The Impact of Transportation Network upon the Potential Supply of Base and Precious Metals from Sonora, Mexico

By DeVERLE P. HARRIS,* and DAVID E. EURESTY †

SYNOPSIS

Potential metal supply refers to metals to be produced from as yet unknown deposits. Naturally, to measure potential supply requires, among other things, inference to the number of deposits, their tonnage, and their grade Thus, probability distributions were constructed for each 400-square mile cell in Northern Sonora, Mexico. A computer system was developed that simulated the occurrence of deposits for each cell, exploration for the deposits, and their exploitation. Evaluation was performed for each deposit by discounted cash flow analysis.

An objective of this study was to explore the effect of the transportation network and required road or railroad construction upon potential supply. Consequently, the evaluation system was designed to consider the spatial location of a simulated deposit to the existing transportation network and the market destination. The construction of linkages to connect the deposit to existing transportation arteries as well as the carrying of the concentrates to the intended market were simulated in terms of cost. The selection of the path took into account the existing terrain. This was done by digitizing topographical maps and representing the topography of the area by an orthogonal grid of elevations. The optimum route was selected by dynamic programming, meeting a prescribed gradient constraint. The potential supply of each cell was determined, allowing for the infrastructure effect.

INTRODUCTION

Simply stated, the objective of this study was to estimate the potential supply of base and precious metals from northern Sonora, Mexico, with consideration given to the cost and efficiency of exploration, capital and operating costs, construction costs of transportation linkages, and the costs of transporting the concentrates to market. Potential supply refers to that quantity of metal expected to occur in deposits which would be discovered in the future and which would be economic to produce. Naturally, estimation of potential supply requires inference about events which are not currently known; consequently, analysis relies heavily upon the application of probability theory and computer simulation.

The first step in the estimation of potential supply is the estimation of metal endowment, the resources of metal that are expected to occur in the area. In this study, metal endowment was estimated by obtaining subjective probability distributions from geologists who were knowledgeable of the resources of Sonora by virtue of their experience in exploration.

The second step was the development of a computer program to simulate exploration for the probabilistic deposits and to simulate the exploitation of those deposits discovered within an accounting scheme that provided a discounted cash flow evaluation for each deposit. Only those deposits that were discovered and could be produced at a profit contributed to potential supply.

Realistic accounting of production costs for a deposit requires consideration of the capital cost of construction of transportation linkages to the existing transportation network and consideration of the carrying costs of transporting the concentrates to market. Therefore, a third phase of this study was the digital representation of the topography of the area to select a feasible path for the required transportation link. Path selection was made by dynamic programming.

METAL ENDOWMENT

Concepts

In order to explore the concept of metal endowment, the term must first be defined. As used here, metal endowment is metal concentration above some established intensity. Thus, in general, it refers to anomalous concentrations of metals. This study concerns concentrations of base and precious metals: copper, molybdenum, lead, zinc, silver, and gold. The following seems a reasonable definition:

$$M = \phi(E_1, E_2, \ldots, E_n), \quad \ldots \ldots \ldots \ldots \quad (1\text{-}1)$$

where M is some measure of metal endowment and E_i is the i-th earth process.

Earth processes can be regarded as activities, such as the intrusion of a magmatic body into host rocks of the earth's crust, as distinct from the geological feature that results from the activity, such as granite rock. Then (1-1) supports a *deterministic model*, implying that if for the area in question we knew the earth processes that had transpired and if we knew the function ϕ, we could describe with certainty the area's endowment.

Of course, we are far from having this complete understanding of the earth and its processes. We can only speculate in retrospect, after the transpiration of the earth processes, as to conditions leading to the formation of the ore deposit. This speculation is based upon observable geological features resulting from the earth processes causing the ore deposit. Although earth science has contributed much to a knowledge of the formation of ore deposits, we do not yet know for *certain* even the *identity* of all of the processes, E_i, let alone the form of the function ϕ. Thus, although a deterministic model may be the appropriate representation of the concepts of metal endowment, because of the lack of information and knowledge of the components of the conceptual model, its usefulness in practical decision-making is limited.

A practical model must be based upon observable information and its relationship to metal occurrence. Experience has proven geological features to be *useful*, but not infallible, in the location of metal deposits. Because of the *imperfect*

* Associate Professor, Department of Mineral Economics, The Pennsylvania State University, University Park, Pennsylvania, 16802, U.S.A.

† Mineral Economist, Direccion de Estudios Economicos, Palacio Nacional 4to. Piso, Mexico 1, D.F., Mexico.

representation by the *observable* geological features of the relationships of the conceptual model, one area that looks identical to a metal-rich area, as determined from an observable subset of geological features, may have little or no metallization. Thus, the metal endowment of an area, conditional upon the geological features, should be considered a random variable:

$$P(M \leqslant m) = F(m|\hat{G}_1, \hat{G}_2, \ldots, \hat{G}_D), \quad \ldots \ldots \quad (1\text{-}2)$$

where $P(M \leqslant m)$ is the probability for metallization, M being larger than m, and $\hat{G}_j$ is the subset of imperfectly-known geological features.

The metal endowment model of this study

The model selected to express metal endowment in this study was a subjected probability model. This selection was based upon the following considerations:

(i) Subjective probabilities provide more precise description of grade and tonnage of deposits *not yet discovered* than can be obtained from basing estimates upon historical grade and tonnage data.

(ii) Significant mineral production has occurred in Sonora, yet no reliable data are available on production by mines. Consequently, the scheme employed must be independent, to a certain degree, of current and past production statistics.

Both of these reasons are important, because it was desired to use the estimated metal endowment in subsequent economic studies, such as the impact of transportation on potential supply from deposits not yet known. Therefore, the inability in this study to adjust estimates for past production (data which were not available) is a serious fault. Furthermore, since the impact of varying carrying rates on potential supply is one of the objectives of the study, grade and tonnage characteristics of future discoveries are important factors.

DATA COLLECTION

The collection of the geologists' opinions was undertaken by a graduate student, Peter Donald, who interviewed a total of ten experts from several major mining firms in Mexico. The participants were asked to express their opinions in the absence of economic considerations, in order to approximate as closely as possible an estimate of what might occur, not what might be economic under current economic conditions. This is not an easy task for a geologist, since his main interest lies in the generation of prospects conducive to the discovery of feasible mineral deposits.

PROSPECTING ZONES AND QUESTIONNAIRES

The procedure followed in the construction of the occurrence model of this study differed somewhat from that employed in a previous study of the Canadian Northwest [Harris, *et al.* (1970)], for in that study the geologists' opinions were obtained on a common reference (a grid of cells). In this study a common reference scheme was not imposed upon the geologists. Each geologist was allowed to select subareas of any size and shape (within the study area) in which he was particularly interested, generating for each one of them probability distributions for the number of deposits, for tonnage and for grade. This flexibility in definition of areas was provided so as to create circumstances that conformed to his subjective reasoning process as it is applied in practice. In addition to providing distributions for each of his subareas, he provided probability distributions for the entire area.

The questionnaire was broken into two parts, which in turn separated metal deposits into two general subdivisions: the bulk, low-grade (high-tonnage) deposits, and the small-tonnage, (high-grade) deposits. For the bulk low-grade deposits a minimum tonnage and a minimum grade were imposed: 25 million tons and 0.46 per cent copper equivalent, respectively. The minimum tonnage for the small high-grade deposits is 200 000 tons and the minimum grade is $16.00/ton. According to the geologists, the separation of the questionnaires in this manner permits an easier subjective estimation of the occurrence probabilities for mineral deposits, because in practice two distinctly different populations of mineral deposits, each with different geological characteristics of occurrence appear to exist. The lower limits for the groups were suggested by the participants; such limits they claimed were necessary because of the low familiarity of the geologists with mineral deposits whose characteristics fell below the indicated limits.

Each of the ten participants was interviewed individually. Initially, the geologist was provided with recent geological maps of the study area so that he was able to analyze and supplement his former knowledge of the area with the extra knowledge summarized by the maps. Once he had familiarized himself with the data and the study area, he was asked to select subareas in which he would conduct mineral exploration; these subareas hereafter are called prospecting zones. The selection of the prospecting zones was made according to priority. A geologist could select as many prospecting zones as he wished, so long as he maintained a priority ordering in his selection.

Having selected various prospecting zones, the participant was asked to provide probability distributions for the number of deposits, for tonnage, and for grade for each of his selected subareas. Verbally, the participant was requested to estimate the probability of at least a given number of deposits occurring in a particular zone.

The probability distributions for the number of deposits provided by the participants consisted of two independent probability distributions, one for the bulk low-grade deposits, and the other for the small high-grade deposits. Besides these distributions for each prospecting zone, the participants were asked to provide a probability distribution for the number of occurrences (deposits) for the entire study area.

Next, the participants were asked to provide the probability for any one deposit possessing tonnage of specified tonnage categories: 200 000 to 1 000 000; 1 000 000 to 5 000 000; 5 000 000 to 25 000 000; 25 000 000 to 100 000 000; 100 000 000 to 300 000 000; and 300 000 000 to 600 000 000. The questionnaire was separated into the two parts described previously, low- and high-grade, the breaking point on tonnage being at 25 million tons. Again, the participants provided two probability distributions per prospecting zone selected, plus two for the entire area.

Finally, the participants were asked to provide conditional probability density distributions for grade, that is, given a deposit of certain tonnage, what is the probability of its grade being in each of six grade categories? Each tonnage class on each subdivision was thus provided with probabilities. Therefore, there were six distributions for grade provided by each participant.

The participants did not provide different probability distributions for grade for each selected prospecting zone, but one set for all of them. According to the participants, the same grade distributions could apply to each of his selected subareas, either because he felt strongly that grades would be similar for a given tonnage, or because he was unable to define a different distribution for each selected subarea because of insufficient information.

DISTRIBUTIONS FOR CELLS

From the ten geologists' responses many zones were generated (some of which overlapped) together with their associated probability distributions. Before these data could be employed

in the analysis of the impact of transportation upon potential metal supply, the multiple responses had to be resolved into one set of probability distributions (number of deposits and tonnage) and a tonnage-grade equation for each cell (20 miles square subdivision) of the study area. The procedure used for the estimation was Monte Carlo sampling, using an electronic computer. A computer program utilized the location of the prospecting zones with respect to the network of cells to generate probability distributions for number of deposits and tonnage for each of the 64 cells of the study area (see Fig. 1). The tonnage and grade output from the program served as the basis for estimating the parameters of the grade-tonnage equation. In any one sampling of the geological opinions (iteration), both of the number distributions (small high-grade and large low-grade deposits) were tested.

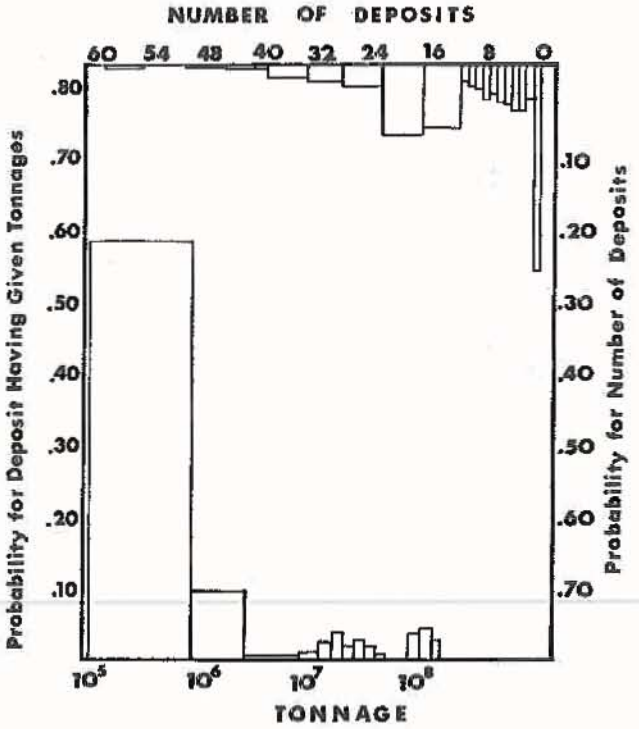

Fig. 1. Computed probability distributions for cell 50: number of deposits, and tonnage per deposit.

The computer program, through Monte Carlo sampling on the cumulative probability distributions for grade, generated a set of grade values, one for each of the deposits in the cell per iteration; thus, every deposit had associated a tonnage and a grade at the completion of each iteration. These data were subjected to regression analysis. The main objective of the regression analysis was to obtain a functional relationship capable of expressing the correlation existing between tonnage and grade. Several functional forms were tested in order to select the one that fitted the data best; among those tested were the following:

(i) $= a+b(\text{tonnage})$

(ii) $\log(\text{grade}) = a+b[\log(\text{tonnage})]$

(iii) $\log(\text{grade}) = a+b[\log(\text{tonnage})] + c[\log(\text{tonnage})^2]$

For most of the cells, the second functional form was the best fit, but for a few, the third form was selected. As usual the cells were maintained as separate units so that each cell had its own functional relationship, based upon the analysis of its own sets of tonnage and grade values.

Metal endowment of Sonora

SPATIAL DISTRIBUTION OF EXPECTED METAL POTENTIAL

In order to examine the spatial distribution of metal endowment of the study area, the average metal potential was computed for each cell. This expectation was calculated by computing for each cell the average number of deposits, $\bar{N}$, and the average tonnage of ore per deposit, $\bar{T}$. This average tonnage divided by 1 000 000 was substituted into the grade equation to generate the average grade, $\bar{G}$. The expected tonnage of metal, $\bar{M}$, for a cell was determined by multiplying these three quantities:

$$\bar{M} = \bar{N}.\bar{T}.\bar{G}$$

Figure 2 shows the spatial distribution by cell of the expected metal (copper equivalent). The most striking feature of the map is the belt of high-expected metal endowment extending from cells 62 and 63 of the southern border northwestward, passing through the cells containing Nacozari and Cananea. There are six cells in the zone that have an expected metal endowment greater than 3 000 000 tons of copper equivalent. Five of these cells are in the Nacozari area. The sixth one is immediately east of Cananea. Along this same belt, situated chiefly between Nacozari and Cananea, are four cells with an expectation of 2 000 000 to 3 000 000 tons of copper equivalent. Obviously, in the opinions of the geologists, this area has a large metal endowment. The traditional producing cells, such as Nacozari and Cananea, have high potential even though these cells have produced considerable metal. Cell 52, the most richly endowed cell, includes La Caridad, the recently discovered copper deposit of over 600 000 000 tons of ore at 0.8 per cent Cu equivalent. It is interesting that cells 25, 50, 61, and 62 have an expected endowment nearly as great as cell 52.

Apart from the cells in the eastern part of the area, there is a cluster of cells in the western part with a low to moderate metal expectation. The highest of these is cell 7, which has an expectation of 1 000 000 to 2 000 000 tons of copper equivalent.

If we were to treat this metal potential as copper and employ a price of 0.50 \$/lb, the richest cells would have an expected gross value of endowment exceeding \$1 500 000 000. Of course, this potential may not all be discovered or all be economic to produce, depending upon local specific rock conditions, depth of occurrence, and so on.

COMPARISON OF METAL ENDOWMENT

Statistical analysis of the expected metal endowment for the 64 cells yields an average endowment per cell of about 1 031 000 tons of copper equivalent. This is equivalent to 2 577 tons per square mile. Let us add to this average endowment the copper equivalent of total past production, giving us an estimate of the total original endowment as currently estimated. Data on the Sonora alone are not available; the data that are available indicate that, based upon 1968 prices, an estimate of total cumulative production of base and precious metals and reserves of that date is about \$31 000 000 000. Arbitrarily, let us assume that about one-fifth of this was derived from the study area, that is, about \$6 000 000 000. This value is equivalent to about 7 000 000 tons of copper, or about 109 000 tons per cell (based upon the 1968 price of copper). Thus, cumulative past production and as yet unknown endowment is equivalent to about 1 140 000 tons per cell or 3 050 tons per square mile.

For comparative purposes, a value per square mile was computed at a copper price of 0.32 \$/lb, giving about \$1 950 000 per square mile. This value is approximately nine times the per-square-mile value of total cumulative production to 1957 plus reserves for an area of 154 800 square miles comprising parts of Arizona, New Mexico, and Utah

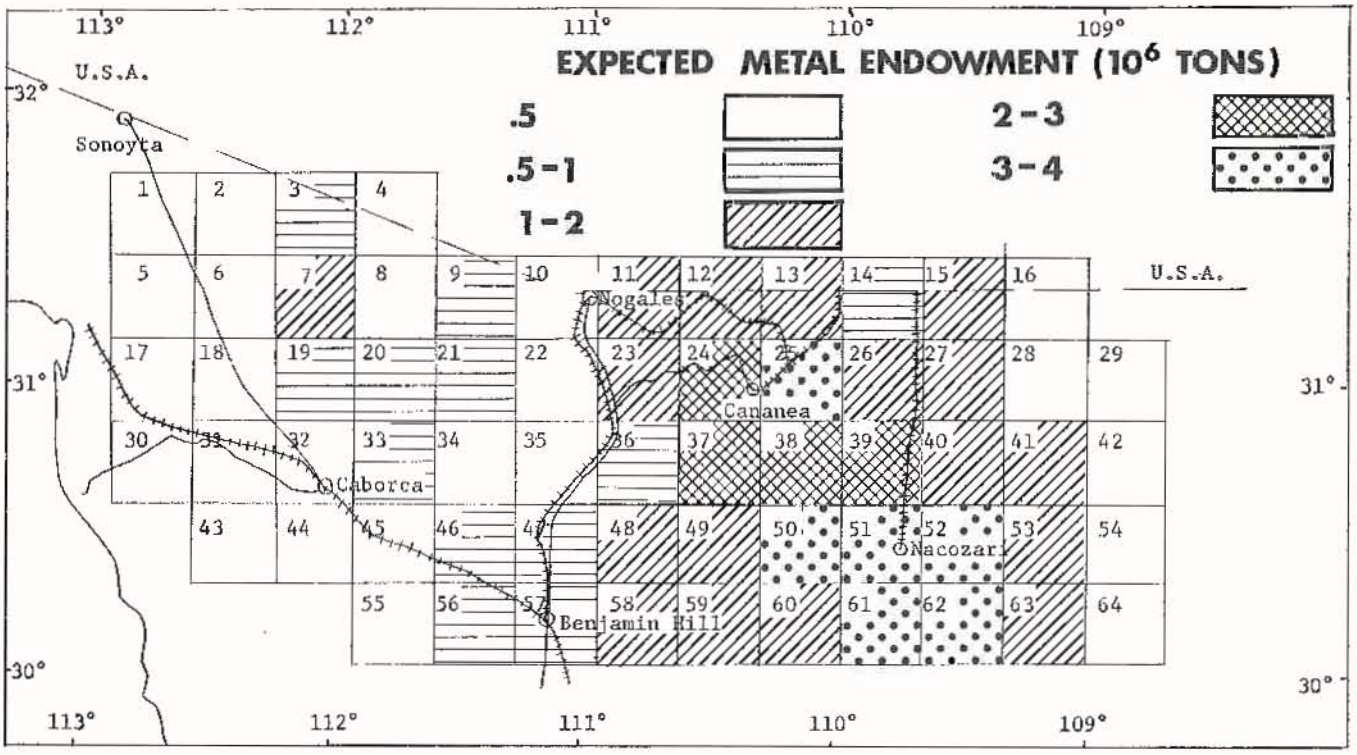

Fig. 2. Map of expectation of metal endowment in terms of copper equivalent, Northern Sonora, Mexico.

(Harris, 1968). Of course, this larger western U.S. area could be subdivided to produce an area equal in size to the Sonora area and with per-square mile value much nearer to that of the Sonora. Nevertheless, such an area would of necessity consist of the richest part of the western U.S., which is one of the richest areas in the world in metal production and reserves. Obviously, the *metal endowment* of the Sonora is *large*. It must, however, be remembered that this study indicates only what is *believed* to exist in the mineable part of the earth's crust in Sonora. In short, what has been generated is believed to be an estimate of *metal endowment*, not *potential supply*.

POTENTIAL SUPPLY

Concepts

Let us define potential supply of the j-th metal as follows:

$$PS_j = K_j T \quad \text{(2-1)}$$

where T is the ore in tons and K_j is the proportion of ore consisting of the j-th metal.

Let us further define T and K_j:

$$T = \beta_1(TRC, CMC, CCC) \quad \text{(2-2)}$$

$$K_j = Z.(h_j/r_j) \quad \text{(2-3)}$$

where h_j is the proportion of value per ton comprising the j-th metal, r_j is the price of the j-th metal (consistent with r_j and Z), and Z is the grade in dollars per ton, based on 1970 prices. Also, let TRC be the multiple of basic transportation construction schedule, CMC be the multiple of basic carrying rate, and CCC be the fraction of government sharing of construction costs.

$$Z = \beta_2(TRC, CMC, CCC) \quad \text{(2-4)}$$

Thus,

$$PS_j = (h_j/r_j)ZT \quad \text{(2-5)}$$

If a different set of prices is to be used, h_j must be adjusted:

$$h_j' = h_j(r_j'/r_j),$$

and

$$PS_j = (h_j'/r_j')Z.T, \quad \text{(2-6)}$$

where h_j' and r_j' are the new value proportion and price, respectively, of the j-th metal.

Let us specify further that β_1 and β_2 are of the following form:

$$Z = A_0(TRC)^{A_1}(CMC)^{A_2}(CCC)^{A_3} \quad \text{(2-7)}$$

$$T = B_0(TRC)^{B_1}(CMC)^{B_2}(CCC)^{B_3} \quad \text{(2-8)}$$

Then, from (2-1) and (2-3), potential supply can be written as follows:

$$PS_j = (h_j'/r_j')A_0B_0(TRC)^{A_1+B_1}(CMC)^{A_2+B_2}(CCC)^{A_3+B_3} \quad \text{(2-9)}$$

In this form, A_1+B_1 is the elasticity of potential metal supply relative to carrying rates:

$$ETRC = \frac{\partial PS_j}{\partial TRC} = A_1+B_1$$

Similarly,

$$ECMC = \frac{\partial PS_j}{\partial CMC} = A_2+B_2 =$$ elasticity of PS_j with respect to transportation construction.

$$ECCC = \frac{\partial PS_j}{\partial CCC} = A_3+B_3 =$$ elasticity of PS_j with respect to government sharing of CMC.

Thus, we can write (2-5) as follows:

$$PS_j = (h_j'/r_j')\, A_0 B_0 (TRC)^{ETRC} (CMC)^{ECMC} (CCC)^{ECC} \quad . \; . \; . \; . \; . \quad (2\text{-}10)$$

However, *a priori* reasoning suggests that these elasticities for any particular cell should vary with the transportation construction distance and the carrying distance; the greater the distance, the larger the elasticities:

$$ETRC = \alpha\,(CD)^{\alpha_1}\,(TD)^{\alpha_2},$$

$$ECMC = \omega\,(CD)^{\omega_1}(TD)^{\omega_2},$$

$$ECC = \gamma\,(CD)^{\gamma_1}\,(TD)^{\gamma_2},$$

where CD is the construction distance and TD is the transportation (carrying) distance.

Finally, we can write potential supply of the j-th metal as a function of h_j, r_j, TRC, CMC, CCC, CD, and TD:

$$PS_j = (h_j'/r_j')\, A_0 B_0 (TRC)^{\alpha(CD)^{\alpha_1}(TD)^{\alpha_2}} (CMC)^{\omega(CD)^{\omega_1}(TD)^{\omega_2}} (CCC)^{\gamma(CD)^{\gamma_1}(TD)^{\gamma_2}} \quad (2\text{-}11)$$

The potential metal supply of an area could be computed by (2-5), (2-9), or (2-11). In the case of having point estimates only of Z, T, and (h_j/r_j), equation (2-5) would be appropriate. If the analysis has yielded elasticities of the economic variables of interest, (2-9) is appropriate and if the analysis has defined the elasticities as functions of distances, (2-11) is appropriate. If the ultimate objective were the charting of a transportation route, expressions of the type of (2-11) computed for each cell would provide useful auxiliary information for selection of the optimum route.

Regardless of which equation is to be used, in order to compute potential metal supply, exploration, exploitation and transportation must be simulated on the metal endowment of each cell of Sonora. The simulation of exploration and exploitation on metal endowment has been described by Harris *et al.* (1970), in a resource and transportation study of the British Columbia Province. The method of analysis employed in this study parallels the Canadian study; in fact, many of the same relationships, some modified appropriately for special conditions of Sonora, were used in this study. Consequently, these relationships will not be reviewed in this paper. The reader is referred to the Canadian study and to Euresty (1971).

The feature of this analysis is the necessity for treating transportation and the construction of transportation explicitly in the estimation of potential metal supply. This means that the analysis must allow the selection of a feasible transportation link for each cell, the costing of its construction, and the costing of transporting the metal concentrate to market.

Estimation of expected potential supply

In the section on metal endowment, it was proposed that for the *estimation* of metal endowment a probability model is appropriate. Since metal endowment was defined probabilistically and since the estimation of potential supply is conditional upon the metal endowment, a probability model is also appropriate for the estimation of potential supply.

$$P(t_0 \leqslant T \leqslant t_1) = \int_{t_0}^{t_1} \psi(T \mid TRC, CMC, CCC)\, dT, \quad . \; . \quad (2\text{-}12)$$

$$P(z_0 \leqslant Z \leqslant z_1) = \int_{z_0}^{z_1} \Lambda(Z \mid TRC, CMC, CCC)\, dZ, \quad . \; . \quad (2\text{-}13)$$

where t_0, t_1, z_0, z_1 are the bounding tonnages and grades specified in the estimation of metal endowment.

The expectations of tonnage and grade can be computed from (2-12) and (2-13):

$$E[T] = \bar{T} = \int_{t_0}^{t_1} \psi(T \mid TRC, CMC, CCC)\, T\, dT \quad . \; . \; . \quad (2\text{-}14)$$

$$E[Z] = \bar{Z} = \int_{z_0}^{z_1} \Lambda(Z \mid TRC, CMC, CCC)\, Z\, dZ \quad . \; . \; . \quad (2\text{-}15)$$

Upon performing the integrations called for in (2-14) and (2-15), expected tonnage and expected grade are each a specific function of TRC, CMC, and CCC. Let us identify these functions as β_1 and β_2 of (2-7) and (2-8):

$$\bar{T} = \beta_1(TRC, CMC, CCC)$$

$$\bar{Z} = \beta_2(TRC, CMC, CCC).$$

Specifically, the form of the functions β_1 and β_2 were specified as equations that were linear in logarithms; their coefficients were estimated by regression of tonnages and grades developed by the simulation upon the transportation variables. Thus, for the estimation of potential supply, we have the following equations:

$$\hat{\bar{T}} = \hat{A}_0 (TRC)^{\hat{A}_1} (CMC)^{\hat{A}_2} (CCC)^{\hat{A}_3}, \quad . \; . \; . \; . \; . \; . \quad (2\text{-}16)$$

$$\hat{\bar{Z}} = \hat{B}_0 (TRC)^{\hat{B}_1} (CMC)^{\hat{B}_2} (CCC)^{\hat{B}_3}, \quad . \; . \; . \; . \; . \; . \quad (2\text{-}17)$$

$$\hat{PS}_j = (h_j'/r_j')\, \hat{A}_0 \hat{B}_0 (TRC)^{\hat{A}_1+\hat{B}_1} \times (CMC)^{\hat{A}_2+\hat{B}_2} (CCC)^{\hat{A}_3+\hat{B}_3} + e_1 \quad . \; . \; . \; . \; . \quad (2\text{-}18)$$

$$\hat{\hat{PS}}_j = (h_j'/r_j')\, \hat{A}_0 \hat{B}_0 (TRC)^{\hat{\alpha}(CD)^{\hat{\alpha}_1}(TD)^{\hat{\alpha}_2}} \times (CCC)^{\hat{\omega}(CD)^{\hat{\omega}_1}(TD)^{\hat{\omega}_2}} (CCC)^{\hat{\gamma}(CD)^{\hat{\gamma}_1}(TD)^{\hat{\gamma}_2}} + e_2 \quad . \; . \; . \; . \; . \; . \; . \quad (2\text{-}19)$$

where e_1, e_2 are errors in the estimation of PS_j by $\hat{\hat{PS}}_j$.

In a subsequent section of this paper, an equation of the form of (2-19) for one of the cells and an estimate of the potential metal supply of the entire northern Sonora area (allowing for transportation costs) are presented.

THE TRANSPORTATION MODEL

The representation of topographic features within the computer

DIGITIZING OF TOPOGRAPHIC MAPS

The major characteristics of the terrain can be readily observed by visual examination of a topographical map. When a decision concerning the path of a road has to be made, a quick scanning eliminates many possible routes and focuses upon paths that are feasible. This scanning is followed by more detailed examination and calculation in order to select the best of the feasible paths. In the present study, the computer made the decisions on the paths of railroads or roads. A means for examining the topographic features of the terrain internal to the computer was therefore required. There are two basic schemes by which this can be done. One of them is by expressing the topography by a mathematical equation (trend surface equation) and the other is by representing the topography by an array of points. Either way, it is necessary to digitize the original topographical maps.

Topographical maps of the study area (Northern Sonora) were digitized; that is, the Cartesian co-ordinates of points on the maps were measured to a hundredth of an inch in relation to a predetermined origin. Points were selected following the contours, and no interpolation was made between the contours; consequently a set of unequally spaced points was produced. The co-ordinates of these points and their associated elevations served as inputs to the trend surface program which calculated the mathematical expression of the topography.

TREND SURFACE INTERPOLATION ON DIGITIZED DATA SETS

The representation of the topography of the area within the computer was first approximated by the use of trend surface equations. The numerical methods required for computing trend surfaces have been described in detail by Krumbein and Graybill (1965) and Harbaugh and Merriam (1968).

The rationale behind a trend surface equation seems to fit the purpose envisioned adequately. However, in practice, its accuracy in selecting transportation routes is dependent upon how well the trend surface equation fits the actual topography. Therefore, the adequacy of representation was investigated.

It was found that for a very simple group of contours the goodness of fit of the twelfth-order trend surface was satisfactory; however, for a large area like that under study, it failed to reflect some of the most striking topographical features. Furthermore, high-degree polynomials induce features not present in the original data (polynomial fluctuations). Therefore, the internal representation of the topography of the study area by a trend surface equation was abandoned and the search for an alternative was initiated.

ORTHOGONALIZATION OF DIGITIZED DATA SETS

The alternative selected consisted of representing the topography by a two-dimensional matrix of elevations. This required generation of elevations on an orthogonal grid from the unequally-spaced data. Again, a point of concern is the adequacy of representation of the original features by the interpolated orthogonalized data.

Orthogonalizing of the set of points makes available the elevation of the terrain at every intersection of the grid. The finer the grid, the greater the number of intersecting points and the better the representation. In practice, a very fine grid is not advisable because computer time increases fourfold when the side of a grid square is halved.

The generation of an orthogonal set of data points from an irregular set of points requires an interpolation procedure. Many different estimating schemes have been applied to this general problem. Two common schemes are simple averaging of adjacent points and estimation by a *local* least-squares polynomial surface fit to the points surrounding each grid intersection.

Sections of a computer program for automatic contouring by Donald McIntyre, *et al.* (1968), were adapted to the system 360 model 67 available at the Pennsylvania State University. The section of the computer program where the orthogonalizing of irregularly-spaced data points takes place was made available for the construction of the grid of this study. For details see Euresty (1971).

The orthogonalized system produced by the method described was plotted and compared with the original topographical maps. The results were highly satisfactory and were improved by a slight reduction in the size of the grid. Thus, the grid computed fulfills the condition of giving a fair representation within the computer of the topographical features of the study area.

Besides the fair representation of the topography, the grid computed has an inherent characteristic of special importance. Every point of the grid has an elevation estimate that is easily referenced and is readily available for further analysis. Such a characteristic becomes of primary importance when a path is being selected, because it facilitates route section computations from point to point within the grid.

Multistage optimization (dynamic programming)

GENERAL PERSPECTIVE

As described in the previous section, the topography of the study area was represented within the computer by the orthogonalized set of elevations. The existing transportation network was represented within the computer either by regression equations or by sets of points. Regression equations were employed to describe the transportation routes when the configuration of the route was simple, and a good fit could be obtained. Others were represented by sets of stored pairs of co-ordinates taken along the full length of the transportation routes.

The selection of a path from the cell to the market (from the cell to the main transportation network) by dynamic programming as employed in this study is restricted to segments of the grid; that is, the route cannot cut across the grid network. Therefore, the selection must be considered as a discrete approximation to a minimum-length path under existing topography. It was limited to a search for the minimum-length path without introducing additional alternatives, such as excavation. To avoid impassable routes a maximum allowable gradient was introduced as a constraint.

In practice, if the tonnage to be shipped from the mineral deposit is large enough, a shorter route that requires excavation may be warranted. That is, shipping cost (total over time) could be reduced by a greater expenditure on construction. However, to simplify the analysis no provision was made for excavation.

The connection of the two points can be made by any one of a set of different paths varying in length and direction. Of all the feasible paths between the two points that overlay the grid, the one with the minimum length and a gradient no greater than four per cent was to be selected as the optimum one. The different feasible paths through the grid connecting the two extreme points and the different directions a path can take after crossing any one of the grid intersecting points complicates the search for the minimum length path. The required distance from one grid intersection point to any other (keeping a gradient of four per cent or less) was the *real distance* by which the path was incremented when any two grid points are connected.

If the distance of all the feasible connections among the intersection points are determined, it is possible to calculate the lengths of all the feasible paths throughout the grid connecting any two points. Finally, the one with the minimum length can be selected from the group as the optimum path. However, this exhaustive search, which requires examining all possible paths, is very time-consuming.

GENERAL DESCRIPTION OF THE PRINCIPLE OF DYNAMIC PROGRAMMING

Let us call the path connecting any two extreme points a policy, and a continuous section of a path a subpolicy. A finite number of policies exist between the two points; among these, there is one whose length is the smallest thus becoming the optimal policy. In order to get an optimal policy, the path from point to point within the grid must be examined so that optimal subpolicies are maintained throughout the path. In other words, in order to get an optimal

policy out of the system it is necessary to work by stages or sequences.

Dynamic programming is a mathematical method for the optimization of systems characterized by stages or sequences. The basis of this method is the theorem of optimality formulated by the American mathematician Richard Bellman. This theorem of optimality has been enunciated in the form of a general principle:

> A policy is optimal if, at a stated period, whatever the preceding decisions may have been, the decisions still to be taken constitute an optimal policy when the result of the previous decision is included (Kaufman, 1967, p. 79).

Let us take a small section of the grid representing the topography of the study area (Fig. 3) and define the following symbols:

$X_0, X_1, X_2, \ldots, X_N$	= State variables
$d_1, d_2, d_3, \ldots, d_N$	= Decision variables
$v_1(X_0, X_1)$	= Distances associated with state variables of stage I
$f_{\mathrm{I}}(X_1)$	= Optimal returns associated with decision variables of stage I
$v_2(X_1, X_2)$	= Distances associated with state variables of stage II
$f_{\mathrm{II}}(X_2)$	= Optimal returns associated with decision variables of stage II
$v_3(X_2, X_3)$	= Distances associated with state variables of stage III
$f_{\mathrm{III}}(X_3)$	= Optimal returns associated with decision variables of stage III
$v_4(X_3, X_4)$	= Distances associated with state variables of stage IV
$f_{\mathrm{IV}}(X_4)$	= Optimal returns associated with decision variables of stage IV

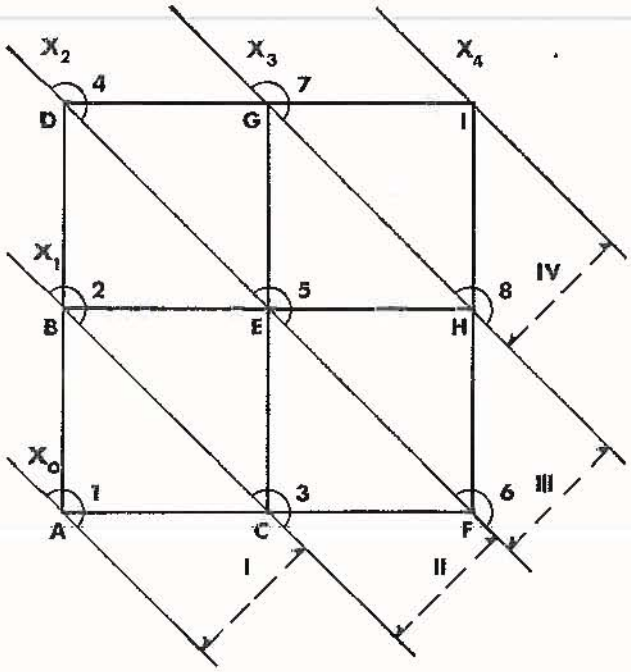

Fig. 3. Stages, decision variables, and associated returns.

The state variables do not acquire numerical values. They are defined at each stage by the appropriate vertices falling on the state variable line, that is, associated with state variable X_1 we have vertices or intersections B and C (Fig. 3). A decision variable implies the transfer from a grid point (vertex) to any other grid point, that is, from A, a path can go to either B or C. Such transfers can be feasible or nonfeasible. A nonfeasible transfer is illustrated by the decision to go from C to D. For every decision variable there exists an associated return. Such returns are represented in this case by the distance required to connect a vertex to any other vertex, keeping a desired gradient. A stage, then, consists of state variables and decision variables, the former defined by vertices and the latter depicted by the choices involved in the transfer from vertex to vertex.

The optimal returns associated with stage I of Fig. 3 are then given by:

$$f_1(X_1) = \text{minimum}\,[v_1(X_0, X_1)], \quad \ldots\ldots\ldots \quad (3\text{-}1)$$

since X_1 can take on vertices B or C, we have

$$f_{\mathrm{I}}(X_1) = \text{minimum}\,[v_1(X_0, B), v_1(X_0, C)], \quad \ldots\ldots \quad (3\text{-}2)$$

or

$$f_{\mathrm{I}}(X_1) = \text{minimum}\,[v_1(A, B), v_1(A, C)]. \quad \ldots\ldots \quad (3\text{-}3)$$

The optimal returns associated with the decision variables of stage II, considering that stage I is operated optimally as a function of its inputs, are given by:

$$f_{\mathrm{I,II}}(X_2) = \text{minimum}\,[f_{\mathrm{I}}(X_1) + v_2(X_1, X_2)], \quad \ldots \quad (3\text{-}4)$$

where X_2 takes on vertices D, E, and F. Substituting we have,

$$f_{\mathrm{I,II}}(D) = \min_{X_1 = B, C}\left[f_{\mathrm{I}}(X_1) + v_2(X_1, D)\right] \quad \ldots \quad (3\text{-}5)$$

$$f_{\mathrm{I,II}}(E) = \min_{X_1 = B, C}\left[f_{\mathrm{I}}(X_1) + v_2(X_1, E)\right] \quad \ldots \quad (3\text{-}6)$$

$$f_{\mathrm{I,II}}(F) = \min_{X_1 = B, C}\left[f_{\mathrm{I}}(X_1) + v_2(X_1, F)\right] \quad \ldots \quad (3\text{-}7)$$

Some of the decision variables in this example are nonfeasible at this stage, such as the ones from C to D, and from B to F. In order to facilitate computations, the nonfeasible decisions are assigned very large values, since we want to minimize distance. By this means we ensure that the nonfeasible decisions will never be selected.

At stage III, the optimal returns associated with the decision variables are given by:

$$f_{\mathrm{I,II,III}}(X_3) = \min_{X_2 = D, E, F}\left[f_{\mathrm{I,II}}(X_2) + v_3(X_2, X_3)\right], \quad \ldots\ldots\ldots \quad (3\text{-}8)$$

since X_3 takes on vertices G and H, we have

$$f_{\mathrm{I,II,III}}(G) = \min_{X_2 = D, E, F}\left[f_{\mathrm{I,II}}(X_2) + v_3(X_2, G)\right] \quad \ldots\ldots\ldots \quad (3\text{-}9)$$

and

$$f_{\mathrm{I,II,III}}(H) = \min_{X_2 = D, E, F}\left[f_{\mathrm{I,II}}(X_2) + v_3(X_2, H)\right] \quad \ldots\ldots\ldots \quad (3\text{-}10)$$

Finally, for the grid of Fig. 3 the optimal returns of the last stage are given by:

$$f_{\mathrm{I,II,III,IV}}(X_4) = \min_{X_3 = G, H}\left[f_{\mathrm{I,II,III}}(X_3) + v_4(X_3, X_4)\right] \quad \ldots\ldots\ldots \quad (3\text{-}11)$$

The state variable X_4 takes on only vertex I, thus

$$f_{\mathrm{I,II,III,IV}}(I) = \min_{X_3 = G,H} \left[f_{\mathrm{I,II,III}}(X_3) + v_4(X_3, I) \right] \quad \ldots\ldots\ldots \quad (3\text{-}12)$$

or

$$f_{\mathrm{I,II,III,IV}}(I) = \min \left[f_{\mathrm{I,II,III}}(X_3) + v_4(G, I) \right] \quad \ldots \quad (3\text{-}13)$$

On the last stage the optimal subpolicy selected becomes the optimal policy; this policy is made up of optimal subpolicies selected throughout the stages. The optimal policy estimated is computed by adding up the optimal subpolicies encountered at every stage. This particular characteristic is conformable to the dynamic programming methodology of optimization. For theory and details of dynamic programming, the reader is referred to Nemhauser (1967).

MULTISTAGE OPTIMIZATION PROGRAM

In general, the Multistage Optimization Computer Program estimates the minimum length path between two points throughout a given grid network, A; where A is a part of B and B is the entire study area grid network. Given the co-ordinates of the center of a cell the computer program determines first the co-ordinates of a point on the means of transportation available. Such point is the one located closest to the center of the cell. Having these two points properly identified, their relative locations within the entire area grid are computed and adjusted, so that the subset grid A can be limited and separated from the entire area grid.

The subset A contains a finite number of paths throughout the sectional grid communicating the center of the cell to a point on the existing transportation network. Among these, there exists one connecting the points at a minimum distance. In order to identify this minimum distance path, it is necessary to examine the lengths of all of the feasible paths. These paths are of different lengths, because the real distances between the elements of A are computed by considering their differences in elevation and the stipulated gradient constraint. The indicated variation of length between the elements reflects the changes in topography or configurations of the terrain.

The computer program is not limited to only one multistage optimization for a given cell, but no more than three minimum-length paths covering different nearby terrain sections are estimated. Every one of the estimations indicated has its own subset grid A, so that the individual paths are projected through different terrain with only a few overlapping. The different paths estimated are independent, so that a comparison among the three to select the one giving the minimum length is required.

The first path estimation covers the subarea between the center of the cell and the closest path to it belonging to the transportation network. The second estimate is done in two steps. First, an intermediate point between the first two, but lying outside the subarea covered by A, is selected (this point is not on the existing network of roads) and then a multistage optimization between the center of the cell and the newly selected point is computed (this selects the path to the new intermediate point). Second, a search is conducted for a point on the transportation network (by shortest straight line distance criterion) to connect with the intermediate point, and another multistage optimization computation is done, covering the subarea enclosed by the intermediate point and the point just selected on the transportation network. The third path estimation is quite similar to the second one, but the intermediate point selected lies in the opposite direction to the one selected for the second estimate. Of these paths, the one of shortest distance is selected as the path for the transportation link.

Thus, for any given cell the computer program provides: (i) the minimum-length path that is required in order to connect the cell to the transportation network, together with its directions throughout the grid, and (ii) the total transport distance to reach the market or trans-shipment point; this includes the transport distance along the path to be constructed plus the distance along the existing transportation network.

ANALYSIS

Conditions

Occurrence of deposits, exploration for deposits, and exploitation of those deposits discovered were simulated under various levels of the pertinent transportation variables, *TRC*, *CMC*, and *CCC*. All other economic variables, such as exploration expenditures, price level, capital costs, discount rate, and overall level of operating costs were held constant. This does not mean, for example, that operating and capital costs were the same for every deposit, nor does it mean that the stochastic properties of the estimated cost functions were ignored. On the contrary, the operating and capital costs were defined, where applicable, to be functions of the tonnage of the simulated deposits, and a random error term consistent with the standard error of each cost equation was added to the estimated cost. When it is stated that capital costs and operating costs were held constant, this means that the relationships of the *basic* cost equations were employed, rather than modifying the costs by multiplying the basic relationships by some selected constant. Similarly, an average exploration expenditure was employed as typical of a regional exploration program. However, the results of exploration for a given deposit varied according to its tonnage and the effect of a stochastic error term (see Harris, *et al.*, 1970; Euresty, 1971).

The basic construction cost employed in this study was that of a crushed stone surfaced road in Mexico, about \$26 000 per mile. The basic carrying rate employed was 0.021 \$/ton-mile.

Elasticities

Inasmuch as the effect of transportation upon potential metal supply was the objective of this study, a sensitivity analysis was performed by varying the transportation variables and observing the output of the simulation model. Of course, the effect of transportation on metal supply varies with the cell, depending upon the metal endowment of the cell, terrain, and location. However, a more general analysis of cells was performed by changing arbitrarily the transportation construction distance and the carrying distances. Statistical analysis of the output of the simulator yielded response functions of tonnage and grade of ore as a function of the transportation variables and the construction and carrying distances. The following equation defines the expected metal supply of cell 39, conditional upon the basic relationships of the economic variables:

$$\hat{PS}_j = \left(7 \times 10^8\right) \left\{ \left(TRC\right)^{-\frac{TD^{2.59}}{7.12}} \right\} \left\{ \left(CMC\right)^{-\frac{CD^{0.41}}{2.14}} \right\} \times \left(h_j'/r_j'\right), \quad \ldots\ldots\ldots\ldots \quad (4\text{-}1)$$

where $\hat{PS}_j$ is the estimated potential metal supply in units compatible with r_j', TD is the transportation distance in units of 100 miles, CD is the construction distance in units of 100 miles and TRC, CMC, h_j', and r_j' are as defined previously.

The variable representing government sharing of construction cost does not appear in the equation because in a statistical sense it was not a significant variable (five per cent level of significance), given the necessary simultaneous estimation of coefficients of *TRC* and *CMC*. In order to explore the properties of this equation, let us assume that carrying distance is 400 miles and construction distance is zero. In this case the potential metal supply is a function only of h_j', r_j', and *TRC*:

$$\hat{PS}_j = \left(7\times10^8\right)\left(TRC\right)^{-\frac{36.2}{7.12}}\left(h_j'/r_j'\right) \quad . \; . \; . \; . \; . \; . \quad (4\text{-}2)$$

Now, suppose that the carrying distance is decreased to 200 miles:

$$\hat{PS}_j = \left(7\times10^8\right)\left(TRC\right)^{-\frac{6.03}{7.12}}\left(h_j'/r_j'\right) \quad . \; . \; . \; . \; . \; . \quad (4\text{-}3)$$

The elasticity of potential metal supply with respect to *TRC* decreases correspondingly from about 5.1 to about 0.85. This is equivalent to saying that given a carrying distance of 400 miles, a 100 per cent increase in transportation rates (charge per ton-mile for concentrate) decreases potential metal supply by 510 per cent. For a carrying distance of 200 miles, the 100 per cent increase in transportation rates decreases the potential metal supply by only about 85 per cent.

Because of the small exponent of *CD*, the elasticity of potential metal supply does not increase as rapidly for increased construction distances as does the elasticity with respect to carrying rates. However, because of the small denominator in the exponent of *CMC*, small to moderate construction distances are seen to have a greater impact upon supply than do small to moderate transportation distances. In the case of $TD=CD=100$ miles, the elasticity of supply relative to *TRC* is $-1/7.12$, while relative to *CMC* it is $-1/2.14$. Because of the definition of *TRC* and *CMC* as multiples of basic cost schedules, equation (4-1) is not meaningful (from the point of view of elasticities) for *TRC* or $CMC = 1.0$, for at these levels potential metal supply is invariant to transportation and construction distances.

Potential supply of cell 39

For comparative purposes, let us evaluate the potential supply of cell 39 under the following states of the variables and parameters: $TRC=1.5$, $CMC=1.5$, $TD=400$ miles, $CD=50$ miles, $h_1=1.0$, $h_2=h_3=h_4=h_5=0$, $r_1=\$0.56$/lb.

In the above set of conditions h_1 and r_1 refer to copper. The implication of setting $h_2=h_3=h_4=h_5=0$ is that potential metal supply will be in terms of copper equivalent. This will allow the comparison of expected potential metal supply to expected metal endowment.

$$\hat{PS}_1 = \frac{(7\times10^8)(1.5)^{-0.85}(1.5)^{-0.62}}{(0.56)} = 690\,600\,000 \text{ lb.}$$

Referring to Fig. 2, the expected metal endowment for cell 39 in terms of copper equivalent is from 4 000 000 000 to 6 000 000 000 lb. Taking the mid-point of this range, it appears that the potential metal supply of cell 39 for the conditions specified is approximately 14 per cent of that expected to occur in the cell. If construction and transportation distances were near zero, the potential metal supply would approach 1 200 000 000 lb.

Adequacy of existing transportation network

To test the adequacy of the existing transportation network, the potential metal supply of the cells of the study area was evaluated by simulating the supply of metals using the transportation links of each cell to the existing transportation network (see Fig. 2). These links are the paths determined by dynamic programming. Concentrates of the simulated deposits were transported south to Hermosillo.

In general, the existing transportation network is fairly well situated, as it passes through the richly endowed areas (see Fig. 2). This was verified by the coefficients of the transportation variables in the potential supply functions for the cells. Only minor construction was required to connect the rich cells to the existing network. Consequently, the elasticity of potential supply relative to the multiple of construction costs was small for most cells. The effect of transportation rates, while greater than construction costs, was also moderate; the elasticity of potential supply relative to *TRC* never exceeded 40 per cent.

Concentrates from deposits in the southeast part of the study area must be transported north across the United States-Mexican border, then west, before they can be transported south to Hermosillo. Naturally this is a much longer route than would be direct shipping south to Hermosillo, but there is no major transportation link south to Hermosillo, except an old unsurfaced road. In view of this indirect shipping route for some of the cells, it seemed appropriate to use the model of this study to determine if construction of a surfaced road from Nacozari to Hermosillo and construction of the links from the pertinent cells to the road could be justified by the benefits of a shorter carrying route. Only three cells were considered: cells 51, 52, and 53. The discounted benefits to potential metal supply from these three cells outweighed the costs of construction by approximately $2 000 000. For details of the analysis, see Euresty (1971).

Potential supply of the study area

The potential supply of the entire study area was determined for the standard economic conditions and for the basic carrying rate and construction cost, given the existing transportation network and required links. Estimation of the expected quantity of each of the five metals that would be supplied was based upon the assumption that the proportions of value of the metals in ore discovered in the future on the average will be similar to the proportion of value of metals in ore produced from Arizona and New Mexico:

Copper	0.263 03	Lead	0.211 15
Zinc	0.140 66	Silver	0.253 21
Gold	0.131 96		

These value proportions are based upon 1968 prices; consequently h_j' and r_j' were determined, based upon 1970 prices, since these were the prices used in estimating *Z*. The results of this analysis of potential supply are as follows:

Copper	7 709 000 metric tons
Lead	10 314 000 metric tons
Zinc	6 951 000 metric tons
Silver	76 498 000 kilograms
Gold	1 764 000 kilograms

CONCLUSIONS

The analysis of metal endowment indicated an expectation of 1 031 000 tons of copper equivalent per 400-square mile cell of the study area, or equivalently, 132 000 000 000 lb of copper equivalent for the entire area. Simulation of exploration and exploitation, with special provision for including costs for construction of required transportation links and the cost of transporting concentrate to Hermosillo, indicated a potential supply of approximately 34 000 000 000 lb of copper

equivalent. Thus, the requirement that only deposits discovered in the simulation that were economic to develop and produce would contribute to potential metal supply reduced potential supply to about 26 per cent of the metal endowment. The remaining 74 per cent of metal endowment either was not discovered by the simulated exploration, or it was not economic to develop and produce.

Analysis showed that the existing transportation network is reasonably well situated with respect to the richly-endowed cells, required linkages from the cells to the network in most cases being short. For many of the cells, additional expenditures on major transportation arteries would not increase potential metal supply greatly. Potential metal supply from the cluster of cells in the southeast quadrant of the study area could be increased by construction of a road from Nacozari to Hermosillo, Mexico. Currently, concentrate must follow a long, indirect route to arrive at Hermosillo. A cost-benefit analysis of the construction of a crushed stone surfaced road to Hermosillo indicated that benefits accruing from only three of the cells in that area more than offset the costs. In practice, additional benefits would accrue from other cells in that area.

Allowing for transportation effects and current economic factors and assuming a metal suite similar in proportions to that of Arizona and New Mexico, potential metal supply was estimated to be 7 709 000 metric tons of copper, 10 314 000 metric tons of lead, 6 951 000 metric tons of zinc, 76 498 000 kilograms of silver and 1 764 000 kilograms of gold.

An analysis of elasticity of potential metal supply indicated that supply is considerably more elastic to carrying charges than to construction costs. The size of the elasticity is a function of many factors, such as the grade and tonnage of the deposits in the cell, operating and capital costs, discount rate, location, and terrain. It was found that the computer model of this study could be used in a sensitivity analysis so as to generate quantities that could be analyzed statistically to yield response functions that define potential metal supply as a function of carrying charges and construction costs. These response functions provide insight into the effect of transportation upon potential metal supply.

Admittedly, because of assumptions and simplifications, there are many distortions of the real world in the model of this study. Actually this study constitutes no more than an initial probe into the problems of modelling potential metal supply with an explicit treatment of transportation.

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