

Simulation of Multicomponent Flotation Plants

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SYNOPSIS

A mathematical model has been developed for flotation circuit simulation. This model was designed to investigate the effect of varying the circulating loads and to determine whether an improvement in overall efficiency could be gained by a size separation prior to flotation.

Model parameters were obtained by regression to plant data. Highly correlated parameters in the model were combined to reduce the number of parameters so that the need for excessive data gathering was eliminated. Mass balance smoothing was used to obtain a consistent data set for regression.

The results of the simulation indicate that for the same grade of concentrate, an increase in copper recovery of 0.6 per cent may be possible by increasing the circulating loads. The results of the simulation also indicated that recleaning of the plus 200-mesh material may be beneficial.

INTRODUCTION

The incentive for increasing recovery by as little as 0.5 per cent is considerable when flotation plants handle large tonnages of ore. However, when circuit re-arrangements are proposed, it is very difficult to obtain conclusive evidence of so small an increase experimentally because of errors in assay and flow determinations and variations in ore composition. In addition, the scope of experimental investigations is often limited by the capacity of plant equipment, necessitating modifications.

An alternative approach to this problem is the use of simulation methods. These methods enable the metallurgist to compare accurately numerous control strategies. However, the conclusions reached by simulation are sensitive to the accuracy with which the system can be described by a mathematical model. Consequently, the best approach is usually a combination of simulation and experimental studies, with each contributing towards a better understanding of the process.

Numerous simulation studies of flotation cells in both batch and continuous operation have been reported in recent years, but relatively little has been published on complete plant simulation. Generally speaking, the philosophy has been that having established a model for a single cell, it is a simple matter to string a large number together to form a plant. As a result, these models usually have too much detail, making it difficult to evaluate parameters from plant data and necessitating many additional measurements. Recently, work on banks of cells has been reported by Kelsall, *et al* (1970). However, the papers by Niemi, *et al* (1969), Smith, *et al* (1969) and Scrimgeour, *et al* (1970) appear to be the only ones in which the interactions between rougher banks and cleaner banks are considered.

This paper describes the methods that were used to simulate a large concentrator for Kennecott Copper Corporation. Model parameters were regressed to plant data and simulation runs were performed to identify the effects of circulating loads and possible circuit re-arrangements (including a size separation).

Kennecott Copper Corporation has recently installed a computer-controlled on-stream X-ray analysis system (Marchant, *et al*, 1971). This paper discusses an investigation that is part of a feasibility study of concentrator control by computer.

THE MODEL

It has been well established that under free-floating conditions as defined by Tomlinson, *et al* (1963), the rate of flotation is proportional to concentration of mineral in the cell, but that individual particles have a range of rate constants which are

related to particle size and mineralogy. Also, most compartmented cells may be regarded as perfect mixers.

When individual cells or a bank of cells are simulated, the recovery curve as a function of flowrate and cell number may be well described by assuming two discrete rates of flotation (Bull, 1967; Kelsall, 1960). However, when it is required to link cleaner performance to the operating conditions of the rougher, it has been shown that a model which uses a continuous distribution of rate constants gives better predictions (Woodburn, *et al*, 1966).

The distribution of rate constants may be approximated by either assuming several discrete values (Scrimgeour, *et al*, 1970) or by assuming a plausible mathematical form. The former approach results in a large number of parameters and the mathematical form of the latter may restrict its applicability. Batch tests on feed material indicated that a two-parameter gamma distribution described the distribution of rate constants of the Utah ore adequately, provided that a correction for unfloatable material was made, that is, if a third parameter was used. Batch tests over extended periods of time indicated that the assumption of an unfloatable fraction was indeed valid and not simply a convenient parameter which made the model fit the data.

The above discussion is related to constant cell conditions with varying flowrates through the cells. In practice, the volumetric flowrate through the rougher and scavenger cells is virtually constant and the circulating loads are usually varied by changing the froth depth and frother concentration, or by varying the air rate to the cells (Eerola, *et al*, 1969). The latter type of control has the effect of shifting the mean of the distribution of rate constants without changing its shape noticeably (Loveday, 1966). A model which Ball, *et al* (1971) have described assumes a distribution of rate constants for flotation and allows for the return of particles from the froth. This model assumes that the rate of rejection of particles from the froth is proportional to the mass of particles in the froth. The model thus assumes implicitly that the probability of particle rejection is independent of the nature of the particles (independent of k) and the grade of the froth is constant over the height range used for froth control (that is, no secondary enrichment occurs beyond the initial separation of pulp and froth). In the Appendix, it is shown that changes in flow of froth and hence changes in the rate of particles are equivalent to changes in the residence time in the cell when this model is used. A special batch test indicated that the return of

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particles from the froth was significant and the simple model discussed above was adopted.

The fractional recovery of a particular species of particles with a rate constant k from a perfectly mixed flotation cell is given by:

$$R = \frac{k\theta}{1 + k\theta}$$

where θ is the average residence time (that is, 'active' volume/volumetric flowrate of tailings).

This is the expression that has generally been used to evaluate rate constants (discrete or distributed) from steady-state data. It should be noted that, for a given value of R , a change in the value of θ is equivalent to an inverse change in the value of k (or the mean value of k , when a distribution of rate constants is considered). Thus, all the controls discussed above have the same effect as changing the average residence time in a flotation cell. It is, therefore, convenient to define the key element (copper in this case) as having a dimensionless distribution of rate constants (with a mean of unity) and to determine the dimensionless residence times from recovery data. This makes it possible to compare data on plants with different types of ore and eliminates the need to calculate cell volumes and volumetric flowrates. The effects of residence time, froth condition and air rate are combined into a single parameter which we call dimensionless residence time.

APPLICATION OF THE MODEL

The concentrator used in this study processes more than 50 000 tons/d of ore. The first stage of flotation in the plant is a separation of copper and molybdenum sulfides from pyrite and gangue. Subsequently, the molybdenite is removed from the copper concentrate. Only the primary separation was considered in this investigation because of its economic significance and the possibility of automatic control via the computer-controlled on-stream X-ray analysis system. The flotation circuit and mill practices have been described by Last, *et al* (1964). Figure 1 illustrates the circuit arrangement for the primary separation at this concentrator.

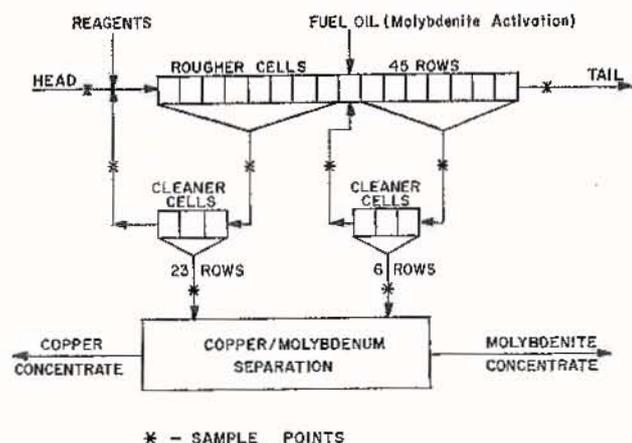


Fig. 1. Flow diagram of the primary separation in the Arthur Concentrator.

Approximately 60 per cent of the copper appears as chalcopyrite with the remainder as bornite, chalcocite and covellite. Quality control samples are assayed routinely for copper, iron, molybdenite and insolubles. Thus, it is not possible to calculate the proportions of the various copper minerals. For simulation purposes, copper, iron, molybdenite and insolubles were regarded as separate components. Copper and iron appear together in the major copper mineral and

physical interlocking with quartz (insolubles) is common. These associations are accounted for by the fact that the distributions of rate constants overlap and it is not possible to separate elements of the distributions with the same value of k . Thus, the parameters which describe the distributions for individual components can take on values which allow for chemical or physical interlocking or equal probabilities of flotation for different particles.

The level of detail of the model determines the quantity and type of data required from the plant. Sampling of individual cells within a row could provide useful rate data, but fluctuations in concentrate flow and maldistribution of the pulp between the rows could result in biased data. Consequently, samples of the combined concentrates and tailings were used. Samples were collected at eight points (see Fig. 1) each day shift for a period of a week. The samples were then combined with the weights proportional to the daily head tonnage.

Only the head, copper concentrate and molybdenite concentrate flows are currently measured and the very large flows made batch flow determination of the other streams impractical. Consequently, it was necessary to estimate flowrates from the assays. Each sample was split into six size fractions and each size fraction was assayed for the four components listed above. Special programs were developed to estimate the flowrates by a weighted least-squares method, using all the assays of the size fractions. When the flowrates had been determined relative to the feed flowrate, the assays and weight fractions were smoothed to obtain consistent mass and metal balances across the plant. Details of these programs will not be described here as the methods used were essentially the same as those used by Nielson, *et al* (1970), and Lees, *et al* (1971).

When model parameters are determined from steady-state data, the model is at least correct at the steady-state conditions and the parameters take on values which compensate for invalid assumptions. It is, therefore, usually possible to use relatively simple models, provided that the form of the model is correct. Using this philosophy, a single dimensionless residence time (per cell), τ_1 , is used to characterize the rougher cells. The fractional recovery from these cells for a given rate constant k is given by

$$r_1 = (1 + \lambda) \left[1 - \frac{1}{(1 + k\tau_1)^6} \right],$$

where λ is the ratio of rougher cleaner recycle to head.

Similarly, the cleaner cells are characterized by a dimensionless residence time τ_{11} . In this case, an arbitrary increase of 20 per cent in residence time per cell was used. The fractional recovery in the rougher cleaner concentrate is given by

$$r_{11} = r_1 \left[1 - \frac{1}{(1 + k\tau_{11})(1 + k\tau_{12})(1 + k\tau_{13})} \right],$$

where $\tau_{12} = 1.2 \tau_{11}$ and $\tau_{13} = 1.2 \tau_{11}$.

The circulating load λ may be eliminated, thus defining any stream in the rougher section in terms of k , τ_1 and τ_{11} . Similarly, flows in the scavenger section are defined in terms of k , τ_1 , τ_{11} , τ_2 and τ_{22} . When a distribution of rate constants is used, the fractional recovery is obtained by integration over the distribution.

For example

$$R_{11} = \int_0^{\infty} f(k) \cdot r_{11}(k, \tau_1, \tau_{11}) dk,$$

where $f(k)$ is the frequency function describing the distribution of rate constants.

A two-parameter gamma distribution was used in this case, with a correction for unfloatable material. The latter passes through the system untouched and the equations developed here apply only to the 'floatable' feed. The frequency function is defined as follows:

$$f(k) = \frac{(\alpha/\beta)^\alpha}{\Gamma(\alpha)} k^{\alpha-1} e^{-\alpha k/\beta}$$

The mean or first moment is equal to β . Parameter α defines the shape of the distribution, β defines the scale. As mentioned above, the value of β is taken arbitrarily as unity for the key component, copper. The value of β for other components is thus relative to copper.

The method that was used to evaluate the four time constants and parameters α and β for each element was as follows:

A number of laboratory batch tests were performed on feed material to determine the ultimate recovery of each element in each size fraction. Mass and metal balances were obtained using the same programs discussed above. These numbers were used to calculate the quantity of floatable material entering the plant (from the head assays).

Assays and flows of individual streams were used to calculate the fractional recovery of floatable material. A non-linear regression package was used to search the parameter space for a least-squares fit of the calculated recoveries to the experimentally determined ones. Every time a theoretical recovery was calculated, an integration over k -space was required. The search routine calculated partial derivatives by perturbation, thus requiring many integrations to be performed. Thus, it was necessary to find an integration package that was both rapid and accurate for the type of functions used in the model. The Gauss-Laguerre Quadrature method with a negative exponent and a suitable scale transformation finally proved suitable (Concus, *et al*, 1963).

Parameter β was defined as unity in the copper stream for all size fractions and parameters α , τ_{11} , τ_2 and τ_{22} were required. Only four unique recoveries were defined by the flows in the plant, thus limiting the number of independent parameters to four. Consequently, it was necessary to assume that τ_2 was equal to τ_{11} , which is approximately true as far as actual residence time is concerned. Having evaluated the time constants and α for copper, the parameters α and β were evaluated for each element (using the same time constants). These parameters were also obtained for size fractions (including copper), or groups of size fractions, to determine the influence of particle size on the kinetics of separation.

Once the parameters α and β for each element had been determined, it was possible to manipulate the time constants, which is equivalent to changing the froth height, air rate or volumetric flow. The effects of these controls are shown in the next section.

It should be noted that the molybdenite is activated only in the scavenger section of the plant (see Fig. 1). A considerable amount of unactivated molybdenite is, however, recovered in the rougher section. Molybdenite thus requires an additional parameter which corresponds to the activation of the particles which enter the scavenger section of the plant. If it is assumed that only the scale of the residual distribution of rate constants is affected, the conditioning has the effect of increasing the time constants in the scavenger section by a factor (that is, the same type of control as considered above). Small-scale tests by Loveday (1966) have shown that the shape of the distribution can be changed by conditioning. It was not practical, however, to use this level of detail during this investigation.

RESULTS

Laboratory rougher batch tests with normal froth columns were conducted for periods of up to 30 minutes. Samples of the head, total concentrate and tailing were split into size fractions and analyzed for the four elements. A mass balance smoothing program was then used to calculate the ultimate recovery of elements in each size fraction. These numbers were used to determine the proportions of floatable material entering the plant. Table I summarizes the ultimate recoveries determined by batch testing. It is interesting to note that because of the low ultimate recovery of gangue (insolubles), the floatable feed is, in fact, about 13 per cent copper for a typical head grade of about 0.7 per cent copper.

TABLE I
ULTIMATE FRACTIONAL RECOVERIES

Size fraction	Copper	Iron	Moly.	Insol.
-100 mesh	0.860	0.230	0.890	0.045
100/150 mesh	0.954	0.305	0.920	0.019
150/200 mesh	0.971	0.310	0.930	0.010
200/270 mesh	0.980	0.470	0.950	0.010
270/325 mesh	0.980	0.450	0.950	0.010
-325 mesh	0.932	0.280	0.870	0.039
Total (plant)	0.943	0.309	0.898	0.031

Having evaluated the time constants and α for copper from plant data (complete range of sizes), the parameters α and β were evaluated for the other elements. (Also complete range of sizes.) Parameters for copper on individual size fractions indicated that the copper in the fractions coarser than 200 mesh was considerably less floatable than that in the finer fractions. Consequently, a hypothetical size separation at 200 mesh was considered to determine whether any advantage was to be gained by floating coarse and fine fractions separately. Table II lists the parameters obtained by regression to plant data.

TABLE II
MODEL PARAMETERS FOR PLANT DATA

$$\begin{aligned} \tau_1 &= 2.27 & \tau_{11} &= 20.68 \\ \tau_2 &= 2.27 & \tau_{22} &= 3.39 \end{aligned}$$

Element	α	β	Fuel oil activation
Copper	0.985	1.000	1.0
+200 mesh	0.579	0.690	1.0
-200 mesh	2.161	0.600	1.0
Iron	0.340	0.400	1.0
+200 mesh	0.250	0.173	1.0
-200 mesh	0.401	0.418	1.0
Molybdenite	0.706	0.363	3.58
+200 mesh	0.590	0.224	5.03
-200 mesh	0.777	0.521	3.77
Insolubles	0.250	0.007 6	1.0
+200 mesh	0.251	0.011 3	1.0
-200 mesh	0.251	0.005 2	1.0

To help the reader interpret these numbers, Fig. 2 shows dimensionless gamma distributions for a range of α values. The scale parameter β shifts the curves along the horizontal axis. It should be noted that the parameters α and β were regressed to four unique recoveries in the plant. Consequently, the fit to the data was slightly imperfect (except for total copper).

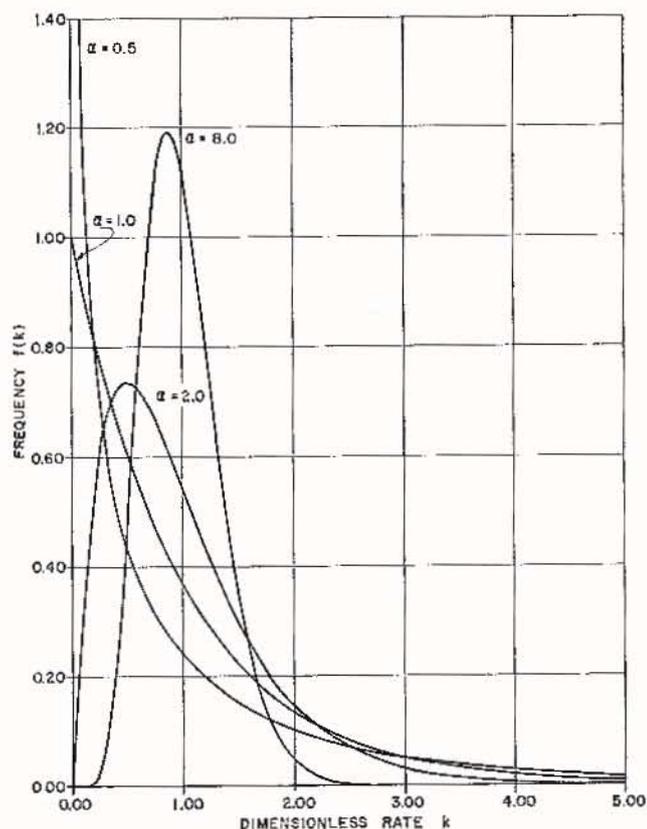


Fig. 2. Dimensionless gamma distributions (mean or $\beta = 1$).

The use of more sophisticated models was evaluated to determine if a better fit to the data could be obtained. A bimodal gamma distribution with two additional parameters was tested, but only negligible increases in accuracy were obtained. In another series of tests, the ultimate recovery was made a variable in the regression, but again only very slight increases in model accuracy were obtained. As a result, the model was left in its original form to avoid meaningless distortions caused by highly correlated, redundant parameters.

Once the model parameters were evaluated, the operating conditions of the plant were changed by manipulating the four time constants. To increase the circulating load, for example, the rougher time constant was increased and the cleaner time constant decreased to maintain the same grade of concentrate as the base case. Figure 3 illustrates the effect of increasing circulating loads in both the rougher and scavenger portions of the plant. When both the rougher and scavenger circulating loads were increased to 500 tons/d, an increase of 0.6 per cent in copper recovery was indicated. By increasing only the scavenger circulating load to 500 tons/d, an increase in recovery of 0.35 per cent was obtained. The indicated increases in molybdenite recovery were 0.8 and 0.55 per cent, respectively.

The reason why an improved recovery is attained may be explained in terms of the separation efficiency. A flotation plant separates the particles with a high rate of flotation from those with a low rate of flotation. Figure 4 illustrates the efficiency curves for the plant over a range of circulating loads. The middling particles have a rate of flotation in the region of the cut point. Scrimgeour, *et al* (1970) have shown how the residence time of these particles is increased, resulting in a more efficient separation. The advantage of closed-circuit operation over open-circuit operation is well known, but the magnitude of the circulating load has generally been limited by the ability of the cleaners to maintain the required

grade of concentrate and possibly by an overloading of the rougher cells.

It is possible to imagine a large number of cleaning stages which would result in a perfect plant efficiency curve (Fig. 4). However, this does not necessarily mean a perfect separation as, for example, particles of pyrite and chalcopyrite with the same rate of flotation cannot be separated. Thus, we define the concept of an infinite plant which provides a convenient upper limit to improvements that may be achieved for a given chemical environment and size distribution.

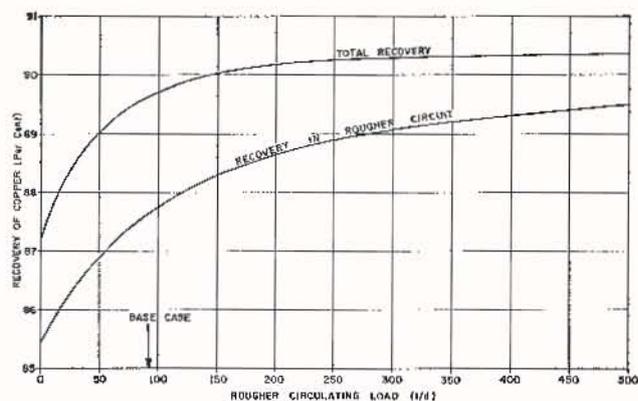


Fig. 3. Effect of circulating loads on recovery at constant grade of product (Scavenger circulating load proportional to rougher circulating load.)

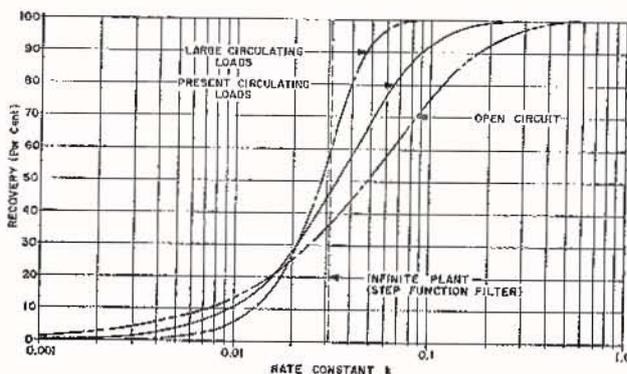


Fig. 4. Plant recovery of individual rate components for a constant grade of combined concentrate.

Figure 5 illustrates the calculated grade-recovery curves for open-circuit operation and an infinite plant. The actual grade-recovery curve is a function of the controls in the plant and thus any number of curves may be generated between the two extreme cases.

Figure 6 illustrates the calculated grade-recovery curves for the size fractions above and below 200 mesh. It is evident that their behaviour is very different. The coarse fraction is also very close to open-circuit operation and a significant improvement in recovery is potentially possible. Assuming that the coarse and fine fractions are floated in separate circuits, it is possible to produce a variety of concentrates which may be blended to yield the same grade of concentrate as the base case. Figure 7 illustrates the influence of the grade of the coarse fraction concentrate on the overall recovery of copper. Again, the two extremes of open-circuit and infinite plant are used. The recovery for the infinite plant is significantly higher when separate size fractions are used (that is, 91.6 versus 90.7). However, the grade of the coarse fraction is in the region of maximum recovery.

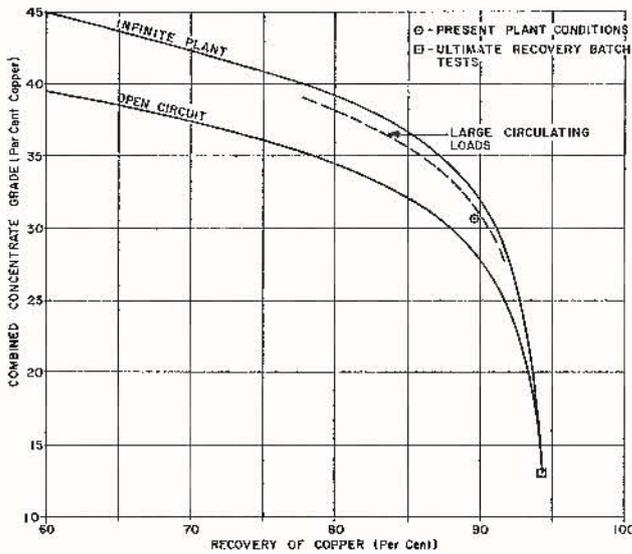


Fig. 5. Calculated grade-recovery curves.

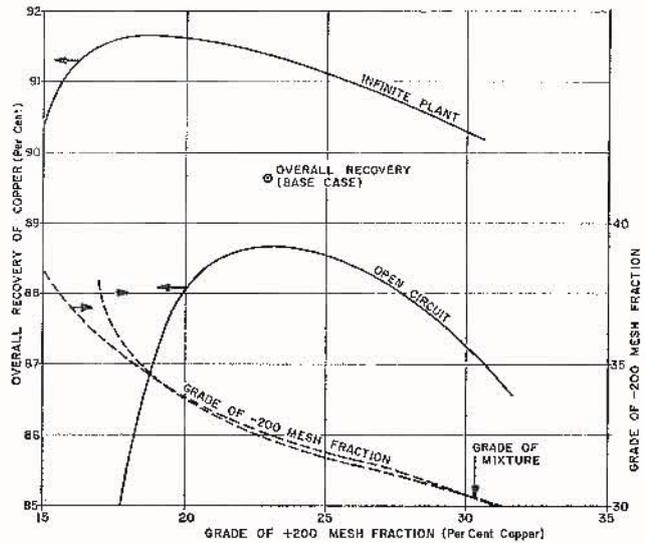


Fig. 7. Overall recovery of copper with separate circuits for coarse and fine fractions.

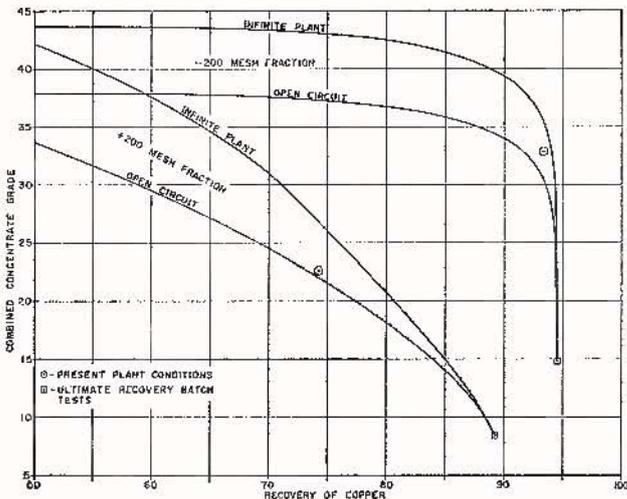


Fig. 6. Calculated grade-recovery curves for coarse and fine fractions.

CONCLUSIONS

It has been shown how a relatively simple model of a flotation plant may be used to investigate the effects of physical controls for a given set of reagent conditions. The purpose of this investigation has been to quantify the potential gain from increasing circulating loads and to investigate the effects of a size separation. The results of the simulation still require confirmation by experiments. Nevertheless, the simulation has provided a useful insight into the mechanisms which control plant efficiency.

In order to implement a policy of increasing circulating loads, it is clear that the grade of the cleaner concentrate should be controlled separately. It may be necessary to control the stability of cleaner froth, and the use of water sprays as described by Klassen, *et al* (1963) seems appropriate. The existence of a computer-controlled on-stream X-ray analysis system provides the capability of regulating the grade of the cleaner concentrate automatically. If this is accomplished, the circulating loads could then be adjusted by varying the froth flow from the rougher cells.

The simulation has also shown that a significant improvement in the recovery of copper in the plus 200-mesh fraction may be possible. A size separation of the concentrate and releaning of the coarse fraction may be advantageous.

Even though the cleaners do not operate under free-floating conditions, a selection of particles with a higher rate of flotation does take place and an 'equivalent' free-floating time constant can be used. Similarly, it is possible that the coalescence of bubbles in the froth results in a selective rejection of the less mineralized particles, which is related to rate of flotation. Thus, the simplifying assumptions used in the model could result in errors in the recovery increases and the grade-recovery curves. Nevertheless, the principles brought to light by this investigation are useful in understanding flotation plant behaviour.

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APPENDIX

Model for the return of particles from the froth

Assume that above the region of separation between pulp and froth the grade of the froth is homogeneous and that particles are rejected randomly from the froth by bubble coalescence. The rate of removal of concentrate is assumed to be proportional to the volumetric flow of concentrate.

Let m = mass flow

M = mass hold-up

Subscripts p, f, c, t = pulp, froth, concentrate, tail

The flow of concentrate is given by:

$$m_c = \frac{Q_c}{V_f} \cdot M_f$$

Where Q_c = volumetric flow of concentrate

V_f = volume of froth hold-up

The rate of return of particles to the pulp is also proportional to the mass hold-up in the froth:

$$\text{Rate of return} = hM_f$$

Consider particles with a rate constant k for flotation.

At steady-state:

Flow of concentrate = rate of flotation - rate of return

$$m_c = kM_p - hM_f = \frac{Q_c}{V_f} \cdot M_f$$

Eliminating M_f :

$$m_c = \frac{kM_p}{1 + h \frac{V_f}{Q_c}}$$

The fractional amount of material leaving in the tailings of a cell in continuous operation is given by:

$$F = \frac{m_t}{m_t + m_c} = \frac{1}{1 + k \cdot \frac{M_p}{m_t} \cdot \frac{1}{1 + h \frac{V_f}{Q_c}}}$$

The ratio $\frac{M_p}{m_t}$ is the average residence time (9). Thus, the return of particles from the froth with a rate constant h has the effect of reducing the apparent residence time. As h is assumed to be independent of k , this effect is the same when a distribution of rate constants is used.

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