Optimization Studies on the Design of a Processing Plant for a Large Mining Venture

By J. C. PAYNTER,* B.Sc.(Eng.), B. K. LOVEDAY,† Ph.D. and C. G. ROBINSON,‡ B.Sc.(Eng.)

SYNOPSIS

A method of optimizing the design of a processing plant has been developed which minimizes capital and operating costs for average stripping ratios and ore characteristics. Models were developed to describe the operating characteristics and costs involved in various operations such as up-grading, leaching and solid-liquid separation.

The options available in up-grading policy and the relation between the cut-off grade and the costs incurred subsequently in the treatment of the ore formed the basis of the optimization. To arrive at an optimum plant configuration and a set of operating parameters, the interrelation between grinding size, leaching temperature, leaching time, and various plant recycle options was investigated.

INTRODUCTION

The profitability of mining low-grade ore is sensitive to cut-off grade, ore reserves, the cost of capital, operating costs, and market constraints. These factors may vary dramatically during the proving phase of a mining venture as knowledge of the deposit and marketing conditions change. Some mining companies have therefore developed computer programs to assist in discounted cash flow calculations so that profitability can be re-assessed as the information improves and its sensitivity to various assumptions can be tested.

The cut-off grade is commonly used in these calculations in which the annual profits and the life of the mine are varied. Process variables such as size of grind and process temperature are held constant, although the choice of process variables is related to the grade of ore being processed and hence to cut-off grade. These interactions are not always obvious, especially if there are alternative methods of waste rejection in the process. The cut-off grade determines the waste-to-ore ratio and hence the mining cost per ton of ore milled. The costs of processing are primarily a function of tons of ore milled, but at some point in a process the costs become dependent primarily on the tons of metal processed.

In this paper consideration is given to the problem of optimal design for a processing plant when the annual rate of production of metal is known (obtained via strategic economic studies). Allowance is made for a number of ore types having different processing characteristics and costs. Separate circuits could be used for each ore type (as in the Kidd Creek Concentrator), as described by Domaas (1969) or the circuits could be combined at some point in the process. At the design stage, concentrator and mining capacities can be regarded as variables and a balanced plant can be designed to produce the required amount of metal.

Capital and operating costs per unit of metal are then calculated in terms of the controls available (that is, cut-off grade, size of grind, plant capacity, etc.). Where independence is reasonable, each type of ore is allowed an independent cut-off grade and independent plant controls. A general-purpose flowsheet is used so that marginal processes can be included, and a search routine is followed for the minimization of costs as a function of all the variables. Processes with a negative marginal contribution to profit are eliminated automatically and an optimal plant design and control structure are obtained. The total cost per unit of metal is taken as the sum of the operating costs and the capital costs discounted at a given rate. Clearly, such a design is very sensitive to the rate at which capital is charged. A few case studies are given to illustrate this effect.

APPLICATION TO AN OPEN PIT MINE

This philosophy was applied to the design of a plant for processing an oxide ore obtained from an open pit mine having a very large ore reserve. Each of the unit operations was modelled in some detail, resulting in a rather complex mathematical structure describing the physical system and the associated capital and operating costs. In Fig. 1, the flow diagram for the process, the variables to be optimized are ringed. These variables are, briefly,

(i) the block cut-off grade for each ore type,
(ii) the block grade above which up-grading is not justified,
(iii) the up-grading controls associated with each ore type,
(iv) the size of grind in milling,
(v) the temperature, residence time and acid requirements in leaching,
(vi) the configuration of the solid-liquid separation system, and
(vii) the number of stages and the ion-exchange inventory.

In a simplification of the problem of optimal design (minimum capital and operating costs per unit of metal), it is convenient for the system to be divided into a series of sub-systems. The sub-systems are then optimized in terms of the variables that link them to the other sub-systems. For example, the controls in the upper portion of Fig. 1 can be used to produce a range of grades of ore for further processing. The higher the grade of ore, the lower the milling and leaching costs, but the higher the mining and up-grading costs. The grade of ore in the coarse-ore stockpile is thus the link variable between the up-grading sub-system and the subsequent processes.

Up-grading

Figure 2 shows the decision variables for up-grading that were considered for each ore type. The cut-off grade (variable 1) used in the mine is related to the cost of up-grading in subsequent processes. Above a certain grade (variable 2), the cost of up-grading cannot be justified. If up-grading is too expensive, the grades specified by variables 1 and 2 will be the same and the tonnage fed to the up-grading plants will be zero. The controls on the up-grading plants (variables 3 and 4) determine the recovery and grade of individual products.

* Head, Process Development Division; † Principal Scientist; ‡ Senior Scientist; National Institute for Metallurgy, Private Bag 6, Cottesloe, Johannesburg.

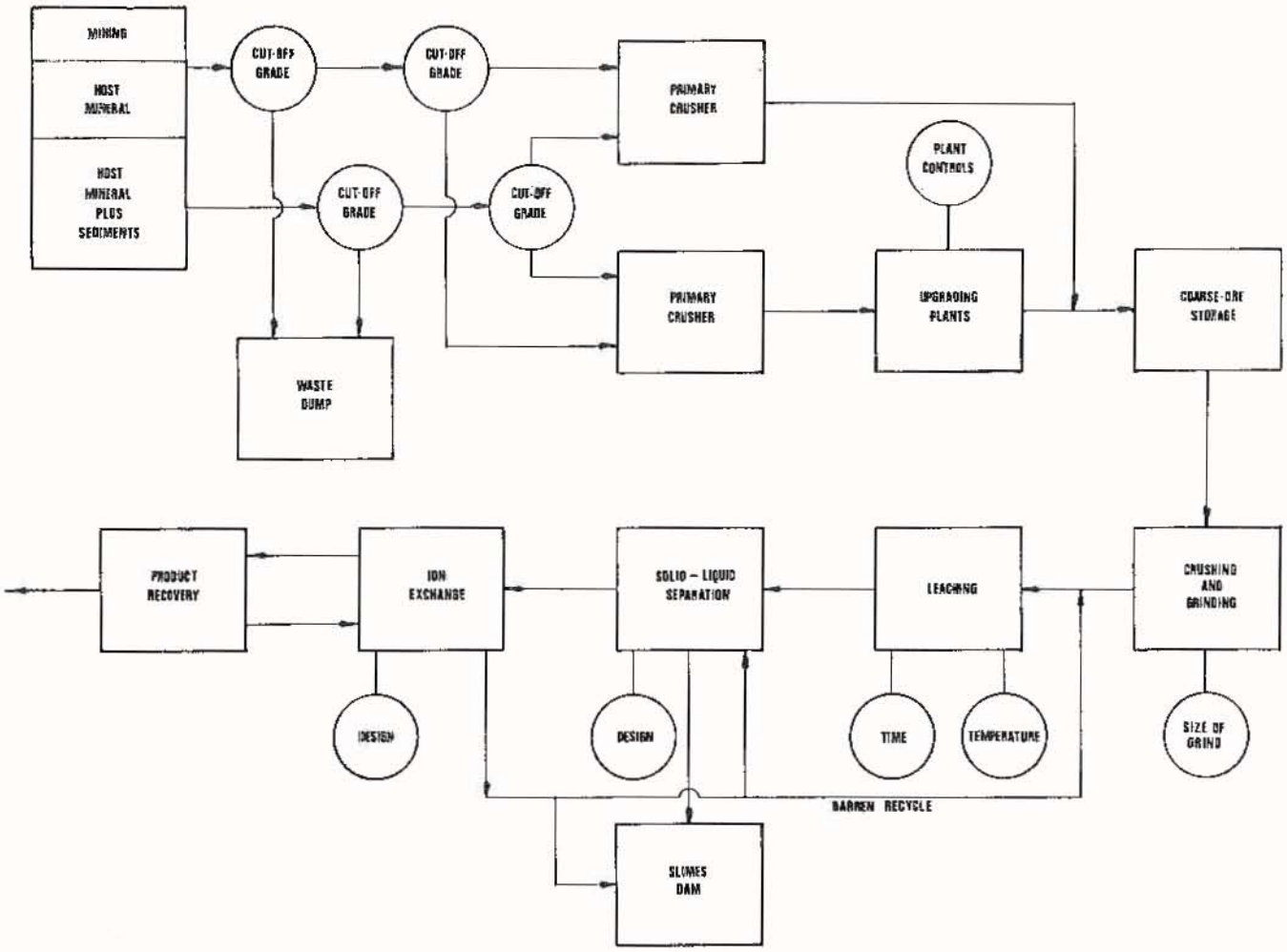
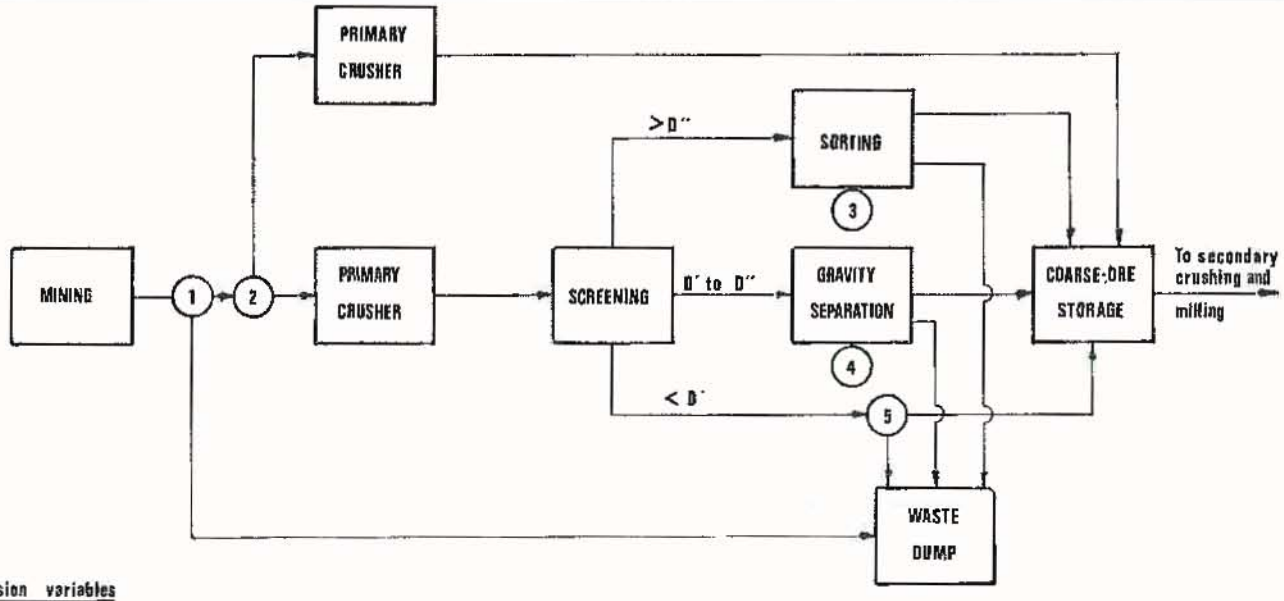


Fig. 1. Flowsheet of proposed process.



Decision variables

1 Block cut-off grade – to waste
2 High block cut-off grade – to milling
3 Detector setting
4 Specific gravity
5 Rejection of fines

Fig. 2. Mining and up-grading flow diagram and decision variables.

Sorting is a direct method of up-grading, based on the concentration of mineral alone. Heavy-medium separation utilizes the difference between the specific gravities of the host mineral and associated sediments. The sediments may contain limestone, which has a marked effect on the acid consumption in leaching. Consequently, the up-grading plants have the dual effect of up-grading and of removing acid-consuming sediments. The amount of acid consumed in the leaching operation is affected directly by up-grading decisions, but it is not affected markedly by the time and temperature of the leaching operation (within the limits of 'normal' operation). The costs of the acid plant are included in the up-grading costs because up-grading controls have a direct influence on these costs; for example, the acceptance of fines (variable 5) is related to the grade and the acid consumption.

The block cut-off grade used in the mine determines the grade of the ore to be processed. Consequently, mining block statistics can be used for the generation of a grade-recovery curve for block cut-off. Blocks of marginal grade are sent to up-grading (variable 2), but a high acid consumption could make up-grading essential, irrespective of grade. Thus, ore with a high acid consumption must have a higher cut-off grade because all the ore incurs additional up-grading costs.

Screening results in up-grading of the fines and a consequent degrading of the coarse material. Thus, for any head grade, a relative up-grading or degrading of the size fractions was defined. Similarly, the grade of concentrate in the up-grading plants was defined in terms of the plant controls and head grade. The operating characteristics of each ore were defined by relative grade as a function of recovery, and relative acid consumption was also related to the recovery of metal.

For n ore types, the objective in the up-grading sub-system is a search for the set of $5n$ controls that minimize the total cost of mining, crushing, up-grading, and acid requirements for a specified grade of ore and production rate. Total capital and operating costs and the set of optimal controls are obtained as a function of the ore grade. Costs and controls for the subsequent processes are also obtained as a function of grade, and hence an optimum is found.

A computer program was written as a simulation of the process illustrated in Fig. 2 and its associated costs. Grade-recovery curves for the up-grading plants were based on information obtained from laboratory tests. Three ore types and estimated block statistics were then used in a test of the operation of the program. Because of the large number of variables (15) and because the final grade (that is, an equality constraint) had to be specified, there was considerable difficulty in finding the required minimum cost by a non-linear search routine. The minimum was found to be rather insensitive to the up-grading plant settings, except for the ore having a high acid consumption. Variables 1 and 2 were the most sensitive (as one might expect), and cut-off grade increased linearly with the grade of final ore. All the high-acid-consuming ore was diverted to up-grading (variable 2), but only a small proportion of the non-acid-consuming ore was treated profitably by the up-grading plants. The proportion of non-acid-consuming ore treated by the up-grading operations increased from zero at low grades to about 20 per cent at high grades.

In the simultaneous treatment of three ore types, it was implicitly assumed that pit development required the mining of all three types. However, if the ore types are in completely separate areas, it may be possible for them to be mined sequentially. It was also assumed that each ore type had separate control settings in the up-grading plants. Such a situation could be achieved in practice if the plants were run on a campaign basis using a single unit.

Although, in practice, each mining block would have a characteristic grade and acid consumption, it was necessary to assume a specific acid consumption for each ore type. This assumption implies a whole range of ore types, and the solution to this problem would require the assumption of a functional relation between cut-off grade and acid consumption. Case studies of the nature described above could be used in the estimation of this relation.

The optimal operating costs as a function of ore grade are shown in Fig. 3. Similar curves are obtained for optimal capital costs. These results were calculated with a discounting rate of 20 per cent per annum. The acid costs decrease, because the tonnage of ore to be leached decreases with an increasing grade. The mining costs increase with increasing grade, because of the higher waste-to-ore ratio. These costs dominate, resulting in a rising curve for total costs.

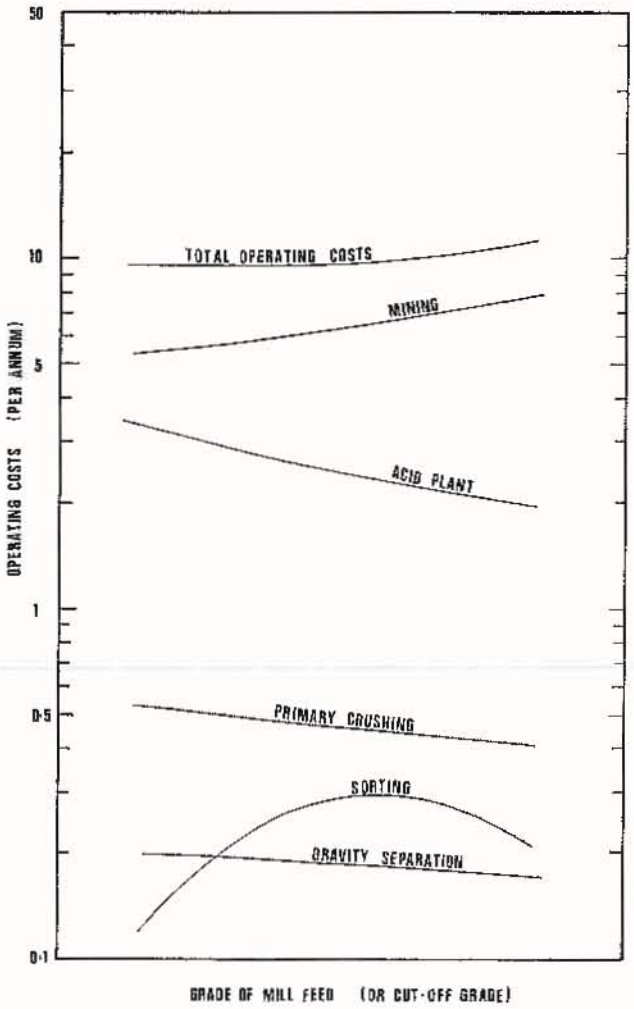


Fig. 3. Optimal operating costs for mining and up-grading as a function of ore grade.

The overall recovery of metal in the up-grading process decreased linearly from 90 per cent to 60 per cent over the range of grades in the investigation. This decrease has important implications for the life of the mine and for the net present value of the project. A constant rate of production of metal was used throughout because the pit was very extensive and could be regarded as an infinite resource. Marketing and contractual obligations dictated the need for a constant production rate at a minimum cost.

Crushing and milling

The energy required in milling and crushing was related to the size distribution of the products by establishment of the

d_{60} size of the product distribution and variation of that size as a function of the energy. Implicit in this treatment is the assumption that the shape of a typical product distribution is constant over a reasonable range of particle sizes.

Leaching

The important variables in leaching are the size to which the ore is milled and the temperature and residence time in leaching. (A further control is the use of mechanical or air agitation.) As shown earlier, costs incurred in the neutralization of the acid-consuming sediments are debited to the up-grading operation.

Agitation costs are a function of the grind size and very coarse grinds are difficult to keep in suspension. More metal can be dissolved from finely-ground ore than from coarsely-ground material and an increase in temperature increases the rate at which the metal can be extracted. An increase in the residence time in leaching results in an increase in the percentage of metal extracted. However, high temperatures and long residence times lead to increased operating and capital costs for steam-raising and agitation equipment.

A model was developed by Robinson *et al* (1972) in which the kinetics of the leaching operations were described as a function of the particle size distribution, temperature, and time. Attempts at fitting the kinetic behaviour to a simple model were unsuccessful, and it was assumed that, because of differing physicochemical environments in the ore, the ore reacts according to zero-order kinetics and has a distribution of effective rate contents. This assumption compensated for the differences in reactivity between the various amounts of oxide, resulting from variations in pore size, ore structure, and chemical environment.

The leaching of the ore proceeds according to zero-order kinetics until all the very fast reacting species have been leached. The rate then begins to decrease. The equations obtained were as follows:

$$f(t,T) = (1-INS)\left(1-\frac{n}{n+1}k_0 T t \exp(-E/RT)\right) + INS \quad \text{for } t < 1/k_{\max}$$

$$= \left(1-INS\right)\left(\frac{1}{n+1}\right)\left(\frac{\exp E/RT}{k_0 T t}\right)^n + INS \quad \text{for } t \geqslant 1/k_{\max},$$

where $f(t,T)$ is the fraction undissolved, k_0 is a frequency factor, E is the activation energy of the zero-order reaction, T is the absolute temperature, R is the gas constant, INS is the insoluble fraction, $k_{\max}$ is the rate constant at which the fastest reacting oxide reacts, n is the shape factor, and t is the residence time in the leach.

The various parameters were ascertained from regressions on a large number of leaching tests. From these tests it was found that E was constant and that n, INS, and k_0 were functions of the particle-size distribution. The relations were evaluated so that the fraction leached could be calculated for any size distribution produced in grinding.

Liquid-solid separation

The liquid-solid separation was modelled simply by sets of simultaneous linear mass-balance equations having coefficients that were functions of the particle size distribution, the performance of the individual units, and the additions of reagent. The dimensionality of the matrix of coefficients depends on the number of units and the system configuration. As these numbers are integer values, the inclusion of the numbers in the optimization would have complicated the search routine.

The best configuration and numbers of units were established by a separate sub-optimization. The choice of configuration and units could then be studied by a few cases so that the correctness of the procedure could be verified. The best configuration was one that minimized the cost function associated with increased capital charges due to an increase in the number of stages and the subsequently lower soluble loss.

Ion exchange

Because the link variable is the concentration of metal in solution, the extraction part of the ion-exchange circuit is related to the grade of mill feed. The cost of the plant is a strong function of the amount of metal produced. Separate sub-optimizations can be made where the purpose of the study is to determine the optimum number of stages in each sub-operation and the optimum inventory of ion-exchange medium.

An increase in the number of stages results in an improvement in extraction efficiency at the expense of increased capital costs, whereas an increase in the inventory of ion-exchange medium improves efficiency at the expense of increased operating costs. The operating costs rise because of losses of the medium and increased capital charges associated with the inventory. An optimization along these lines has been done for a copper solvent extraction process by Robinson *et al* (1971).

Costing of the individual operations

Capital and operating costs for the operation were developed and were expressed in terms of the tonnage of material processed in each operation. The general forms of the equations used were as follows:

$$\text{Capital} = K_1(\text{tons/day})^n + K_2,$$

where K_1 is a scale factor and n is a power law factor (usually in the range 0,5 to 1). The value of n can take into account decreased processing costs per ton as the units used increased in size. Constant K_2 represents fixed costs that are independent of throughput. Operating costs were expressed in a similar manner.

This analysis assumes that the costs vary as a smooth function of capacity. This is not always true where large units are used. Primary crushing, for example, may require two large primary crushing units that are insensitive to capacity within the limits set in the optimizing of the design.

Marginal costing

Metal is lost at various points in the process and in order to optimize the sub-systems which determine the quantity of metal lost, it is necessary to assign a cost to this loss. This cost is that of producing an additional unit of metal up to that point in the process. Thus, at each stage in the process, a marginal cost per unit of metal may be calculated, for use in subsequent sub-optimizations. Marginal costs are less than average costs when average costs decrease with production rate. A marginal capital and a marginal operating cost are used for design optimization. However, care must be exercised in the use of marginal capital when the system contains large units, that is, the marginal cost may just be a working cost.

OPTIMIZATION TECHNIQUE

Fletcher *et al* (1963) and Fletcher *et al* (1964) searches were used in the up-grading program. Powell's (1967) non-derivate search technique was used for the optimization of milling and concentrating. The two searches were embedded in a simple search for the correct grade of feed from up-grading to milling that would minimize the total costs. So that the up-grading

program would ensure that the equality constraint on the rate of production was met, an efficiency for extraction was assumed before the up-grading plant was designed. The efficiency for the optimum milling and concentrating plant was then calculated, based on the grade and tonnage fed. Finally, iteration ensured that the assumed and calculated efficiencies were the same.

Up-grading program

The inputs are as follows:

(i) the assumed efficiency for the milling and concentrating operation,
(ii) the rate at which capital is charged,
(iii) waste proportions in the ore,
(iv) the number, proportion, acid consumption, and metal content of the ore types,
(v) the distribution of blocks in the grade categories for each ore type,
(vi) the grade of feed to the mill, and
(vii) an initial guess at the optimum set of controls.

The operating characteristics at each stage and, hence, the grade of mill feed and the tonnages at each stage are computed. The capital and operating costs and partial derivatives calculated for each control indicate the direction in which each control must be changed for the total cost to be minimized. The search then proceeds until the minimum cost is obtained subject to the equality constraint on the grade of feed to the mill.

Milling and leaching program

The milling and concentrating program uses a Powell search for the optimum size distribution in milling and the optimum temperature and time in leaching.

The objective function to be minimized is:

$F =$ operating costs + capital costs + unextracted metal × (marginal operating and capital costs for mining and up-grading).

The operating and capital costs include those incurred in milling, leaching, and liquid-solid separation, and the term for unextracted metal takes into account the metal not extracted in leaching or lost as a solute with the solids residue.

Computational problems

Most of the computational problems related to difficulties in the handling of constraints or steep functions or both. A penalty-function approach was not completely satisfactory for the up-grading program, and Lagrange multipliers likewise had their problems. It is hoped that the use of a new program developed by Haarhoff *et al* (1970) will simplify matters. The Powell search was used in the milling and concentrating program because of problems with Newton-Raphson and other searches, and it proved fast and reliable.

CONCLUSIONS

Figure 4 shows the effect of variations in cut-off grade on the amount of ore that must be milled and mined for the required metal production figures to be realized.

Figure 5 shows the effect of variations in cut-off grade on the total optimum capital costs and the total optimum operating costs. As the cut-off grade increases, the metal recovery drops, and this has important effects on the mining policy. It can be seen that the minimum in the total capital curve does not correspond to the minimum in the curve for total operating costs. The best grade of mill feed will lie somewhere between the two minima and will depend on the rate at which the capital is discounted. A high discount rate for the capital will shift the minimum in the direction of that grade which corresponds to the minimum of the capital cost curve. The individual sub-optimizations were performed at a particular discount rate. A change in this rate could change the total capital and operating cost curves.

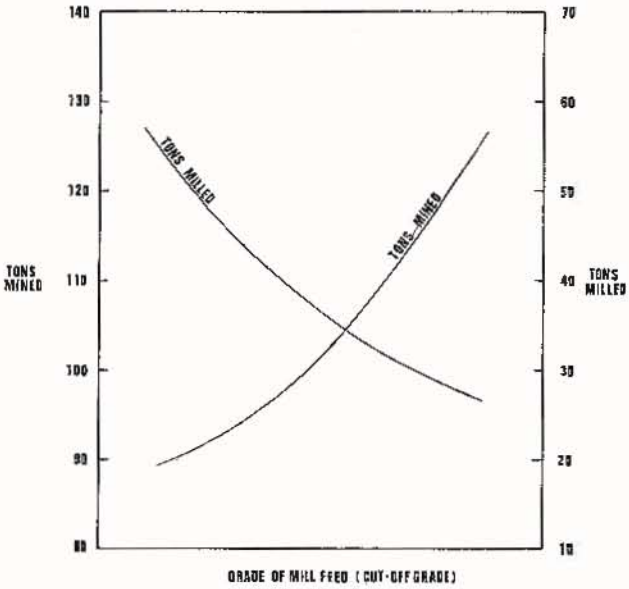


Fig. 4. The effect of ore grade on the tonnage milled and mined.

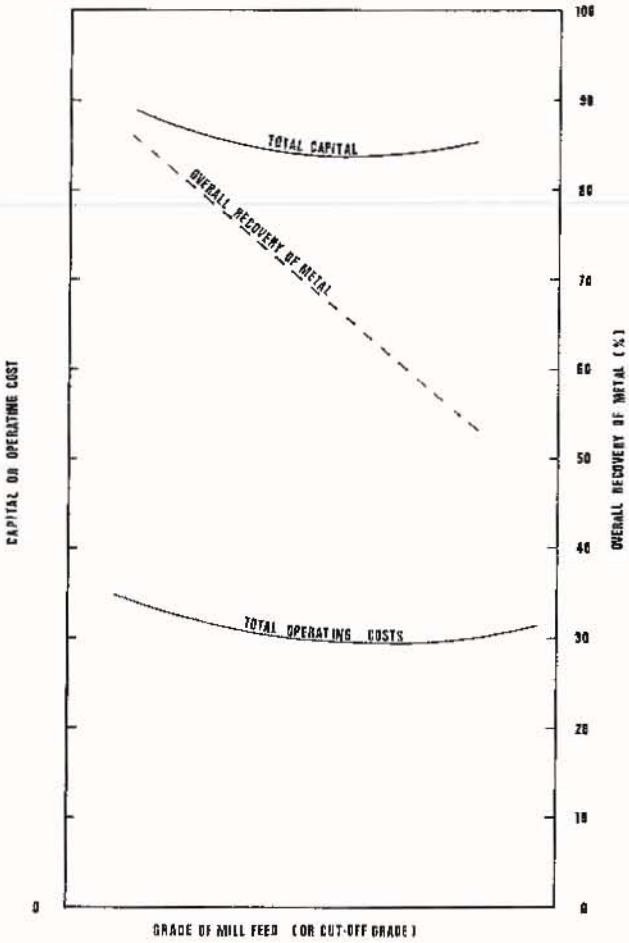


Fig. 5. Effect of ore grade on capital and operating costs and overall recovery of metal.

Experience with the modelling and optimizations of the system has shown that too much time can be spent in modelling complex operations that have a small influence on the costs. A simplified analysis of the kind outlined above should therefore be made before a detailed analysis is done.

ACKNOWLEDGEMENT

Acknowledgement is made to the Director of the National Institute for Metallurgy for permission to publish this paper.

REFERENCES

DOMAAS, F. B. (1969). Kidd Creek Mine. *Engng Min. J.*, vol. 170, no. 4, pp. 87-108.

ROBINSON, C. G., LOVEDAY, B. K., and PAYNTER, J. C. (1972). A model for the prediction of leaching curves and the determination of activation energies of a low grade ore. *To be published.*

ROBINSON, C. G., and PAYNTER, J. C. (1971). The optimization of the design of a countercurrent liquid-liquid extraction plant using LIX-64N. *International Solvent Extraction Conference*, The Hague, 1971.

FLETCHER, R., and POWELL, M. J. D. (1963). A rapidly-convergent descent method for minimizations. *Computer J.*, vol. 6, no. 2, pp. 163-168.

FLETCHER, R., and REEVES, C. M. (1964). Function minimization by conjugate gradients. *Computer J.*, vol. 7, no. 2, pp. 149-154.

POWELL, M. J. D. (1964). An efficient method for finding the minimum of a function of several variables without calculating derivatives. *Computer J.*, vol. 7, no. 2, pp. 155-162.

HAARHOFF, P. C., and BUYS, J. D. (1970). A new method for the optimization of a nonlinear function subject to nonlinear constraints. *Computer J.*, vol. 13, no. 2, pp. 173-184.