Heat Flow Models in Ventilation Planning

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SYNOPSIS

This paper reviews the progress made to date in the development of computer models of heat and moisture transfer from rock surfaces to the ventilating air in deep-level mines. The application of these programs to ventilation planning and design is discussed.

INTRODUCTION

There are two major systems to be studied in the ventilation of underground workings in hot mines. The first is the air flow system and the second, which depends on the first, is the heat and moisture transfer system. The objective is to ensure, as efficiently as possible, an acceptable thermal environment. The subject of gas and dust control is not dealt with in this paper.

This paper will be concerned primarily with heat and moisture transfer processes and with the computer programs which are being developed or are already available to aid in the design and analysis of ventilation and refrigeration systems. Since the computer programs themselves have been described in various publications and reports. the emphasis in this review will be on what these programs can do, how valid they are, what the ventilation engineer can hope to learn from them and what still remains to be done in this field.

From the thermal point of view, the minimum requirement in any underground environment is that the human body should be able to reject heat at a rate which is at least equal to the rate of production of metabolic heat. This heat production rate depends on the type of manual work being performed. In hot, damp environments the rate at which a human body can in fact reject heat has been found to depend essentially on the wet-bulb temperature of the air, and the air velocity past the body, and to a lesser extent on the dry-bulb air temperature and the radiant temperature of the surroundings. Recently, Mitchell, *et al* (1971) established guidelines for suitable combinations of air temperatures and velocities to meet the minimum requirements for various types of manual work done underground. A computer program has been developed to produce tables of the temperature and velocity combinations which would meet specified conditions. Used in conjunction with the heat and moisture transfer programs described below, these tables should enable the ventilation engineer to balance the air flow rate and refrigeration requirements in a working area.

Since heat stress and heat pick-up criteria are both functions of air velocity as well as air temperature, the design or analysis of a ventilation system must involve considerations of both the air flow as well as the heat flow characteristics of a mine. In addition it is frequently necessary to calculate air velocities in situations when the heat flux is unimportant. A number of programs have been developed by, for example, Trafton, *et al* (1964) and McPherson (1966) to predict air flow rates given the topology of the network, resistances of the network components, and fan locations and characteristics.

HEAT FLOW IN AIRWAYS

Heat flow in tunnels and airways has been studied more extensively than heat flow in the working areas of a mine. The reasons for this are, firstly, that the air flow through an airway is defined simply and air flow rates are predicted or measured fairly easily. Secondly, the heat flow into an airway is relatively simple to model and temperature measurements are obtained more easily and are more consistent in airways than in stopes or other working areas of a mine. Thirdly, in South African gold mines, where, for example, the ventilating air may quite commonly travel down 7 500 feet of shaft and then along 4 000 feet of airway through hot rock before reaching the working areas, control of heat pick-up in airways is in fact, a significant part of the ventilation problem.

Once the air flow rate in an airway has been specified, heat flow into the airway depends on:

(i) the virgin rock temperature,
(ii) the thermal properties of the surrounding rock,
(iii) the psychrometric and thermal properties of air,
(iv) the inlet wet- and dry-bulb temperatures of the air,
(v) the shape, size and age of the airway,
(vi) the wetness of rock surfaces, presence of open drains and discharge of water into the airway, and
(vii) the machinery in the airways.

The number of variables influencing heat flow is so large that empirical studies based on large numbers of measurements can be of only limited value. Where an established pattern of mining and ventilation conditions exists, a regression analysis of a large number of *in situ* measurements, as has been done by Lambrechts (1967a), serves to quantify and categorize current experience. However, regression analyses do not lend themselves to extrapolation to new conditions (e.g., higher rock temperatures) or innovation in the ventilation system. Hence the need arises for a mechanistic model, that is, a model that simulates the detailed physics of the heat and moisture transfer processes occurring in the mine.

Mechanistic models usually consider a cross-section of an airway, neglecting heat flow in the rock parallel to the axis of the airway. The rock is assumed to be homogenous and isotropic. Heat is conducted through the rock in accordance with Fourier's law of heat conduction. At the surface of the airway the transfer of heat from rock to air depends on the surface heat transfer coefficient, which is a function of the roughness of the airway and the velocity of the air.

Heat flux = (surface transfer coefficient) × (rock surface temperature minus dry-bulb temperature).

At wet surfaces part of the heat in the rock is used to evaporate water at a rate proportional to a surface moisture transfer coefficient and the difference between the vapour pressure of the moisture in the air and the saturated vapour pressure of air at the temperature of the wet rock surface.

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The simplest model of all is that of a perfectly dry tunnel of circular cross-section. If the rock surface temperature is assumed to be equal to the dry-bulb air temperature, then the heat flux into the airway can be calculated from tables by Goch, *et al* (1940). More realistic assumptions about the heat transfer coefficient at the rock surface lead to extended tables by Starfield (1966a) and Jaeger, *et al* (1966) for dry airways.

Given the heat flux into a cross-section of the tunnel it is then easy to calculate the change in air temperature over a short length of the airway and the new flux over the next short section, and so on along any length of airway, Jordan (1965), Starfield (1966b). This method is readily extended to airways that are uniformly wet around their entire perimeter.

Theoretically the rock temperatures around an airway at any time depend upon the entire ventilation history of the airway up to that time. The models of Jordan (1965) and Starfield (1966b) are both designed to take into account changes of the inlet temperature of the air with time. However, Starfield (1966b) has shown that where the air temperature changes relatively slowly with time, it is reasonable to assume that rock temperature and heat fluxes at a given time depend only on the prevailing air temperature. This simplifies the mathematics considerably. Amano, *et al* (1969) made use of this and other simplifying assumptions in the development of a combined air flow and heat flow computer program for airways.

Neglecting the temperature history of the ventilating air enables one to put what would otherwise be very specific results for a single cross-section of an airway to more general use.

This prompted the development by the present authors, Starfield and Dickson (1967), of a more detailed model of heat flow into the cross-section of an airway. The model uses finite difference techniques to analyze heat flow into airways of rectangular cross-section where the hanging and side walls are usually dry but the footwall is usually wet. This accords well with most airways in deep-level gold mines where the footwall is kept wet either through repeated wetting-down to allay dust or else through leakage from drains. The difference between rectangular airways and circular airways is not significant, but the difference between a model with uniformly wet perimeter and one with dry walls and wet footwall results in noticeably different patterns of moisture pick-up.

This model introduces a 'wetness factor' which varies from zero for a perfectly dry surface to unity for a thoroughly wet surface to account for evaporation taking place from a damp rock or mud surface rather than a free water surface. A factor of 0,2 was deduced for a wet footwall from isotherms measured around a tunnel under typical conditions. The results from the computer program were summarized in charts which show wet-bulb and dry-bulb measurements for wetness factors of 0, 0,2, and 1,0. Since a new set of charts has to be calculated for any change in air velocity, virgin rock temperature or size of airway and since computation time is high, this is not a program that can be used easily on a routine basis.

A rapid computer program was developed subsequently by Starfield (1969) for a limited range of parameters. This program interpolates between stored solutions from the authors' model and relies on the independence of heat flux on temperature history mentioned above. It has been programmed in a conversational mode for inter-active use on a remote terminal and interrogates the user for the required parameters. It then prints out wet-bulb and dry-bulb temperatures at specified distances along an airway. A typical problem is solved in the order of a minute of real time and the user can change parameters easily and so experiment with alternative ventilation designs. A small sacrifice of accuracy for speed and ease of use thus leads to a highly flexible design tool for the ventilation engineer. A typical print-out is shown in Fig. 1. Comparisons between wet-bulb temperature increases, predicted by the program and actual measurements in some 80 airways reported by Unsted (1971) are shown in Fig. 2. The footwall wetness factor was taken as 0,2 whenever the airway was described as 'dry', 'damp' or 'wet', a factor of zero being used only when the airway was described as 'very dry'. This accorded well with measured dry-bulb temperature gradients which are considerably more sensitive to footwall wetness than are wet-bulb temperature gradients.

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              TEMPERATURE INCREASE ALONG A MINE AIRWAY

  1 -WET BULB TEMPERATURE (DEG.F)
70
  2 -DRY BULB TEMPERATURE (DEG.F)
84
  3 -AIR QUANTITY (c.f.m.)
55000
  4 -VIRGIN ROCK TEMPERATURE (DEG.F)
112
  5 -THERMAL CONDUCTIVITY (Btu/hr.ft.DEG.F)
3.1
  6 -THERMAL DIFFUSIVITY (ft.ft./hr.)
0.1
  7 -AGE (years)
6
  8 -PERIMETER OF AIRWAY (ft)
44
  9 -BAROMETRIC PRESSURE (Ins.Hg.
31.5
  10 -LENGTH OF AIRWAY (ft)
2000
  11 -PRINTOUT INTERVALS (ft)
500

    INPUT DATA CORRECT ? ANSWER yes OR no
yes
    TURN TO A NEW PAGE AND PRESS "RETURN"


              TEMPERATURE INCREASE ALONG A MINE AIRWAY

    55000.00 AIR QUANTITY (c.f.m.)
      112.00 VIRGIN ROCK TEMPERATURE (DEG.F)
        3.10 THERMAL CONDUCTIVITY (Btu/hr.ft.DEG.F)
         .10 THERMAL DIFFUSIVITY (ft.ft./hr.)
        6.00 AGE (years)
       44.00 PERIMETER OF AIRWAY (ft)
       31.50 BAROMETRIC PRESSURE (Ins.Hg)

DISTANCE(FT)          DRY                    DAMP                   WET
                  WB        DB           WB        DB           WB        DB
     0.0        70.00     84.00        70.00     84.00        70.00     84.00
   500.0        70.61     85.76        70.62     84.23        70.88     82.22
  1000.0        71.17     87.40        71.23     84.47        71.39     81.08
  1500.0        71.69     88.92        71.83     84.71        72.12     80.41
  2000.0        72.16     90.34        72.42     84.96        72.85     80.10
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Fig. 1. Typical computer print-out for airways.

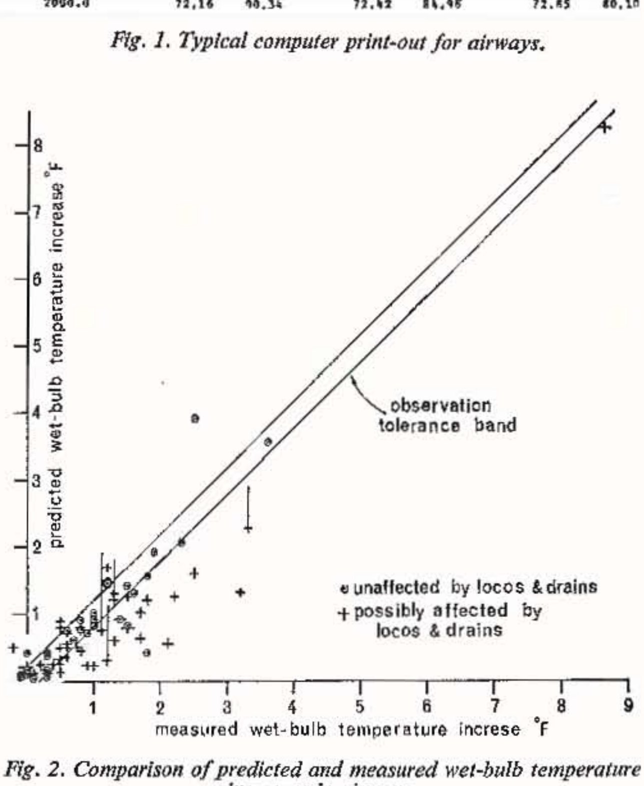

Fig. 2. Comparison of predicted and measured wet-bulb temperature increases in airways.

In plotting the data, distinction has been made between those airways having a minimum of heat sources in addition to rock heat and those in which, for example, there were open drains and/or diesel or electric loco tramming at the time of the survey, a significantly better correlation being obtained in the former case. The presence of open drains and locos, particularly diesel locos, would result in calculated increases in wet-bulb temperature which are too low, as is in fact indicated in the figure. Although it is not clear from the original data whether locos in the airway actually contributed directly to the heat pick-up in the test section at the time of the survey, a number of wet-bulb temperature increases have been recalculated assuming the presence of an 'average' diesel loco. The results are shown as the vertical lines in the diagram. It is similarly possible to make allowances for the presence of open drains, using a nomogram prepared by Dickson (1968). It is worth noting that while diesel locos may affect local temperature increases considerably, their contribution to the total heat pick-up in the average South African gold mine was not thought to be significant (Cook, *et al*, 1966).

The rapid computer program thus appears to be reasonably well validated and could become a useful computational tool for the analysis of ventilation systems. Further validation and probable refinement will result from feedback obtained from its wider use by industry.

With simple modifications computer programs for airways can be extended to take autocompression effects into account. Modified programs can be used to model heat flow in inclined and vertical shafts (Gooch, 1970). In South Africa the pick-up in shafts of heat from rock tends to be small due to the similarity of the autocompression and the geothermal temperature gradients.

The effect of seasonal variation of intake air temperatures is significant in shallow mines and has been analyzed by Sharp, *et al* (1965) for conditions obtaining in British coal mines.

HEAT FLOW INTO WORKING AREAS

The working areas of a mine, that is, the stopes and development ends, differ from airways in that new and relatively hot rock surfaces are continually being exposed as the working face advances into virgin rock. The working areas are thus subject to higher rates of heat and moisture pick-up than neighbouring airways, while the problem of maintaining the quality of the environment is made more urgent by the concentration of working personnel in these areas. Models of the transfer process in working areas have, however, lagged behind models of airways for a number of reasons.

Firstly, the heat conduction problem to be solved is more complex. The working face advances in a stepwise cyclic pattern into the hot rock mass. In South African gold mines there are stope faces that advance in steps of up to about 1,5 ft every 24 hours. Development ends advance at a greater rate with up to four blasting cycles in 24 hours. Provided that the rock is homogeneous and the same blasting pattern is repeated for a number of cycles, the heat flow pattern in one cycle will eventually be indistinguishable from the heat flow pattern in the next cycle. It is this quasi-steady heat flow pattern which must be modelled.

Secondly, the air flow patterns in stopes and to a lesser extent in development ends are less well defined than in airways. In particular, in stopes the worked-out areas are often sealed off incompletely from the working area and parallel airflows develop and intermix. A computer program has recently been developed to simulate air flow patterns through stopes in gold mines (McPherson, 1970). Although the program has yet to be validated, it appears likely to be a major contribution in this area.

Finally, the activities and variables influencing heat flow in working areas are more difficult to measure or describe than those in airways. Steeper temperature gradients exist in the surrounding rock so that surface heat and moisture transfer coefficients play a more significant role in controlling heat flow from rock to air. The values of these coefficients and their dependence on air velocity thus become more critical. Heat flow during a cycle is complicated by the presence of broken rock in the area for part of the cycle and by repeated wetting-down of rock surfaces. In addition, temperature measurements taken in stopes have been shown by Hemp (1969) to fluctuate considerably. This makes experimental validation of any model difficult and also raises the question of how these fluctuations affect criteria for evaluating thermal stress on humans in the working area.

The difficulty of obtaining and interpreting temperature measurements in working areas mitigates against empirical analysis of the heat flow problem in these areas. A mechanistic model does at least offer the hope of accurate simulation of gross effects and hence the possibility of making an overall comparison of the relative efficiency of various ventilation practices. Examples illustrating this approach are given later.

The development end

Shuttleworth (1964) has studied air flow patterns in development ends in British coal mines. Jordan (1965) has developed a heat flow model of an advancing heading which, however, ignores heat flow through the actual development face. More recently a simplified computer program has been introduced for predicting temperature and humidity increases in a development end and this work is being continued with the aim of producing a more realistic simulation model (Whillier, *et al*, 1971).

The stope

Starfield (1966c) developed a computer model for predicting wet- and dry-bulb temperature increases in stopes. The model concentrates on the air flowing along the stope between the face and a scatter barricade which, it is assumed, seals off the worked-out area effectively. The flow of air through worked-out areas and the heat picked up by this air are thus ignored.

The stope face is advanced at regular time intervals corresponding to the blasting cycle. At each blast the scatter barricade is assumed to jump forward so that the distance between the barricade and the face remains constant. Rock broken off the face by the blast is assumed to lie in the stope for some proportion of the blasting cycle.

As in the authors' airway model, a wetness factor ranging from zero to unity is introduced. The wetness of the hanging and face can be varied independently of the footwall, and wetnesses can be varied with time during the blasting cycle. Heat and moisture transfer coefficients are calculated on the same basis as in the airway model. The computer program has also been adapted for use in conversational mode for interactive use on a remote terminal.

Comparisons between predictions obtained from the model and empirical analysis in stopes by Lambrechts (1959) have been encouraging. The difficulties of checking out the program carefully by *in situ* experiments are, however, probably insurmountable. It is only through a cautious use of the program that confidence in its predictions can be built up and any required modifications introduced. To illustrate this cautious use, some preliminary results obtained from the program are discussed below.

APPLICATIONS OF THE STOPE MODEL IN VENTILATION PLANNING

The following variables were specified for the examples used in this discussion:

Virgin rock temperature (VRT)	110°F or 130°F
Atmospheric pressures	31 in. Hg or 33 in. Hg
Stope span from face to scatter barricade	12 ft
Stoping height	3,5 ft
Face advance rate	1,5 ft every 24 hrs
Air velocity	140 or 240 ft/min
Inlet air temperatures	70/80°F or 70/100°F
Rock conductivity	3,1 Btu/hr ft °F

In the computation the 24-hour cycle was divided into eight equal time steps. It was assumed that the blast took place at 15.00 hours, the beginning of the first time step, and the working shift began at 06.00 hours and ended at 15.00 hours, that is, to last through the sixth, seventh and eighth time steps. Broken rock was removed from the stope at the end of the sixth time step, that is, three hours after the beginning of the working shift.

Wetting-down cycles

Two wetting-down cycles were simulated. In the first the footwall was assumed to be wet for the whole working shift while the hanging and face were wet for the first six hours of the shift. During the off-shift period the stope was dry. Since this approximates conditions presently obtaining in practice, it is referred to as the 'standard wetting-down cycle'. In the second or 'full wetting-down cycle' it was assumed that all rock surfaces were wet throughout both working and off-shift periods.

There is at present no reason to assume that the wetness factors in stopes are the same as the wetness factors in airways. In airways the moisture is generally evaporating from a uniformly damp compacted footwall while in stopes evaporation takes place from a wet or damp rock surface. In addition, the rock surfaces may not be uniformly wet. Calculations were made using wetness factors of 0,2 0,5 and 0,8. A summary of the results is given in Table I which shows wet-bulb temperature increases over a 350-foot length of stope at the beginning of the working shift and six hours after the start of the shift.

TABLE I

Comparison of wet-bulb temperature increases in stopes for various values of wetness factor.
Inlet air temperature = 70/80°F; VRT = 110°F
Air velocity = 140 ft/min; stope length = 350 ft.

Time	Wetness factor	Temperature increase °F	
		Standard wetting-down cycle	Full wetting-down cycle
Beginning of shift	0,2	10,9	8,3
	0,5	15,3	9,7
	0,8	25,4	10,5
Six hours after beginning of shift	0,2	6,9	7,9
	0,5	8,4	9,0
	0,8	13,7	9,7

The results in the table show the need for a careful investigation into the correct value of wetness factor to be used in the stope model. Because, however, of the controlling influence exerted by the conductivity of the rock, the significance of the wetness factor in the case of the full wetting-down cycle is considerably less than in the case of the standard cycle.

Table I also suggests that a considerable saving in increase in wet-bulb temperature could be effected (depending, of course, on the correct value of wetness factor) by using the full wetting-down cycle to liberate as much heat as possible during the off-shift period.

Further computer 'experiments' covering a range of variables were performed to investigate the influence of the wet-bulb depression (that is, dry-bulb temperature minus wet-bulb temperature) on wet-bulb gradients in stopes. The difference in wet-bulb increases for the two cases of inlet air temperature (70/80°F or 70/100°F) was usually less than 20 per cent, in most instances considerably less. This is in accord with the conclusions reached by Lambrechts (1959) in his empirical analysis. In the case of a standard wetting-down cycle a low wet-bulb depression is actually beneficial in the early hours of the shift. A high depression is always beneficial for the full wetting-down cycle. The wet-bulb depression of the inlet air in a stope depends on heat and moisture pick-up in the airways leading to the stope, so that where the depression is significant a systems analysis using both airway and stope programs should be made.

Refrigeration cycles

At present the majority of the refrigeration plants installed underground in the South African gold mines are operated continuously, but this policy may need revision. If wetting-down occurs during the working shift only, heat transfer coefficients in the off-shift period are relatively low and one might well be able to make better use of the refrigeration system by storing the coolth produced during the off-shift period in, say, a reservoir of water. The stored cold water could then be used in the stope coolers to boost the output from the refrigeration plant during the working shift. For the same cooling capacity available during the working shift one could then make do with a smaller refrigeration plant.

In a series of computer experiments it was assumed that cooling of the air was resumed three hours before the start of the shift. Coolth was thus stored for 12 hours of the 24-hour cycle. A wet-bulb temperature of 86°F at an air velocity of 140 ft/min was found from Mitchell, *et al* (1971) to be the maximum allowable condition for heavy manual work. The maximum wet-bulb temperature at an air velocity of 240 ft/min was similarly found to be 88°F. Air was re-cooled in these experiments whenever it reached the maximum allowable condition. The quantity of cooling supplied was always such as to reduce the wet-bulb temperature to 75°F at the beginning of the working shift.

A sample of the results obtained for the length of stope downstream from the first cooling point is presented in Fig. 3. As would be expected, the air heats up more quickly in the case of periodic cooling, but for the same conditions one might nevertheless save on capital outlay and running costs by using periodic cooling and a smaller plant. A more detailed analysis of this problem will have to be made, including the interaction of refrigeration cycles and wetting-down cycles. An example of what to expect in this case is given in Fig. 4.

The balance between air quantity and refrigeration capacity

Consider a stope in which the air velocity is 140 ft/min so that the air must be re-cooled on reaching a wet-bulb temperature of 86°F to, for the sake of argument, a wet-bulb temperature of 75°F. If the air flow rate is increased to

240 ft/min, the air must be cooled on reaching a wet-bulb temperature of 88°F. If the capacity of the cooler is unaltered the decrease in air temperature will be less than in the previous case because of the increased mass flow rate. Figure 5 shows the corresponding wet-bulb air temperatures at the beginning of the working shift and subsequent to the first cooling point, for one set of parameters.

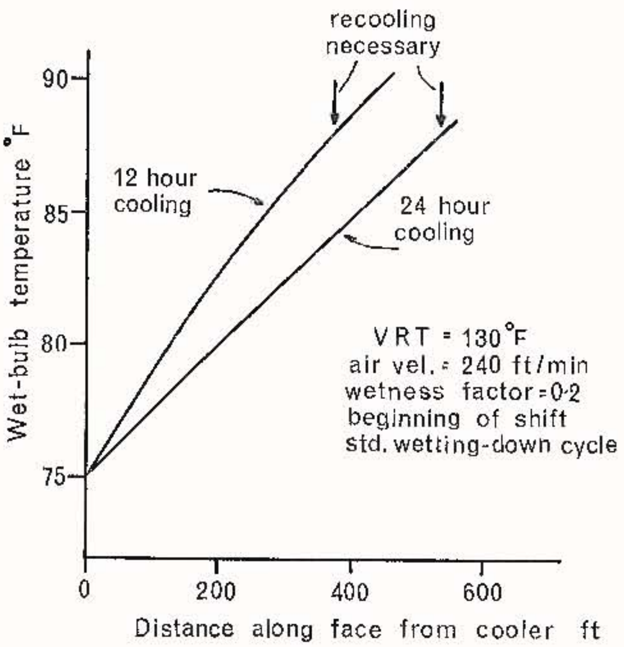

Fig. 3. Computed wet-bulb temperatures in a stope illustrating the effect of 12 and 24-hour cooling.

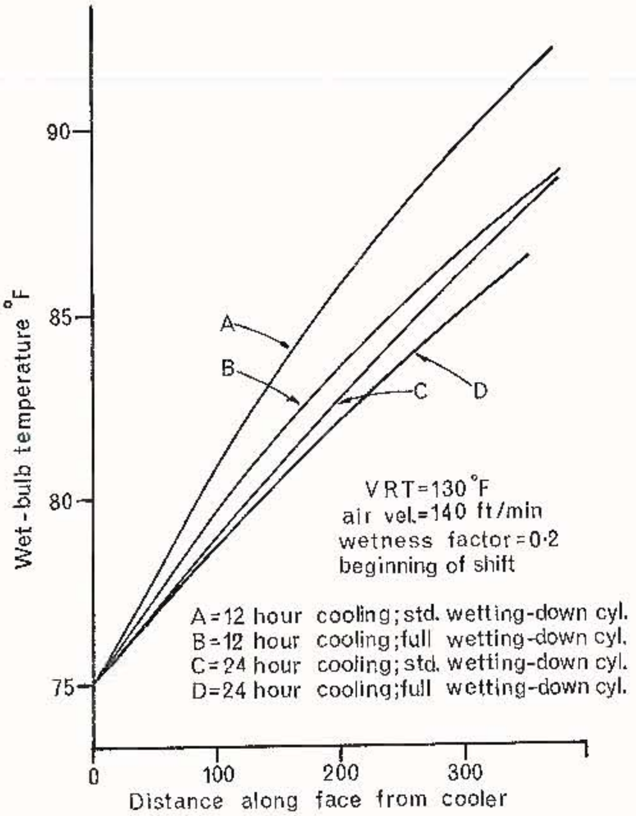

Fig. 4. Computed wet-bulb temperatures in a stope illustrating interaction of refrigeration and wetting-down cycles.

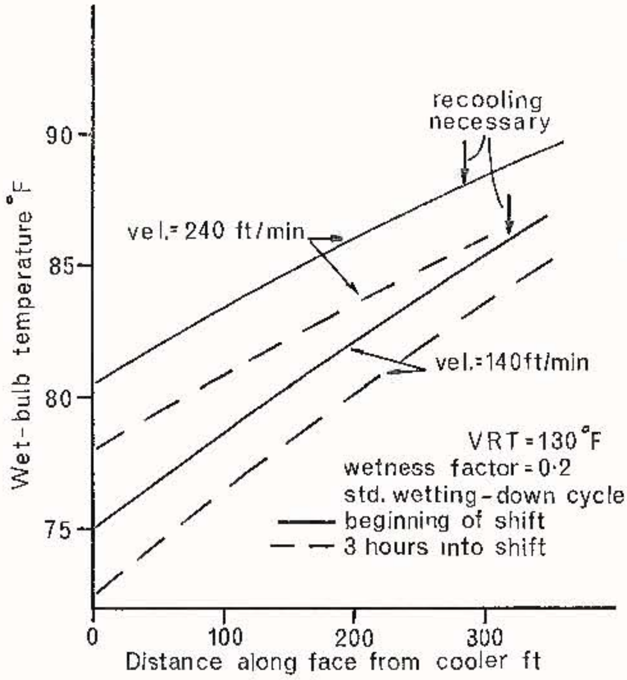

Fig. 5. Computed wet-bulb temperatures in a stope illustrating velocity-refrigeration interdependence.

One might have expected intuitively that an increase in air quantity is always beneficial. Fig. 5 shows, however, that the higher air flow rate in this case requires more frequent cooling and hence more refrigeration per unit length of face in a long stope. It follows that the air quantity and refrigeration capacity must be mutually balanced and one would expect that the computer model could be used with advantage to explore the particulars and significance of this balance. A substantially similar conclusion was reached by Lambrechts (1967b) from an empirical analysis of the situation.

CONCLUDING REMARKS

Computer programs can be, and have been, written to simulate the heat and moisture transfer processes in underground mines. These processes are complex and are influenced by a large number of variables. It is precisely for this reason that computer programs should be based on mechanistic models rather than regression analyses of underground temperature measurements.

The mechanistic models are based on assumptions. It is very often difficult and unrewarding to try to test these assumptions *in situ*. The model should rather first be taken to its logical conclusion and then analyzed for its sensitivity to the assumptions that have been made. If the model is insensitive to an assumption there is little point in trying to test the validity of the assumption rigorously, while if the model *is* sensitive to an assumption, experimentation with the computer program should suggest ways of confirming this underground. To illustrate these remarks: heat flow into airways is often controlled by the thermal conductivity of the surrounding rock rather than the surface transfer coefficients from rock to air. It is therefore unnecessary to conduct detailed experiments to evaluate the transfer coefficients and the wetness factor precisely. On the other hand, the results obtained in stopes indicate the need for a careful study of 'wetness' in a stope.

Some indication has been given of how these programs may be used to make predictions for new conditions or to evaluate different ventilation systems. It is through using the programs in planning situations that the engineer can gain simulated experience that would be virtually impossible to gain underground. Where computer models are weak in their assumptions a clash between simulated and real experience should soon be noticed and the model modified. One aspect of the use of computer models could be in the training of ventilation engineers.

Finally, the importance of rapid and easily used programs should be emphasized. The interactive use of conversational programs on remote terminals makes it possible for an engineer to concentrate on his problem — the ventilation problem — rather than on preparing and running a program on a computer.

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