An Open Pit Design Model

By R. H. ROBINSON and N. B. PRENN

SYNOPSIS

The model described is a design and economic planning tool for analyzing surface mineral deposits. Mineralization, topography, costs and significant geologic features are input to the model. The results are:

(i) final pit limits yielding the maximum total profit,
(ii) annual cut-off grades and plant sizing yielding the maximum present value, and
(iii) annual maps of the pit and annual production statistics for the mine, concentrator and smelter.

Additionally, summaries are printed of the block mining sequence and cash flow. A special feature is an option to include dump-leaching operations. Stockpiling of material can also be simulated by the model. The model is built around theories of dynamic cut-off grades and a pit design algorithm. The dynamic cut-off grades maximize present value by examination of all economic and physical constraints for the optimum combination. The pit design algorithm is a set of rules formulated to find the maximum value from a special graph. The graph is different from graphs of analytical geometry, being made up of points and arrows connecting some of the points. These graph elements describe the relationship between any point in the deposit and the material which must be mined to get at that point. The model was designed to bring together the interdependent theories of economics, pit design and production scheduling.

INTRODUCTION

The GROPE model is a design and economic planning tool for analyzing surface mineral deposits. GROPE is an acronym representing the functions of the model which are: Grade and reserve estimation, Revenue and cost computations, Open pit design, Production scheduling and plant sizing and Evaluation.

The problem is to find the solution for exploiting the deposit which will maximize present value. Before formulating the solution, the first job is to define the deposit. This definition is done through the familiar block concept.

DESCRIPTION OF THE MODEL

The block concept

The deposit is divided into blocks by constructing a three-dimensional grid. A block representation of a surface deposit is illustrated in Fig. 1. Only the grid lines that outline the surface and boundaries of the deposit are shown.

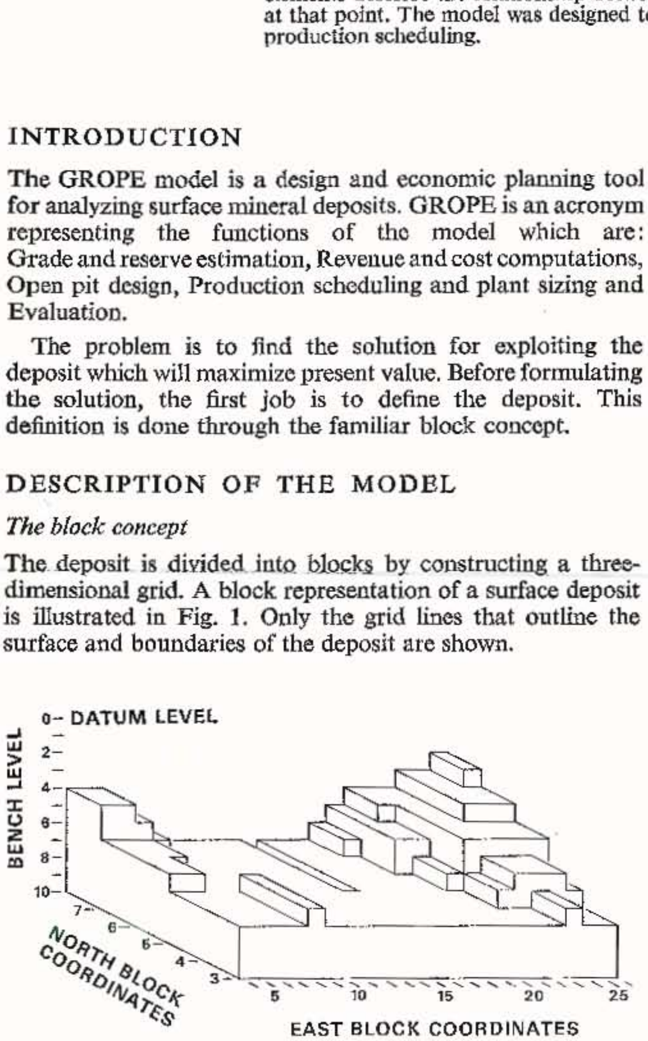


Fig. 1. Grid-block representation of a surface deposit.

The block dimensions are specified to conform with mining equipment and deposit size, and to approximate topographic relief and irregularity of the shape of the orebody. The coordinates are stated as the number of blocks from an origin to a particular block in each of three directions.

The blocks form units for evaluation. The material of each block is considered to act at the geometric center of the block. The mineral distribution, topography, important geological features and other spatial characteristics of the deposit are described through the blocks. The distribution of mineralization is defined by estimating the grade of, and tonnage in, each block. The GROPE model acts upon the blocks as a data set, thus it is sensitive to spatial changes which no averaging techniques can duplicate.

The blocks are also used to describe geological or physical features with no mass and only presence. The topography, a fault or a property boundary are examples of features that are simulated by GROPE. Pointers are given to blocks to indicate the presence of various features. For example, in the case of topography, a zero pointer is given to the blocks representing air space between the surface of the deposit and the datum level of the grid. A pointer value from 0.01 to 1.00 is assigned to the subsurface blocks. A pointer value of 1.00 is given to blocks not intercepted by the surface. Pointer values of less than unity are given in proportion to the rock-filled volume of blocks that are intercepted by the surface.

The pointers and mineralization data are easily stored in a computer and are instantly accessible. GROPE is provided with the capability to branch to computations appropriate to different physical conditions with those spatial data sets.

Maximum present value concept

The objective is to find the cut-off grades, plant capacities, production schedule, and the pit volume that will maximize the present value of the mine. The present value of the mine, PV, is the sum of the net value, $C_{r,t}$, of the blocks mined, discounted for the year they are mined.

$$PV = \sum_R \sum_T \frac{C_{r,t}}{(1+d)^t},$$

where R is the closed three-dimensional region encompassing the deposit, T is the life of the mine, $C_{r,t}$ is the net value of block r, mined in year t, and d is the interest rate.

The net value of each block, $C_{r,t}$, is a function of various parameters, that is,

$C_{r,t} = f$ (location, grade, costs, prices, plant capacities).

The domain of the function $C_{r,t}$ is restricted to the family of pit surfaces whose walls are flatter than the safe wall angles at any point and to the excavation sequence limited by the mining equipment. Further restrictions are imposed by geological and legal boundaries and by processing constraints. Unfortunately, no simple relationship exists for the function, since there are too many unknowns. The given data and some of the information desired are:

Given	*Unknown*
Mineralization	Cut-off grades
Costs	Mining sequence
Prices	Pit volume and shape
Plant capacities (if assumed)	Mine life
	Plant capacities (if not assumed)

The *unknowns* cannot be defined entirely in terms of the *given* data. The *cut-off grades* are not known and are influenced by the *mining sequence*. The *mining sequence* is not known and is influenced by all the other *unknowns*. Similarly, *pit volume* and *shape*, *mine life* and *plant capacities* are interdependent with each other, the other *unknowns* and the *given* data.

Traditional pit design

In the traditional approach to pit planning (Soderberg, *et al*, 1968), two broad assumptions are made to overcome the problem of too many unknowns. Firstly, the cut-off grades are set at the break-even point between profit and loss. This is a static cut-off grade in that the grade changes with time, and the capacities of the processing units and other constraints are ignored. The dynamic cut-off grades used in GROPE will be discussed later.

The second simplifying assumption in traditional pit planning is in the design of pit limits. The deposit is divided into large vertical sections as in the example shown in Fig. 2. There are usually 10 to 20 sections per deposit as compared with 10 000 to 20 000 blocks in the grid concept. The sections are assumed to be two-dimensional and the fact that no real increment of removal has vertical sides is ignored. An economic limit is found independently for each section by moving its end boundaries to the break-even point between profit and loss. Adjacent sections are then smoothed so that the safe wall angle is not exceeded.

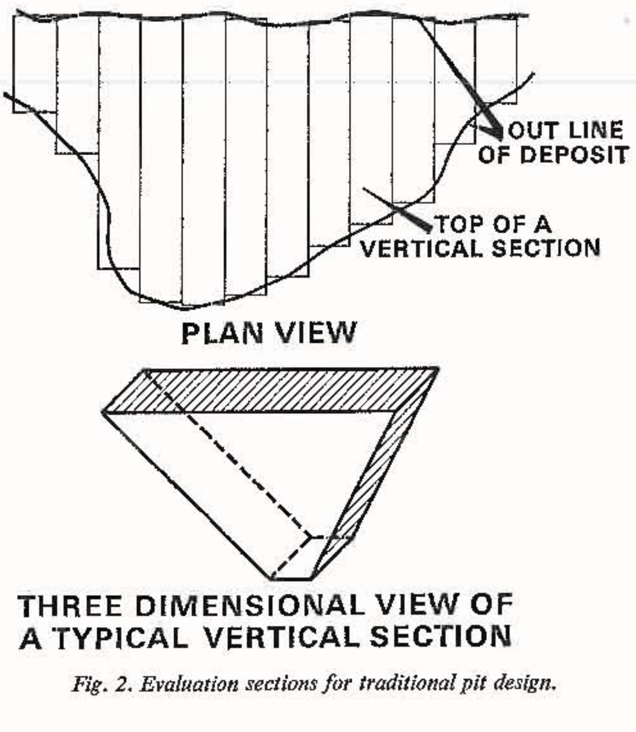


Fig. 2. Evaluation sections for traditional pit design.

The traditional pit design method does not use a total system approach. The vertical sections, or, in fact, sections of any shape, which first of all do not simulate the shape of the pit, are evaluated independently and the results then modified to smooth the surface. Also, these sections are bulky and do not reflect changes in the mineral distribution, topography and other physical features. In addition, the break-even criterion for setting pit limits is not the ideal objective, which is to achieve maximum present value.

A new pit design algorithm

The key to overcoming the difficulties of traditional pit design is an algorithm that was evolved from the work done by Lerchs, *et al* (1965). For pit design, the grid system described earlier becomes a graph (Fig. 3). Each block is a vertex in the graph, and each vertex has a value. The value equals the revenue of the block minus the operating, capital and fixed costs. Vertices in the graph with a positive value are ore blocks and negative vertices are waste. Thus, the graph is four-dimensional. Each vertex, or block, has three dimensions for its location and a fourth dimension for its value.

The vertices are connected by arrows. The arrows are drawn from a specific vertex to all adjacent vertices if the mining of the specific vertex is dependent on the removal of the adjacent vertex or vertices. The arrows describe paths to the surface of the deposit, when they are connected from a terminal vertex of one arrow to the initial vertex of a succeeding arrow. The arrows are flexible and can be drawn from a vertex to any adjacent vertex to describe variable pit slope angles. The graph in Fig. 3 shows only two vertical sections, perpendicular to each other, since a three-dimensional view would be masked by a profusoin of arrows.

The material that must be mined to expose any specific vertex is found by following all paths leading away from that vertex. A closure is the vertices and paths so described. A more rigorous definition of a closure is any set of vertices such that, if a vertex belongs to the closure and an arrow exists from the vertex to an adjacent vertex, the adjacent vertex must also belong to the closure. Thus, a closure is any surface satisfying the constraints on the safe wall angle of the pit.

The arrows are depicted quite easily; programming them, however, presents a difficulty. It has been suggested (Hartman, *et al*, 1966, Johnson, *et al*, 1971, Lerchs, *et al*, 1965) that the arrows be defined through the construction of the grid system, and then stepping up and out one block at a time to form the slope. Theoretically, a grid construction could be found to furnish any desired slope angle. Unfortunately, the grid blocks may not correspond to a convenient size in terms of bench dimensions or equipment limitations. Additionally, the slope angles would be fixed and incapable of being varied through the pit. This difficulty is overcome by simulating the arrows using analytical geometry to describe the set of vertices satisfying all the paths to the surface from a specific vertex.

To elaborate, the dependence of one block (vertex) on another for removal is defined by the arrows. This dependence is described by geometrical shapes made up of planes and the upper portions of circular or elliptical cones or hyperboloids of one sheet. These surfaces define the boundary between those blocks which must be removed and those which may remain to expose a specific block. Three-dimensional data sets keep track of the blocks within the closures defined by the geometrical shapes.

The closures are programmed by deriving FORTRAN expressions from the equations for the geometrical shapes, the coordinates of the grid system and the maximum safe slope angles. The safe slope angle can be varied as often as required by the stability conditions of the particular deposit. This is done by modifying the constants in the FORTRAN expressions for various parts of the pit. Additionally, elliptical expressions are formulated for directionally-dependent stability conditions. For example, pit slopes can be simulated which must vary according to their orientation with respect to the weak strength direction of a bedding plane. The geometrical formulation of closures is actually more flexible than the arrows since the arrows are confined to a vertex-to-vertex relationship. The arrows are limited somewhat by the block dimensions. However, the geometrical formulation can cut through blocks, if required, by the maximum safe slope angle.

The objective is to find the maximum closure, in other words, the set of vertices or blocks yielding the maximum total value of the deposit. The maximum closure for a deposit is found by operating on the graph with a set of rules. The rules are shown in the network diagram, Fig. 4. The blocks represent computations and arrows indicate the computational sequence. The procedure starts by setting the block locator, r, equal to zero. A block locator references the value, C_r, of a block with its location in the deposit. The deposit R is searched for the positive or ore value blocks by incrementing the locator, $r = r + 1$. If the block r is waste, then $C_r < 0$, and the procedure leads to the next block. When a block is found with ore value, then $C_r > 0$. This block is checked first for having been included in the maximum closure in an earlier cycle of the procedure. The procedure leads back to the locator incrementing step if the block C_r is already in the maximum closure. Otherwise, the procedure branches into a routine for identifying the blocks that must be mined to expose the ore block, C_r.

Another block locator is initialized, $q = 0$. A second search of the deposit is then made for blocks in the path from C_r to the surface. When a block C_q is found to be in the path, or removal closure, for C_r, it is first checked for having been included in the maximum closure in an earlier cycle. If so, the routine returns to the locator incrementing step, $q = q + 1$. However, should C_q be in the removal closure, its value is added to the accumulated value S of the other blocks in the removal closure. The search for blocks exposing C_r ends when all blocks in its removal closure have been identified. The value C_r and its removal closure are part of the maximum closure if its total value is positive, $S > 0$.

The routine then branches back for the next unexposed ore block. The search of the unexposed ore blocks is repeated until there are no more positive removal closures added to the pit. The set of positive removal closures, so identified, will be a maximum closure.

A simple illustration of the pit design algorithm is given in Fig. 5. The graph represents a vertical section through a surface deposit. The vertices represent blocks 1 through 13 and the value C of each block. The arrows identify which blocks must be removed to expose a lower block. The example is two-dimensional, but this is merely a convenience for simplifying the illustration. The number of dimensions is immaterial to the algorithm.

The procedure begins with block C_1. This block is waste since its value is negative. The rules of the algorithm then lead to the next block C_2, and it is also waste. The routine continues to loop through the incrementing step, $r = r + 1$, until an ore block is found. The first ore block is $C_6 = +2$. The next step is to identify the removal closure for C_6. In this case, the removal closure is C_6 by itself since it is on the surface. Obviously, C_6 must be part of the maximum closure and so it is included in the pit.

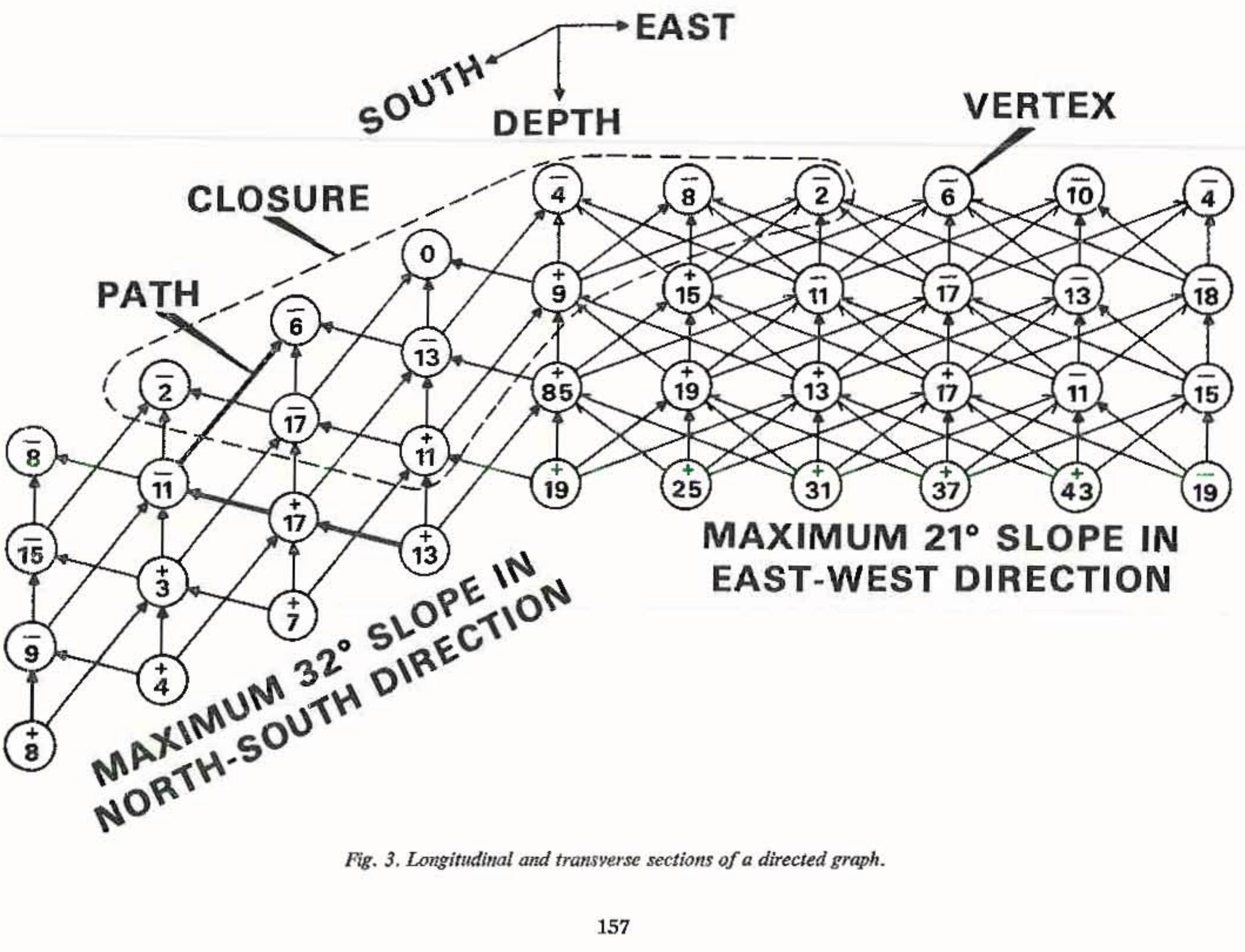


Fig. 3. Longitudinal and transverse sections of a directed graph.

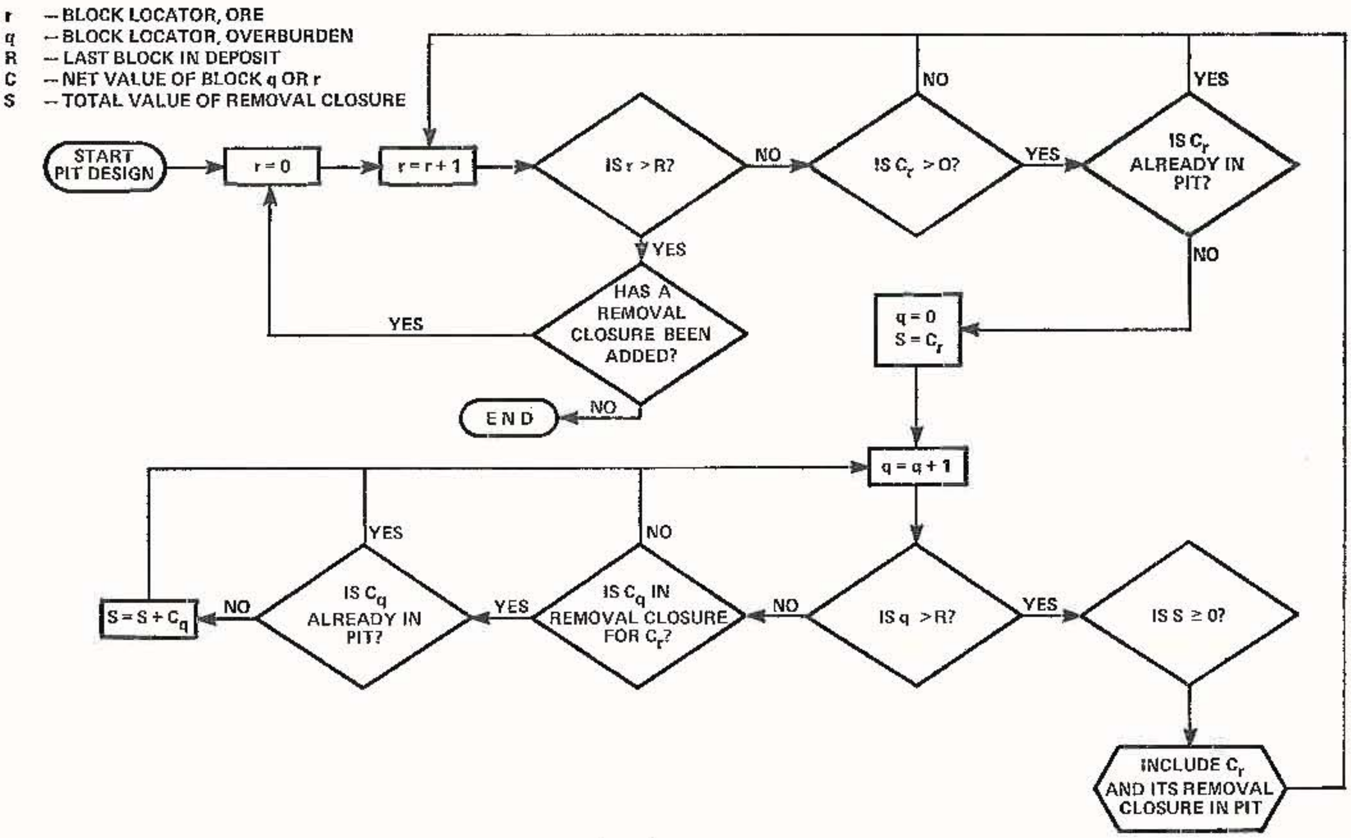


Fig. 4. Pit design algorithm.

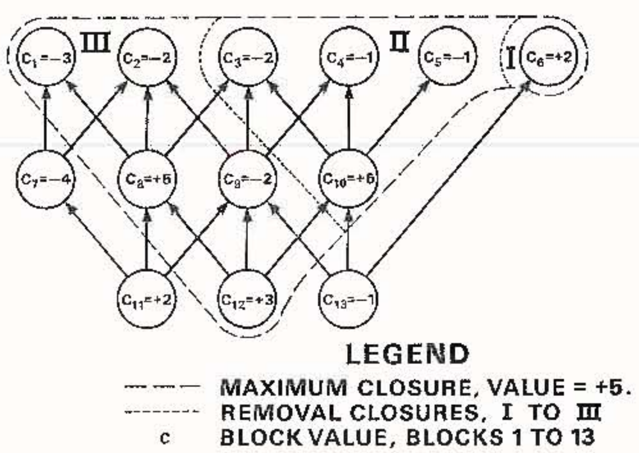


Fig. 5. An example pit design problem.

The search of the graph for ore blocks continues and the next one is C_8. The routine then branches into the steps for identifying the removal closure of C_8. This closure includes blocks C_1, C_2, C_3, and, of course, C_8, and its total value is $S = -2$. The C_8 removal closure is not part of the maximum closure since it has a negative value. Block C_{10} is the next to be tested for a positive removal closure. The closure is blocks C_3, C_4, C_5 and C_{10} and has a total positive value of $S = +2$. Therefore, this removal closure becomes the second to be included in the pit.

The next ore block is C_{11}. Its removal closure has blocks C_1, C_2, C_7, C_8, C_9 and C_{11}. The removal closure does not include blocks C_3 and C_4 because they are already part of the pit. The value of the removal closure is negative, $S = -4$, and so is not in the maximum closure.

The removal closure for C_{12} is positive and is included in the pit. The closure has blocks C_1, C_2, C_8, C_9 and C_{12}. The blocks that are not in the removal closure but are on the paths from C_{12} to the surface are already in the pit. Note how the algorithm handles the mutual support of ore blocks with common overburden. Block C_8 could not outweigh the overburden in its own closure. However, it is eventually included in the pit with support from C_6 and C_{12}.

The search is repeated and there are no more positive removal closures; therefore, the routine ends. The resulting closure is the maximum for the graph. It has the highest value possible from the blocks as constrained by the arrows. The exclusion of any subclosure from the maximum closure will subtract from the value of the maximum closure.

Dynamic cut-off grades

The value of each block must be known in order to construct the pit design graph. Consequently, block values are computed beforehand, as if they were all mined. This computation requires a decision on cut-off grades to distinguish ore from waste. Frequently, the decision is more complex if some material must be classified further for a leaching treatment or temporarily stockpiled.

Traditionally, the cut-off grade is set where revenue from the ore, less its processing and overburden stripping costs, is equal to zero. This break-even cut-off grade is static. It is determined independently of the mineral distribution, mining sequence, capital charges and processing capacities. For a particular break-even cut-off grade the processing plants may be starved or glutted as mining moves between lean and rich ore. Serious bottlenecks crop up during rich years. In lean years, the plants operate below capacity. Actually, operators do adjust their cut-off grades. In general, in rich years, cut-off grades are adjusted upwards to avoid bottlenecks and to maximize profits. Also, in lean years, cut-off grades are adjusted downwards if the marginal cost is more than offset by additional revenue.

The dynamic cut-off grades calculated by GROPE are an attempt to incorporate actual practice into pit planning. The formulation of these cut-off grades was taken from the work of Lane (1964) who explained how to calculate cut-off grades that will maximize the present value of the mine. This is done by expressing the present value, V, resulting from processing a unit of material, in terms of cut-off grade, COG. The present value may be expressed as

> $V = f$ (cut-off grades, processing costs, selling prices, plant capacities, overhead costs, mineral distribution, and time).

For example, the curve shown in Fig. 6a is a typical relationship between the present value and cut-off grade. At low cut-off grades, V increases as more material that cannot support its processing costs is discarded. The curve reaches a maximum that is limited by production capacity, and the size and richness of the deposit. The curve then falls off as decreasing quantities of product become less able to support the investment costs on capital equipment.

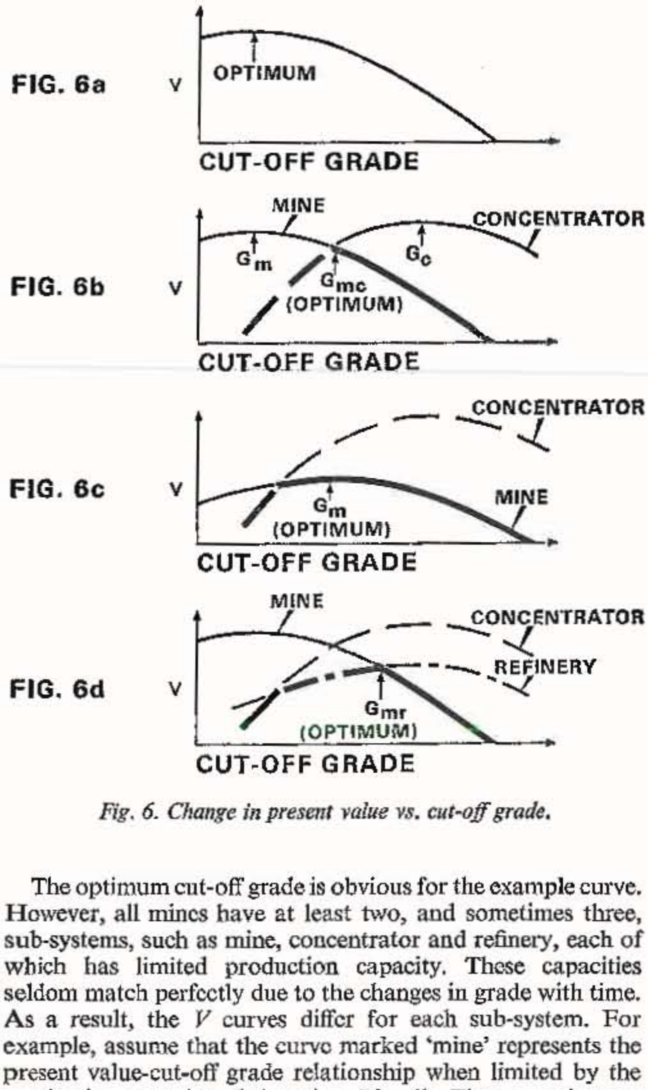


Fig. 6. Change in present value vs. cut-off grade.

The optimum cut-off grade is obvious for the example curve. However, all mines have at least two, and sometimes three, sub-systems, such as mine, concentrator and refinery, each of which has limited production capacity. These capacities seldom match perfectly due to the changes in grade with time. As a result, the V curves differ for each sub-system. For example, assume that the curve marked 'mine' represents the present value-cut-off grade relationship when limited by the production capacity of the mine, Fig. 6b. Then superimpose a second curve, marked 'concentrator', for the same relationship but based on the production capacity of the concentrator.

The actual present value, which can be realized by linking the two sub-systems, is always shown by the lower of the two curves. This is marked heavily in the diagram. The maximum V occurs at the intersection of the curves. This cut-off grade, G_{mc}, balances the production of the mine and the concentrator at their maximum capacities. The cut-off grade is such that the mined material-to-ore ratio is the same as the ratio of mine-to-concentrator capacities. At the balancing cut-off grade, there will be no accumulations or interruptions in material flow.

The system should not operate at either of the optimal cut-off grades, G_m or G_c, for the sub-systems. This would unbalance production so seriously that the V realized would be very low.

There are times when present value is not maximized by balancing plant production. For example, Fig. 6c shows a situation where the mine is a bottleneck. In this case, the optimum cut-off grade is G_m at the peak of the curve marked 'mine'.

The examples in Figs. 6b and 6c illustrate mines with only two sub-systems. However, the analysis is similar for three sub-systems. For example, the curves shown in Fig. 6d include one marked 'refinery'. The optimum cut-off grade is at the peak of the curved polygon formed by the lowest parts of the curves. In this case, the optimum is the cut-off grade, G_{mr}, that balances mine and refinery production.

The V-COG curves represent one point in time since the relationship changes with the spatial dispersion of minerals. Therefore, the cut-off grades must be re-evaluated periodically to maximize present value. The GROPE model calculates cut-off grades annually using the concepts given above.

Annual production scheduling

The time dimension is important to pit planning. To calculate the annual cut-off grades, it is necessary to know what material is exposed for mining at any specific time. The time dimension is determined by the sequence in which the deposit is mined and the production rate. Scheduling the annual production is done by simulating the excavation sequence with a computer. The excavation sequence is programmed into GROPE from the set of rules that define the scheduling peculiar to the various mining methods.

The scheduling routine is programmed for yearly cycles, Fig. 7. Each cycle is in two parts. The first part schedules the ore that must be mined to meet concentrator feed requirements of grade and quantity. The second part identifies the overburden that must be mined to expose the ore. At the option of the user, GROPE will divert some of this overburden to leach dumps or stockpiles if it has marginal value. The stockpiled material is picked up later if the cut-off grades drop down low enough to make the material ore. Stockpiled material is processed at the end of the life of the mine, if any remains. The network diagram in Fig. 7 summarizes the scheduling routine.

The routine directs the computer to scan the deposit for the next ore block to be mined in a particular year. When the ore block has been found, it is tagged with its production year. Then, the tonnage in the ore block is summed with the other ore mined in the same year. The cumulative ore production is compared with the target. The ore scheduled for mining in a specific year is completed when the cumulative tonnage equals the target. The yearly cycle is finished by scheduling the overburden required to expose the ore. The yearly cycle continues until all material within the optimum pit has been scheduled for mining.

The scheduling rules noted in the fourth step of the diagram are tailored for each application of the program. The rules are quite simple. They merely require defining:

(i) the number of exposed ore benches,
(ii) the number and capacity of ore production faces,
(iii) the direction of advance for each face,
(iv) the working area limits of each face, if any, and
(v) whether any portion of the deposit is to be hi-graded.

This definition of the excavation sequence provides GROPE with a flexibility to adapt to different mining methods. The flexibility is important to the computation of cut-off grades. In this way, the cut-off grades are made sensitive to spatial changes in the mineral distribution. If the scheduling routine were replaced by broad time assumptions, it would not be possible to maximize the present value of the deposit. Further, alternative scheduling rules can be substituted to study their effect on the deposit. In addition, the scheduling rules can be programmed to reflect the effect of physical features on production. For example, a large fault displacement could force kinks in the ore benches, or the pit walls could require variable slope angles, and so on.

Interdependent variables

GROPE executes a converging iterative procedure towards a final solution. The iterative procedure is necessary since all the variables are interdependent. Recall that the pit design algorithm must be preceded by the cut-off grade computations. However, the cut-off grade computations depend on knowing when each unit of material is mined, and so production scheduling must come first. Unfortunately, it is impossible to schedule any production unless the pit limits and cut-off grades are known beforehand. The whole process can be conceived as a closed loop in which all computations depend upon each other. Obviously, it is necessary to break into this loop to initiate the solution. The closed loop is opened with the assumption that the entire deposit is mined in the first year. Gradually, the assumption is replaced as the solution converges. In the end, the iteration converges on the final pit surface, cut-off grades and annual production that will maximize present value. Convergence usually occurs in four iterations, the last iteration producing results exactly like those of the preceding iteration.

Advantages over earlier pit design models

The GROPE model introduces a number of new developments into pit planning. These will be discussed in some detail.

PIT DESIGN

The pit design algorithm produces the unique optimum solution for the final pit limits. The solution is the pit yielding maximum total value. This is a significant improvement over the trial and error pit design techniques. There is no need to select a plan from a number of trial plans. Consequently, the element of doubt has been removed from whether the trial plan is the best possible solution.

The pit design algorithm is also three-dimensional. Thus, it is not necessary to pre-select a pit bottom as in some two-dimensional algorithms (Johnson, *et al*, 1971). These two-dimensional algorithms are 'forced' into three dimensions by pre-selecting the pit bottom. Further, the pit does not have to be simulated by complex geometric volumes as in other algorithms (Plewman, 1970). The complex volumes are adaptations of wedges, cylinders, cones and prismoids. They never quite approach the real pit shape, and they are not sensitive to spatial changes in grade, topography, safe wall angles, and so on.

The grid blocks are not evaluated on an individual basis. Any potentially mineable block is evaluated for inclusion in the pit as a part of all the other blocks on which it depends for removal.

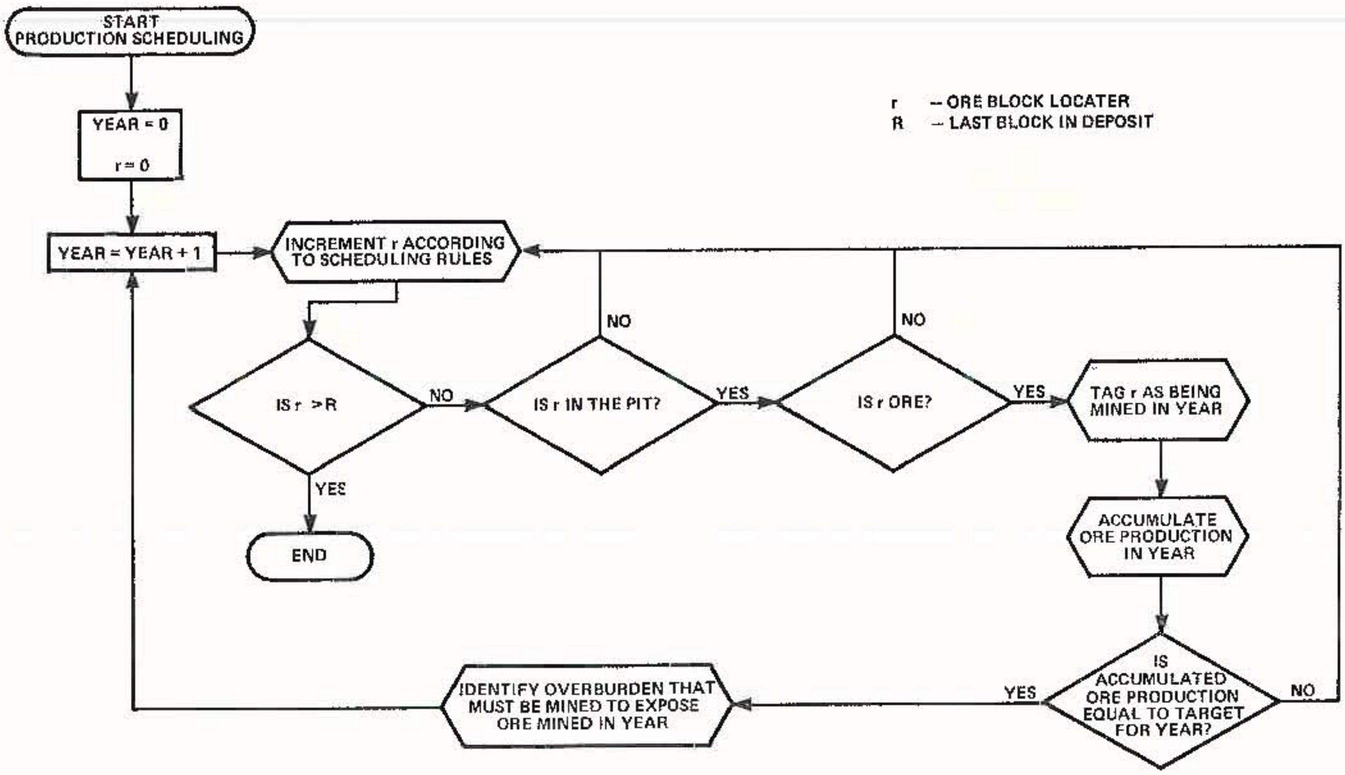


Fig. 7. General flow diagram of production scheduling.

TOTAL SYSTEMS PLANNING

GROPE is a 'total systems' model for planning and evaluating open pit ventures. The model incorporates grade estimation, costing, pit design, cut-off grades, production scheduling, plant sizing and financial evaluation into one model. The synthesis of all these planning elements into a single model results in a plan that maximizes present value. This maximization is accomplished by introducing the time dimension. Another significant aspect of the total systems planning is that there is no pre-determination of the distinction between ore and waste.

VARIABLE PIT SLOPE ANGLES

The GROPE model is not limited to particular angles for the pit walls (Hartman, *et al*, 1966; Johnson, *et al*, 1971; Lerchs, *et al*, 1965; Plewman, 1970; Soderberg, *et al*, 1968). Any number of safe wall angles may be specified for changing slope stability conditions through the deposit. In addition, there is the option to specify working wall angles. These working angles define flatter intermediate slopes. The flatter slopes are maintained in the pit bottom until they intersect with the final pit limits. From the intersection, the slope turns upward to the maximum safe angle. The intermediate working slopes ensure sufficient operating space for mining equipment in pits with steep sides.

SPATIAL SIMULATION

Physical features and legal boundaries are simulated in GROPE by recording their presence in special three-dimensional data sets. The model consults these data sets to branch the routines to computations appropriate to the changing environment. Some examples of spatial data sets are: mineralization, topography, property lines, metallurgical penalties and costs that are a function of location. In this way, GROPE is made sensitive to spatial changes and the dangers of generalizing the attributes of a deposit by averaging are minimized.

GROPE therefore brings together a number of new pit planning ideas with some earlier contributions to the field and some innovations in the use of computers.

OPERATIONAL FEATURES

The remainder of this paper is devoted to the operational features of the model programs. The major portion of the programs was developed originally by Robinson, 1969. The writers have made many major changes since early 1970 to reduce running time and to add leach and stockpile options to the programs. The model has been operational since late 1970.

Grade prediction program

PROGRAM FUNCTIONS

The functions of this program are to

(i) calculate grid level assays,
(ii) calculate grid block grades based on drill hole assays weighted by the inverse distance raised to a power method,
(iii) calculate total deposit ore reserves based on input cut-off grades,
(iv) produce files storing grid block grade data and grid block topography, and to
(v) produce scale plan and cross-section maps showing grid block value.

Program accuracy

The choice of the weighting parameters is very important to the accuracy of the results. The size of the grid blocks and maximum hole influence are usually chosen to ensure that grid blocks will be influenced by four to eight holes. Computer-calculated grid block grades have agreed with production records when the number of holes influencing the block is in the four to eight range, and when limiting parameters placed on fringe blocks are based on a minimum distance from drill hole and a minimum number of holes influencing the grid block. The power to which the inverse distance is raised is low for homogeneous-type deposits (that is, about 1.5 for typical large porphyry copper deposits) and is increased as homogeneity decreases. Grid blocks can be tipped for dipping deposits. Ore reserve totals tend to be slightly low in grade and slightly high in tonnage when compared with mine records.

Costs and running time

The grade prediction program is usually run in a time-sharing environment on a Burroughs 5500 computer. The running time of the program varies with the number of drill holes and grid blocks of the deposit. The number of cut-off grades, different metals and geological parameters generally have only a slight effect on running time.

Running times have been in the range of a few minutes up to an hour of central processor time.

A typical orebody, say, 2 000 ft × 4 000 ft × 800 ft depth with 100 to 200 drill holes, takes between 20 and 40 minutes of central processor time and costs in the range of $200 to $400 per run. Keypunching and error checking of input data generally has a similar cost. A deposit of similar size was calculated several years ago by Cyprus Mines staff, using the inverse distance raised to a power method and a mechanical calculator. The hand calculations took about 10 man-months to complete.

Grope programs

PROGRAM FUNCTIONS

The functions of these programs are to:

(i) design the final pit surface that will maximize present value,
(ii) produce a mine schedule that will maximize present value,
(iii) choose plant capacity yielding the greatest present value from input of a variety of plant sizes,
(iv) choose yearly cut-off grades to maximize one of three economic parameters, which are:
  (a) total profit,
  (b) present value,
  (c) immediate profit, and to
(v) calculate value or cost of each grid block.

The model works in an iterative process as shown by the flow chart in Table I. Examples of the results of the calculations are given in Figs. 8 and 9.

Costs and running time

The model programs are generally run in a remote batch mode on a Burroughs 5500 computer. The running time depends on the number of blocks. Approximately 80 to 90 per cent of the central processor time is spent equally divided between the loops to optimize pit design and the loops to obtain the stripping required for the ore scheduled to be mined. With the aid of a mathematician with programming knowledge the efficiency of the program can probably be greatly increased. The running times of the programs are now in the range of 1 to 1.5 hours per iteration for a medium-size orebody. Usually, three iterations are required for the program to converge. The cost for a medium-size orebody, that is, 25 000 000 tons, containing 32 000 grid blocks was in the range of $500 per iteration and about $1 500 in total.

(A) PROCESSING SUMMARY

	LEACHING			MILLING		SMELTING		
YEAR	LEACH TONS	CU TONS	$ YEAR	TONS	$/YEAR	PRODUCT TONS CU	S	$/YEAR
1	2860000.	0.	0.	1820000.	2639000.	12973.	0.	1452942.
2	3900000.	1279.	319800.	1820000.	2639000.	10100.	0.	1131166.
3	0.	2544.	636025.	1820000.	2639000.	15647.	0.	1752442.
4	2080000.	2046.	511550.	1820000.	2639000.	12752.	0.	1428190.
5	5200000.	2596.	649025.	780000.	1131000.	4089.	0.	457912.
6	0.	4960.	1240038.	0.	0.	0.	0.	0.
7	0.	3643.	910813.	0.	0.	0.	0.	0.
8	0.	2405.	601250.	0.	0.	0.	0.	0.
9	0.	1764.	441025.	0.	0.	0.	0.	0.
10	0.	749.	187200.	0.	0.	0.	0.	0.
TOTALS	14040000.	21987.	5496725.	8060000.	11687000.	55559.	0.	6222653.

(B) PRODUCT SUMMARY

	MILL FEED						PRODUCTS			NET SALES	
	FROM MINE			FROM STOCKPILE			LEACH	SMELTER			
YEAR	CU GRADE	S GRADE	TONS	CU GRADE	S GRADE	TONS	CU TONS	CU TONS	S TONS	CU $/YEAR	S $/YEAR
1	0.84	5.73	1820000.	0.00	0.00	0.	0.	12973.	0.	14425642.	0.
2	0.65	4.05	1820000.	0.00	0.00	0.	1279.	10100.	0.	12653336.	0.
3	1.01	2.56	1820000.	0.00	0.00	0.	2544.	15647.	0.	20229280.	0.
4	0.82	1.25	1820000.	0.00	0.00	0.	2046.	12752.	0.	16455264.	0.
5	0.62	0.25	780000.	0.00	0.00	0.	2596.	4089.	0.	7433275.	0.
6	0.00	0.00	0.	0.00	0.00	0.	4960.	0.	0.	5515686.	0.
7	0.00	0.00	0.	0.00	0.00	0.	3643.	0.	0.	4051294.	0.
8	0.00	0.00	0.	0.00	0.00	0.	2405.	0.	0.	2674360.	0.
9	0.00	0.00	0.	0.00	0.00	0.	1764.	0.	0.	1961679.	0.
10	0.00	0.00	0.	0.00	0.00	0.	749.	0.	0.	832665.	0.
TOTALS	0.81	3.10	8060000.	0.00	0.00	0.	21987.	55559.	0.	86231481.	0.

(C) YEARLY OPERATING COST SUMMARY

YEAR	STRIPPING	MINING	LEACHING	STOCKPILING	MILLING	SMELTING	TOTAL
1	2562625.	477360.	0.	0.	2639000.	1452942.	7131927.
2	3102450.	482040.	319800.	0.	2639000.	1131166.	7674456.
3	636350.	484120.	636025.	0.	2639000.	1752442.	6147937.
4	1846000.	487760.	511550.	0.	2639000.	1428190.	6912500.
5	1879150.	209040.	649025.	0.	1131000.	457912.	4326127.
6	-0.	0.	1240038.	0.	0.	0.	1240038.
7	-0.	0.	910813.	0.	0.	0.	910813.
8	-0.	0.	601250.	0.	0.	0.	601250.
9	-0.	0.	441025.	0.	0.	0.	441025.
10	-0.	0.	187200.	0.	0.	0.	187200.
TOTALS	10026575.	2140320.	5496725.	0.	11687000.	6222653.	35573273.

(D) CASH FLOW SUMMARY

YR	CU SALES	S SALES	OP COSTS	PROFIT	DEPRECIATION	GROSS INCOME	TAXES	NEW INCOME	CASH FLOW	DCF AT 15.%
1	14425642.	0.	7131927.	6793715.	345000.	6448715.	2901922.	3546793.	3891793.	3384168.
2	12653336.	0.	7674456.	4478880.	345000.	4133880.	1860246.	2273634.	2618634.	1980063.
3	20228280.	0.	6147937.	13580343.	345000.	13235343.	5955905.	7279439.	7624439.	5013192.
4	16455264.	0.	6912500.	9042764.	345000.	8697764.	3913994.	4783770.	5128770.	2932391.
5	7433275.	0.	4326127.	2607148.	345000.	2262148.	1017967.	1244181.	1589181.	790104.
6	5515686.	0.	1240038.	4275649.	0.	4275649.	1924042.	2351607.	2351607.	1016664.
7	4051294.	0.	910813.	3140482.	0.	3140482.	1413217.	1727265.	1727265.	649343.
8	2674360.	0.	601250.	2073110.	0.	2073110.	932900.	1140211.	1140211.	372737.
9	1961679.	0.	441025.	1520654.	0.	1520654.	684294.	836360.	836360.	237746.
10	832665.	0.	187200.	645465.	0.	645465.	290459.	355006.	355006.	87752.
SUM	86231481.	0.	35573273.	48158208.	1725000.	46433208.	20894944.	25538265.	27263265.	16464160.

Fig. 8 (a) Yearly processing summary. (b) Yearly product summary. (c) Yearly operating cost summary. (d) Cash flow summary.

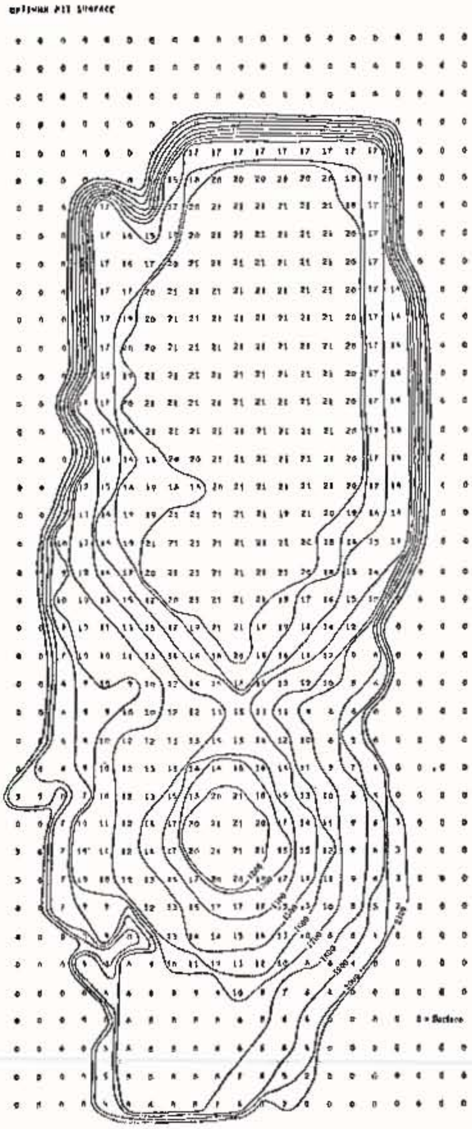


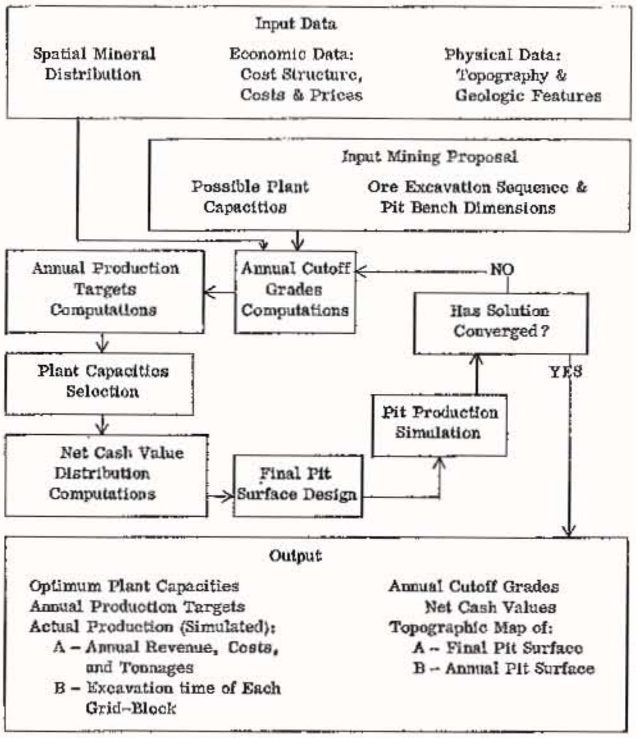
Fig. 9. Optimum pit map.

Program accuracy

The accuracy of the GROPE programs has not been tested completely. The pit design portion of the program will design an optimum pit if drill hole assays, estimated operating costs and metal recoveries are reasonably accurate. The computer-designed pits have arrived at a more economic pit design than trial-and-error hand-designed pits on the several properties that have been tested to date.

TABLE I

COMPUTATIONAL SEQUENCE



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