Model for the Design and Control of Flotation Plants

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SYNOPSIS

A model of the flotation process can be used effectively for design and control. For design purposes it can be used to predict the performance of a large plant from laboratory and pilot plant data. It can be used for control purposes to assess the utility of any proposed operating modification to an existing plant, and it can also be used to determine optimal control settings for best operating performance.

A model has been developed that will allow for variations in feed properties and for variations in fineness of grind. The model can be used to assess the effects of reagent addition rate, aeration rate and pulp level on plant performance and these are the control variables that are used individually or together in all operating flotation plants. The model provides for a distribution in the values for flotation rate constants, particle size and mineral fraction, and, because of its complexity, requires a large computer when it is to be applied to a typical plant configuration. The computational method that is used to apply this model to such configurations is described in this paper, and particular attention is paid to the problem of allowing for feedback of tailings from cleaners and recleaners.

INTRODUCTION

Individual pecularities among the particles and bubbles in a flotation cell complicate the formulation of a reliable model for flotation kinetics. The need for such models for both the design and control of operating plants is well established, and much useful progress has been made. The analogy between the flotation process and a chemical reaction, which until fairly recently was explored extensively as a possible mechanism for a viable model, is disappointingly weak because of the significant differences between individual particles. However, usable mathematical models have been developed by consideration of groups of particles and bubbles with similar characteristics. The formulation of the model in this report is based on the hypothesis that the rate at which a sufficiently homogeneous assembly of particles floats can be modelled by the equation normally used for the modelling of the first-order chemical reaction. All interactions between groups of different particle types are neglected when an average is taken over the entire heterogeneous population for the modelling of the rate in a cell containing many different particle types. Mika, *et al* (1969) have reviewed much of the work leading to this point of view. The greatest difficulty associated with models generated in this way is the necessity for a large number of variables; both state variables such as particle concentrations, and model parameters of which there must necessarily be a large number. This difficulty can be avoided by the consideration of only a very few groups with widely differing properties. Bushell (1962) has achieved some success along these lines and has shown that in some circumstances no more than two groups need be considered for each mineral type. In an alternative method, use is made of a mathematical device that not only allows a large reduction in the number of variables but at the same time brings the model closer to a true description of reality. This device is based on the assumption that particle properties are distributed continuously over all the particles and was first used by Imaizumi, *et al* (1965). It was shown by Woodburn, *et al* (1965), to be very effective for the analysis of data from batch- and continuous-flotation cells. In this report the use of this device is extended to include particle mineralogy, surface activity, particle size, and particle mineral fraction.

The establishment of a useful model is complicated further by the need for the modelling of several small-scale processes, each of which contributes to a greater or lesser extent to the rate at which material is removed from the pulp and finally removed from the cell in the stream of froth. Although several distinct processes can be distinguished clearly, they are difficult to describe quantitatively. Thus, the process of particle-bubble collision and of particle adhesion and retention during the rising of the bubble through the pulp, and, finally, of particle retention in the froth layer, all contribute to the average rate of flotation of particles within any property group. The model formulated here does not require detailed specification of models for these sub-processes and merely utilizes various functional forms that require subsequent specification by reference to a detailed mechanistic description of these sub-processes. Such a formulation has the advantage of remaining intact while allowing considerable freedom in the choice of sub-process models.

THE MODEL

It is assumed for model development that the flotation cell consists of a perfectly mixed pulp through which a cloud of bubbles rises. Particles that are floated are transported at the bubble-pulp interface into the layer of froth that is removed from the top of the cell.

Three distinct properties of the particles that influence the rate of flotation significantly are identified, and values of these properties are distributed over the individual particles in the particle population. These properties are the particle size, D, the mineral composition of the particle, g, and the surface activity, k. The mineral composition, g, is a vector of values $(g_1, g_2 \ldots g_s)$ representing the fraction of each mineral type in the particle. The gangue is excluded. The assignment of numerical values to k is difficult, and it should properly be regarded as an empirical rate constant that carries quantitative information about the rates of many sub-processes contributing to the overall rate of flotation.

To accommodate the distribution of particle values in a mathematical structure, a function $F'(k,\mathbf{g},D)$ is defined as the fraction of particles in the system that are smaller than or equal to D in size and have a mineral-composition vector less than or equal to g, and surface activity less than or equal to k. The particle population is necessarily finite, and consequently $F'(k,\mathbf{g},D)$ is not a continuous function. However, for even moderately small populations, the size of discontinuities will be small, and some continuous function $F(k,\mathbf{g},D)$ is substituted for $F'(k,\mathbf{g},D)$ and $F(k,\mathbf{g},D)$ approximates $F'(k,\mathbf{g},D)$ very closely. In many instances it is convenient for the derivative

$$f(k,\mathbf{g},D) = \frac{\partial^{M+2}}{\partial k \partial \mathbf{g} \partial D} F(k,\mathbf{g},D)$$

to be used.

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The analogy with a probability density function is very close, but care must be taken in the use of the analogy if significant mathematical errors are to be avoided. A precise formulation of a stochastic model for the flotation process is given in the literature (Katz, *et al*, 1969).

To the fundamental hypothesis concerning the proportionality between the rate of flotation of solids of a particular type and the amount of that type present, the further hypothesis is added that this rate is proportional also to the amount of free bubble surface area. Some such additional hypothesis is necessary so that the model can allow correctly for the heavy loading experienced by the bubbles at high production rates and also for the effect of aeration rate in the cell. The assumption of direct proportionality has been criticized with some justification by Tomlinson, *et al* (1965), because the particles collected on the advancing front of the bubble are swept to the back, leaving free surface available for collision at the front. Insufficient experimental data are available for the more detailed model demanded by such a complex phenomenon. The rate is accordingly written as

Rate of flotation of particles characterized by property values $k,g,D = k\phi(D)\,ASWf(k,g,D)$ in kg/s.

In this expression, k is a rate constant and should allow for all small-scale effects that influence the rate of flotation, but the effects of particle size and available bubble surface area are excluded specifically. Function $\phi(D)$ is supposed to account for all the influence of the particle size, A is the total bubble-surface area per unit of pulp volume, S is the fraction of the surface area of a single bubble that is not covered by adhering solid particles, and W is the mass of solids in the cell.

Modelling of the variable S is a straightforward task (Pogerly (1962) and Zaidenberg (1964)) if it is assumed that the bubbles rise without coalescence through a perfectly mixed pulp. If τ represents the bubble lifetime and V the cell volume,

$$VA\frac{dS}{d\tau} = -WAS\int_0^\infty\int_0^1\int_0^\infty \eta(D)\,k\phi(D)f(k,g,D)\,dk\,dg\,dD \qquad .(1)$$

In equation (1), $\eta(D)$ is a factor representing the surface area covered by 1 kg of solid. It is assumed to be independent of the mineralogy and surface activity of the adhering particle. Factor $\eta(D)$ should vary approximately as $1/D$. This would be consistent with the fact that particle mass varies as the third power of the size, and the area of an equatorial section varies as the second power of the size.

Integration of equation (1) gives

$$S = \exp\left(-\frac{WK\tau}{V}\right), \qquad .(2)$$

where

$$K = \int_0^\infty\int_0^1\int_0^\infty k\,\eta(D)\,\phi(D)\,f(k,g,D)\,dk\,dg\,dD \qquad .(3)$$

The average value of S throughout the cell can be calculated from a knowledge of the spread of bubble residence times. If this is denoted by $f_\tau(\tau)$, the average is given by

$$S_{av} = \int_0^\infty \frac{f_\tau(\tau_b)}{\tau_b}\int_0^{\tau_b}\exp\left(-\frac{WK\tau}{V}\right)d\tau\,d\tau_b \qquad .(4)$$

Under normal operating conditions, only a small spread in bubble residence time is expected so that

$$f_\tau(\tau_b) = \delta(\bar{\tau} - \tau_b)$$

where $\bar{\tau}$ is the average residence time of the bubbles. Equation (4) becomes

$$S_{av} = \frac{V}{WK\bar{\tau}}\left[1 - \exp\left(-\frac{WK\bar{\tau}}{V}\right)\right] \qquad .(5)$$

THE DESIGN VARIABLES

Flotation plants always consist of several stages which are usually called roughing, scavenging or cleaning stages, depending on their position in the plant. Each stage usually consists of several flotation cells connected so that the tailings pass sequentially through the cells of each stage. Each stage is fed from one or more other stages and a concentrate and tailings stream are produced in each stage. The collection of stages with their interconnecting feed, concentrate and tailings streams is called the plant configuration. The plant configuration depends on the type of ore to be treated. A plant that must extract and separate two minerals will have a configuration different from one that must separate one mineral from the gangue. However, there are a very great many possible configurations for a given job and the first task facing the design engineer is the selection of a suitable plant configuration. This is usually done on the basis of laboratory batch experiments but the model that has been developed allows the design engineer to investigate any number of plant configurations before the final design is chosen. Such an investigation can be very rewarding in the design of plants that must separate minerals because selectivity in a continuously operating plant can be significantly different from that found in a laboratory batch cell.

Once the plant configuration has been determined, the design engineer must allocate sizes to the various stages in the plant so that the plant can recover sufficiently large amounts of the valuable minerals in the feed at the rates demanded by the production capacity of the mine. There will always be two decisions required of the design engineer: he must determine the total capacity of the plant to be installed and he must allocate flotation cell volume to the individual stages to the best effect. Too much total cell volume is wasteful in capital expenditure and too little will mean the loss of valuable mineral in the plant tailings. An optimum exists and this optimum can be found by the use of the model described in this paper.

Two other design variables are important but have not been included in the computational scheme as yet. These are the provision and location of a regrinding mill in the circuit and the provision of a sand-slimes separation stage in the circuit.

THE CONTROL VARIABLES

Those variables that are available for manipulation on an operating plant are called control variables and these are available to the plant operator for control purposes. Four such variables are available on any particular plant: the classifier setting in the milling circuit which determines the fineness of grind, the addition rates of the conditioning chemicals, the pulp levels in each of the stages and the aeration rate in the individual cells. The effect of these variables on the plant performance can be predicted by the model which may therefore be used by plant operators to determine optimum settings of these variables and so to gain improved performance.

Control variables are of two kinds: those that influence the performance of individual cells and can be varied at any cell as required and those that can be controlled at various points in an integrated flotation circuit. Among the first the pulp

level and the aeration rate will be considered, and, among the second, the classifier setting and the conditioning chemical addition rates.

The aeration rate acts through the associated variation of the bubble surface area (Edwards, 1971). An adequate model for this effect is a simple proportionality between the bubble area and the gas rate:

$$A = \frac{\sigma\bar{\tau}G}{V}$$

where σ is the bubble surface area per unit volume of bubble, and G is the aeration rate (m^3/s). A secondary effect is the variation of effective cell volume with aeration rate, and this is modelled adequately by allowing for the gas hold-up in the cell equal to $G\bar{\tau}$, but this is usually small compared with the cell volume.

The dominant action of pulp-level variation is through their effect on froth flowrate. Low pulp levels relative to the froth overflow lip permit a large proportion of the froth to break and return to the pulp with a resulting decrease in the amount of froth that comes over the lip. A secondary, but very important, effect is the enrichment of froth that occurs as the froth height increases. This enrichment results from the selective return of gangue materials from the breaking froth. At present no adequate quantitative model for this phenomenon is available but it is the subject of an experimental program by this Research Group, and provision has been made for the incorporation of a quantitative model into the computational scheme that is developed here. On the flotation plant, the pulp level can be controlled by manual adjustment of the tailings overflow weir, the sand-gate opening or, on some modern flotation machines, by automatic control of pulp level in individual cells.

Let M_R equal the mass flowrate of solids from the froth returning to the pulp in the flotation cell. This rate will be a fraction, $(1 - \gamma)$, of the total rate, M_F, at which solids leave the pulp by flotation. Thus

$$M_R = (1 - \gamma) M_F \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and γ is a function of the pulp level, the particle size and the mineralogy of the particle **g**.

The action of the conditioning chemicals is essentially a variation of the rate constants, and this action is modelled by allowing the distribution function $f(k|\mathbf{g},D)$ to vary with conditioner concentration. It might not be possible to specify this variation quantitatively for some ores but it will be possible to infer this by observations on an operating plant — either sequentially or in a parallel pilot installation (Woodburn, 1970). Secondary effects of conditioner concentrations are their influence on the bubble surface area, A, and on the draining and breaking properties of the froth.

THE MATERIAL BALANCE EQUATIONS FOR THE SOLID

A single perfectly mixed cell

The fundamental equation describing the flotation of a specific particle type is

$$M_F f_F(k,\mathbf{g},D) = k\phi(D)AS_{av}Wf_T(k,\mathbf{g},D) \quad \ldots\ldots (7)$$

where subscript T indicates material in the tailings and subscript F indicates material that is floating.

If M_I represents the rate at which solids enter the cell and M_T the rate at which solids leave in the tailings stream, a total material balance over the pulp gives

$$M_I + M_R = M_T + M_F \quad \ldots\ldots\ldots\ldots (8)$$

Substitution of equation (6) gives

$$M_I - M_T = \gamma M_F \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and a balance for a specific particle type gives

$$M_I f_I(k,\mathbf{g},D) - M_T f_T(k,\mathbf{g},D) = \gamma M_F f_F(k,\mathbf{g},D)$$
$$= \gamma\phi(D)\, k\, AS_{av}W f_T(k,\mathbf{g},D) \quad \ldots\ldots (10)$$

from which

$$M_T f_T(k,\mathbf{g},D) = \frac{M_I f_I(k,\mathbf{g},D)}{1 + \gamma k\phi(D)\, AS_{av}\, W/M_T} \quad \ldots\ldots (11)$$

The cell is assumed to be perfectly mixed, and this assumption makes it possible for the ratio W/M_T to be estimated:

$$\frac{W}{M_T} = \frac{VC_T}{q_T C_T} = \frac{V}{q_T} = \theta_T \quad \ldots\ldots\ldots\ldots (12)$$

Here q_T is the volumetric flowrate of the tailings, C_T the concentration of solids in the cell, and θ_T the mean residence time of the tailings.

Equation (11) is in fact a non-linear integral equation because q_T and hence θ_T depends on the performance of all particle species in the cell as follows:

$$q_T = W_T + M_T/\rho s$$
$$= W_T + 1/\rho s \int_0^\infty \int_0^1 \int_0^\infty M_T f_T(k,\mathbf{g},D)\, dk\, d\mathbf{g}\, dD \quad \ldots (13)$$

where W_T is the flowrate of water in the tailings stream. In addition, S_{av} depends on the performance of all the particle types through equations (5) and (3).

Multi-cell plant of arbitrary configuration

Let $\upsilon_i(k,\mathbf{g},d) = \gamma_i k\phi(D)\, A_i Sav_i \theta_{Ti}$

and

$$x_i(k,\mathbf{g},D) = M_{Ti} f_{Ti}(k,\mathbf{g},D)$$

for stage i in an arbitrary configuration. The problem of determining the material balance for solids in the plant is solved completely when x_i has been determined for each stage and for each value of the particle variables k,**g** and D, because the mass flowrates in the concentrate streams can be recovered from x_i by the use of equation (10):

$$M_{ci} f_{ci}(k,\mathbf{g},D) = M_{Ii} F_{Ii}(k,\mathbf{g},D) - M_{Ti} f_{Ti}(k,\mathbf{g},D)$$
$$= \upsilon_i(k,\mathbf{g},D)\, M_{Ti} f_{Ti}(k,\mathbf{g},D)$$
$$= \upsilon_i x_i \quad \ldots\ldots\ldots\ldots (14)$$

and any other streams in the plant with the exception of the known feed stream, will be linear combinations of the concentrate streams and tailings streams.

The computational method that is used here is based on a penetrating analysis of a linearized version of the model that was made by Deift (1970).

Let $-b_{ik}$ be the fraction of the tailings stream that leaves stage k and enters stage i, and let $-a_{ik}$ be the fraction of the concentrate stream that leaves stage k and enters stage i. If **D** is the matrix having off-diagonal elements $D_{ik} = b_{ik} + \upsilon_k a_{ik}$ and diagonal elements $D_{ii} = 1 + \upsilon_i$, the plant material balance for particles of a particular type is defined by the simultaneous equations

$$\mathbf{Dx} = \mathbf{F} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

where **x** is the vector of stage tailings flowrates x_i and **F** is a vector with element F_i equal to the rate at which material of the given type is fed to stage i. For most plant configurations **F** will have only one non-zero element. For example, if the feed enters the plant at stage n, the nth element of **F** is equal to $Ff_{feed}(k,\mathbf{g},D)$ and all the other elements of **F** are zero. It must be remembered that equations (15) hold for each

combination of the variables k,g and D. Notwithstanding the apparent linearity of equations (15), they represent a set of non-linear equations because the elements of **D** depend on **x** through the dependence of v_t on θ and S_{av} which in turn depend on **x** through equations (11), (12), (13), (5) and (3). This non-linearity causes some difficulty in the solutions of equations (15) and an efficient method of solution is essential to the usefulness of the model.

THE MATERIAL BALANCE EQUATION FOR THE WATER

The flowrates of water in the various plant streams are important for the calculation of plant performance because of the large dependence of stage-holding time on the water flowrate in equation (13). The water flowrates in the plant are related to the mass flowrates and it is reasonable to assume that the specific gravities of the pulp in the concentrate streams from all the cells can be predicted for design purposes or measured for control purposes. Such an assumption is sufficient to fix uniquely the water flowrates throughout the plant. Let W_{ci} be the water flowrate in the concentrate stream from stage i and S_{pi}, the specific gravity of the pulp in that stream. Then

$$W_{ci} = \frac{S_s - S_{pi}}{S_{pi} - 1} \frac{M_{ci}}{\rho_s} \quad \ldots\ldots\ldots\ldots (16)$$

where S_s is the specific gravity of the solid. A water balance over any stage i gives the equations

$$\sum_k b_{ik} W_{Tk} + W_{Ti} = -W_{ci} - \sum_k a_{ik} W_{ck} + F_{wi} \quad (16)$$

where F_{wi} is the rate at which water is fed to stage i in feed streams or as additions of water to adjust pulp densities. Equations (16) can be written in vector form

$$\mathbf{E}\mathbf{w} = -\mathbf{G}\mathbf{W}_c + \mathbf{F}_w \quad \ldots\ldots\ldots\ldots (17)$$

where $E_{ik} = b_{ik}$ and $G_{ik} = a_{ik}$ are typical elements of the matrices **E** and **G**, and **W** is the vector of water flowrates in the tailings from the stages.

The solutions to equations (17) are given by

$$\mathbf{w} = -\mathbf{E}^{-1}\mathbf{G}\mathbf{W}_c + \mathbf{E}^{-1}\mathbf{F}_w \quad \ldots\ldots\ldots\ldots (18)$$

THE NUMERICAL SOLUTION

A Newton-Raphson scheme was developed for the numerical solution of the material balance equations. This scheme was based on finding the vector θ of stage-holding times that satisfied the equations

$$\psi_k = F_k - \theta_k q_{Tk} = 0 \quad \ldots\ldots\ldots\ldots (19)$$

The procedure started at an initial guess $\theta^{(0)}$ and successively better estimates were calculated recursively from

$$\theta^{(n+1)} = \theta^{(n)} - \mathbf{H}^{-1}\psi \quad \ldots\ldots\ldots\ldots (20)$$

where **H** is the matrix of partial derivatives $\frac{\partial \psi_k}{\partial \theta_i}$. The evaluation of these partial derivatives was no easy task and the necessary relationships are given in Appendix B. Implicit in this solution are the vectors of flowrates of solid material of each particular particle type in the tailings and concentrates. All quantities of interest can be computed from these flowrates.

Computation of the grade of any mineral

In any stream the grade can be computed from the values of x for that stream as follows:

$$\text{Grade of mineral type } m = \frac{\int_0^\infty \int_0^i \int_0^\infty g_m f(k,\mathbf{g},D)\, dk\, d\mathbf{g}\, dD}{\int_0^\infty \int_0^1 \int_0^\infty g_m M f(k,\mathbf{g},D)\, dk\, d\mathbf{g}\, dD} = \frac{\int_0^\infty \int_0^1 \int_0^\infty M f(k,\mathbf{g},D)\, dk\, d\mathbf{g}\, dD}{\int_0^\infty \int_0^1 \int_0^\infty g_m x\, dk\, d\mathbf{g}\, dD} = \frac{\int_0^\infty \int_0^1 \int_0^\infty x\, dk\, d\mathbf{g}\, dD}{\quad} \quad \ldots (21)$$

where x refers to an appropriate element of **x** or $\mathbf{x}_c$ according to the stream that is under consideration.

Computation of fractional recovery of any mineral

Let F be the total rate at which solids are fed to the plant. Then recovery of mineral type m

$$= \frac{\int_0^\infty \int_0^1 \int_0^\infty g_m x\, dk\, d\mathbf{g}\, dD}{\int_0^\infty \int_0^1 \int_0^\infty g_m F f_{feed}(k,g,D) dk\, dg\, dD} \quad \ldots\ldots (22)$$

Computation of the particle size distribution

The particle size distribution of the solid in any stream is given by

$$f(D) = \frac{\int_0^\infty \int_0^1 x\, dk\, d\mathbf{g}}{\int_0^\infty \int_0^1 \int_0^\infty x\, dk\, d\mathbf{g}\, dD} \quad \ldots\ldots\ldots\ldots (23)$$

Consideration of the material presented above and a glance at Appendix B will indicate that a digital computer must be used very skilfully in order to make the computation possible. With this in mind, a general purpose program has been prepared that can be used routinely by design and control engineers for plant calculations. This program merely requires data to specify the number of stages and configuration of the plant and data that characterize the feed material. The program provides as output the grade and recovery of each mineral, the particle size distribution, the total solid and water flowrates in each of the process streams and, in addition, the average holding time in each stage is provided. The program, together with detailed and precise instructions for the preparation of input data, can be obtained from the author.

SPECIFICATION OF THE DISTRIBUTION FUNCTIONS

The best way to specify the distribution function $f_{feed}(k,\mathbf{g},D)$ is through the conditional distribution functions

$$f_{feed}(k,\mathbf{g},D) = f_{feed}(k|\mathbf{g},D)\, f_{feed}(\mathbf{g}|D)\, f_{feed}(D).$$

Here $f_{feed}(D)$ is the particle size distribution in the feed material. Function $f_{feed}(\mathbf{g}|D)$ is the distribution of mineral types in particles of size D in the feed. If no chats are present it can be obtained directly from a chemical analysis of the particular size fraction. If chats are present further mineralogical data are required to obtain the fraction of the particles of size D that falls within narrow ranges of the variable **g**. The

measurement of $f(g|D)$ requires the separation of particles of size D into different classes of grade. This separation can be done by microscopic point-count methods, or the density-gradient column or fractional separation in a heavy medium can be used for ores containing minerals of different densities (Smale, 1970). Function $f_{feed}(k|g,D)$ is the distribution of k values for particles of a given mineral composition, g, and size D. This is assumed to be independent of D because the variation with D should be taken care of by the function $\phi(D)$, so that $f_{feed}(k|g,D) = f_{feed}(k|g)$. The function $f_{feed}(k|g)$ can be determined by the graphical technique of Imaizumi, *et al* (1965), or by any other parameter estimation technique from batch or continuous cell operating data.

It is very convenient to specify these distribution functions as discrete fractional distributions. In that case all integrals in the theory become finite sums which suits the digital computation very well. An example is given in Appendix C to show how the distribution functions can be calculated from experimental data.

NUMERICAL RESULTS

To illustrate the type of result that can be obtained from the model, a set of hypothetical data has been processed. The plant configuration that was chosen for study is the conventional four-stage scavenger-rougher-cleaner-recleaner circuit that is shown in Figs. 1 and 2. A single mineral was separated from the gangue.

Standard operating conditions for this plant were taken as: feed rate = 100 *kg/s*, scavenger volume 100 m^3, rougher volume 300 m^3, cleaner volume 75 m^3, recleaner volume 25 m^3, $\bar{\tau} = 10$ *s*, $\sigma = 1{,}2 \times 10^3\ m^{-1}$, $\varepsilon = 2{,}0 \times 10^{-10}\ m^2$. Concentrates from each stage were taken to contain 50 per cent solids and sufficient water was added to adjust the feed to cleaner and recleaner to 20 per cent solids. The plant feed was assumed to contain 15 per cent solids.

The ore was characterized by three values of k, three values of g and five values of D and the various distribution functions are presented in Tables I, II and III.

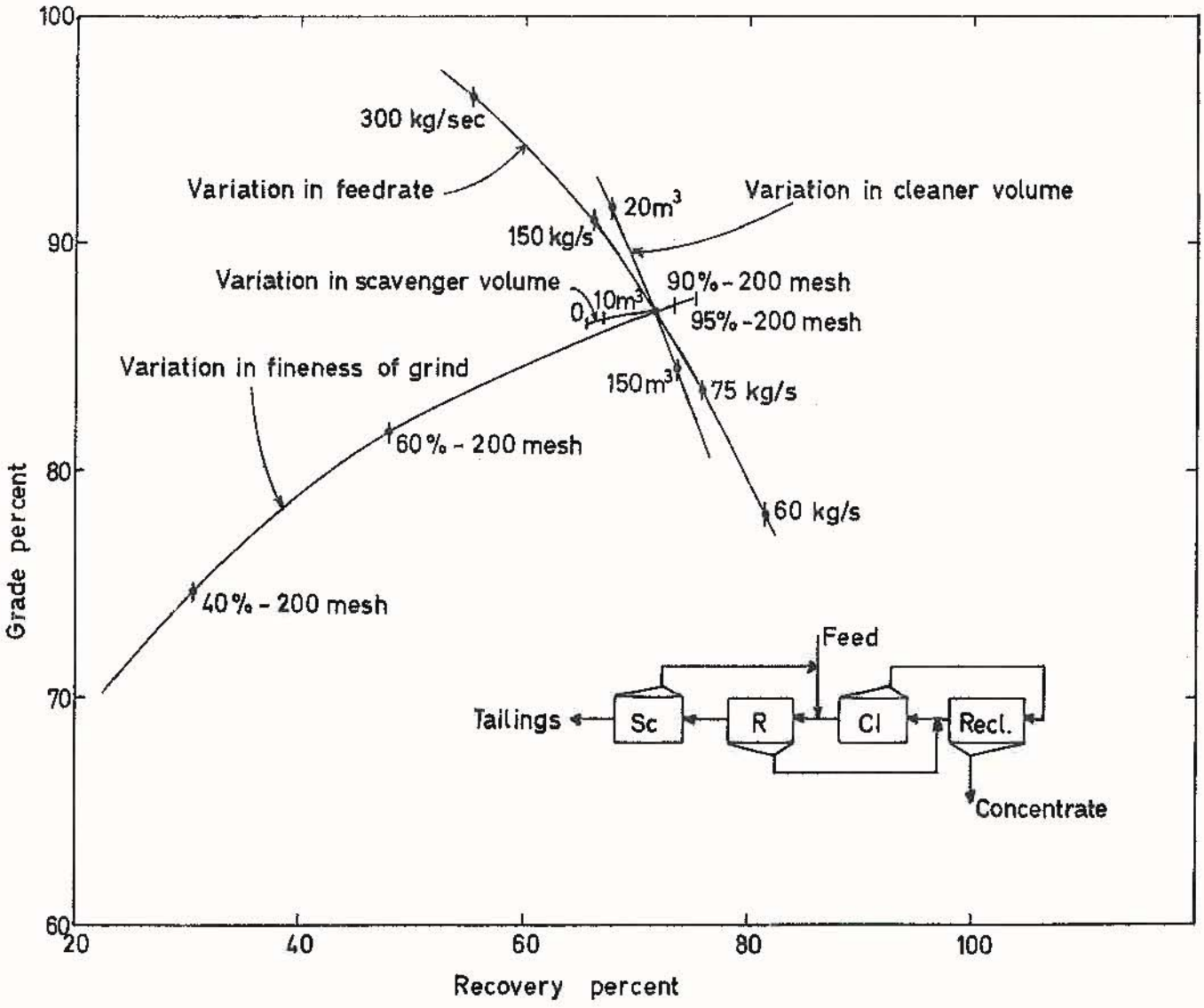

Fig. 1. Grade-recovery curves showing variation with design variables.

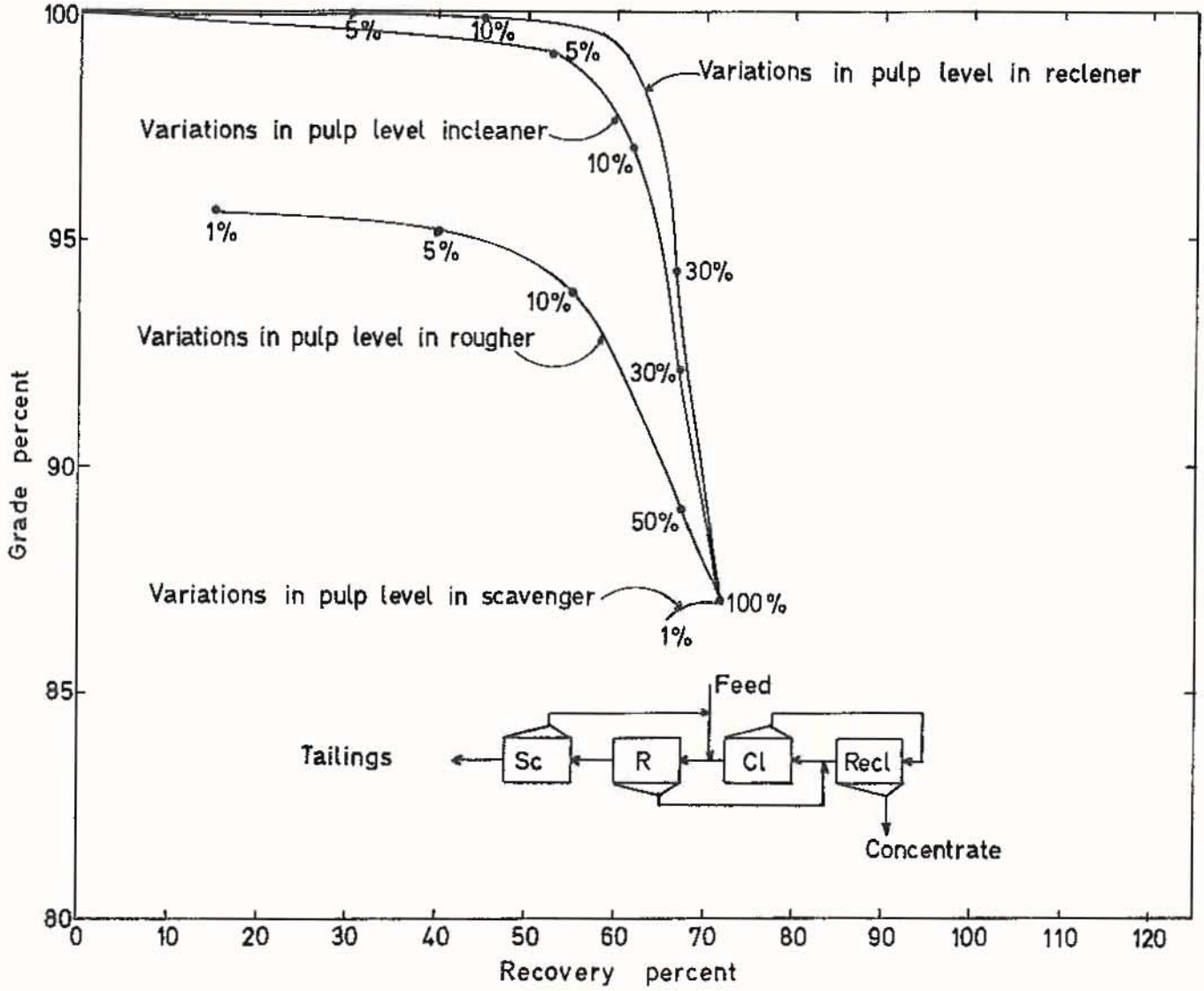

Fig. 2. Grade-recovery curves showing variation with control variables.

TABLE I

THE DISTRIBUTION OF PARTICLE SIZES $f(D)$

Fineness of grind	$f(D=1\times10^{-5})$	$f(D=3\times10^{-5})$	$f(D=5\times10^{-5})$	$f(D=7\times10^{-5})$	$f(D=10\times10^{-5})$
40% passing 200 mesh	0,02	0,11	0,11	0,24	0,49
60% passing 200 mesh	0,03	0,17	0,21	0,36	0,23
*80% passing 200 mesh	0,05	0,22	0,28	0,45	0,00
90% passing 200 mesh	0,05	0,25	0,32	0,38	0,00
95% passing 200 mesh	0,06	0,26	0,34	0,34	0,00

*The standard condition.

TABLE II

THE DISTRIBUTION OF MINERAL FRACTIONS

	$g = 1,0$	$g = 0,5$	$g = 0,0$
$f(g \mid D = 1 \times 10^{-5})$	0,1	0,0	0,9
$f(g \mid D = 3 \times 10^{-5})$	0,08	0,02	0,9
$f(g \mid D = 5 \times 10^{-5})$	0,05	0,05	0,9
$f(g \mid D = 7 \times 10^{-5})$	0,05	0,10	0,85
$f(g \mid D = 10 \times 10^{-5})$	0,00	0,40	0,60

TABLE III

THE DISTRIBUTION OF RATE CONSTANTS

	$k = 1 \times 10^{-4}$	$k = 1 \times 10^{-5}$	$k = 1 \times 10^{-6}$
$f(k \mid g = 1,0)$	1,0	0,0	0,0
$f(k \mid g = 0,5)$	0,0	1,0	0,0
$f(k \mid g = 0,0)$	0,0	0,0	1,0

The numerical results are presented as a series of grade-recovery curves in Figs. 1 and 2. The grade and recovery for the standard operating conditions as presented above are plotted as the point of convergence of the four curves in Fig. 1. The four curves are grade-recovery curves that are generated by varying individually the fineness of grind, the volume of the scavenger, the volume of the cleaner and the feed rate to the plant. These represent the design variables and, of course, several others could be added. The appropriate values of these variables are indicated along each line and the curves contain the quantitative information that is required to assess the economic value of these variables. For example, the value of increased grade and recovery can be balanced against the cost of increased fineness of grind. Usually a decrease in feed rate would be associated with increased fineness of grind and it would be a simple matter to generate a grade-recovery curve with fineness of grind and feed rate varying simultaneously.

Grade-recovery curves are shown in Fig. 2. These were generated by variation of four of the possible control variables in the plant. These are the froth flowrates from each of the stages. The curves are graduated in percentages of maximum froth flowrate. Variations in froth flowrate can be effected by variation in pulp level or aeration rate. The former was used here for the illustrative calculation.

The curves in Fig. 2 are interesting because they indicate that the curve that represents variations in recleaner pulp level lies to the right and above all other curves. It thus represents the upper boundary of the attainable region (Woodburn, 1970) and the plant must always be operated somewhere on this line with maximum froth withdrawal rate from all but the recleaner stage. It is consequently fruitless to regard pulp levels in the other stages as plant control actions and the provision of costly pulp level controllers in those stages is hardly likely to be economically justifiable.

The curves in Figs. 1 and 2 are presented only as illustrations of the type of information that can be obtained from the model and do not begin to exhaust the possible scope of this information. Computation time on a CDC SC1700 was approximately two minutes for individual points on these curves.

CONCLUSIONS

An efficient computational scheme has been developed for the distributed parameter flotation model in a plant of arbitrary complexity. This computation permits the calculation of plant performance over a wide range of possible design and control variables and will prove to be of great benefit to design engineers and plant operators in achieving maximum profit from the flotation plant.

Notwithstanding the complexity of the model, the computational technique is sufficiently efficient to permit rapid computation of plant performance. This makes the method suitable for both optimal design calculations and on-line control calculations and the use of the computation to provide classifier setpoints in a grinding circuit control loop is an obvious application.

The computer program, together with detailed instructions for its use can be obtained from the author.

NOTATION

In order to simplify the algebra for the multi-stage plant, a vector notation has been adopted. Vectors and matrices are printed in bold face. With the exception of **g**, all vectors have dimension N (the number of stages in the plant). Each element of the vector pertains to an individual stage. For example, the vector $\mathbf{W}_c$ is made up of the water flowrates in the concentrates from each of the stages. The vector **g** has s elements and the integral $\int_0^1 \mathbf{dg}$ should be interpreted as the s-fold integral $\int_0^1 \ldots \int_0^1 dg_1\, dg_2 \ldots dg_s$.

A bubble surface area per unit volume of pulp, m^{-1}

a_{ik} — fraction of concentrate from stage k that enters stage i.

b_{ck} — fraction of tailings from stage k that enters stage i.

D particle size, m.

D_m particle size at which flotation rate is a maximum, m.

d_b bubble diameter, m.

F Solids feed rate to the plant, kg/s.

$\mathbf{F}$ vector of feedrates of particular type, kg/s.

$\mathbf{F}_w$ vector of water feed rates, m^3/s.

$f(k,\mathbf{g},D)$ distribution of particle types. $f(k,\mathbf{g},D)\ dk\mathbf{dg}dD$ is the fraction of particles that are characterized by k, **g** and D.

$f(k \mid \mathbf{g})$ distribution of k values in a particular g class.

$f(\mathbf{g} \mid D)$ distribution of mineral fractions in a particular narrow particle size fraction.

$f(D)$ particle size distribution.

G aeration rate, m^3/s.

g_m fraction of a particle that is mineral type m.

$\mathbf{H}$ matrix of partial derivatives $\frac{\partial \psi_k}{\partial 0_i}$.

K integral defined in equation (3).

k specific rate constant, m/s.

M total mass flowrate, kg/s.

N number of stages.

q pulp flowrate, m^3/s.

S fraction of bubble surface that is uncovered.

S_{av} average value of S in a stage.

s number of minerals.

S_s	specific gravity of solid.
S_p	specific gravity of pulp.
V	volume of stage, m^3.
W	mass of solid material in a cell, kg.
w	water flowrate, m^3/S.
x	mass flowrate of material characterized by particular values of k, g and D.
γ	1— fraction of floated material of particular type that is returned from the froth to the pulp.
ε	parameter in ϕ function, m^2.
η	bubble surface area covered per kg of solid, m^2/kg.
θ	stage holding time, s.
υ	product of specific flotation rate of particular particle type and holding time in a stage.
ρ_s	solid density, kg/m^3.
σ	bubble surface area per unit volume of bubble, m^{-1}.
τ	bubble holding time, s.
$\bar{\tau}$	average bubble holding time in a stage.
ϕ	fractional efficiency of impaction, adhesion and levitation for particles of size D.
ψ	function defined in equation (19).

Subscripts

C	concentrate stream.
F	material floated from pulp.
I	material feed to the cell.
T	tailings stream.
i,k	designate a particular stage.
m	designates a particular mineral
feed	designates the feed to the plant.

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APPENDIX A

The effect of particle size

Colborn (1969) has analyzed the microscopic factors influencing the collision, adhesion and levitation processes that are important and concluded that a function of the form

$$\phi(D) = \frac{\text{const.}}{D}\{1 - (D/\Delta)^{1.5}\}\exp(-\varepsilon/D)$$

described the fractional efficiency of flotation of particles of size D. ε is a parameter that depends on the hydrodynamic conditions in the cell and Δ is the largest sized particle that can be retained on a bubble. This function has essentially two parameters ε and Δ. The function is relatively insensitive to Δ and the bracketed term can be neglected in the absence of precise experimental data on the variation of k with D. The function $\frac{\text{const.}}{D}\exp(-\varepsilon/D)$ has a form consistent with experimental observations (Tomlinson, *et al*, 1965) in that it is very small at small D and has a single maximum before decaying to zero for large D. The normalizing constant is evaluated so that $\phi(D)$ has a maximum value of unity. Thus

$$\phi(D) = \frac{2{,}33\,\varepsilon^{\frac{1}{2}}}{D}\,e^{-\varepsilon/D^2} \qquad \text{(A1)}$$

The value of D that maximizes $\phi(D)$ is related to ε by

$$D_{\max} = (2\varepsilon)^{\frac{1}{2}} \qquad \text{(A2)}$$

and this leads to a convenient method for the estimation of ε from plant data. A plot of experimentally-determined rate constants against D will identify D_{max} from which ε can be evaluated from equation (A2).

APPENDIX B

Evaluation of the matrix **H**

$$\psi_k = V_k - \theta_k q_{Tk}$$

$$\frac{\partial \psi_k}{\partial \theta_i} = -\theta_k \frac{\partial q_{Tk}}{\partial \theta_i} - q_{Tk}\,\partial_{ik}$$

From equation (13)

$$\frac{\partial q_{Tk}}{\partial 0_i} = \frac{\partial W_{Tk}}{\partial \theta_i} + \frac{1}{\rho_s}\int_0^\infty \int_0^1 \int_0^\infty \frac{\partial x_k}{\partial \theta_i}\, dk\, dg\, dD$$

From equation (18)

$$\frac{\partial \mathbf{W}_T}{\partial \theta_i} = -\mathbf{E}^{-1}\,\mathbf{G}\,\frac{\partial \mathbf{W}_C}{\partial \theta_i}$$

From equation (16)

$$\frac{\partial W_{ck}}{\partial \theta_i} = \frac{S_s - S_{pk}}{S_{pk} - 1}\,\frac{1}{\rho_s}\int_0^\infty \int_0^1 \int_0^\infty \left(\upsilon_k \frac{\partial x_k}{\partial \theta_i} + x_k \frac{\partial \upsilon_k}{\partial \theta_i}\right) dk\, dg\, dD.$$

From the definition of υ_k

$$\frac{\partial \upsilon_k}{\partial \theta_i} = \frac{\upsilon_k}{\theta_i}\,\delta_{ik} + \frac{\upsilon_k}{S_{av}}\,\frac{\partial S_{av\,k}}{\partial \theta_i}$$

From equation (5)

$$\frac{\partial S_{av\,k}}{\partial \theta_i} = \frac{V_k\,\delta_{ik}}{W_k K_k \bar{\tau}\,\theta_k}\left[\exp\left(-W_k K_k \bar{\tau}/V_k\right)\left(\frac{1}{\theta_k} + \frac{W_k K_k \bar{\tau}}{V_k}\right) - \frac{1}{\theta_k}\right]$$

From equation (15)

$$\mathbf{x} = \mathbf{D}^{-1}\,\mathbf{F}$$

$$\frac{\partial \mathbf{x}}{\partial \theta_i} = -\mathbf{D}^{-1}\,\frac{\partial \mathbf{D}}{\partial \theta_i}\,\mathbf{D}^{-1}\,\mathbf{F} = -\mathbf{D}^{-1}\,\frac{\partial \mathbf{D}}{\partial \theta_i}\,\mathbf{x}.$$

With the use of the assumption that

$$\frac{\partial S_{av\,k}}{\partial \theta_i} = 0$$

if $k \neq i$ it is possible to show that

$$\frac{\partial \mathbf{D}}{\partial \theta_i}\,\mathbf{x} = x_i\,\frac{\partial \upsilon_i}{\partial \theta_i}\,\mathbf{a}_i$$

where $\mathbf{a}_i$ is the column vector $(a_{1i}\ a_{2i} \ldots a_{Ni})_T$.

Therefore

$$\frac{\partial \mathbf{x}}{\partial \theta_i} = -x_i\,\frac{\partial \upsilon_i}{\partial \theta_i}\,\mathbf{D}^{-1}\,\mathbf{a}_i$$

and all the necessary derivatives have been determined.

APPENDIX C

Estimation of the distribution functions

In order to demonstrate the meaning of the various distribution functions more clearly and to demonstrate the method of estimation of these functions, the data presented by Imaizumi, *et al* (1965) is used as an example.

After microscopic examination, these authors reported the particle types shown in Table I in a narrow size range centred at 45×10^{-6} *m*. The appropriate values for the elements g_1 (chalcopyrite fraction) and g_2 (pyrite fraction) of the vector g are shown as well as the (assumed) percentage of the sulphide particles that were of the indicated type. Although these latter data were necessarily determined by these authors, they unfortunately did not report it. Arbitrary values were assumed for the purpose of this illustration.

TABLE I

PARTICLE TYPES

	g_1	g_2	Assumed percentage of sulphide particles that were of this type in the feed material
Free chalcopyrite	1	0	40%
75-100% chalcopyrite 0-25% pyrite	0,87	0,13	5%
50-75% chalcopyrite 25-50% pyrite	0,63	0,37	5%
25-50% chalcopyrite 50-75% pyrite	0,37	0,63	10%
0-25% chalcopyrite 75-100% pyrite	0,13	0,87	15%
Free pyrite	0	1	25%
Gangue	0	0	

In addition, Imaizumi, *et al* report the fractional recovery of these particle types in a cascade of eight cells. These data show the characteristic convex shape when plotted as the logarithm of the recovery against the stage number. This indicates that at least two values of k are necessary to describe the floatability of each particle type. For example, it was easy to use the data to show that 78 per cent of the particles having $\mathbf{g} = (0{,}37,\ 0{,}63)$ floated with $k\ \phi\ A\ S_{av} = 0{,}4 \times 10^{-3} s^{-1}$ and 22 per cent with $k\ \phi\ A\ S_{av} = 8{,}4 \times 10^{-3} s^{-1}$.

Values of $\phi\ A\ S_{av}$ must be estimated.

$$A = \frac{\sigma \bar{\tau} G}{V} \doteqdot \frac{h 10^{-5}}{d_b^3} \frac{G}{V}$$

where h is the depth of pulp in the cell. For the cells used $h \doteqdot 0{,}75\ m$, $d_b \doteqdot 10^{-3}\ m$ and $G/V \doteqdot 0.015\ s^{-1}$.

$A \doteqdot 110\ m^{-1}$.

If ϕ and S_{av} are taken to be unity the two values of k are $0{,}36 \times 10^{-5}\ m/s$ and $7{,}6 \times 10^{-5}\ m/s$, respectively. Thus the k-distribution for these particles is discrete and we write $f(0{,}36 \times 10^{-5} | g_1 = 0{,}37,\ g_2 = 0{,}63) = 0{,}78$ and $f(7{,}6 \times 10^{-5} | g_1 = 0{,}37,\ g_2 = 0{,}63) = 0{,}22$ for the distribution function $(k|\mathbf{g})$.

The estimation of A is not critically important if the data are to be used for the prediction of the plant consisting only of cells of the type from which the data were obtained because the pair kA always appears as a product in the model and it is only this value that is important except under conditions where very high bubble loadings will exist. On the other hand, if the data are to be scaled up to cells of different size or if the data are obtained from laboratory batch cells, the estimation of A is crucial. It will usually be worthwhile to measure the aeration rate and to measure the bubble residence time directly. The latter measurement can be made easily by making use of a radioactive tracer in the air stream.

In order to compute the distribution $f(\mathbf{g}|D)$, the head grade must be known and because this value was not reported by Imaizumi, *et al*, a value of five per cent was assumed for the purposes of this illustration. Because chalcopyrite contains 34.6 per cent Cu, the percentage of the feed that is sulphide is

$$\frac{5}{(0{,}4 + 0{,}05 \times 0{,}87 + 0{,}05 \times 0{,}63 + 0{,}1 \times 0{,}37 + 0{,}15 \times 0{,}13)\ 0{,}346} = 27{,}2 \text{ per cent.}$$

Thus $0{,}4 \times 27{,}2 = 10{,}9$ per cent of the feed is pure chalcopyrite and the discrete distribution for $f(\mathbf{g}|D)$ is given by

$f(1\ 0|D = 45 \times 10^{-6}) = 0{,}109$
$f(0\ 1|D = 45 \times 10^{-6}) = 0{,}068$
$f(0\ 0|D = 45 \times 10^{-6}) = 0{,}728$
$f(0{,}87\ \ 0{,}13|D = 45 \times 10^{-6}) = 0{,}014$
etc.