be considered as approximate at best.

Value of additional data

The previous discussion has demonstrated clearly that the

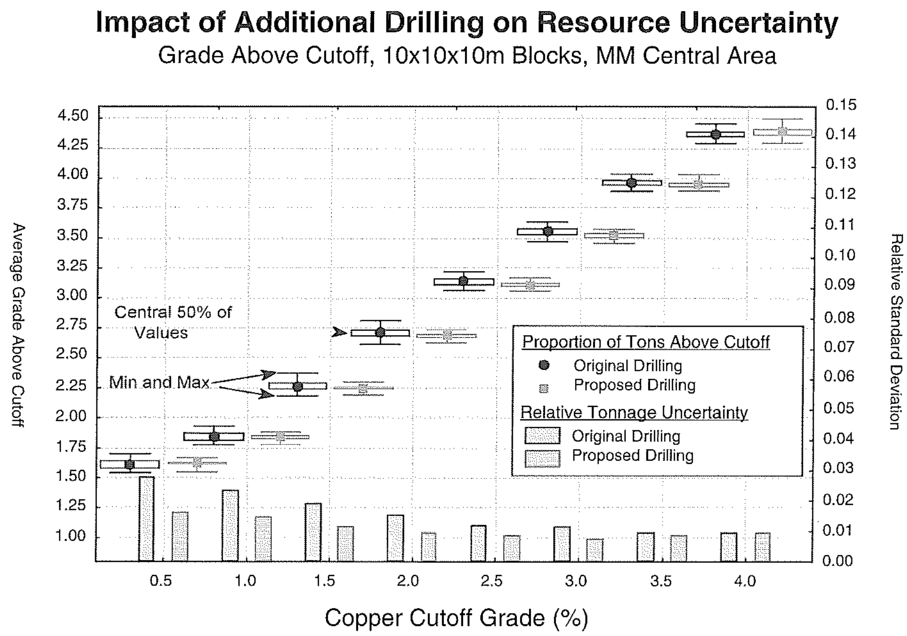


Figure 5. Uncertainty in the grade above cutoff as a function of cutoff grade

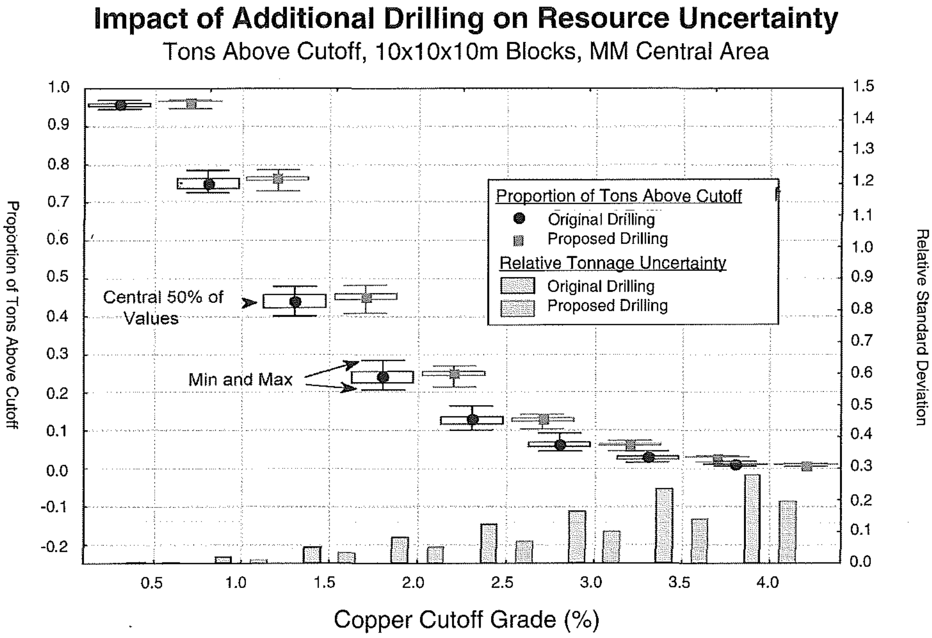


Figure 6. Uncertainty in the tonnage above cutoff as a function of cutoff grade

additional data planned for the study area will reduce the uncertainty in the resource by a significant amount. The reduction in uncertainty clearly adds value to the project by reducing risk. Through the use of standard financial analysis tools, the value of this risk reduction can be quantified.

The approach used to quantify the value of the additional data is based on common financial logic. If two projects offer the same expected rate of return, an investor will always choose the less risky project. In other words, to justify an investment in a riskier project the investor expects a greater rate of return. The increase in the expected rate of return necessary to compensate for the risk is termed the risk premium. To define the risk premium associated with not performing the additional drilling, the distribution of project net present value (NPV) will be considered under the two drilling scenarios.

The volume simulated comprises about 5 years of production at the expected production rate. Given the project mine plan, the uncertainty in the quantity of metal produced at a 1% copper cutoff can be defined for each year. As has been shown previously, the distribution of the uncertainty in the quantity of metal above cutoff is approximately normal. The probability distribution of NPV can be determined by simply evaluating each simulation against the mine plan. This will yield one NPV value for each simulation and hence an approximation of the NPV distribution. To examine the risk premium, however, it is necessary to investigate the computation of NPV in some more detail.

Project NPV is a weighted sum of yearly cash flows where the weights are the yearly discounting factors $(1/(1+r)^i$ where r is the discount rate and i is the project year). If it is assumed that uncertainty in the yearly cash flow is equal to the uncertainty in the yearly quantity of metal (i.e. the resource uncertainty is the major source of uncertainty), NPV becomes a random variable dependent only on a model of the variability in the metal output and the discount rate (the interest rate r in the previous formula). Since the yearly cash flows are normally distributed, the project NPV is also normally distributed.

Knowing that NPV is normally distributed simplifies the analysis. The expected NPV is simply the NPV in the absence of uncertainty. The only remaining parameter needed to specify this distribution is the NPV variance. This is obtained through the well-known formula for the variance of a sum of random variables:

$$s_{npv}^2 = \sum_{j=1}^{n} \sum_{i=1}^{n} \left(\frac{C_{ij}}{(1+r)^i (1+r)^j} \right)$$

$s_{npv}^2 = NPV \text{ variance}$

$r = discount\ rate$

$C_{ij} = covariance\ for\ years\ ij$

As shown, the uncertainty in the NPV is a function of the discount rate and the yearly uncertainty in the quantity of metal output modified for any correlation in the yearly output. Thus if the quantity of metal produced in year 1 is correlated with the quantity produced in year 2, then the variance is modified (usually reduced) by the amount of this correlation. In the event that there is no correlation between the yearly production of metal, then the above formula simplifies to:

$$s_{npv}^2 = \sum_{i=1}^{n} \left(\frac{s_i}{(1+r)^i} \right)^2$$

Where s_i is the standard deviation in the quantity of metal for year i.

To determine which of these formulas are appropriate (and model the covariance if necessary), the correlation between the yearly quantity of metal was measured by computing experimental variograms on the yearly simulated quantities of metal. That is for each drilling program there are 30 simulations of the metal output for each of the 5 years. There are therefore 30 data from each simulation that can be used to measure the correlation between the quantity of metal produced in year 1 and year 2 for example. Similarly the correlation between the quantity of metal

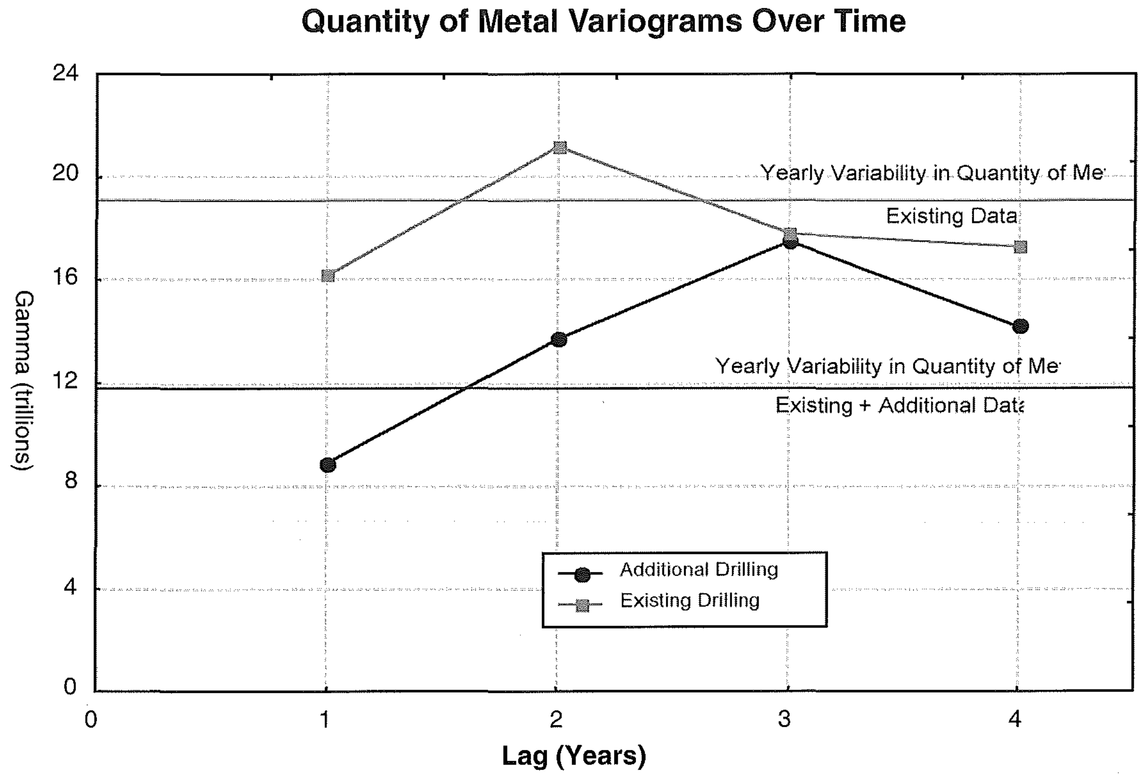


Figure 7. Correlation in the yearly quantity of metal output

produced in year 1 and year 3 can be measured. Using an assumption of temporal stationarity of the squared differences, experimental variograms of the yearly quantity of metal can be produced (Figure 7). These variograms show very minor correlation between the quantity of metal produced in subsequent years and no correlation over larger time periods. Based on this experimental data, it is appropriate to assume that the yearly quantity of metal is essentially independent and the simplified method of computing NPV variance is appropriate.

The assumed independence of the yearly resource is a project-specific decision. For this deposit it is reasonable because the variograms of grade show very little to no correlation and the variograms of the UG indicators showed only weak correlation. If grades and UG indicators (essentially tonnage) are poorly correlated, then the resource produced can be expected to be uncorrelated in time. For other deposits, this might not be the case and the experimental variograms of the yearly quantity of metal must be checked.

Knowing the distribution of NPV for the projects before and after additional drilling, the risk premium can be computed as follows.

- Determine the NPV distribution for the less risky project.
- Compute a statistic that can be used to define the value of the project in the presence of uncertainty. The p10 statistic is selected for this application. There is a 90% chance that the project will exceed the p10 NPV value.
- The p10 for the project after completion of the additional drilling is larger than the p10 of the current project due to the reduction in uncertainty.
- NPV is dependent on both the yearly uncertainty in the output of metal and the discount rate. Since the uncertainty cannot be changed, to compensate for uncertainty, a larger discount rate is required for the current project (less drilling) to make the p10 values equal. The difference between the initial discount rate and the rate that makes the p10 values equal is the risk premium.
- Discounting the expected yearly cash flows to the present at the higher interest rate (initial rate plus the risk premium) will reduce the NPV of the project. The difference between the initial and reduced NPVs is the value of the additional data. Depending on the size of the risk premium, the value of the additional data can be quite significant.

The individual distributions of NPV are shown in the Figure 8. In creating these NPV distributions, the following assumptions were applied.

- Uncertainty in silver grades is not considered. All cash flow uncertainty is due to the uncertainty in the quantity of copper per year.
- The uncertainties in the yearly quantity of metal are defined by determining the blocks located within each production volume (approximating a year) and determining the standard deviation of the quantity of metal. This uncertainty translates directly into the uncertainty in the yearly cash flow.
- Tonnage from outside the study area enters the cash flow without uncertainty.
- Assuming independence between the yearly cash flow, the NPV uncertainty is defined for the current project and the project following the additional drilling.

Considering a 10% discount rate for both projects, the expected NPV differs by $11 million. This is the difference in capital investment (in this case, the cost of additional drilling). The two graphs of potential NPV at the 10% discount rate show that the project based on only the existing drilling is clearly more variable. At the lower tail

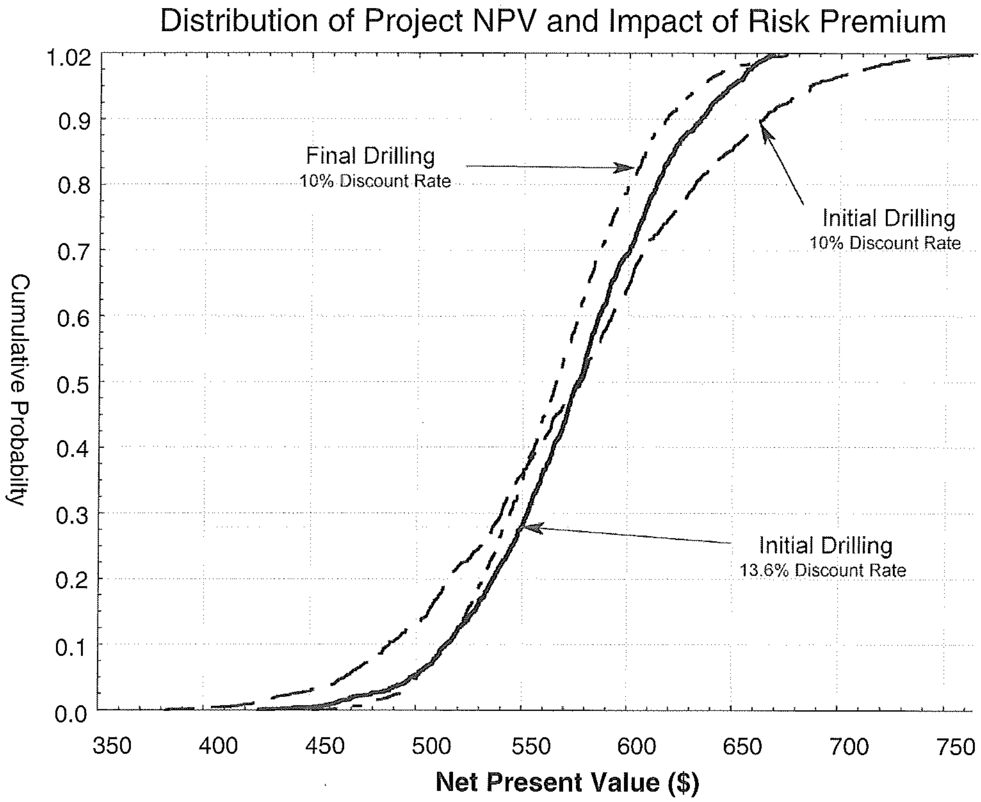


Figure 8. Distribution of project NPV

of the distribution, the two curves cross so that the p10 of the project after additional drilling is greater than the project based on only the current drilling.

As discussed previously, holding other variables constant, an increase in the discount rate decreases the variability in the project NPV. Thus there is a discount rate that makes the p10s of the two distributions equal. By trial and error, it was determined that a 13.6% discount rate sets the two p10s equal. The risk premium associated with not performing the additional drilling is thus 3.6%. In other words, a project with the current level of drilling must be discounted at a 13.6% discount rate to compensate for the additional level of risk. Discounting the project at a 13.6% rate reduces the project NPV by nearly $150 million. Clearly the additional drilling is money well spent.

An examination of Figure 8 shows an important feature of the distribution of project NPV. It is clear that it is possible to over-drill a project. As more holes are added, the NPV variance decreases and the slope of the line on the probability plot increases. At the same time, however, the line shifts to the left reflecting the cost of the additional drilling. At some level of drilling expense, the two NPV distributions will no longer overlap indicating that accepting the risk associated with the project with less drilling is the best economic alternative. Stated another way, an examination of the project NPV distribution for the available drilling quickly shows the maximum amount of drilling expenditure for purposes of defining the resource.

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