An Application of Linear Programming to Investment Analysis

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SYNOPSIS

The crude ores from the LKAB mines are sorted and refined into a number of products. As there are a number of different plants the flow of material is often complex.

In connection with studies of a proposed extensive investment program it was necessary to get a good measure of the changes which would be caused in the total profit by the different possible investments. A linear programming model was set up to accomplish this, and to cover the consequential effects on the production system. The objective function of the model maximized the total profit, taking into account restrictions in materials balances, blending conditions, production capacities and market. The changes in total profit resulting from the different investment alternatives, together with investment and fixed costs, were used in a series of investment analyses. The application has proved to be very successful and the results come very near to what was expected.

INTRODUCTION

Luossavaara-Kiirunavaara AB (LKAB) is the biggest Swedish producer of iron ore and one of the major exporters on the world iron ore market. LKAB's two mining divisions in Kiruna and Malmberget, both north of the Arctic circle, exploit the huge iron ore deposits at Kiirunavaara, Luossavaara, Svappavaara and Malmberget. The total annual production of about 28 to 30 million metric tons breaks down into some 20 different quality grades, of which eight are pellets and concentrates.

Most of the production is shipped via the two harbour divisions at Narvik (Norway) and Lulea (Sweden). The major part of material shipped via Narvik comes from Kiruna, and the major part shipped via Lulea from Malmberget.

The locations of the four divisions are shown in Fig. 1.

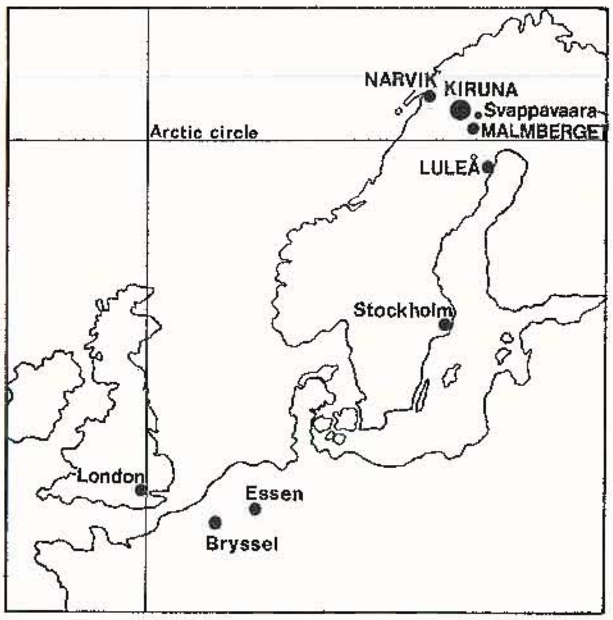

Fig. 1. Location of divisions.

During the last 15 years LKAB has expanded substantially. From 1958 to 1971 the production has increased from 13 to 27 million metric tons per year. During the same period the requirements for improved metallurgical and mechanical properties have become more and more stringent. The rate of investment in benefication and pelletizing equipment has consequently been very high during the period.

FORMULATION OF THE PROBLEM

Of the crude ores only a small fraction is shipped directly, while the major part is sorted to lump ores and fines, or further refined to concentrates and pellets. Owing to the market conditions and the different capacities of mines and plants, the proportions of the different products fluctuate. Furthermore, various mines can sometimes deliver crude ore to more than one benefication plant and, also, some plants can draw their raw materials from more than one mine. The flow of material can often be complex, so that it is difficult to evaluate different changes in the system. When, therefore, investments in different parts of the system are discussed, it is usually difficult to determine how the cash flow will actually change as a result of the investments made.

Between 1971 and 1974, LKAB will increase its production capacities, with the emphasis on benefication and pelletizing. The main investments will be concentrated in Malmberget division. Before the decisions to do so were taken, extensive studies were carried out. Among other aspects, these studies included market potential and profitability analysis. In this connection, two essential questions had to be answered:

(i) How would the total profit vary with each of the different investment possibilities?

(ii) What effects would the investments have on the whole system? For instance, should there be any changes in the supply of crude ore? Would it be necessary to invest in increased raw material production?

Because of the difficulties in obtaining correct answers to these questions and the necessity for having an overall picture of the problem, it was decided to set up a model describing the system. The model was intended for use as a tool in some of the profitability studies.

SELECTION OF THE MODEL AND THE OPTIMIZATION MODEL

The following requirements were laid down as a basis for the selection of the model and the optimization method:

(i) The model should be capable of describing such factors as production capacities, yield and market demands.

(ii) It should be possible to extend the model subsequently and fit it into future corporate models.

(iii) The solution method should optimize the total profit.

(iv) The model should be flexible to permit the analysis of different changes and investments.

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(v) The solution method should, if possible, be available as a standard computer program.

After comparing the different techniques of simulation and mathematical programming, it was soon evident that these requirements would be met best by a model based on the principles of linear programming. This technique is assumed to be well known and, therefore, it will not be described in this paper.

FORMULATION OF THE MODEL

Flow of material

As an introduction to the model, the material flow is described briefly. Figure 2 shows the actual flows and plants at the time the study was started. The three mining areas are largely independent of each other, except for some internal transfers of raw materials to one of the ore-dressing plants.

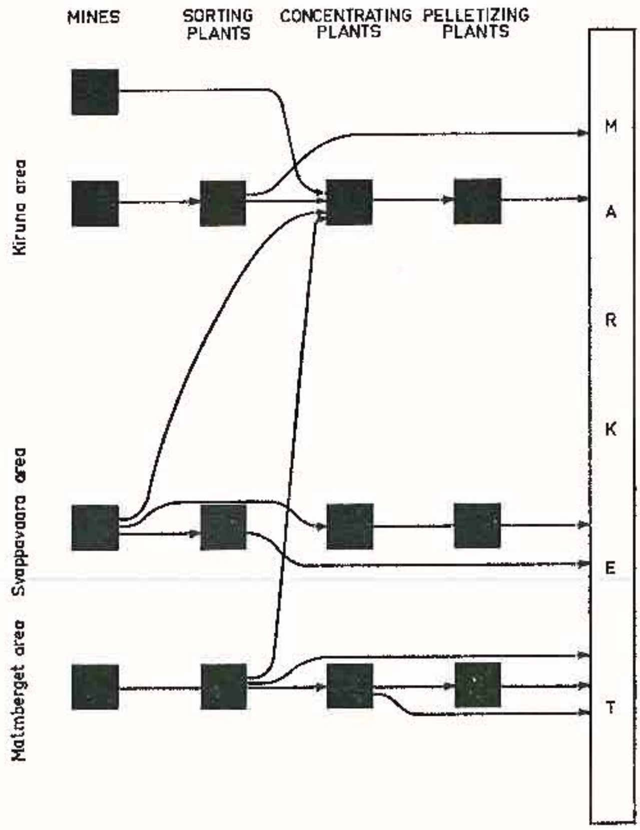

Fig. 2. Flows and plants at the time the study was started.

Figure 3 shows the possible flows and plants if each of the suggested investments was made without considering market demands. As can be seen from this illustration, the flow of material here is much more complex than that shown in the previous figure.

In order to be able to answer the two questions set out above, and to analyze the different investments, the linear programming model was based on the flow chart shown in Fig. 3.

Model

Two different variables (besides slack variables) are represented in the model:

(i) Quantity of a given raw material (crude ore) which is used in a given product at a given plant.

(ii) Product quantities from a given plant.

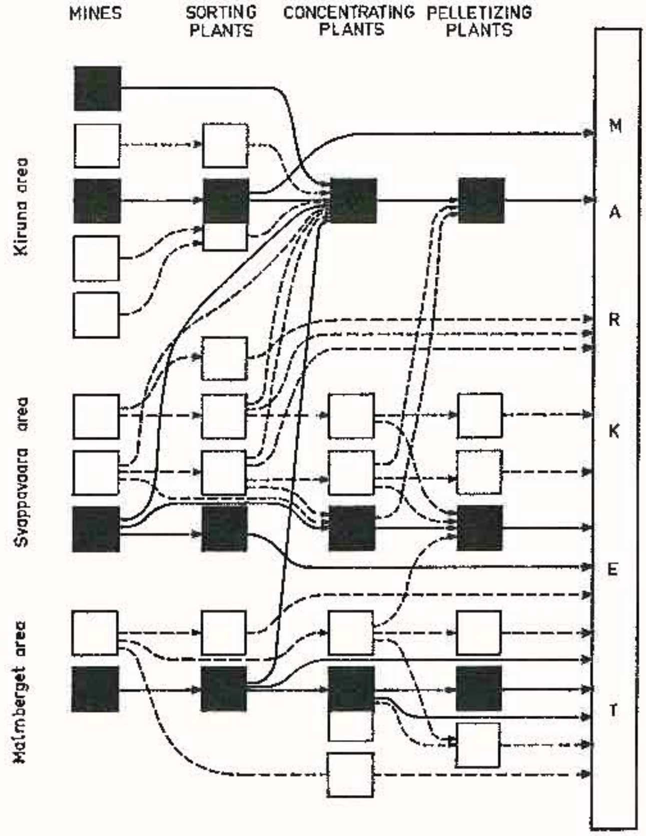

Fig. 3. Possible flows and plants considered in the analysis.

This means that the primary results provided by the model will be the optimum product program and the optimum raw material supply. This has the advantage that the model can subsequently be adapted easily to different corporate models.

The objective function of the model maximizes the total profit from the production, that is,

$$\text{Max } Z = \sum^{i,j,k} X_{ijk} \cdot P_{ijk},$$

where P_{ijk} is the profit on raw material i which is refined into product j at plant k, Z is the total profit and X_{ijk} is the quantity of raw material i used at plant k for product j. This means that the function to be maximized consists of the sum of the profits from the individual raw materials.

It should be noted that profit is defined here as income minus direct variable costs. Investment costs, direct fixed costs and changes in overhead costs are taken into consideration subsequently in the investment analysis.

The restrictions in the model can be divided into four different groups. These are material balances, blending requirements, production capacities for mines and plants, and, finally, market restrictions.

The material balances consist of balances for yield, iron (Fe), phosphorus (P) and silica (SiO_2). The four balances are necessary for such plants, for example, concentrating plants, since these can accept more than one raw material and/or produce more than one product. In the case of plants with only one raw material and one product, only the balance for yield was taken into account.

The balances for yield for each of the plants are given by the following formula:

$$\sum^{i} a_{ijk} \cdot X_{ijk} - \sum^{j} Y_{jk} = 0,$$

where a_{ijk} is the yield by weight when raw material i is refined into product j at plant k, X_{ijk} is the quantity of raw material i used at plant k for product j and Y_{jk} is the quantity of product j from plant k.

The balances for iron (Fe) are given for each of the plants by the following formula:

$$\sum^{i} a_{ijk} \,.\, b_{ijk} \,.\, X_{ijk} - \sum^{j} b_{jk} \,.\, Y_{jk} \geqslant 0,$$

where b_{ijk} is the content of Fe in product j which is made at plant k from raw material i and b_{jk} is the minimum average content of Fe in product j from plant k.

The balance for phosphorus (P) is given by the following formula:

$$\sum^{i} a_{ijk} \,.\, c_{ijk} \,.\, X_{ijk} - \sum^{j} c_{jk} \,.\, Y_{jk} \gtreqless 0,$$

where c_{ijk} is the content of P in product j which is made at plant k from raw material i and c_{jk} is the maximum (minimum) average content of P in product j from plant k.

The formula for the balances for silica (SiO_2) is as follows:

$$\sum^{i} a_{ijk} \,.\, d_{ijk} \,.\, X_{ijk} - \sum^{j} d_{jk} \,.\, Y_{jk} \gtreqless 0,$$

where d_{ijk} is the content of SiO_2 in product j made at plant k from raw material i and d_{jk} is the maximum (minimum) average content of SiO_2 in product j from plant k.

The blending restrictions primarily govern the mixing of magnetite and hematite since in some pelletizing plants these raw materials cannot be mixed freely. The restrictions are expressed by the following formula:

$$\sum^{i} e_{ijk} \,.\, X_{ijk} - X_{hjk} \gtreqless 0,$$

where e_{ijk} is the proportion limit when raw material i is refined at plant k to product j and X_{hjk} is the quantity of raw material h which cannot be mixed freely when refined to product j at plant k.

The restrictions of mine capacities and the supply of external raw materials are all measured in annual quantities. The restrictions are all in accordance with the following simple formula:

$$\sum^{i} X_{ijk} \leqslant P_i,$$

where P_i is the maximum annual production (or supply) of material i.

The capacities of the different sorting and pelletizing plants are defined either as input quantities of raw materials or as output quantities of products. The restrictions are given by the following formulae:

$$\sum^{i} X_{ijk} \leqslant I_k,$$

where I_k is the maximum annual raw material input at plant k, or by

$$\sum^{j} Y_{jk} \leqslant 0_k,$$

where 0_k is the maximum annual product output at plant k.

For the concentrating plants, the capacities are defined in terms of grinding energy. The restrictions are given by the following formula:

$$\sum^{i} f_{ijk} \,.\, X_{ijk} \leqslant E_k,$$

where f_{ijk} is the grinding energy required for raw material i when refined to product j at plant k and E_k is the total available grinding energy at plant k.

The market restrictions are given by the following simple formulae:

$$\sum^{k} Y_{jk} \leqslant M_j$$

where M_j is the maximum demand for product j and/or

$$\sum^{k} Y_{jk} \geqslant L_j$$

where L_j is the minimum demand for product j. The last restriction is needed when there are contracts to be honoured.

Some products are substitutional; this is allowed for by summing their market restrictions.

A summarizing schematic picture of the model is shown in Fig. 4.

The formulation of the model is, on the whole, fairly conventional. The primary purpose is, however, not to analyze the raw material supply but to study changes in the system, so as to provide a basis for investment analyses.

OUTPUT FROM THE MODEL

A standard computer program was used for the solution of the model (ICL Linear Programming Mark 2). The essential outputs from this program are

(i) a primary solution consisting of optimal product and raw material quantities,

(ii) total profit from the optimal solution,

(iii) dual solution which states what changes will occur in the total profit if those restrictions which are fully used are increased by one unit,

(iv) the value of the slack variables, that is, how much remains of the restrictions which are not fully used,

(v) 'shadow' prices for those products and raw materials not included in the primary solution, which indicate how the individual profits must be changed in order to ensure that these raw materials and products do figure in the solution, and

(vi) sensitivity analyses of the solution in respect of changes in the individual profits and in the restrictions.

APPLICATION OF THE MODEL TO INVESTMENT ANALYSIS

Method of application

As mentioned previously, the total flow in Fig. 3 (with all the proposed investments) was taken into account in the model. However, the basis for the investment analysis was the existing system and flows, as shown in Fig. 2. Hence, the analysis and optimization started with the existing system. This was done by assigning zero value to the capacity restrictions for the new plants. The total profits from the different alternatives were compared subsequently with the total profit from this basic alternative.

The first stage in the analysis started with all raw materials available in the model. The model was then run stepwise, the different plants being introduced into the model one by one. The changes in total profit revealed which alternatives were dominated by others. As the model allowed only for the direct variable costs, and did not include, for example, investment and fixed costs, the dominations were not necessarily unambiguous. Consequently, this stepwise analysis was taken primarily only as an indication for further analysis, where

Variables	X_{111} ····	X_{221} ····	X_{122} ····	X_{212} ····	X_{ijk} ····	X_{lmn}	Y_{11}	Y_{12} ····	Y_{21}	Y_{22} ····	Y_{jk} ····	Y_{mn}	
Profit	p_{111} ····	p_{221} ····	p_{122} ····	p_{212} ····	p_{ijk} ····	p_{lmn}							
Yield	a_{111} ····	a_{221} ····					-1		-1				$= 0$
			a_{122} ····	a_{212} ····				-1		-1			$= 0$
					···· a_{ijk} ···						-1		$= 0$
						···· a_{lmn}						-1	$= 0$
Fe	$a_{111} \cdot b_{111}$ ···	$a_{221} \cdot b_{221}$ ···					$-b_{11}$		$-b_{21}$				≥ 0
			$a_{122} \cdot b_{122}$ ···	$a_{212} \cdot b_{212}$ ···				$-b_{12}$		$-b_{22}$			≥ 0
					··· $a_{ijk} \cdot b_{ijk}$ ···						$-b_{jk}$		≥ 0
						··· $a_{lmn} \cdot b_{lmn}$						$-b_{mn}$	≥ 0
P	$a_{111} \cdot c_{111}$ ···	$a_{221} \cdot c_{221}$ ···					$-c_{11}$		$-c_{21}$				$\lesseqgtr 0$
			$a_{122} \cdot c_{122}$ ···	$a_{212} \cdot c_{212}$ ···				$-c_{12}$		$-c_{22}$			$\lesseqgtr 0$
					··· $a_{ijk} \cdot c_{ijk}$ ···						$-c_{jk}$		$\lesseqgtr 0$
						··· $a_{lmn} \cdot c_{lmn}$						$-c_{mn}$	$\lesseqgtr 0$
SiO_2	$a_{111} \cdot d_{111}$ ···	$a_{221} \cdot d_{221}$ ···					$-d_{11}$		$-d_{21}$				$\lesseqgtr 0$
			$a_{122} \cdot d_{122}$ ···	$a_{212} \cdot d_{212}$ ···				$-d_{12}$		$-d_{22}$			$\lesseqgtr 0$
					··· $a_{ijk} \cdot d_{ijk}$ ···						$-d_{jk}$		$\lesseqgtr 0$
						··· $a_{lmn} \cdot d_{lmn}$						$-d_{mn}$	$\lesseqgtr 0$
Blend	e_{111}						-1						$\gtreqless 0$
					e_{ijk}						-1		$\gtreqless 0$
Mine capacities	1		1										$\leq P_1$
		1		1									$\leq P_2$
					1								$\leq P_i$
						1							$\leq P_l$
Plant capacities	1	1											$\leq I_1$
			1	1									$\leq I_2$
					1								$\leq I_k$
						1							$\leq I_n$
							1		1				$\leq O_1$
								1		1			$\leq O_2$
											1		$\leq O_k$
												1	$\leq O_n$
	f_{111}	f_{221}											$\leq E_1$
			f_{122}	f_{212}									$\leq E_2$
					f_{ijk}								$\leq E_k$
						f_{lmn}							$\leq E_n$
Market							1	1					$\leq M_1$
									1	1			$\leq M_2$
											1		$\leq M_j$
												1	$\leq M_m$
							1	1					$\geq L_1$
									1	1			$\geq L_2$
											1		$\geq L_j$
												1	$\geq L_m$

Fig. 4. Schematic picture of the model.

the dominations were more completely investigated. It soon became clear that when total costs and returns were considered, certain alternatives could be excluded forthwith. The remaining investments were investigated thoroughly, the model being run for most of the possible combinations.

The total profits from the different investment alternatives were used in calculations of present value ($D\ C\ F$) and internal rate of return. The present value of alternative s, V_s, is given by

$$V_s = \left[(Z_s - Z_o) - C_F - C_H\right] \frac{1-(1+r)^{-t}}{r} - R_s + S_s(1+r)^{-t}$$

where Z_s is the total profit from investment alternative s, Z_o is the total profit from the basic alternative, C_F is the direct fixed cost and C_H is the change in overhead costs. Also, R_s denotes investment costs for investment alternative s, S_s is the end value of investment alternative s, r is the rate of interest and t is the economic life.

The internal rate of return, from investment alternative s, that is, q_s, is obtained by solving the following equation:

$$\left[(Z_s - Z_0) - C_F - C_H\right] \frac{1-(1+q_s)^{-t}}{q_s} - R_s + S_s(1+q_s)^{-t} = 0$$

The results of these calculations were used to categorize the different investment alternatives in terms of profitability.

Results obtained

The decision making in the actual situation was based on very comprehensive data. In addition to the above-described investment analysis and various supplementary economic analyses, the data also included short- and long-range studies of, for example, production, ore reserves, transportation and social environment.

It was finally decided to invest in

(i) two new mines, one for magnetite and one for hematite,

(ii) two new dressing sections in one of the existing concentrating plants,

(iii) increased capacity in one of the existing sorting plants,

(iv) one new concentrating plant, and in

(v) one new pelletizing plant.

The total investment will amount to about U.S.$ 100 million and the plants will be completed at the beginning of 1974. When the installations covered by the new investments are in production, the flow of material will be as shown with Fig. 5.

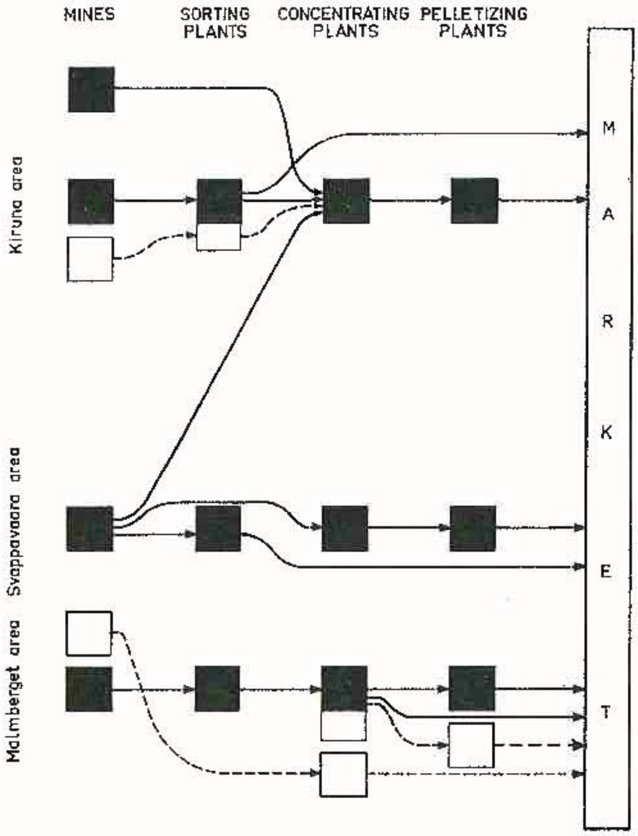

Fig. 5. Decided flows and installations.

EXPERIENCE GAINED

This application was in the main highly successful. The model makes it possible to discuss and analyze investment proposals in a manner very different from that possible before. It also gives a comprehensive picture of the flow pattern and of the different problems which are part of this picture. The technique has great advantages as a tool for investment analysis, although it sometimes involves a considerable amount of work. The results of this kind of analysis are, however, much more accurate than those of ordinary investment analysis.

The application required far more work than was expected at the outset. The data retrieval operation, in particular, was sometimes rather complicated.

As a summary of our experience, we can, however, say that the results of the application came very largely up to our expectations and were decidedly worth the effort involved.