

# Capacity Calculations, Investment Allocation and Long-Range Production Scheduling in German Coal Mines

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## SYNOPSIS

A model for long-term production scheduling as well as capacity and investment allocation for mining enterprises is introduced by means of an example.

A deposit is assumed to be subdivided into blocks each associated with a certain mineral content, discrete levels of possible output, direct mining cost and expected revenue. The blocks are served by facilities that may be installed and operated at various discrete levels of capacity. Both the capital investment necessary for installation as well as mining costs of these facilities are represented in the model.

The objective is to maximize the total discounted cashflow over the planning period. The paper describes a heuristic approach for arriving at a 'good' solution. True optimality is not claimed.

Some sample results are given.

## INTRODUCTION

In 1969 all mining companies of the Ruhr district (except two) were combined into one enterprise, the Ruhrkohle AG, which was then divided into seven divisions. The previous boundaries of the coal fields lost their importance and the total deposit of the Ruhr district may now be looked upon as a coherent unit. Subdivision into geological blocks is now effected purely on the basis of faults or coal quality.

Figure 1 outlines the region of the 'Westfalen' division. As shown, a virgin field is included in the middle of the concession area. This is expected to bear considerable quantities of high-quality coal. The reserves of some of the adjacent mines will be exhausted in the near future. The previous field boundaries often prevented exploitation of parts of the field due to the smallness of these parts or because of their great distance from hoisting shafts.

After the abolition of previous field boundaries, the fundamental replanning of the development and exploitation of the total deposit became possible, together with the possibility of increasing the profitability of the enterprise considerably. This work was conducted during 1969 and 1970 by the Operations Research Group of our Institute in close cooperation with the engineering staff of the mining company. The resulting computer model was employed successfully at the end of 1970. While the model was developed with special regard to its applicability to this particular problem, its scope is quite general so that it can also be used in similar other contexts.

## THE TASK

The object of the study was to set up a long-term (20 years) production schedule for the 'Westfalen' division as a whole in such a manner that the discounted cash flow would be maximized. To achieve this the following main variables had to be brought into optimal balance:

- (i) The daily output from each geological block. Blocks are defined as such parts of the deposit which can be looked upon as homogeneous as regards mineral content, geological conditions, direct mining costs and earnings. Generally, they are delimited by geological structures.
- (ii) The capacities of all facilities for exploiting the deposit, such as beneficiation plants, hoisting and ventilation shafts, haulage roads, development work, etc.

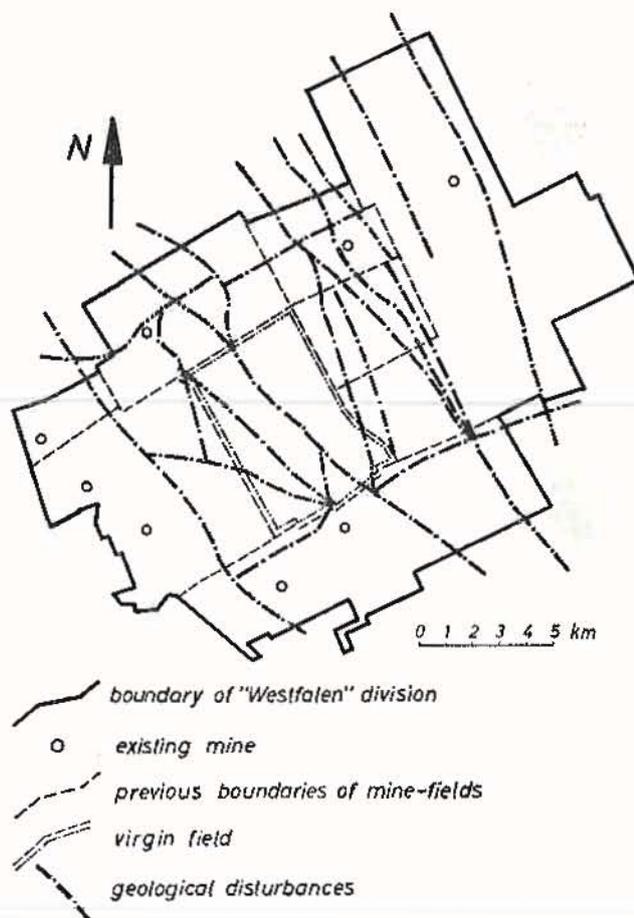


Fig. 1. Concession area of the Westfalen division of Ruhrkohle AG.

- (iii) The allocation of each block to certain of these facilities.

Obviously this is a dynamic problem, so that all factors of influence with their mutual interdependence, and all variables must be assumed to be time-dependent. The aim was to plan

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the optimal procedure for exploiting the deposit rather than to determine a certain capacity. For instance, block allocations and block outputs are subject to alterations during the planning period. Capacities of hoisting shafts and all other facilities can be enlarged or reduced as time progresses. Even a complete mine might be shut down or newly opened whenever the overall profitability of the total system could be increased by doing so.

Some rigid restrictions were to be observed. The total output had to meet market demand which was expected to increase up to 10 000 t/d of steam coal and 35 000 t/d of coking coal, but no certain prognosis was available. The capital investment was limited both to a total amount and to a certain sum per year.

Restricted manpower availability was also to be taken into account. Last, but not least, all geological and technological constraints were to be taken into consideration, for instance, upper and lower limits for face production, sequence of working the faces, time requirement for setting up facilities, etc. The main challenge, however, was to treat the problem in such a way that the results could immediately assist management in its decision making. Theoretical sophistication was not aimed at.

## THE MODEL

### Formulation of the problem

Each geological block can be described in terms of a number of variables. For instance, with respect to block  $i$ , the following parameters need be defined for the present purpose:

- $M_i$ , the mineral content of the block ( $t$ ),
- $Q_{ni}$ , discrete steps in daily output (t/d), for example,  $Q_{1i} = 0$ ,  $Q_{2i} = 2\ 000$  and  $Q_{3i} = 4\ 000$  (none, one and two longwall faces are working in the block, respectively).
- $IMP_{ni}$ , required manpower (man-shifts/day) when output equals  $Q_{ni}$ ,
- $C_i$ , mining costs, including all costs incurred in the block ( $DM/t$ ), and
- $P_i$ , selling price of the coal mined in the block ( $DM/t$ ).

Similarly, all facilities may be expressed in terms of another set of parameters. These parameters, with regard to facility  $j$ , which may, for example, be a hoisting shaft, are

- $K_{mj}$ , discrete steps in possible capacities (t/d),
- $JMP_{mj}$ , manpower required to work this facility at capacity  $K_{mj}$  (man-shifts/d),
- $I_{mj}$ , capital required to provide capacity  $K_{mj}$  ( $DM$ ),  $I_{mj} = 0$  if  $K_{mj}$  is already available,
- $R_{mj}$ , the cost of running the facility at capacity  $K_{mj}$ , ( $DM/t$ ), and
- $T_{mj}$ , the time required to construct the facility with capacity  $K_{mj}$  or to enlarge capacity  $K_{m-1,j}$  to  $K_{mj}$ .

The task is to determine for all reasonable block facility allocations and for each time interval  $t$  within the planning period the daily output from all blocks,  $Q_{nit}^x$ , and capacity of all facilities to achieve this output,  $K_{mjt}^x$ . If all values of  $Q_{nit}^x$  and  $K_{mjt}^x$  are available then it is possible to derive  $\sum_i Q_{nit}^x C_i$  and  $\sum_j K_{mjt}^x R_{mj}$ , which are the total daily mining and running costs during time interval  $t$ , respectively. Also, the sums  $\sum_j I_{mj}^x$  and  $\sum_i Q_{nit}^x P_i$  are the total capital investment and daily revenue, respectively, during period  $t$ . The investment figures are obtained by discounting all items of expenditure incurred in the construction of a particular facility to the beginning of the period.

The values of  $Q_{nit}^x$  and  $K_{mjt}^x$  must be determined so as to have the total cash-flow,  $\phi$ , maximized without violating any restrictions. Thus, the calculations should result in the maximum value of

$$\phi = \sum_t q^{-t} \{ [\sum_i Q_{nit}^x (P_i - C_i) - \sum_j K_{mjt}^x R_{mj}] - \sum_j I_{mj}^x \}.$$

Here, daily expenditure and revenue within each interval are discounted and accumulated by applying the usual factor  $\delta_t = (q^n - 1)/(q - 1)q^n$ , where  $n$  is the length of period  $t$  and  $q = 1 + r$ , where  $r$  is the rate of interest.

It should be noted that this formulation is not specific to the particular case under study. It may be used also to describe a broad variety of production scheduling problems.

## METHOD OF SOLUTION

With regard to the unsteady and, generally speaking, non-linear characteristics of even the most important functions on the one hand, and to the difficulties of mixed integer programming on the other, application of linear or nonlinear programming did not seem to be promising enough to be applied. Dynamic programming, too, was rejected because it proved impossible to define steps of the process which would be constant and not affected by the process itself. The question of possible application of mathematical programming to this problem is, however, under study at our Institute. In the meanwhile, considering the short time available in which to achieve practical results, another solution had to be found, even if it was less efficient and, as it proved, computer time-consuming.

We decided to handle first the main alternatives, namely, block facility allocations, consecutively by restricted enumeration. This meant setting up a model to treat production scheduling as well as capacity calculations and investment allocation relative to only one specific and defined block facility allocation. If one determines the optimum solution for each of the main alternatives, the overall optimum can be found by comparison of these solutions. Setting up a decision tree of all reasonable block facility allocations, we found that only a limited number of about 20 of such main alternatives would have to be investigated.

### Determining time intervals

As set out before, the problem must be treated as being time-dependent. Therefore, time intervals  $t$  must be determined for which the calculations have to be executed. These time intervals are defined by the occurrence of at least one of the following events:

- (i) The mineral content of at least one block is exhausted. This time is conditional on  $M_i$ , the mineral content of the block at the start of period  $t = 1$ , and on all  $Q_{nit}^x$   $t \leq t^x$ , that is, the planned outputs of the block for all periods from the very beginning up to the time  $t^x$  which is presently under consideration.
- (ii) Alteration in market demand meaning that the total output of the enterprise can be enlarged or must be reduced for at least one coal product. A redistribution of the block outputs becomes necessary.
- (iii) At least one new facility becomes available. The end of period  $T_{mj}$  for this facility coincides with  $t^x$ , the time under consideration. The availability of the facility at this moment, for instance, the fact that a new hoisting shaft becomes operational, may permit an increase in profitability. This in turn may affect the outputs of the blocks and/or the block-facility allocation.

It is apparent that  $t^x$ , the length of the time interval to be considered, depends largely on all decisions taken at all  $t < t^x$ . Hence this interval cannot be determined without

taking into account these decisions which are obviously unknown at the start of the calculation. It seemed appropriate, therefore, to construct the model in such a way that simulation is used to reproduce the chronological evolution of the system. The time intervals  $t$  are determined by considering all decisions prior to  $t$ . The algorithm for taking these decisions which forms the most essential part of the model, is embedded in the simulation.

#### Scheduling production and investment allocation

When scheduling production and investment allocation for the next time interval  $t$  the following important questions must be answered:

- (i) Which blocks should produce what quantities? This means determination of  $Q_{nt}^x$ .
- (ii) What capacities of the available facilities should be employed? This means determination of  $K_{mjt}^x$ . Particularly, if any facility is newly available at this time, should it be used, and if so, at what capacity level? Which other facilities are affected by this decision and how?

These decisions must be taken so that all restrictions are met and the overall profitability of the system is maximized. The principles employed in the algorithm resemble somewhat Bellman's optimality criterion: at any time  $t^x$  an attempt is made to take decisions that will ensure optimality for all remaining time intervals  $t > t^x$  irrespective of the path leading to the state of the system at this time. Because at this stage there are unknown interactions between the decisions just taken and all future decisions, certain assumptions as to the future development must be established which must be met later on if overall optimality is to be reached. This in turn implies that, whenever a new decision is taken, one has to see whether any such condition established at any previous stage of the process would be upset. If so, this condition must be changed and all calculations repeated, starting at the stage at which the condition in question was established. This procedure must be repeated as often as necessary.

The algorithm which we developed and used in solving our problem may be explained in its basic concepts considering the example as sketched in Fig. 2. The computation

utilizes a so-called selection matrix, which is indicated schematically in Table I.

Assume that the deposit consists of six blocks  $i = 1, 2, \dots, 6$ , which are listed in column 1 of the matrix. The  $Q_{nj}$  discrete steps of possible daily output of each block are given in column 2, for instance, block 3 can produce 2 000, 4 000 6 000 or 8 000 t/d, but not, for example, 1 500 t/d due to the restriction that if one, two, three or four faces are opened in this block they should work at their optimum utilization of capacity to ensure maximum profit. Block 3 must be exhausted before any production in block 5 can start (blocks 4 and 6 similarly). The figure '3' in column 3, rows 15 through 18, excludes block 5 from selection. It will be set to zero in the simulation part of the computer program as soon as the mineral content of block 3 has been mined out completely, that is, when  $M_3$  becomes zero. Selection of  $Q_{nt}$  is made only from those rows containing a zero in column 3.

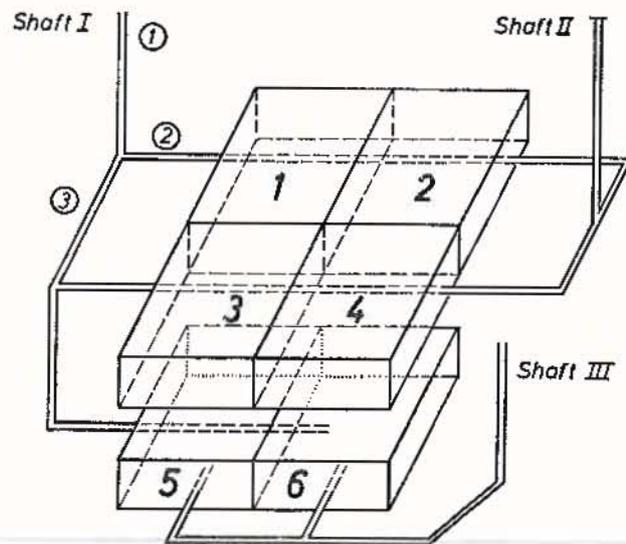


Fig. 2. Schematic sketch of block facility allocation.

TABLE I  
SCHEMATIC EXAMPLE OF SELECTION-MATRIX

Column	1	2	3	4	5	6	7	8	9	10...k	k+1	k+2	k+3	k+4	k+5	k+6...1	l+1	l+2	l+3
Symbol	$t$	$Q_{ni}$		$P_i - C_i$															
row	delimiting quantities		$j$	1	1	2	2	3	....	1	1	2	2	3	.....				
			$m$	1	2	1	2	1	....	1	2	1	2	1	.....				
			$DQ$	0	10 000	0	4 000	0	....	0	10 000	0	4 000	0	.....	20 000	10 000	35 000	
1	1	2 000		46.40	1	1	1.50*	2.30*			1.20	-0.40	0.80	-0.50			1	1	
2		3 000		46.40	1	1	1.34*	2.02*			1.20	-0.40	0.80	-0.50			1	1	
3		4 000		46.40	1	1	1.19*	1.80*			1.20	-0.40	0.80	-0.50			1	1	
4	2	2 000		44.00														1	
5		3 000		44.00														1	
6		4 000		44.00														1	
7	3	2 000		41.80	1	1			1	....	1.20	-0.40			1.30	.....	1	1	
8		4 000		41.80	1	1			1	....	1.20	-0.40			1.30	.....	1	1	
9		6 000		41.80	1	1			1	....	1.20	-0.40			1.30	.....	1	1	
10		8 000		41.80	1	1			1	....	1.20	-0.40			1.30	.....	1	1	
11	4	2 000		38.80															1
12		4 000		38.80															1
13		6 000		38.80															1
14		8 000		38.80															1
15	5	4 000	3	42.80	1	1			1	....	1.20	-0.40			1.60	.....	1	1	
16		5 000	3	42.80	1	1			1	....	1.20	-0.40			1.60	.....	1	1	
17		6 000	3	42.80	1	1			1	....	1.20	-0.40			1.60	.....	1	1	
18		7 000	3	42.80	1	1			1	....	1.20	-0.40			1.60	.....	1	1	
19	6	4 000	4	44.00															1

\*After updating.

Column 4 contains the contribution margin  $P_i - C_i(DM/t)$ , which is of course equal at all production levels within a block at the start of the calculation.

Columns 5 through  $k$  and  $k + 1$  through  $l$  are concerned with the facilities, their capacities and running costs, respectively. Let us consider only facilities 1 through 3 (encircled numbers in Fig. 2), a hoisting shaft (facility 1) and two haulage roads (facilities 2 and 3), say. In the top row of columns 5 through  $k$  and  $k + 1$  through  $l$  'delimiting quantities'  $DQ_{mj}$  are entered representing the discrete steps of possible capacity of each facility. For instance,  $DQ_{11}$  (column 5) = 0 means that  $K_{11}$ , the first step of capacity at facility 1, must be available if the total output of all blocks requiring this facility exceeds zero,  $DQ_{21}$  (column 6) = 10 000 means that an enlargement of the capacity  $K_{21}$  becomes necessary as soon as the output of those blocks exceeds 10 000 t/d. At the beginning of each calculation a sign ('1') in the pertinent row and column marks the blocks that do require the particular facility, for instance, facility 1 is engaged by blocks 1, 3 and 5 (columns 5 and 6, rows 1 through 3, 7 through 10 and 15 through 18, respectively) whereas facility 2 is required only by block 1 (columns 7 and 8, rows 1 through 3). The matrix elements in columns  $k + 1$  through  $l$  contain the running costs (or alteration in running costs, respectively) of the facility  $j$  at capacity  $K_{mj}$  to be applied when the total output of all relevant blocks (associated with non-zero costs) exceeds  $DQ_{mj}$ , that is, when an alteration of capacity becomes necessary. This alteration of running costs may be, but need not be, equal for all blocks.

In the last columns of the matrix ( $l + 1, \dots$ ) upper limits of output are defined by the respective  $DQ$  which must not be exceeded. For instance, let column  $l + 1$  represent the capacity limit of the hoisting shaft, then  $DQ = 20\ 000$  means that the total output of all the blocks which require this shaft and hence are marked by the sign '1' in the pertinent rows (blocks 1, 3 and 5) cannot exceed 20 000 t/d. Here too, market demand may be fixed so that, for example, the output of steam coal from blocks 1 through 3 does not exceed 10 000 t/d (column  $l + 2$ ) while the output of coking coal stays below 35 000 t/d (blocks 4 through 6, column  $l + 3$ ). Some or all of these upper limits can be subjected to changes during the planning period, for instance when the simulation has reached the  $h$ -th year, market demand may be increased by a certain amount. Here any other technological and/or market restriction can also be formulated.

The task is now to select from all  $Q_{ni}$  listed in column 2 and not excluded by a mark in column 3, those  $Q^{x_{nit}}$  for the next time interval that will ensure maximum profit for all remaining intervals. The calculation proceeds as follows: selection criterion is the maximum contribution margin as listed in column 4. However, before a selection can be made, this contribution margin must be updated. The running costs of the facilities engaged by the respective block must be considered. In addition, any capital investment due to enlargement or construction of the respective facility must be taken into account. To achieve this the investment is evaluated and discounted to time  $t^x$  and then converted into an annuity factor ( $DM/t$ ) taking into account only the mineral content of the specific block. For instance, consider block 1 and facility 2. If production from block 1 exceeds zero, capacity  $K_{12}$  is needed. Therefore, the capital investment for constructing  $K_{12}$  must be distributed to the actual mineral content of block 1. This figure is put down in the relevant column (7) and rows (1 through 3) of the matrix. As the recovery period for this investment decreases when the output increases, it is evident that the annuity factor ( $DM/t$ ) will decrease as  $Q_{ni}$  increases. Similarly, the respective annuity factors for all other capacity steps of this facility and all other blocks are determined and put into the selection matrix. In so doing,

the capital investment associated with a certain facility is obviously charged twice, three times, etc., if the facility belongs to more than one block. This is not a mistake but a deliberate effect as pointed out below.

Selection of the first  $Q^{x_{nit}}$  can now be made. Taking into account the contribution margin from column 4 as well as the running costs and annuity factors of all facilities that are required by a certain  $Q_{ni}$ , one can determine the one with the best profitability, say, for instance,  $Q_{31} = 4\ 000$  (column 2 row 3). Having chosen the first  $Q^{x_{nit}}$ , some alterations in the selection matrix must be carried out:

- (i) The capacities of all facilities to which this block belongs are engaged up to 4 000 t. Hence this figure has to be deducted from all delimiting quantities  $DQ_{mj}$  in the top row of these facilities. For instance,  $DQ_{31}$  (column 6), which was 10 000, has to be set to 6 000 meaning that the second capacity stage of this facility becomes necessary if production from any other block requiring this facility exceeds 6 000 t/d.
- (ii) Similarly, the  $DQ$  of all absolute limits belonging to this block must be reduced. For instance, as the maximum shaft capacity was 20 000 t/d (top row, column  $l + 1$ ), after having chosen  $Q_{31} = 4\ 000$  t only an additional 16 000 t/d can be hauled through that shaft. Accordingly, only 6 000 tons of steam coal (10 000 - 4 000) must be produced from other blocks (column  $l + 2$ ), etc.
- (iii) If capital investment is required for this choice of  $Q^{x_{nit}}$ , for instance, because a delimiting quantity  $DQ$  of any facility is exceeded, its disbursement is completely accounted for by the relevant annuity factor. Therefore, all other blocks participating in that facility must be relieved of the annuity factor associated with it.

This reduction leads to a preference of these blocks when the next  $Q^{x_{nit}}$  is chosen. In other words, as soon as a certain capacity of any facility is installed, the selection algorithm attempts to ensure full utilization of that capacity unless installing a new facility would be more economic.

In this way the second, third, etc.,  $Q^{x_{nit}}$  is determined until the market demand is met or all upper limits of capacity are reached. Obviously, all restrictions mentioned above have to be considered, for instance, the availability of a certain capacity at the time of selection. This can be achieved simply by setting or removing blocking marks within the selection matrix in the simulation part of the computer program similar to those given in column 3. Manpower restrictions may be taken into consideration in a similar manner as the absolute upper limits in columns  $l + 1$  and greater.

It should be noted, however, that the selection has to be repeated several times, if a  $Q^{x_{nit}}$  requires any additional capacity in which other blocks not chosen in this time interval, are participating. More details are given by Klien (1972).

Having determined all  $Q^{x_{nit}}$  for the time interval under consideration, it may turn out that some conditions established earlier are violated. For instance, the output of a certain block often differs greatly from the output of the same block during previous time intervals. This means that its lifetime is different from the time taken for the determination of the annuity factor to account for an investment at a previous stage. If this deviation is serious, it becomes doubtful whether the decisions taken at this previous stage are really optimal. Hence, taking into account the revised information as to the proper value of the annuity factor in question, possibly other capacities or, in the extreme, other facilities and even block outputs would have to be chosen at the previous time intervals. To avoid substantial wrong decisions, therefore, the calculation has to be repeated starting at the relevant time interval bearing in mind the corrected information as to lifetime of blocks, etc. This replanning cycle may sometimes be

necessary twice, three times, etc. In fact, we found that in most cases up to 10 iterations of the calculation were required. They are caused not only by changing block outputs as described above, but also by the fact that the running costs of facilities must be treated as stepwise depending on the utilization of their capacity. Furthermore, if for instance a mine that means the total of all facilities at this mine, is found to have a very low utilization, say below 10 per cent of the available capacity, it may be economical to shut this mine down and distribute the respective output to others. Obviously the calculation has to be repeated in such a case, too.

#### Computational experience

Calculations were carried out mainly on a medium-size computer (Type TR4, Telefunken AG) comparable with an IBM 360-40. The storage capacity required depends largely on the number of blocks, steps of output and facilities. Peripheral storage (disc or tape) is necessary in any case.

The calculations for one main alternative (one block facility allocation) took between 20 minutes and two hours of computer time with about 45 minutes as the average. The time is occupied by the numerous replanning iterations during nearly every run of the program and any substantial reduction of this time seems to be impossible.

As to the convergence of the algorithm, there is no proof in the proper sense. The only statement we can make is that

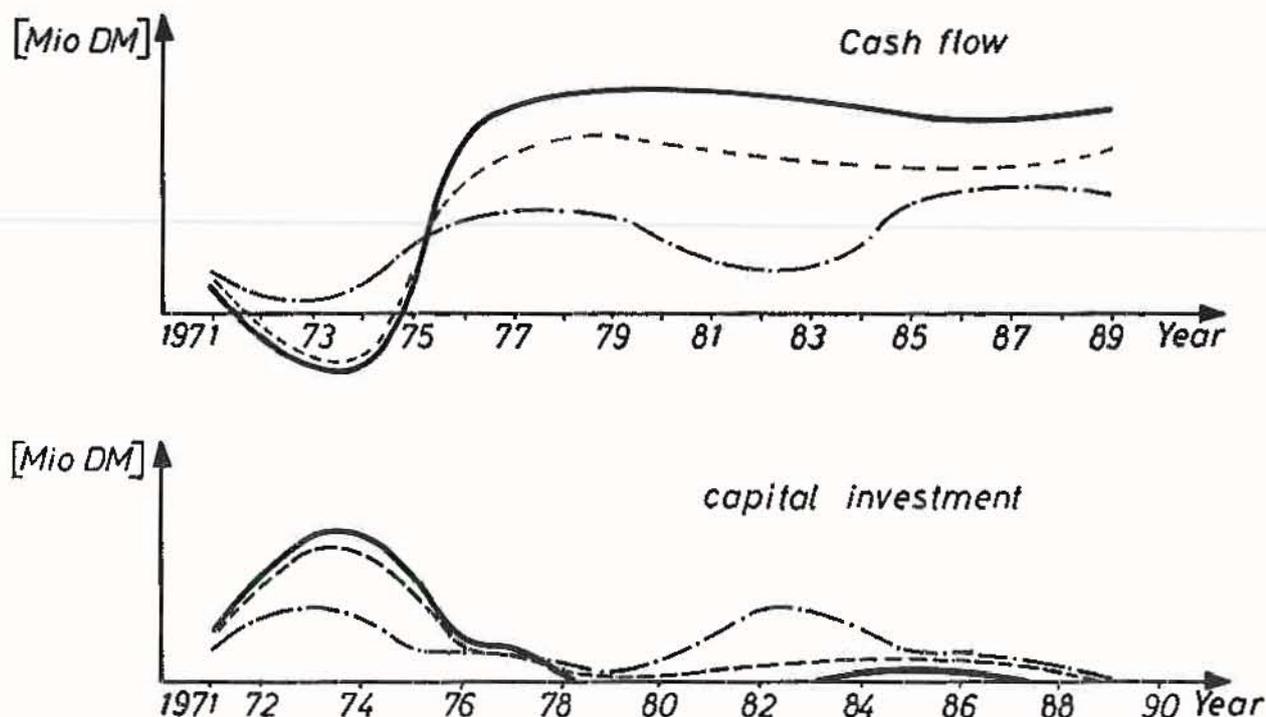
each practical example which we have run has converged. So far we have computed about 100 cases. It should be mentioned, however, that there is also no proof as to the optimality of the final solution. This is why the term 'optimization' has been avoided throughout this paper. The solutions obtained by the method, nevertheless are substantially more economic than any others obtained by usual means of planning, therefore, and with due regard to all uncertainties involved in the problem, they can, for practical purposes, be looked upon as optimal.

#### RESULTS

Obviously the results are specific to the particular case under study, so that only a short glance will be sufficient here.

The value of the discounted cash flow is the most important figure for decision making, but besides this there are other results which may be helpful. They can be divided into two groups.

The first one contains statements to judge the economic value of the alternatives. Figure 3, for instance, shows the development of cash flow and capital investment as a function of time for three alternatives differing in total output. It may be used to estimate the risk of the capital investment against uncertainties in market demand, or, for instance, as a guide for negotiations with top management of the mother company to determine the overall production allocation.



alternative No1 ——— total output 40.500 t/d (average)  
 alternative No2 - - - - - total output 35.500 t/d (average)  
 alternative No3 - · - · - total output 31.500 t/d (average)

Fig. 3. Development of cash-flow and capital investment (example).

The second group of results is associated with technological questions, such as print-outs of manpower development, schedules for constructing new facilities, and so on. Besides this, real production schedules for the whole division and each mine are produced; an example referring to one alternative is given in Fig. 4. As will be seen from this figure, the algorithm produces a feasible scheme with quite a steady development of production in total, at each mine and each block.

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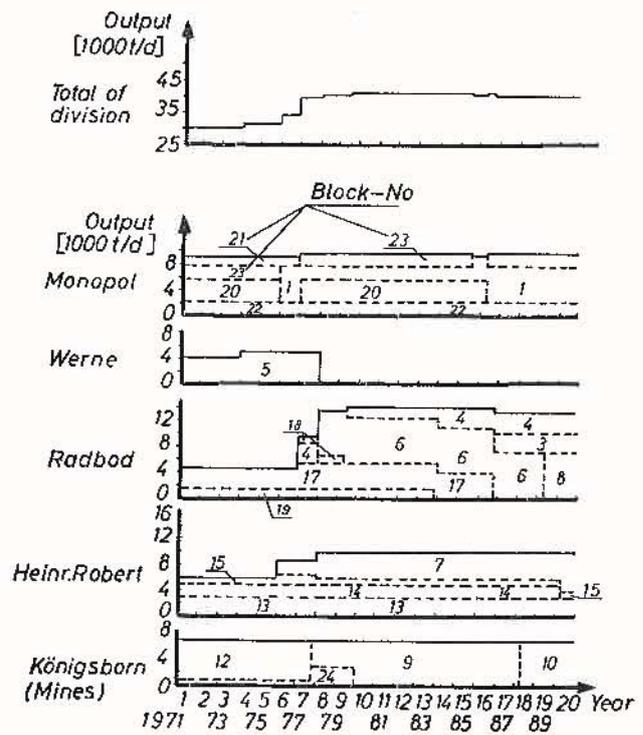


Fig. 4. Production schedule as calculated by the model (example).