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Estimation of optimum exploitation life of a bucket wheel excavator: Through the prism of dynamic programming

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The problem of determining the optimum exploitation life of machinery, namely the optimum time for machinery and equipment replacement, represents a complex and highly responsible engineering task. Taking into consideration the situation prevailing at coal pit mines in Yugoslavia, the tasks of this class are very complex and difficult. To make a decision on replacement of capital equipment and machinery, such as bucket wheel excavators within the mentioned systems, understands a management task of utmost responsibility. It requires high professional and analytical knowledge as well as the reliable arguments, based on multidisciplinary professional approach. In this paper, the authors present their views on the problem of establishing the optimum exploitation life of bucket wheel excavators, offering an algorithm, based on dynamic programming, as a solution.

Keywords: Bucket wheel excavators, exploitation life of machinery, optimization, operations research, theory of replacement, dynamic programming

Introduction

The reasons to replace bucket wheel excavators in mines may be various and are generally classified into four groups: physical ageing of bucket wheel excavators, 'moral' wearing, technical-technological and functional obsolescence.

As the criterion of success for any production system, including open pit mines, is determined by economic indicators, the problems of this group of tasks are generally reduced to finding out the optimum time when, according to the selected economic criterion, it is advisable to replace the existing bucket wheel excavators by new ones. To define the proper time means to determine the optimum exploitation life of a bucket wheel excavator. It would be useful if such information be available even at the time when the assembly of machinery is carried out, thus providing the policy of its adequate exploitation. It is obvious that the economy of utilization represents a factor that clearly indicates the necessity of putting a bucket wheel excavator out of use from the production process.

The criterion for write off the value of machinery according to the standard IEC 60300-3-11, representing the original implementation procedure of the maintenance program as per reliability, says 'that a means must show a functional degradation of characteristics at the certain age, and that the majority of components should confirm the same age'. The criterion of efficiency, relating to the direct write off costs says 'that economically limited life should be costs efficient, namely the costs should be less than the expences to prevent failures'.

The past experience indicates acceptability of two different optimization criteria to find out the optimum time of replacement for a bucket wheel excavator. The first approach is based on the maximum net income resulting from production engagement of a bucket wheel excavator during its exploitation. The net income may be equalized, if necessary, with the difference between production result value of bucket wheel excavator and direct proportional production costs, namely presented by adequate portion of profit or accumulation resulting from excavator operation. The second optimization approach is based on the minimum exploitation costs of bucket wheel excavator during its exploitation life. From theoretical aspect, both criteria are of same importance. In practice, however, a significant advantage has been given to the criterion of minimum exploitation costs. Both explanation and justification for this 'inclination' towards the criterion of minimum exploitation costs are primarily found in rationality, namely in less acquisition and processing of data required for such analyses.

The exploitation life of bucket wheel excavators at open pit mine lasts for many years. In those long-lasting periods, numerous changes, occurring in the world's and domestic economy, different in intensity, effect on economic systems, changing the frameworks and relations of economy. Directed effects of such changes reach the mines forcing them do adjust the business policy to new conditions. Within the scope of the analysed problem, the mentioned factors cause changes of the value of parameters that are used as a quantitative inputs in the process of establishing the optimum exploitation life of bucket wheel excavators. The variety of input elements and, very often, their small information reliability, as well as the permanent increase in bucket wheel excavator maintenance and overhaul costs underline the necessity of great consideration and criticism when selecting the input data for the process of finding out the optimum exploitation life of bucket wheel excavators.

Identification of the period, the end of which represents the optimum time for replacement, is the objective information on economic justification for replacement of bucket wheel excavators. Those who are responsible for the business policy of the mining company may, but should not, replace bucket wheel excavators within that optimum time. It is obvious that the replacement of bucket wheel excavators prior to optimum time would miss the opportunity to achieve greater material values without any additional investments. However, it is advisable to replace the bucket wheel excavators after elapse of optimum time as then (by the increased additional investment) the operating effects are reduced and, at the same time, the possibility of sudden failures of bucket wheel excavators increase.

The analysis shows that the problem of estimating the exploitation life of capital mining equipment, and consequently bucket wheel excavators, still remains open and that any developed (based on available data) and generally accepted pragmatic approach to solving this class of complex engineering tasks, does not exist.

The operations research methods, first and foremost, the dynamic programming through its transferred theory of replacement, offer guidelines of possible directions for solving the problems of optimum exploitation life of bucket wheel excavators.

The above-mentioned explanations, sufficiently enough supported by arguments, indicate both the complexity of the subject problem and the importance of its study. The electric power industry of Serbia has given priority to this problem and asked the Department of Computer Application, Faculty of Mining and Geology to make the study 'Establishment of exploitation life of capital mining equipment at coal open pit mines of the Electric Power Iindustry of Serbia: Phase I—bucket wheel excavators'. Some of the results of this study are presented in this paper.

Decisive factors and selection of a model

The factors, having effects on exploitation life of capital mining machinery, such as bucket wheel excavators, were analysed and the most significant selected. The group of factors, greatly affecting the duration of bucket wheel excavators exploitation cycle and determination of their optimum exploitation life include the following (Figure 1):

- working environmental conditions
- climatic conditions
- technological factors
- economic conditions
- construction of bucket wheel excavator
- logistics and maintenance
- bucket wheel excavator management.

According to both natural and physical effects on duration of bucket wheel excavators, the mentioned decisive factors greatly differ. From aspect of function, however, they have mutual features such as: time variable, fluidity, difficult measuring and poor correlative evidence (at coal open pit mines of Electric Power Industry of Serbia).

It was concluded that, due to the mentioned characteristics, it is very hard or almost impossible to quantify the decisive factors separately and define directly the intensity and the regulations of activity, as well as the duration of their effects on exploitation life of bucket wheel excavators. Thus, their explicit building into the models of replacement can hardly be done: in fact, it is almost impossible. However, they directly effect on exploitation costs of bucket wheel excavators, meaning that a functional dependence exists as well as a complete correlation between decisive factors and exploitation costs of bucket wheel excavators. Exploitation costs as an econometric value, easy and steadly to measure and follow, bind for itself the results of all factors that effect on exploitation life of bucket wheel excavators, thus opening an alternative way in searching for an answer to the question, 'How to introduce decisive factors into the model of replacement?'

The authors' determination to find out the final solution within the sphere of dynamic programming, is strongly supported by arguments and justification by introducing cost criteria, as alternatives to 'real' decisive factors, into the model of replacement. All more important components of each and every alternative model of replacement were considered and it was established that all the limiting

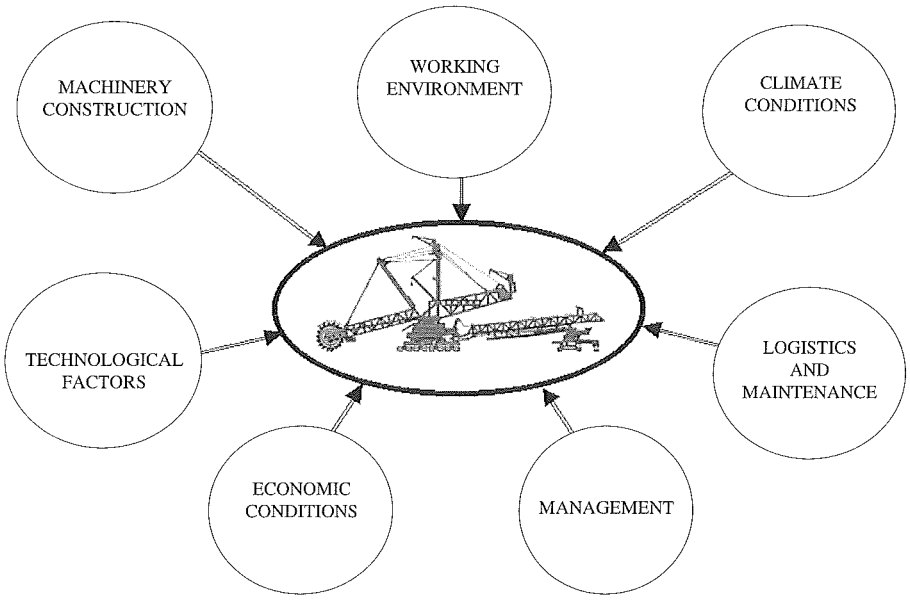

Figure 1. Factors having predominant effect on exploitation life of bucket wheel excavators

factors can be solved outside the model and therefore should not be indispensably included in the model to have effects on selection of the optimum solution.

The criterion of optimization was thoroughly analysed. Different categories of costs and profit were studied in order to find the appropriate criterion for optimization. It was concluded that exploitation costs, namely the costs of owing the bucket wheel excavators, due to their reliability, should have advantage over all other categories of profit.

The basic models of replacement may be adjusted, through correction and supplementations, to every alternative regime of exploitation, namely to any valid hypothesis the model of replacement is grounded on. This statement is grounded on the fact that the study suggests two adaptive base models of replacement for establishing the optimum exploitation life of machinery: with unlimited and limited interval.

In this paper we shall discuss the models with unlimited time, as advantageous for estimation of exploitation life of long-lasting machinery, such as bucket wheel excavators.

The optimum solution for the model of replacement is expressed by quantitative standards, in form of one of two possible decisions: a machinery, namely a bucket wheel excavator should, or should not, be replaced at the beginning of the observed period. Those alternative decisions, named the management policy of exploitation of bucket wheel excavator, are represented by the following symbols:

u_1 - bucket wheel excavator should not be replaced, but its exploitation to be continued in the current period;

u_2 - bucket wheel excavator should be replaced at the beginning of the observed period.

Selection of the optimum management policy on exploitation of bucket wheel excavator throughout any period is based on adequate values of the corresponding function. Therefore, the optimum policy of bucket wheel excavator management (u_o), including one of the two alternative decisions, may refer to the k-period (u_o^k), as well as to the age of the given bucket wheel excavator in that period [$u_o^k(t)$] where:

u_o^k - optimum policy of exploitation management of bucket wheel excavator in k-period;

$u_o^k(t)$ - optimum policy of exploitation management of bucket wheel excavator of t-age in k-period.

By connecting the optimum policies of exploitation management of bucket wheel excavators, as per the periods of sequential course (commencing with the first), the strategy of optimum exploitation managing the bucket wheel excavator is formed. Possible selection of one among alternative decisions on managing the bucket wheel excavator at the end of every period, within the limited period, may require more than one replacement of the observed bucket wheel excavator. On the contrary, within unlimited interval of observation, the process of searching terminates in the very moment when the first replacement is found.

Models with unlimited interval

Variant A: A new bucket wheel excavator

Model hypotheses—At the beginning of the first period, a new bucket wheel excavator is put into operation and therefore an optimum exploitation life should be established so that its optimum exploitation regime during this life may be set in due time—at the beginning of its usage. It results from this hypothesis that the purchase value for a bucket wheel excavator (A) is known at the moment of making a model, and that the expected costs of regular maintenance and periodic repairs $h(t)$, connected with the change in life of the observed bucket wheel excavator, may be defined. As the replacement is done at the beginning of the first period, the age of a bucket wheel excavator (t=1,2,..n...,) is equalized according to duration and changes simultaneously with periods (k=1,2,..., n...) thus making no difference whether the dependence is connected to the age of bucket wheel excavator or to the belonging period. It is assumed that the observed bucket wheel excavator will be in function until its liquidation value, at the end of n-period, equalizes with the costs of disassemby and removal of the rest of the bucket wheel excavator, and therefore the liquidation value of a bucket wheel excavator $Ag(t)$ is not taken into consideration. Also, the opinion prevails that a properly organized maintenance and optimum regime of exploitation would prevent unexpected failures of higher costs, and therefore the possible costs of sudden urgent repairs $s(t)$ may be neglected.

Minimum exploitation costs for a bucket wheel excavator during its life are selected as an *optimization criterion.*

The optimum value of variable is n-period, calculating from the beginning of the observed time interval. In order to find out its value it is necessary to form a model of replacement that is defined by the system of recurrent equations.

Recurrent functions—The model of the described problem may be established by the following system of recurrent functions:

$$z_o(1) = A + h(1) + az_o(2), \quad t = 1 \qquad [1]$$

$$z_o(t) = h(t) + az_o(t+1), \quad 2 \le t \le n-1 \qquad [2]$$

$$z_o(n) = h(n) + az_o(1), \quad t = n, \qquad [3]$$

requiring additional explanation.

Total exploitation costs of the given bucket wheel excavator during its life of n-periods are represented by the function $z_o(1)$ in the relation (1), and the costs include the purchase value of a bucket wheel excavator (A), the costs of regular maintenance and periodic repairs during the first (k=1) period ($h(1)$) and minimum discounted exploitation costs accrued from the beginning to the end of the second period ($az_o(2)$). By discounting, the values of different following periods are equalized with the existing values for the first period.

Upon completion of the first period, only the costs of regular maintenance and periodic repairs are formed in every (relation 2) as well as in the last period (relation 3). These costs also incorporate the discounted minimum costs of identical contents for the rest of time until the following period terminates. At the end of the last n-period, the existing bucket wheel excavator is replaced by a new one, and therefore the discounted minimum exploitation costs for a new bucket wheel excavator ($az_o(1)$) that will accrue during its life, also for the n-period, should be included. This theoretical process keeps on repeating during unlimited time interval in order to provide a value for the variable n.

Should a liquidation value of bucket wheel excavator exist at the end of n-period, then the relations (1) and (2) will remain unchanged, while the relation (3) would have the following form:

$$z_o(n) = h(n) - Ag(n) + az_o(1), \qquad [4]$$

thus enabling formation of the adequate model of replacement:

$$z_o(1) = \frac{1}{1-a^n}\left[A + \sum_{t=1}^{n} a^{t-1}h(t)\right] = F(n) \qquad [5]$$

with functions of total exploitation costs of bucket wheel excavators within the interval of n periods, noting that n and t-1 represent an exponent degree of a discount factor: a^n and a^{t-1}.

A more easier way to find the value for n than calculating it from the relation (5) is by a dual unequation:

$$F(n-1) > F(n) < F(n+1) \qquad [6]$$

valid for the very nature of setting the problem. The function of total exploitation costs for the relative bucket wheel excavator within the interval of n periods $F(n)$ from the relation (5), will reach the minimum value in the n-period only if the conditions from the relation (6) are fulfilled. Instead of repeating the same process it is quite enough to state that the following relation is valid

$$1 - a^n = (1-a)\sum_{t=1}^{n} a^{t-1} \qquad [7]$$

and enables exchange of $(1 - a^n)$ in the relation (6), thus forming the following relation

$$F(n) = \frac{1}{1-a}\frac{1}{\sum_{t=1}^{n} a^{t-1}}\left[A + \sum_{t=1}^{n} a^{t-1}h(t)\right] = \frac{1}{1-a}G(n), \qquad [8]$$

namely, by introducing the costs of urgent repairs $s(t)$ as well as the costs caused by idle time of bucket wheel excavator $k(t)$, the relation (8) gets the following form

$$F(n) = \frac{1}{1-a}\frac{1}{\sum_{t=1}^{n} a^{t-1}}\left\{A + \sum_{t=1}^{n} a^{t-1}\left[h(t) + s(t) + k(t)\right]\right\} \qquad [8a]$$

namely,

$$G(n) = (1-a)\cdot F(n), \qquad [9]$$

where

$$G(n) = \frac{1}{\sum_{t=1}^{n} a^{t-1}}\left[A + \sum_{t=1}^{n} a^{t-1}h(t)\right]. \qquad [10]$$

or

$$G(n) = \frac{1}{\sum_{t=1}^{n} a^{t-1}}\left[A + \sum_{t=1}^{n} a^{t-1}\left[h(t) + s(t) + k(t)\right]\right] \qquad [10a]$$

Transformation of recurrent functions from relations: (1), (2), and (3) into a common function of total exploitation costs for the bucket wheel excavator from the relation (5) denotes completion of forming the adequate model of replacement. Then, the following task is to find out the unknown value of the variable n on the grounds of the function of average exploitation costs for bucket wheel excavators $G(n)$ from the relation (10) or the main function of total costs $f(n)$ from the relation (5). Namely, in both cases it is necessary, to find out the value of functions $G(n)$ or $F(n)$ by gradual adding the value n to every following period (n=1...) and to follow the tendency of their changes. The bucket wheel excavator should not be replaced until the tendency of costs reduction of functions $G(h)$ and $F(n)$ exists. The replacement should be done at the end of the period which is followed by costs increasing tendency, representing at the same time the criterion for determining the values of the variable n (Figure 2).

Should the liquidation value of bucket wheel excavator exist, as stated in the relation (4), then instead of the relation (5), the following relation should appear:

$$z_o(1) = \frac{1}{1-a^n}\left[A + \sum_{t=1}^{n} a^{t-1}h(t) - a^{n-1}Ag(n)\right] = F(n) \qquad [11]$$

with the corresponding value of the function $G(n)$.

The following example illustrates the application of the previous model of replacement for calculation of optimum exploitation life of many-year lasting mining machinery such as bucket wheel excavators.

Let us suppose that a new bucket wheel excavator, the purchase value of which is 11 millions USD, is in question. According to the estimates based on the experience of 'Kolubara Metals', the average annual investment for the maintenance of bucket wheel excavators, amounts to about 3.3% of the purchase value of the new machinery. On the basis of this information, the function of change in maintenance costs for the mentioned bucket wheel excavator is determined by a trend analysis:

$$h(t) = 0.174 + 0.0642\,t - 0.00645\,t^2 + 0.000196\,t^3$$

with the coefficient of correlation K_k=0.997. Duration of 35 period interval is taken on the age basis of the existing bucket wheel excavators in the Kolubara coal basin. It is estimated that the liquidation value for the bucket wheel excavator should not be taken into consideration as it is approximately equal to the costs of disassembling. The analysis covers three discount rates of 4%, 6% and 8%. Table I shows the results of calculations of the change of function values of total $F(n)$ and average costs $G(n)$.

It is evident from Table I and graphs in Figures 3 and 4. that the fall of functional values for total and average exploitation costs of bucket wheel excavator up to 26 years is v=0.04, up to 27 years is v=0.06 and up to 28 years is v=0.08. It is also evident that the changes of average exploitation costs for bucket wheel excavator are insignificant: between 25 and 28 years v=0.04, between 26 and 29 years v=0.05 and between 26 and 30 years v=0.08. Accordingly, the optimum exploitation life of bucket wheel excavator is between 26 and 28 years, namely 30 years.

Should the optimum solutions be accepted, the annual average discounted (reduced to the first period value) exploitation costs for excavators amount to 1.06 for v=0.04; 1.18 for v=0.06 and 1.31 for v=0.08.

This example is based on the assumed data which are correlated with the mentioned experience, and, as such, is used only to illustrate the philosophy of the method and the way of calculating the optimum exploitation life of machinery according to the previous model.

Variant B: The existing bucket wheel excavator

Model hypotheses—At the beginning of the first period within unlimited interval, the exploitation of the existing bucket wheel excavator is carried out..

As the minimum exploitation costs are selected as a criterion of optimization, all the components of such costs should be known at the time of model formation. The following information are indispensable for the analysis: the purchase values for the existing bucket wheel excavator, established on the grounds of its condition at the beginning of the first period of analysis: the costs of regular maintenance and periodic repairs within every period k= 1,2...n,... as the age of the existing bucket wheel excavator

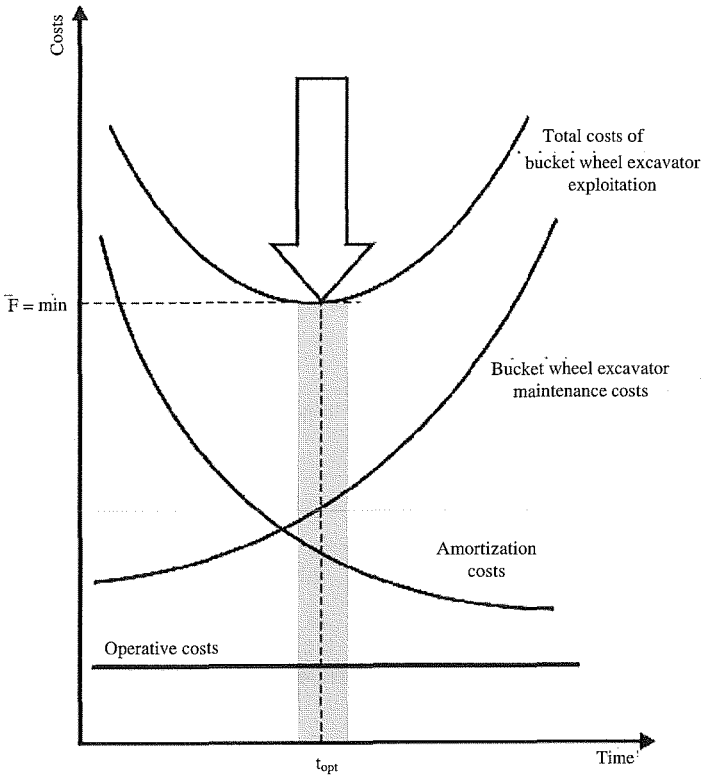

Figure 2. Costs relations

Table I

Optimum solution	Period	$v = 0.04$		$v = 0.06$		$v = 0.08$		$h(n)$
		$G(n)$	$F(n)$	$G(n)$	$F(n)$	$G(n)$	$F(n)$	
	1	11.23	292.03	11.23	198.43	11.23	151.63	0.23
	2	5.86	152.42	5.91	104.49	5.97	80.54	0.28
	3	4.08	106.21	4.16	73.41	4.22	57.03	0.31
	4	3.20	83.28	3.28	58.00	3.36	45.40	0.34
	5	2.68	69.62	2.76	48.84	2.85	38.48	0.36
	6	2.33	60.58	2.42	42.77	2.51	33.92	0.37
	7	2.08	54.14	2.18	38.46	2.27	30.68	0.37
	8	1.90	49.32	2.00	35.25	2.09	28.27	0.38
	9	1.75	45.58	1.85	32.75	1.96	26.41	0.37
	10	1.64	42.57	1.74	30.76	1.85	24.93	0.37
	11	1.54	40.11	1.65	29.13	1.76	23.73	0.36
	12	1.46	38.06	1.57	27.77	1.68	22.73	0.35
	13	1.40	36.31	1.51	26.63	1.62	21.89	0.35
	14	1.34	34.82	1.45	25.65	1.57	21.18	0.35
	15	1.29	33.53	1.40	24.82	1.52	20.57	0.35
	16	1.25	32.42	1.36	24.09	1.49	20.05	0.35
	17	1.21	31.45	1.33	23.47	1.45	19.60	0.36
	18	1.18	30.61	1.30	22.93	1.42	19.22	0.38
	19	1.15	29.89	1.27	22.46	1.40	18.89	0.41
	20	1.13	29.27	1.25	22.06	1.38	18.60	0.45
	21	1.11	28.76	1.23	21.73	1.36	18.37	0.49
	22	1.09	28.34	1.21	21.45	1.35	18.17	0.55
	23	1.08	28.01	1.20	21.23	1.33	18.01	0.62
	24	1.07	27.76	1.19	21.06	1.32	17.88	0.71
	25	1.06	27.60	1.19	20.94	1.32	17.79	0.81
→	26	**1.06**	**27.52**	1.18	20.86	1.31	17.72	0.93
→	27	1.06	27.53	**1.18**	**20.83**	1.31	17.68	1.06
→	28	1.06	27.61	1.18	20.84	**1.31**	**17.67**	1.22
	29	1.07	27.77	1.18	20.89	1.31	17.68	1.39
	30	1.08	28.01	1.19	20.98	1.31	17.72	1.59
	31	1.09	28.33	1.19	21.11	1.32	17.77	1.80
	32	1.10	28.73	1.20	21.27	1.32	17.84	2.05
	33	1.12	29.20	1.22	21.47	1.33	17.93	2.31
	34	1.14	29.75	1.23	21.71	1.34	18.04	2.60
	35	1.17	30.38	1.24	21.98	1.35	18.17	2.92

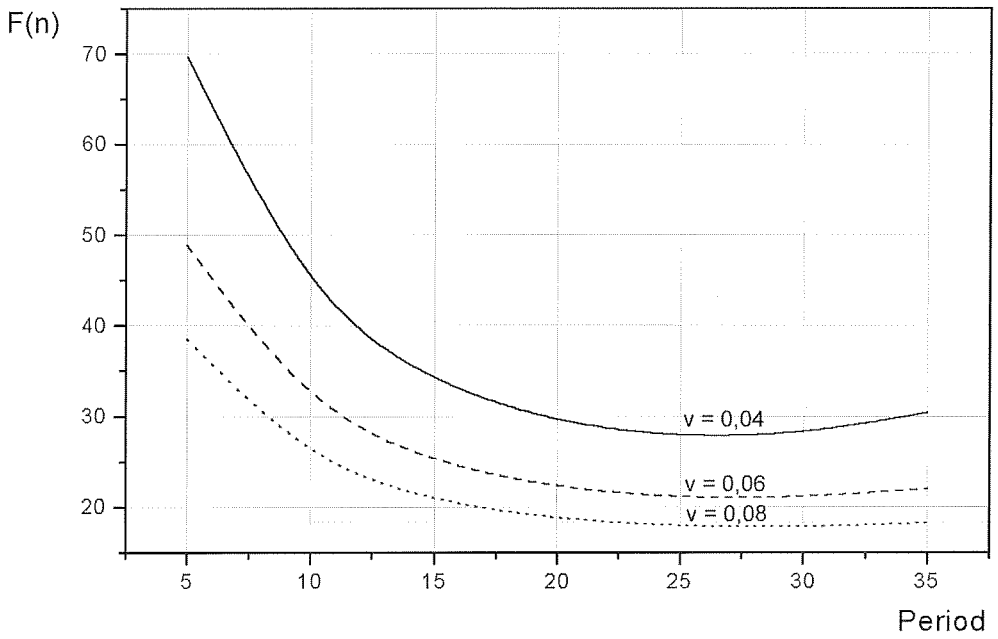

Figure 3. The functions of total costs for bucket wheel excavator (for the given example)

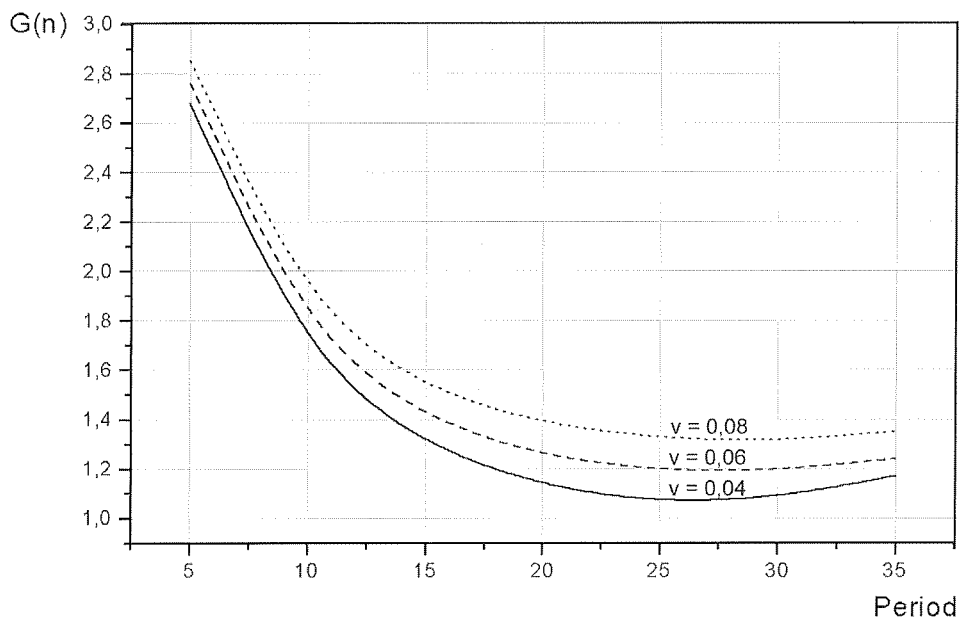

Figure 4. The functions of average costs for bucket wheel excavator (for the given example)

is not equalized with the observation period. Within the interval from the beginning of the first until the end of the n-period, the purchase value of the existing bucket wheel excavator reduces and therefore it is necessary to establish the value of function $G(k)$ by which the liquidation value of the bucket wheel excavator at the moment of its replacement might be determined. It is quite clear that the purchase value of the new bucket wheel excavator (A), to replace the existing one at the end of the n-period, should also be known.

Bearing in mind that the relations from the relation (6) and the connection demonstrated by the relation (7) are valid for this model of replacement, the following function might be established:

$$G(n) = \frac{1}{\sum_{k=1}^{n} a^{k-1}} \left\{ \sum_{k=1}^{n} a^{k-1} h(k) + a^{n} \left[A - A_s g(n) \right] \right\} \qquad [12]$$

It shows the same dependence on the function $F(n)$, defined by the relation (9). The functions $G(n)$, in both models, can equally be interpreted from economical aspect.

The mode of finding out the optimum value for the variable n, does not differ from the previously described one. It is necessary to assign, step by step, the value for the variable n according to the formation sequence of periods (n=1,2...) commencing from the first, and then to calculate the corresponding values for the functions $G(n)$ and/or $F(n)$.

The ordinal number of periods, wherein those two functions have the minimum value, represents the optimum solution for the variable n.

Final estimate

According to the results reached during our research work, the possible solution for the methodology of establishing the exploitation life of the capital mining equipment is grounded on the philosophy of the dynamic programming, namely, on the models of replacement with unlimited interval.

It was concluded that this approach is characterized by a high pragmatic value when determination of the optimum exploitation life is concerned. However, the procedure of estimating the optimum exploitation life of machinery and passing the management decisions requires a systematic team-experts analysis. The suggested methodology, assuming correct data input, offers a reliable determination of exploitation life of capital machinery and represents an avoidable mark in decision-making, but the final decision on writing off and replacement of the bucket wheel excavator with a new one, or on its general overhaul, depends on the overall systematic understanding of technical, technological, economic, logistic and other causal-consequential connections and effects within the post-optimization period.

The data represent the hypothesis for successful implementation of the established methodology, as well as for every sophisticated engineering analysis. If the model is provided with incorrect data input, it is impossible to expect from the model to offer correct solution of the set task. While estimating the exploitation life of capital machinery as a 'long-living' one, the data problem is characterized by two dimensions, one representing accuracy and preciseness, the other-time continuation. The knowledge experienced during elaboration of this study indicates a conclusion that no adequate and accurate information monitoring of exploitation costs of capital machinery exists at coal open pit mines within the Electric Power Industry of Serbia.

The general conclusion, based on the above-mentioned observations, states that the adequate information support is required for implemention of both the replacement model when estimating the exploitation life of capital machinery as well as all other engineering analyses and passing of management decisions.

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