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Solving of determinate mining—Economic models by the method of Monte Carlo

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In mining the method of statistical tests is used by solving of problems, exposing to influence of random factors. Such problems are: rational selection of equipment set; co-ordination in time of different processes; interaction between some processes and kinds of works. Usually it is considered that Monte Carlo method proves its value in the first place in that problems, which allow theoretical-probabilistic description.

Analysis of numerical solution of mining-economic problems shows that the method of statistical tests may be used successfully by solving of determinate problems, which are solved usually analytically or by the methods of mathematical programming. We are talking about the problems of optimal use of resources, expedient placing of equipment, effective use of investments, rational planning of stripping and extracting works, groups of mining enterprises and about different variants of transport problem.

In the report possibility of the method use is studied at an example of typical technical-economic problems solving. By solving of these problems the variant of the method with uniform distribution of random quantity had been applied and built-in random number generator of visual modelling system VisSim had been used.

By algorithms elaboration it seems very perspective to use combination of the Monte Carlo method with traditional methods of numerical solving of the problems (methods of linear, nonlinear and dynamic programming and so on).

Keywords: method of statistical tests, mining-technical problems, diapason of random number sample, efficiency function and its extremum.

Discussion and new observations

Optimizational mining-economic models may be divided into the deterministic and probabilistic ones depending on character of information, used by modelling. Usually it is considered that Monte Carlo method adoption proves its value in the first place in such problems, which permit probability-theoretic description, according to Sobol[1], Ermakov[2], Reznichenko[3]. By this after analysis of the object or process the probabilistic model of its functioning (process imitation) is built. As the main blocs of algorithms random number generators with uniform distribution are used.

General scheme of the method is based on the central limit theorem of probability theory, according to which any unknown quantity Z may be considered as the expectation of some random quantity ξ, i.e. $M[\xi]=Z$. After calculation of N independent values $\xi_1,\ldots\xi_N$ of quantity ξ one can consider $Z\approx(1/N)(\xi_1+\ldots+\xi_N)$. For this the random quantity, uniformly distributed in the interval [0,1], is modelled. The random quantity is built from the sequences of so-called pseudo-random numbers. There are numerous computer programs for generation of pseudo-random numbers for any laws of distribution.

Experience of numerical solving of great number of mining-economic problems according to Rakishev[4,5] shows that the method of random search, employing a body of mathematics of Monte Carlo method, may be used successfully in optimization of deterministic models, usually solved by analytical method or by methods of mathematical programming. We say about the problems of resources optimal use, expedient placement of equipment, effective use of investments, rational planning of work of mining enterprises group and transport.

We shall show possibility of broad application of the methods of random search in examples of solving of typical mining-economic problems. By this the variant of the method with uniform distribution of random quantity (in principle the law of distribution may be arbitrary) and built-in generator of system of visual modelling VisSim are used. For uniform distribution probability of value getting into the given interval is proportional to the length of this interval.

Characteristics of uniform distribution:

$$M[\xi]=(X_{\max}-X_{\min})/2, D[\xi]=(X_{\max}-X_{\min})^2/12,$$

where X_{max}—is maximum value of parameter, X_{min}–minimum value of parameter, $D[\xi]$—variance.

Therefore, the main principle, causing efficiency of the method, consists in narrowing of interval of random quantities sample or, in other words, in reduction of variance.

General algorithm of the problem solving is adduced at the Figure 1. Procedure of algorithm realization may be divided into the stages.

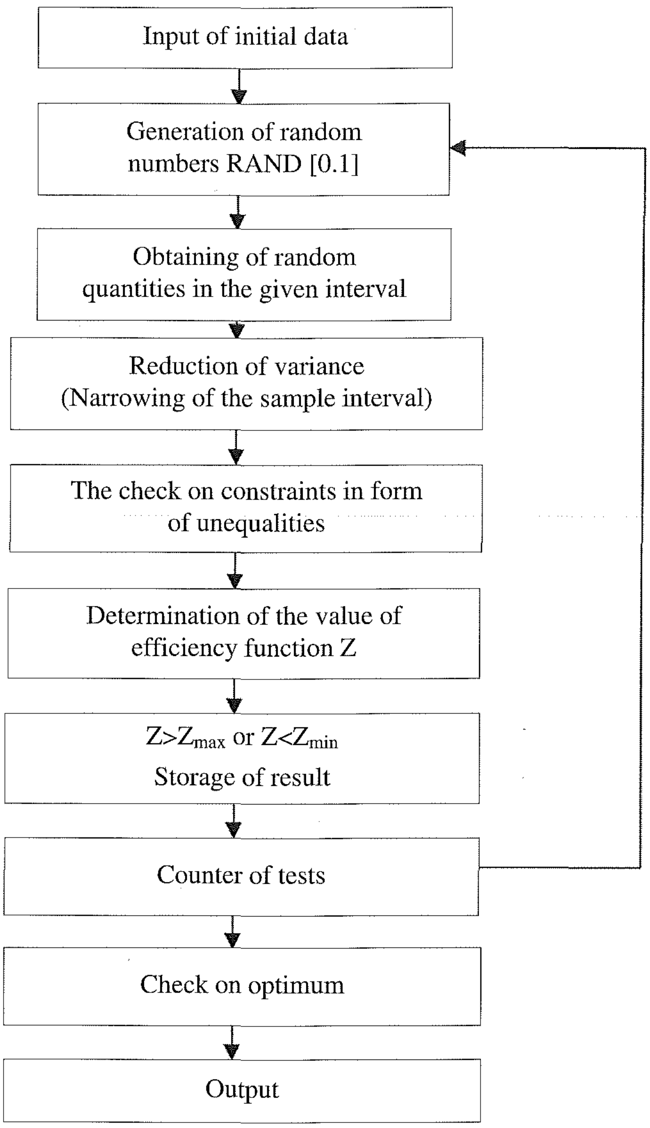

Figure 1. Block-scheme of algorithm of deterministic models optimization by the method of random exhaustion with reduction of variance

The 1st step— random numbers, uniformly distributed in the interval [0,1], are generated and, proceeding from conditions of the problem, the interval of sample of random quantities, corresponding to the process parameters, is designated. For this constraints of model, given in form of equations or equalities, are used.

The 2nd step—reduction of variance. For this the different generally adopted methods, narrowing interval of sample and raising accuracy of the final result (method of least values, method of potentials, rounding off, etc.), are used.

The 3rd step—the obtained combinations are checked up on their conformity to the problem's constraints, given in form of inequalities. If all the constraints are given in form of equalities, part of them serves for precise assignment of sample diapason, which makes for the method efficiency (look at the step 1). Other equalities are reduced to inequalities by the way of assignment of the confidence interval, on which one or another parameter may deviate. The values of combinations of random quantities, which don't answer to constraints, are cut off.

The 4th step—value of efficiency function and its extremum are defined by the method of successive comparison in pairs of obtained values.

The proposed variant of algorithm, using Monte Carlo method, may be called the method of random exhaustion with reduction of variance (narrowing of the sample interval). Necessary amount of tests depends on demanded accuracy of final result and is determined by well-known dependences. Such algorithm, more simple than Monte Carlo method in classical form, doesn't demand reduction of the results by the methods of mathematical statistic.

Typical mining-economic problems from the known sources are considered as an example.

(i) Planning of extraction of ore of prescribed quality according to Protosenya[6]. It is necessary to draw up a plan of the quarry's work under condition that total expenses for ore transportation can't exceed 8690 roub. by maximum possible extraction of ore, and P_2O_5 content must be within 6.8—7.0 per cent. Data on the quarry are adduced at the Table I.

Let's denote with x_1, x_2, x_3 the volume of extraction at the 1st, 2nd and 3rd sections correspondingly. So the efficiency function the total volume of ore extraction—will be written as $u=x_1+x_2+x_3 \rightarrow \max$.

Constraints on:

transportation expenses $25x_1+20x_2+15x_3,\leq 8690$;

ore quality $6.8(6x_1+8x_2+6.8x_3)/(x_1+x_2+x_3)7.0$;

maximum allowed extraction from the sections $x_1\leq 160$, $x_2\leq 160$, $x_3\leq 320$;

non-negativity of extraction volumes $x_1\geq 0$, $x_2\geq 0$ $x_3\geq 0$.

According to Protosenya[6] this problem had been solved by simplex-method of linear programming. The following values of volumes had been obtained: $x_1=58$, $x_2=122$, $x_3=320$, which gave in sum $u=500$. Expenses for transportation were 8690 roub., average content of P_2O_5—7 per cent.

Algorithm of the problem solving by Monte Carlo method is the following. As there is no restriction on maximum extraction from the quarry, diapason of sample of extraction random volumes from every face can be adopted as $0\leq x_{i\,rand}\leq x_{imax}$ or $x_1=160$ RAND, $x_2=160$ RAND, $x_3=320$ RAND, where RAND is value of random number in diapason from 0 till 1, produced by generator of random numbers; $x_{i\,rand}$ is the meaning of random volume of extraction from the i-th face, obtained by testing. By formula of weighted average values content of P_2O_5 in total volume of extraction is $(6x_1+8x_2+6.8x_3)/(x_1+x_2+x_3)=\psi_p$.

Let's check obtained random set of volumes on conformity to planned quality $|\psi_{av} - \psi_{pl}|\leq\Delta\psi_{pl}$ and to expenses for ore transportation. Cut off the volumes, not satisfying these constraints. Define maximum value of efficiency function by the method of comparison in pairs. Volumes of ore extraction from the sections, corresponding to this value, are solutions of this problem. They are: $x_1=58$, $x_2=122$, $x_3=320$ and in sum they give $u=500$. Expenses for transportation are 8690 roub., average content of P_2O_5– 7

Table I
Planned indices on the sections

No. of the section	P_2O_5 content	Specific transportation expences, roub.	Maximum possible extraction, ths.t
1	6.0	25	160
2	8.0	20	160
3	6.8	15	320

per cent. Obtained results coincide completely with the results in research[6].

(ii) Distribution of capital investments between the mines according to Reznichenko[3]. For development of three mines 29 mln roub. are allotted. Efficiency of capital investments use is expressed for these mines by dependences: $q_1 = 0.2\sqrt{x_1}$, $q_2 = 0.3\sqrt{x_2}$, $q_3 = 0.4\sqrt{x_3}$ (where x_1, x_2, x_3 is the volume of investments, allotted for the first, second and third mine correspondingly). It is necessary to distribute investments so that total efficiency of their use be maximum.

Total efficiency of investments is expressed by efficiency function

$$Z = 0.2\sqrt{x_1} + 0.3\sqrt{x_2} + 0.4\sqrt{x_3} \rightarrow \max.$$

In research[3] this problem had been solved by Lagrange's the method of undetermined multipliers and by the method of dynamic programming. The following value of investments had been obtained (mln roub.): $q_1=4$, $q_2=9$, $q_3=16$, the total efficiency of investments would be 29.

Algorithm of the problem's solving by the method of random search. With a help of the method of successive reductions we assume the interval of random number sample of investments for every mine: for the first mine—all the sum of investments $x_1=\text{int}(29\div\text{RAND})$, for the second one the whole sum of investments with subtraction of random meaning of investments, obtained for the first mine, $x_2=\text{int}[(29-x_1)\text{RAND}]$, correspondingly for the third one—the whole sum of investments with subtraction of random meanings, obtained for the first and second mines, $x_3=29—x_1—x_2$ (Int means rounding off till whole number). We determine value of efficiency function Z and find its extremum. In first 100 tests we have the answer, identical with the one, adduced in research[3].

(iii) It is necessary to determine extremum values of separable quadratic efficiency function in research[3]

$$Z = (x_1 - 5)^2 + (x_2 - 4)^2$$

by constraints: $x_1+x_2\geq 2$, $3x_1+2x_2\leq 12$, $x_{1,2}\geq 0$.

In research[3] the problem had been solved by semigraphical method. The following values of the searched quantities had been obtained:

$$Z_{\max} = 29, \text{ when } x_1 = 0,\ x_2 = 2$$

$$Z_{\min} = 9{,}32, \text{ when } x_1 = 2{,}4,\ x_2 = 2{,}4.$$

Algorithm of the problem solving by the method of random search is the line, which is realized in cycle. Using the second constraint, we assume the interval of the sample of random meanings x_1, x_2 with a help of the method of successive reductions: $x_1=12\div\text{RAND}/3$, $x_2=(12-3x_1)/2$. Determine the value of efficiency function Z and its extremum values. The following results are obtained: $Z_{\max}=28{,}998$, when $x_1=0{,}000122074$, $x_2=5{,}99982$. Rounding off till whole numbers gives the values $Z_{\max}=29$, when $x_1=0$, $x_2=6$, $Z_{\min}=9{,}307$, when $x_1=2{,}45772$, $x_2=2{,}31343$. As is obvious, when Monte Carlo method use more precise values of extremums have been obtained.

(iv) It is necessary to determine minimum of quadratic function in research[7]

$$Z = x_1 \cdot x_2^2$$

by constraint: $2 - x_1^2 - x_2^2 \geq 0$.

In research[7] the problem had been solved by the method of quadratic penalty function. The following values had been obtained: $Z_{\min}= - 1{,}08866565$ when $x_1= - 0{,}81650$, $x_2= - 1{,}1547$.

Diapason of sample of random meanings x_1, x_2 is determined, proceeding from the restriction. Algorithm of the problem solving by the method of random search is the next: $x_1=2\div\text{RAND} - 2$, $x_2= |2 - x_1^2|^{0.5}$. Cut off the values of x_1, x_2, not satisfying the restriction. Determine the value of efficiency function Z and its extremum values. Obtained results of 300 tests are identical to the ones, adduced in research[7].

Transportation problem of opened type in research[3]. It is necessary to determine the volumes V_{ij} of transportation from the i-th supplier to the j-th consumer, by which minimum total cost of transportations is achieved. Conditions of a problem are adduced at the Table II.

In research[3] this problem had been solved by the simplex-method of linear programming. When $Z_{\min}=2010$ the following values of the volumes had been obtained:

$$V_{11} = 30 \quad V_{12} = 20 \quad V_{13} = 0 \quad V_{14} = 0$$

$$V_{21} = 0 \quad V_{22} = 90 \quad V_{23} = 0 \quad V_{24} = 0$$

$$V_{31} = 0 \quad V_{32} = 70 \quad V_{33} = 70 \quad V_{34} = 130.$$

Direct application of the method of random search (look at the algorithm of the problem (i) solving) gives the result, differing on 3 per cent from the accurate one, which is essential when large volumes of transportations. Combination of this method with elements of the method of least costs and the results subsequent rounding-off give the accurate solution of the problem.

Algorithm of the problem solving. Realize only the first step of the method of least costs. Using of the data from the Table II gives the following, maximum allowed volume of transportations: $V_{11}=30$, $V_{22}=90$, $V_{33}=70$. Therefore, $V_{21}=V_{31}=V_{23}=V_{24}=V_{13}=0$. Then Monte Carlo method is applied. All the volume of resources of the first and third suppliers is adopted as the interval of the sample of transportation random volumes V_{12}, V_{32}. For determination of transportation random volumes V_{14}, V_{34} the interval of the sample is defined as difference between all the resources volume of corresponding supplier and determined yet volumes of transportations:

$$V_{12} = \text{int}\left[(50 - V_{11} + V_{13}) * RAND\right], \quad V_{14} = \text{int}\left[(50 - V_{11} + V_{13}) - V_{12}\right],$$

$$V_{32} = \text{int}\left[(250 - V_{33}) * RAND\right], \qquad V_{34} = \text{int}\left[(250 - V_{33}) - V_{33}\right].$$

Cut off the volumes, not satisfying the constraints: $V_{11}+V_{21}+V_{31}\leq 30$, $V_{12}+V_{22}+V_{32}\leq 200$, $V_{13}+V_{23}+V_{33}\leq 70$, $V_{14}+V_{24}+V_{34}\leq 130$. Determine the value of efficiency function Z, find min(Z). The found values of the searched quantities coincide completely with the solution in research[3].

(vi) Large amount of mining-economic problems are bound with calculations of areas and volumes. That is why in conclusion let's adduce comparative estimation of

Table II
Resources, necessities and costs of transportations

Supplier	Consumers				Resources
	B_1	B_2	B_3	B_4	
A_1	4	6	5	7	50
A_2	6	4	8	9	90
A_3	5	7	4	6	250
Necessity	30	200	70	130	

results, obtained by the method of random search, on an example of calculation of area of a circle and volume of a sphere. The main principle of solving algorithm consists in the fact that all the tests fall into circumscribed unity square and cube.

By Monte Carlo method area of a circle may be estimated as product of area of unity square (a=1m) by ratio of amount of tests, fallen into the circle, to the total amount of fallings into the square, i.e. $S_{sq\ m.c.}=S_{sq}(N_{cl}/N)$. Control area of inscribed circle $S_{cl}=\pi D^2/4=0{,}785397$ m^2. Changes of ratio $S_{cl}/S_{sq\ m.c.}$, depending on the tests amount (the method's error), are adduced at the Figure 2a. As is obvious, when 2500 tests the error is a tenth of a per cent, and by subsequent increase of the tests amount it tends to zero.

Analogically, volume of the sphere, inscribed into the unity cube (a=1m), is determined by Monte Carlo method as the product of unity cube volume by ratio of amount of the tests, fallen into the sphere, to the total amount of fallings into the cube, i.e. $V_{sp\ m.c.}=V_{\text{cube}}(N_{sp}/N)$. The sphere volume $V_{sp}=4/3\ (\pi r^3)=0{,}523598$ m^3 is adopted as the control one. Changes of ratio $V_{sp}/V_{sp\ m.c.}$, depending on the tests amount (the method's error), are adduced at the Figure 2b. When 5000 tests the error is the tenth of a per cent and by subsequent increase of the tests amount it tends to zero.

Considered examples show that the proposed variant of Monte Carlo method has universality and may be applied for solving of majority of mining-economic models, both the determinate and probabilistic ones, and also it may be applied in the cases, when it is impossible to use methods of linear and non-linear programming on the problem's conditions.

The method has simple, clear and effective algorithm by lack of rigid constraints for application. For considerable number of models the algorithm will consist of 3–4 main steps (stages).

It gives possibility of result obtaining practically with any preset accuracy. Accuracy of the solutions depends on number of tests and accuracy of representation of the sample diapason. If the problem (model) hasn't precise solution, it is possible to receive solutions, the most approximate to the precise one.

By applying of the method it is possible to find definite number of alternative solutions, which, with the simple algorithm, promotes adaptability, efficiency and dynamicness of the method.

The method can be combined with majority of generally used methods of the models solving, which also broadens the sphere of its application and improves efficiency of traditional methods. Using in the method of random search comparatively simple algorithms allows to create dynamic models, reflecting real technological processes more completely and precisely. Slow convergence, which has been considered the main shortcoming of the method, is of no importance by intensive development of computer technique.

Conclusions

The method of random search gives a possibility of obtaining of solutions set, practically equivalent to though single solution, which is achieved by application of different methods of linear and non-linear programming. Possibility of choice of alternative variants of distributions of loads on the faces, volumes of deliveries and consumption, rates of investments and so on allows to say

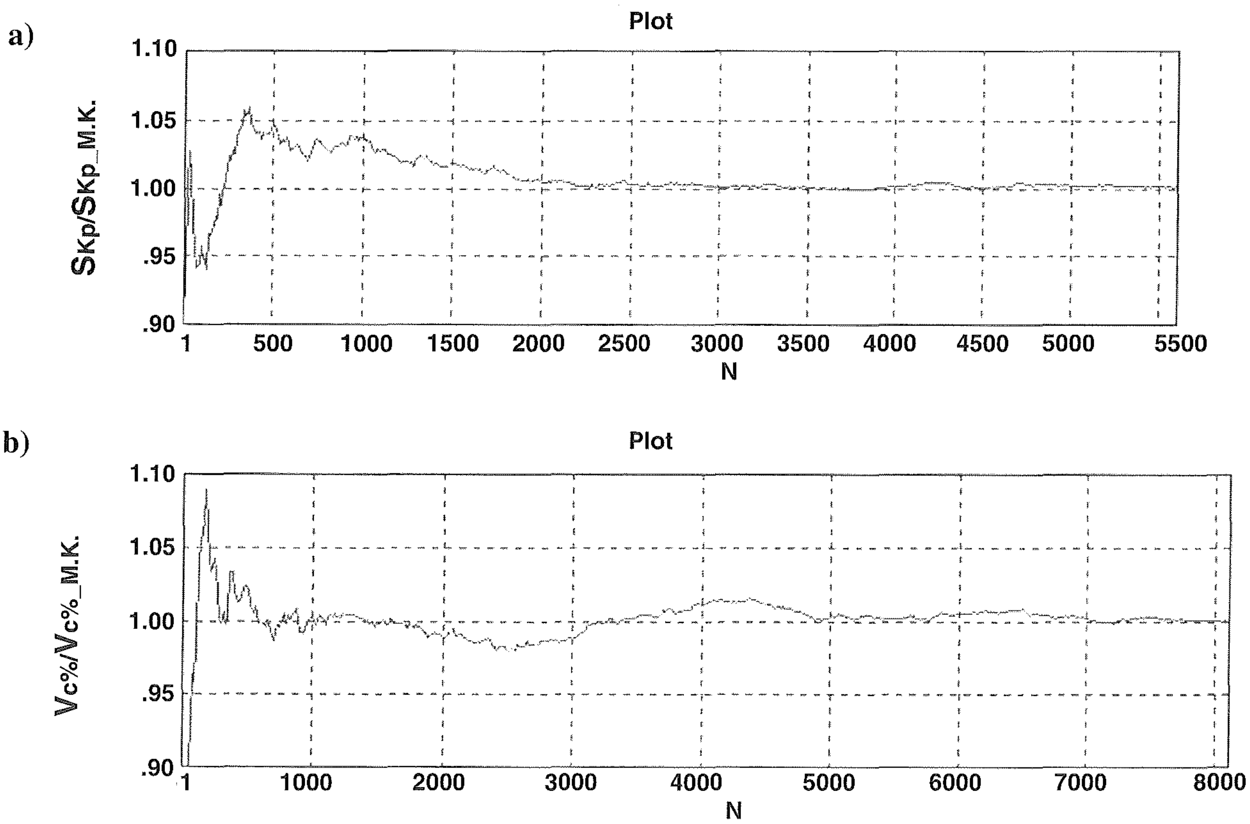

Figure 2. Dependence of accuracy of results of the circle area determination (a) and the sphere volume determination (b) from the tests amount

about large adaptability of this algorithm for variable conditions of problem. And there are no rigid constraints so characteristic of the methods of linear and non-linear programming.

By application of the method of random search accurate enough result is achieved. Only in separate cases deviation is a tenth of per cent. Amount of the tests, depending on the problems conditions, may be within 2–5 thousands. Their realization is of no difficulty by modern level of computer technique.

The method efficiency depends to a large extent on precision of assignment of the interval of random numbers sample, corresponding to the problem's constraints. Ways of assignment of the sample precise interval may be varied, depending on the type of the problems.

By algorithms development it seems very perspective to use combinations of the method of random search with elements of traditional methods of numerical solving of the problems of linear, non-linear and dynamic programming and so on.

All foregoing confirms simplicity and efficiency of the proposed algorithm.

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