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# Ore Reserve Estimation at Western Deep Levels Gold Mine

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The auriferous Carbon Leader reef which is the subject of this study can be subdivided into two types of sedimentological units, each with its characteristic spatial variability. For the longwall mining method applied at Western Deep Levels gold mine, subdivision of the orebody improves the estimation of the actual error variance, but not the grade estimates themselves. This can only be achieved by choosing oreblocks of a size and shape that are maximal parallel to the longwall and minimal at right angles to it, within the limits of mining restrictions. While simple kriging is the only method that gives unbiased results, the difference between untransformed and logtransformed kriging is negligible.

## Introduction

Western Deep Levels Gold Mine is situated near Carletonville in the West Witwatersrand goldfield, Republic of South Africa (see Figure 1). Gently dipping tabular deposits are mined at different levels for gold, with the Carbon Leader Reef being the main deposit (see Figure 2).

The Carbon Leader is one of the richer gold reefs in the Witwatersrand basin[1] and is currently mined to a depth of 3 700 m below surface. It consists of conglomerates and quartzites overlying an angular unconformity.[2,3]

## Geology

This reef horizon is made up of two geological subfacies[2,3]:

- Subfacies 1, formed by fluvial action, which is characterized mainly by multiple conglomerate bands with intercalated quartzites, and a maximum cumulative thickness of 1,50 m. This subfacies occurs in north-south stretching, elongated zones throughout the mined area and represents palaeochannels. Their width varies from 200 m to 500 m.
- Subfacies 2 consists of a single conglomerate band with a thickness of between 0,02 m and 0,25 m. In large areas this conglomerate overlies the name-giving carbon seam, which consists of relics of proterozoic, primitive algae mats. The genesis of this subfacies is doubtful; sheetflood origin[2,4] or deposition as a basal conglomerate of the Witwatersrand Inland Sea transgressing over a fluvial plain are only two of the possibilities discussed.

Figure 3 shows an idealized geological section through the Carbon Leader reef and the resulting two subfacies.

## Problem and methods

The ore reserve estimation procedure which has been applied at the mine is based on an extrapolation of a trend in the average gold grade as observed.

Because the procedure is not conditionally unbiased, it was decided to try out various kriging methods.

At the same time, the influence which the subdivision of the mine area into sedimentological zones has on the estimation results was investigated.

The chip sample data commonly used in the Witwatersrand gold deposits[5] and taken over a mined out area of about 3,5 sq km at Western Deep Levels, were digitized (n = 36 160). As the orebody is a tabular deposit, and all of the reef thickness is mined, it was treated as two-dimensional, and hence gold accumulations (cm g/t values, i.e. gold per horizontal area) were used throughout. The data follow a three-parameter lognormal model.[6]

The mining method applied at Western Deep Levels is longwall mining because of the great depth. Estimating an oreblock therefore is based on data from one side of the block only, as illustrated in Figure 4a. Any valuation method applied has to extrapolate, rather than interpolate as is the usual case with other mining methods.

The size of the oreblocks that are estimated at the mine is = 100 m$^2$, as 100 m is about the average stope advance per year. The oreblock sides run parallel to strike and dip of the deposit, so that the oreblock grid is tilted 15° against north.

It was decided to try various kriging methods on this oreblock size. This was done over the mined-out area, using only data from an area of 500 m × 500 m on one side of the oreblock, thus simulating the longwall mining situation (Figure 4a). Although the longwalls actually con-

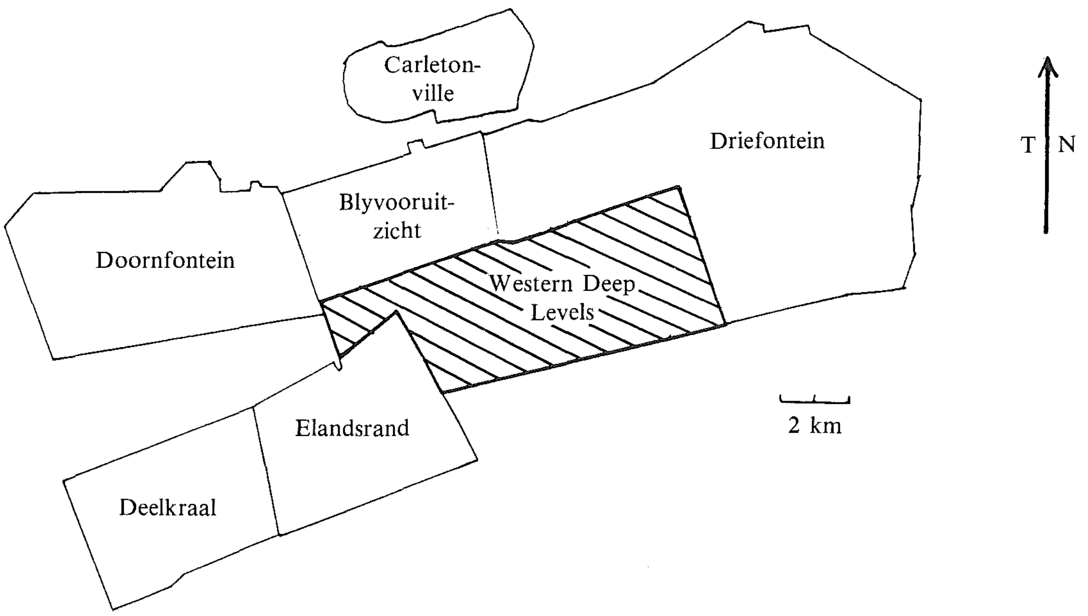


FIGURE 1. Mine lease areas in the Carletonville Gold Field. The study area is hatched.

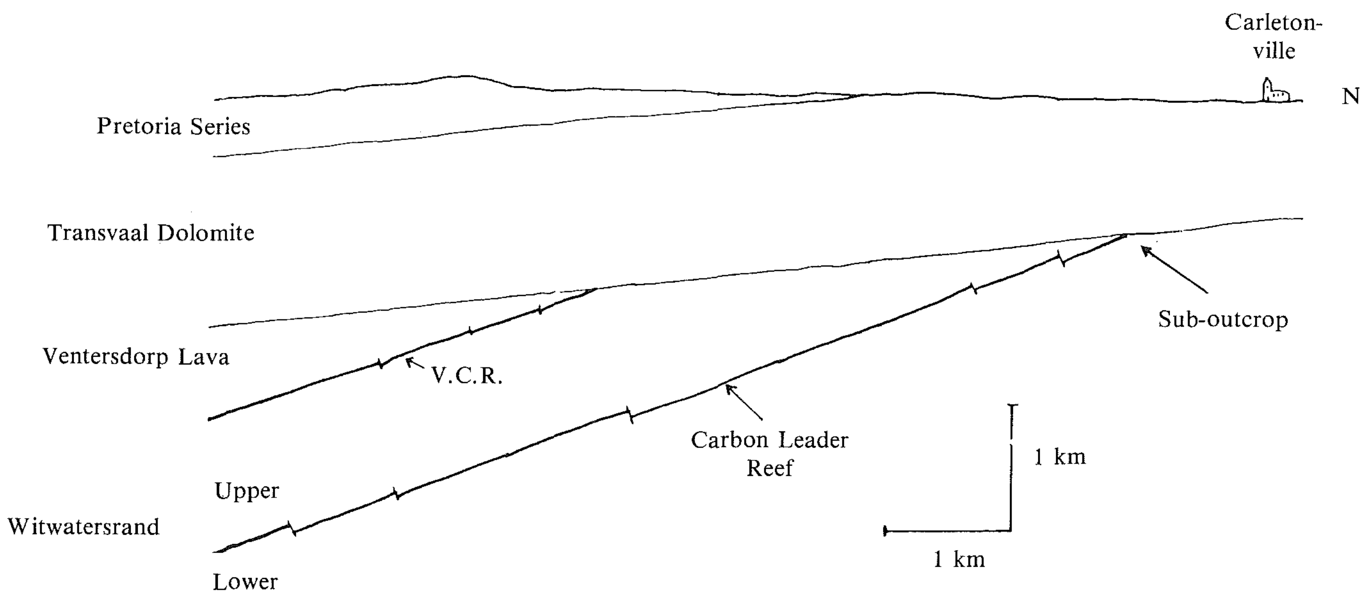


FIGURE 2. Geological section through the Carletonville Gold Field.

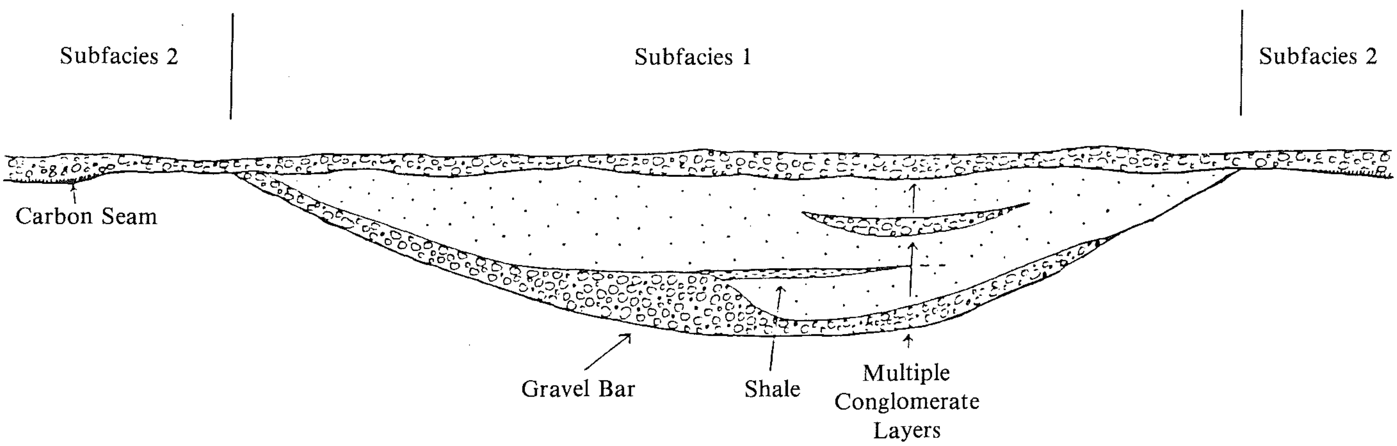


FIGURE 3. Idealised sedimentological section through the Carbon Leader Reef.

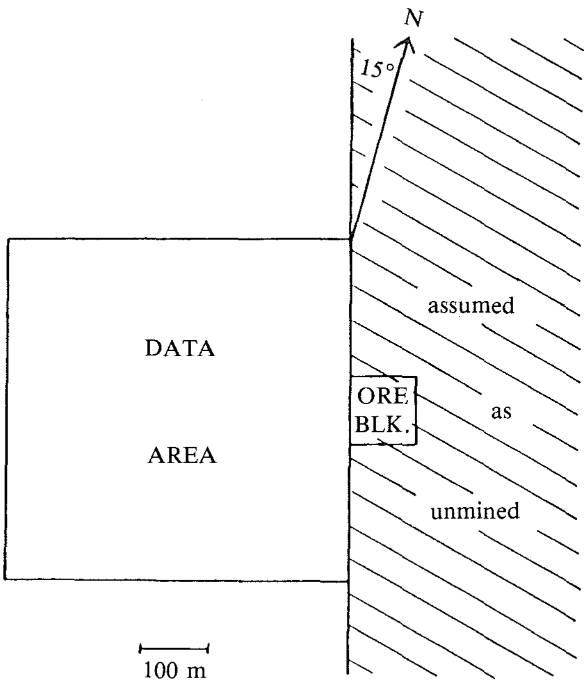


FIGURE 4a. Simulated longwall mining situation.

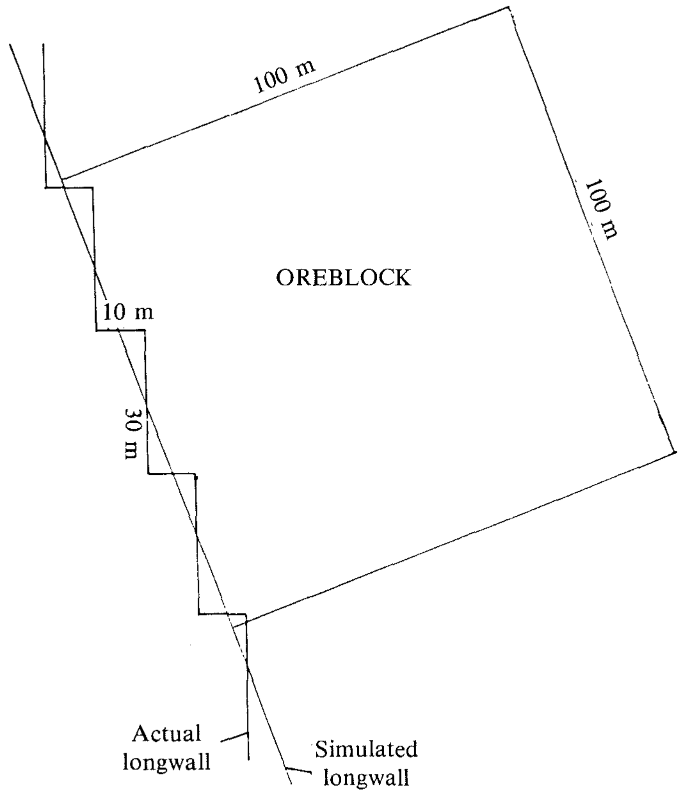


FIGURE 4b. Difference between actual and simulated longwall mining.

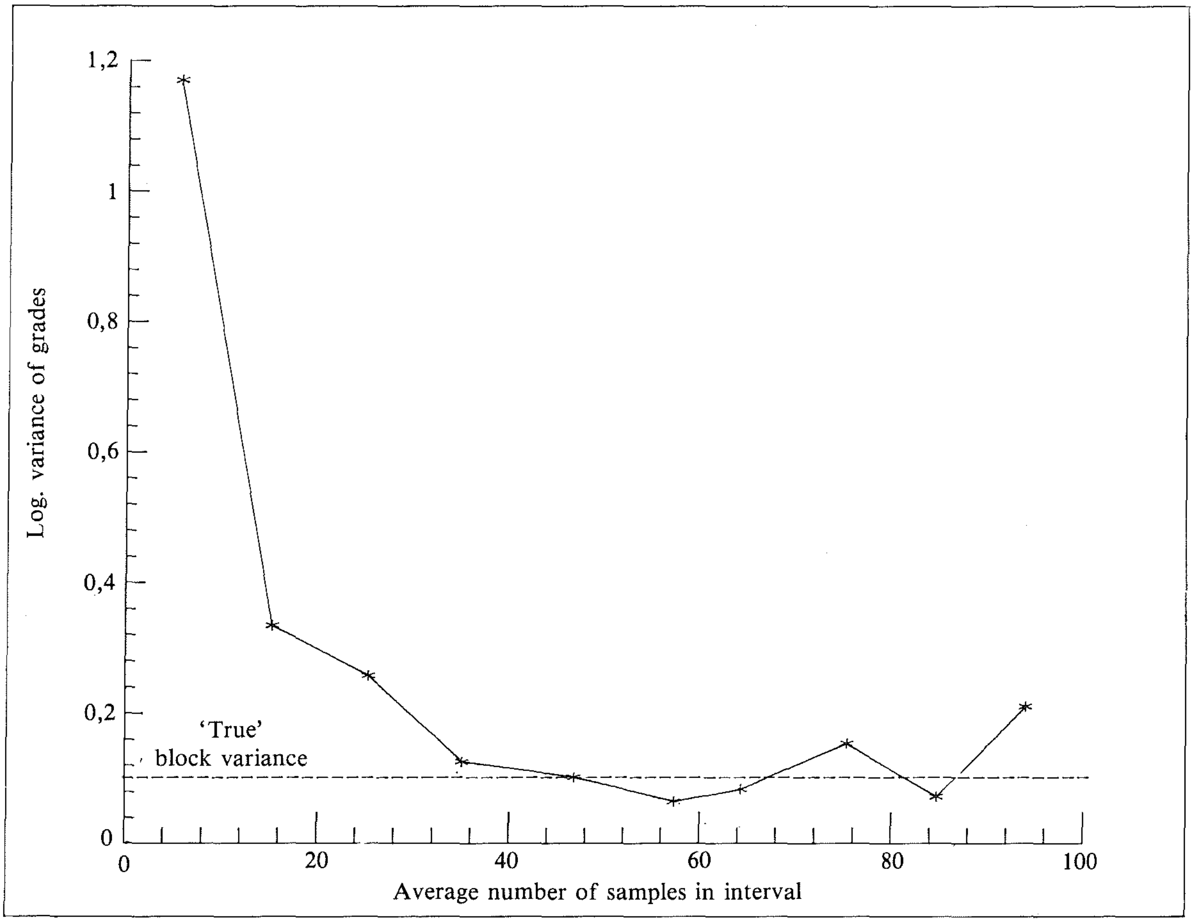


FIGURE 5. Plot of variance vs number of samples, class interval = 10 (100 m × 100 m) blocks

sist of 30 m long stopes which are displaced stepwise by about 10 m, the longwall is simulated as a straight line because of advantages in computing time (see Figure 4b). It is felt that the difference is negligible.

Because the area is mined, follow-up data are available for the estimated blocks.

Blocks containing more than a minimum number of follow up samples were then included in a regression analysis of the results. This 'cut-off' minimum number of samples was determined by grouping the blocks according to the number of samples they contain. The logarithmic variance of the block averages in each group was then plotted against the average number of samples in the group. These logarithmic variances lie between the variance of the samples (if n = 1) and the variance of the 'true' block values as read off the variance-size of area graph[7] or calculated from the logtransformed semivariogram (n = infinite[8]). With increasing number of samples the block variance will decrease and eventually approach the 'true' block variance. The minimum number of samples a block must contain so that the average of the follow-up samples gives a reliable estimate of the true block grade was chosen to be a level where the block variance reduces to within 10% of the calculated 'true' block variance.

Figure 5 illustrates this relationship for a subarea of the mine. In this case oreblocks with more than 40 follow-up samples were included in the regression analysis.

## Subdivision of study area

As the ore control of the Carbon Leader reef is sedimentary,[2,3] a geostatistical analysis should consider the sedimentology.[9–11] Because qualitative and semi-quantitative geological information cannot be included in the numerical estimation process, geology is used to subdivide the area under investigation into geostatistically homogeneous zones with similar variability of ore in each zone.

The criteria for subdividing are selected with practicality in mind. It will be of advantage if the subdivisions could be determined on a straightforward routine basis; it is essential that extrapolation of the zones into the unmined areas can be effected with reasonable accuracy.

Furthermore, the zones should neither be too small, as the data will then be inadequate for the semivariogram calculation, nor should the zones be so large that an overall semivariogram does not allow for local changes in variability.

In the case of the Carbon Leader reef a subdivision into Subfacies 1 areas (S1) and Subfacies 2 areas (S2) is advisable. Because of different depositional processes the two reef types are expected to have different variability patterns; as the palaeochannels seem to be fairly linear, their courses can be extrapolated into unmined areas. With widths of 200 m and more the zones are large enough to allow calculations of reliable semivariograms for use in the block estimation process.

## Data analysis

Table 1 gives the basic statistical parameters of the grades in the study area and the subareas. For reasons of confidentiality no true grades are given, but only the difference in grades of the subareas from the overall mean grade.

All subarea grade populations follow a three-parameter lognormal model. There is no evidence that the grade in one subfacies is consistently higher than in the other. The variability as expressed through the coefficient of variation, however, is higher in Subfacies 1 than in Subfacies 2.

A logtransformed semivariogram calculated on all data (hereafter called overall semivariogram, see Figure 6) shows the presence of a geometric anisotropy. The main axes coincide with the palaeoflow direction of the channels, with the higher continuity of the grades in the general flow direction of north to south.

Logtransformed semivariograms of the individual subareas (local semivariograms) show that for Subfacies 1 the spatial variability pattern is anisotropic, again with the high continuity in the palaeoflow direction. For Subfacies 2, the semivariograms are isotropic in all sub areas.

All fitted models are nested structures consisting of a nugget effect, a spherical model with a short range to accommodate a steep rise in variability for shorter lags, and an exponential model with a larger range. The overall sill in each case equals the estimated logarithmic population variance.

Table 2 shows for comparison some characteristics of the semivariograms. The local semivariograms of Subfacies 1 display the above-mentioned anisotropy, a higher overall sill and a ratio of the sill of the exponential structure (C2) to the sill of the spherical structure (C1) in the

TABLE 1. Basic statistics of study area and subpopulations

| | Mean grade difference % | b | Skewness | n | $LV_b^2$ | CoV |
|---|---|---|---|---|---|---|
| Mine | 0 | 170 | 0,00040 | 36160 | 1,0350 | 1,4080 |
| S1.1 | −29,6 | 112 | 0,00080 | 3996 | 1,0559 | 1,4161 |
| S1.2 | 8,8 | 211 | −0,00007 | 1575 | 1,1378 | 1,5309 |
| S2.1 | 14,9 | 171 | 0,00050 | 9404 | 0,9360 | 1,2941 |
| S2.2 | −10,6 | 230 | −0,00050 | 4611 | 0,6800 | 1,0543 |
| S2.3 | 19,0 | 289 | −0,00050 | 5519 | 0,9520 | 1,3426 |

S1.1: Subfacies 1, subarea 1; S1.2: Subfacies 1, subarea 2; S2.1: Subfacies 2, subarea 1; S2.2: Subfacies 2, subarea 2; b: additive constant; n: number of samples, CoV: coefficient of variation = standard error/mean; $LV_b^2$: log. variance of three-parameter log. model.

TABLE 2. Semivariogram features

| | % CO | C1/C2 | A1 N−S | A1 E−W | A2 N−S | A2 E−W | Log. variance (overall sill) |
|---|---|---|---|---|---|---|---|
| Mine | 53 | 2,10 | 27 | 15 | 90 | 45 | 1,0928 |
| S1.1 | 53 | 3,31 | 20 | 10 | 100 | 20 | 1,1004 |
| S1.2 | 40 | 3,60 | 20 | 15 | 60 | 30 | 1,2071 |
| S2.1 | 49 | 0,98 | 15 | 15 | 50 | 50 | 0,9838 |
| S2.2 | 50 | 1,12 | 10 | 10 | 48 | 48 | 0,7474 |
| S2.3 | 61 | 0,86 | 15 | 15 | 40 | 40 | 1,0305 |

C0: Nugget effect
C1,A1: Sill, Range of spherical structure
C2,A2: Sill, Range (1/3) of exponential structure

range of 3,5.

For the Subfacies 2 subareas, there is no anisotropy, the overall sill is lower, and the ratio of the nested sills is close to 1,0.

The overall semivariogram parameters fall between the local semivariograms of the different subfacies.

The similarities within and differences between the two subfacies types justify the subdivision based on geology.

## Estimation of $(100\ m)^2$ oreblocks and regression analysis

Simple and ordinary kriging were tested both on an (SK, OK) and a logtransformed (SLK, OLK) basis. Initially universal kriging was tried as well, but the results were so discouraging (actual error variances were eight times those of OK/SK) that it was excluded from the analysis.

The error variances for untransformed kriging were transformed to the equivalent logarithmic error variances using the formula:

$$\text{variance} = (\text{mean})^2 * (\exp(\text{log. variance}) - 1)$$

The logarithmic variances were corrected for the additive constant 'b' through the relationship[7]:

$$(\text{mean})^2 * (\exp(\text{log. variance}) - 1) = (\text{mean} + b)^2 * (\exp(\text{log.variance}_b) - 1)$$

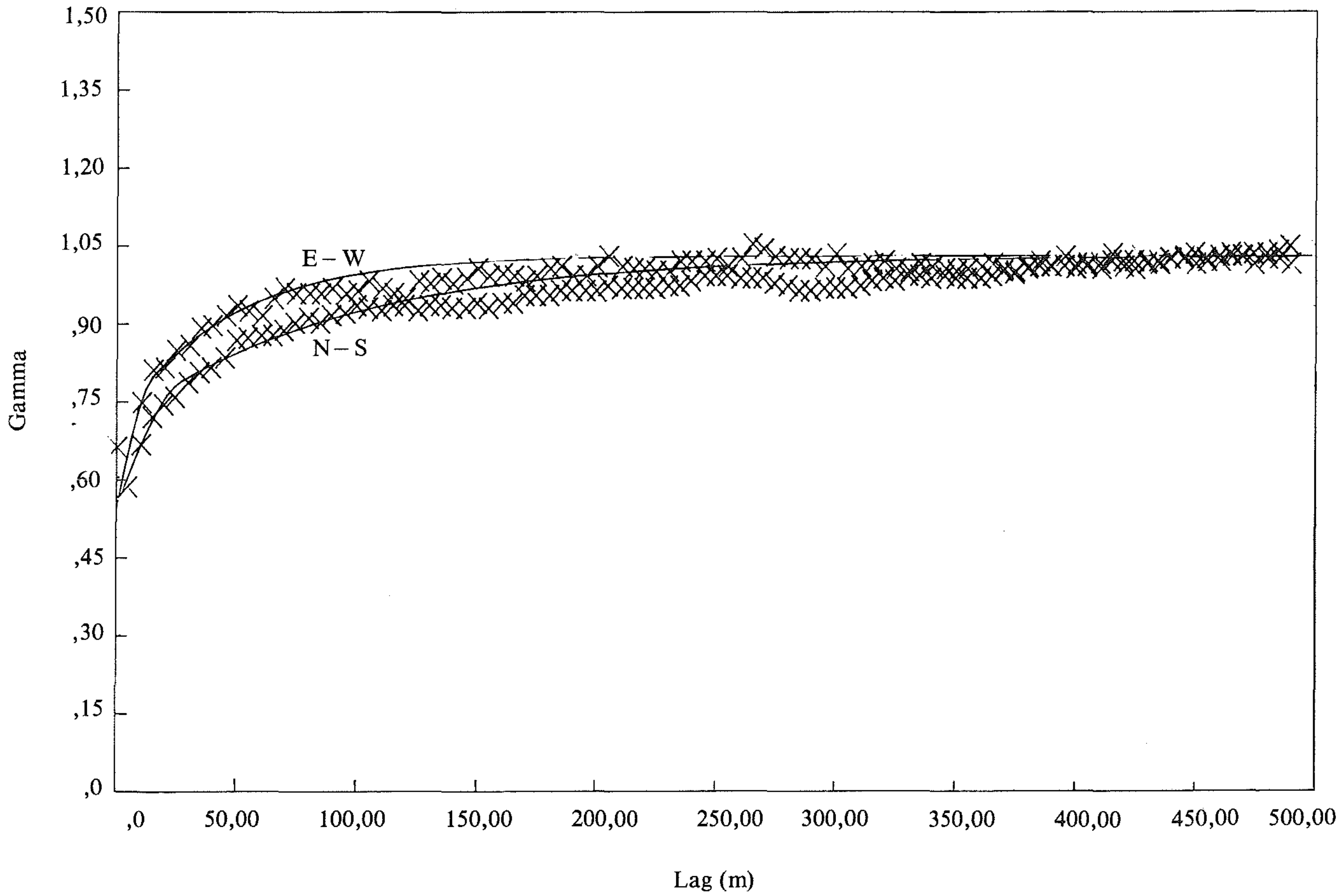


FIGURE 6. Overall logtransformed semivariogram.

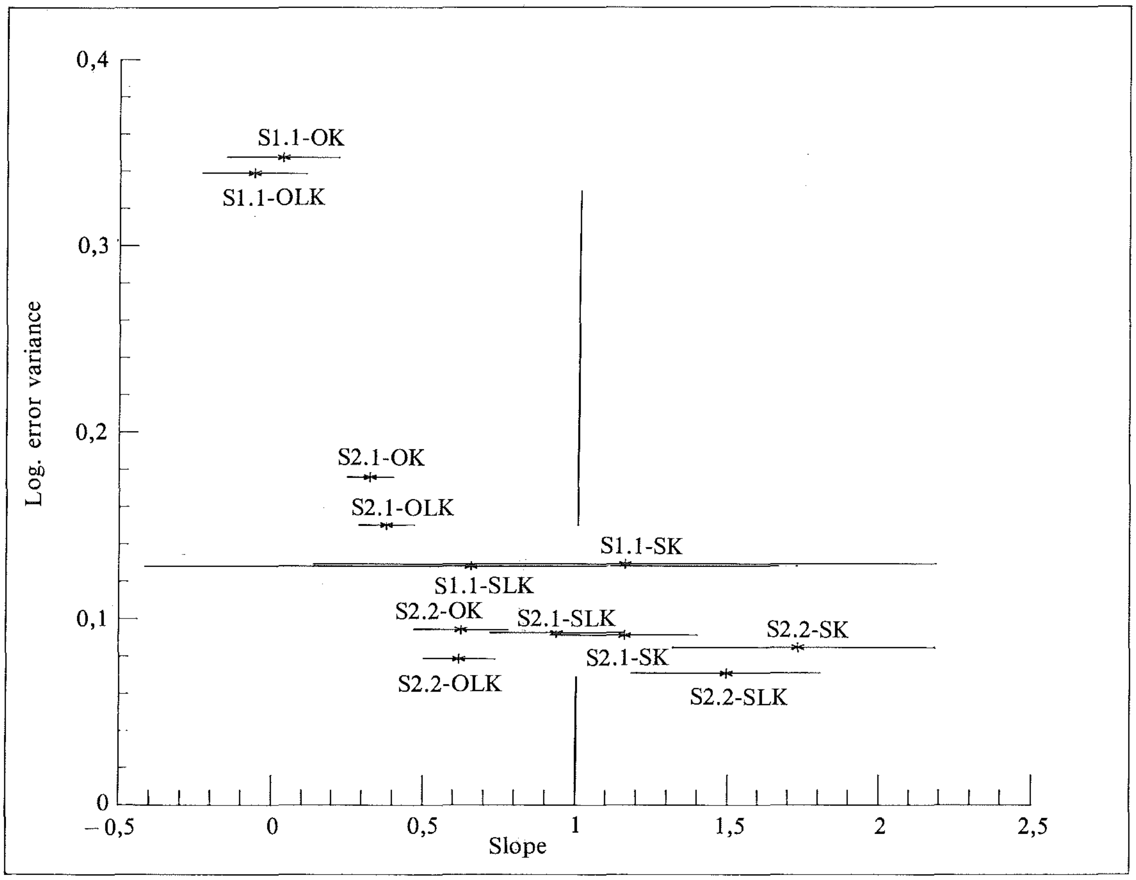


FIGURE 7a. Plot of log. error variance vs slope, local semivariograms (100 m × 100 m) oreblocks.

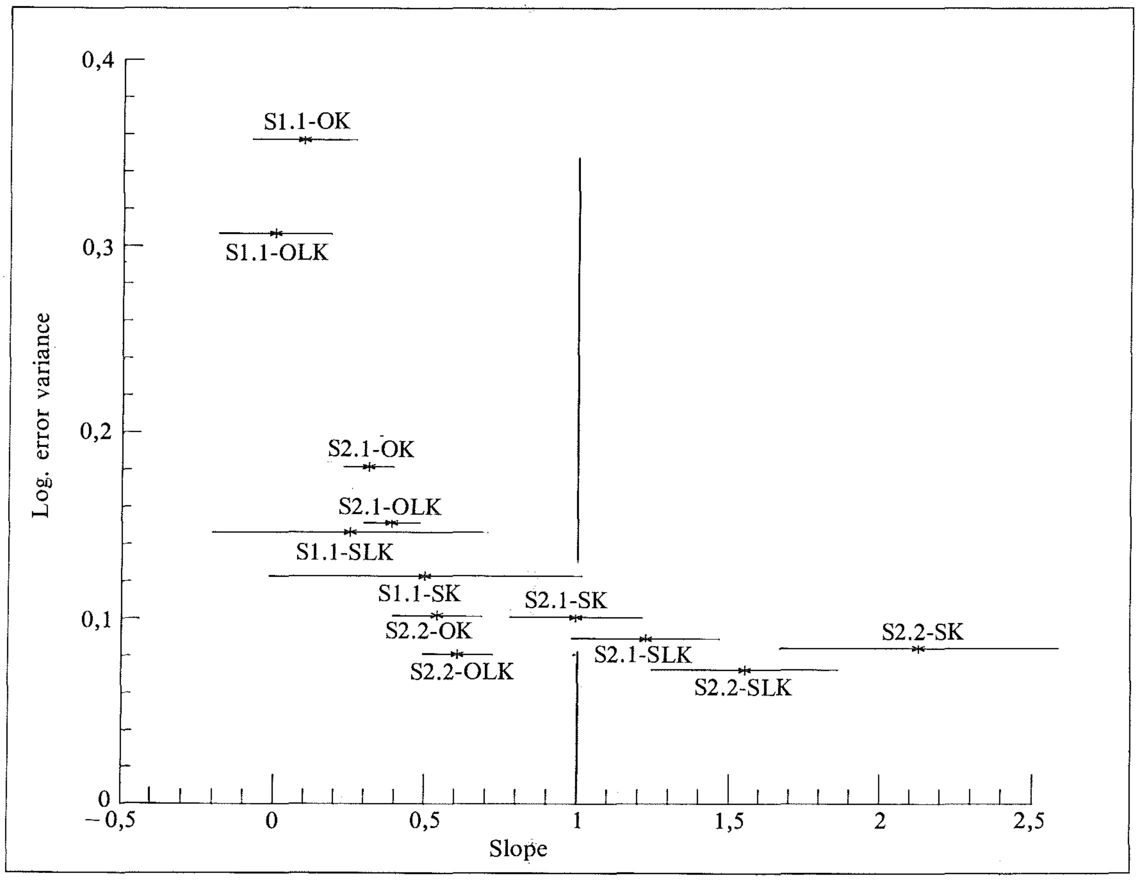


FIGURE 7b. Plot of log. error variance vs slope, overall semivariogram (100 m × 100 m) oreblocks.

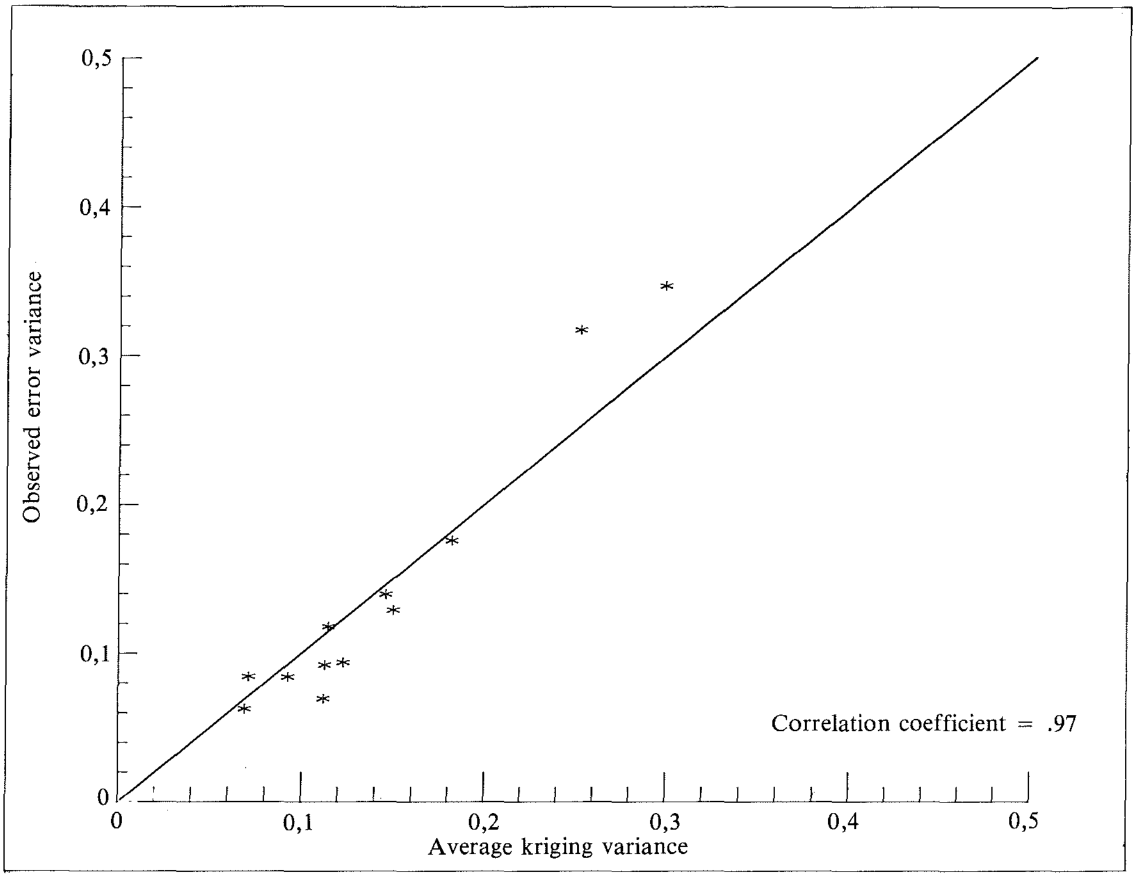


FIGURE 8a. Plot of average kriging variance vs observed error variance, local semivariograms (100 m × 100 m) oreblocks.

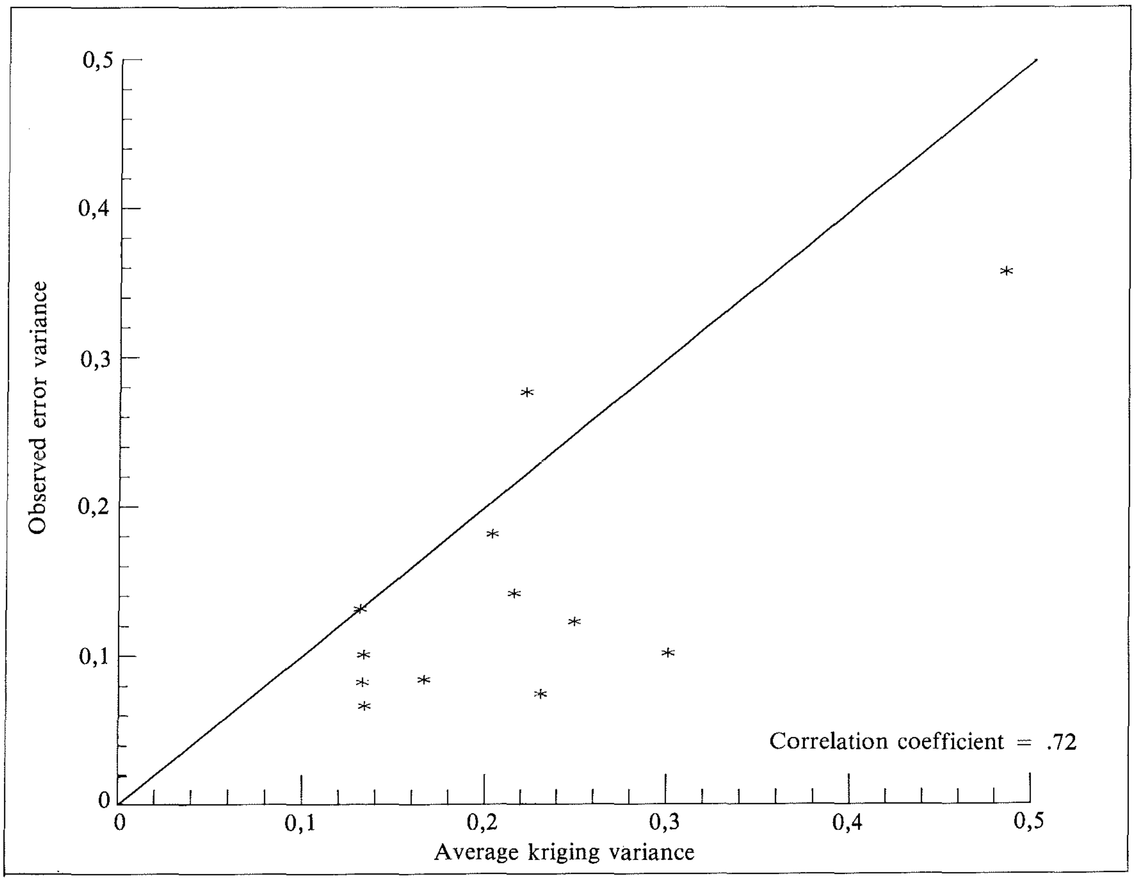


FIGURE 8b. Plot of average kriging variance vs observed error variance, overall semivariogram (100 m × 100 m) oreblocks.

These transformations made the variances comparable.

To investigate the improvement that subdivision has on the estimation, the overall and local semivariograms were used for kriging. For untransformed kriging, a back-transformed semivariogram[12] was used.

For three of the sub-areas, the regression analysis of the 'true' follow-up grades on their estimated grades was done; the other areas are too small to give a sufficient number of follow-up blocks. The regression for subarea S1.1 is based on 34 pairs, for S2.1 on 85 pairs, and for S2.2 on 53 pairs.

In Figures 7a and 7b the slope of the linear regression line is plotted against the logarithmic error variance for the three subareas and the different kriging methods. The horizontal bars measure the standard error of each slope. This error is larger for simple kriging, as its estimates group closely around the mean, and the slope therefore is very sensitive to larger differences.

This comparison shows that:

- there is no significant difference between using the overall or local semivariogram, although the local semivariogram tends to give slightly better results:
- there is no significant difference between untransformed and lognormal kriging;
- simple kriging in this case is less biased than ordinary kriging, and gives lower error variances.

Since for mining reasons the longwalls are subparallel to the palaeochannels, in Subfacies 1 subareas the position of the samples is roughly east or west of the oreblock – the direction in which the spatial variability is highest. The error variances are therefore higher for Subfacies 1 oreblocks than for Subfacies 2 oreblocks.

In Figures 8a and 8b the observed logarithmic error variance (variance of the differences of estimate and true grade) of each method is plotted against the average kriging error variance calculated theoretically on the basis of the semivariogram. (Because of a pseudo-regular sampling grid, the kriging error variances are very similar for the oreblocks in each subarea. The average kriging variance should therefore represent the actual error variance.) The results of this comparison are:

- local semivariograms provide reasonable theoretical estimates of the actual errors;
- the overall semivariogram overestimates the actual errors.

The overall bias of the estimates (mean of estimates minus mean of follow-ups) is listed in Table 3. The local semivariograms in most cases give mean errors closer to zero than does the overall semivariogram.

In Figure 9 the different kriging methods are compared for the combined data. Except for ordinary lognormal kriging, the local semivariograms give slightly lower error variances. Kriging without the mean is still conditionally biased. Table 4 gives the deviation of the mean of the estimates from the mean of the follow-ups in percent of the last. The overall bias is only considerable for ordinary lognormal kriging.

This comparison shows that the advantages of subdivision do not show up clearly in a combined analysis, as the overall positive or negative bias, which the use of the overall semivariogram, and the overall mean in the case of SK and SLK, produces in the subareas is compensated.

TABLE 3. Mean error of different kriging methods (%) (100 m × 100 m) oreblocks.

| | Local SV | Overall SV |
|---|---|---|
| S1.1: | | |
| SLK | 1,4 | 34,0 |
| OLK | 33,8 | 26,1 |
| SK | 1,0 | 30,3 |
| OK | 8,7 | 9,4 |
| S2.1 | | |
| SLK | 0,2 | −10,3 |
| OLK | 5,6 | −11,9 |
| SK | 1,9 | − 7,7 |
| OK | 1,2 | 0,8 |
| S2.2 | | |
| SLK | −1,7 | 4,6 |
| OLK | 2,4 | 21,0 |
| SK | −1,2 | 0,5 |
| OK | −6,4 | − 5,5 |

## Choosing a blocksize for longwall mining

As seen above, simple kriging gives the best results. But still the error variances in the subareas vary between 85% and 105% of the 'true' block variances. This is due to the extrapolation as far as 100 m ahead of the face. The influence of the distance from the face on the estimation error is shown in Table 5. The (100 m × 100 m) oreblocks were subdivided into four sub-blocks of (100 m × 25 m) each, the sub-block boundaries lying parallel to the face. Instead of one block with its centre 50 m ahead of the face, there are now four sub-blocks with their centre 12,5 m, 37,5 m, 62,5 m and 87,5 m ahead of the face, as illustrated in Figure 10. A row of samples with 5 m spacing along the face, and another row 10 m back from the face was used to calculate the logarithmic estimation variance for each sub-block, using SLK. These estimation variances and their proportion of the 'true' block variance of (100 m × 25 m) blocks in percent are shown in Table 5 for the five subareas. The local semivariograms were used, as they give realistic estimation variances. The rapid increase of the error with increasing distance from the face is obvious, and the sub-block centred 37,5 m ahead of the face is already difficult to estimate, especially in the Subfacies 1 subareas. Beyond that distance accurate estimation seems impossible.

It was therefore decided to estimate only 50 m ahead

TABLE 4. Mean error of different kriging methods (%) (100 m × 100 m) oreblocks, combined data.

| | Local SV | Overall SV |
|---|---|---|
| SLK | −2,0 | −1,3 |
| OLK | 5,8 | 15,3 |
| SK | 0,8 | 0,2 |
| OK | 0,0 | 0,3 |

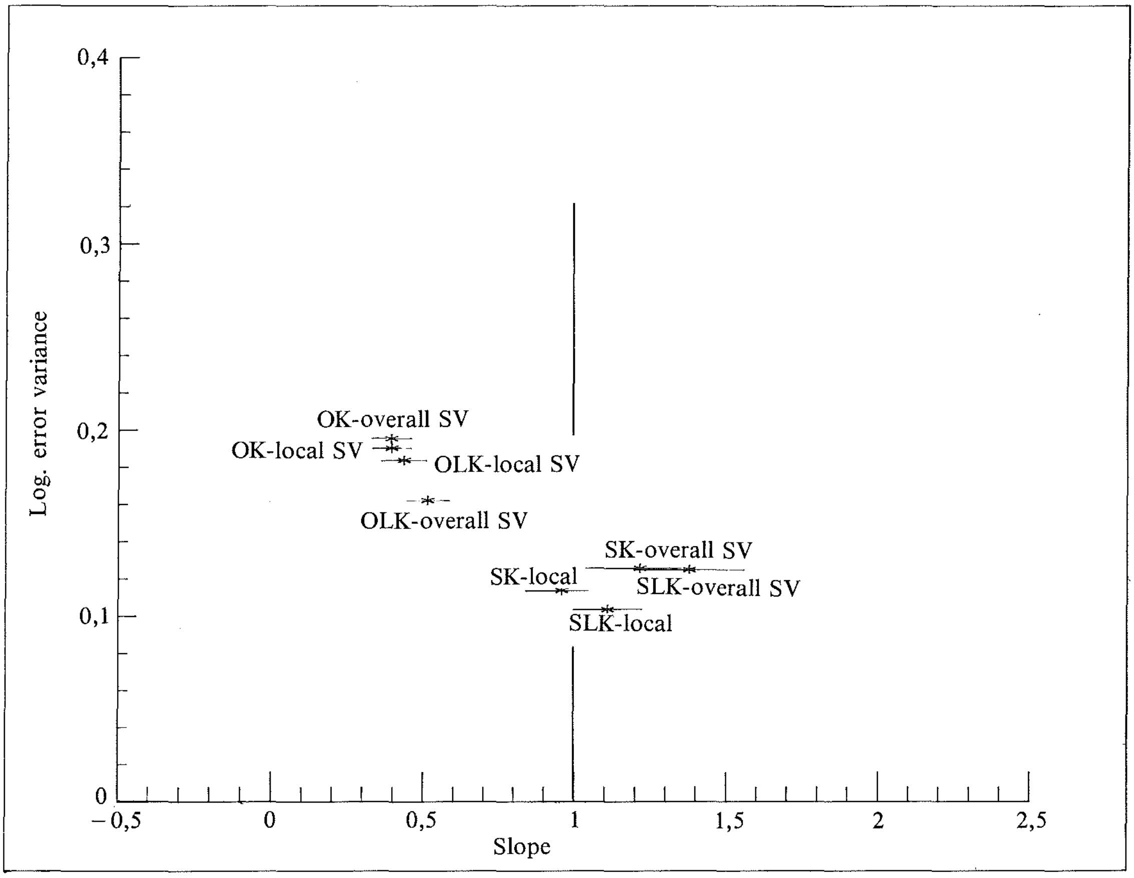


FIGURE 9. Plot of log. error variance vs slope, subareas combined (100 m × 100 m) oreblocks.

of the face. As the mining layout plans pillars each 200 m on a longwall, the block size was chosen as 200 m parallel to the face and 50 m ahead of it.

## Estimation of (200 m × 50 m) oreblocks

The kriging of (200 m × 50 m) blocks was then done for the three subareas. For the regression analysis, 27 pairs of blocks were available in subarea S1.1, 75 pairs in S2.1, and 36 pairs in S2.2. In Figure 11 the various kriging methods are compared.

It can be seen that the error variances decrease and, for simple kriging and simple lognormal kriging, now fall between 56% and 69% of the 'true' block variances.

Ordinary kriging is still conditionally biased. Again there is not much to choose between untransformed and logtransformed kriging.

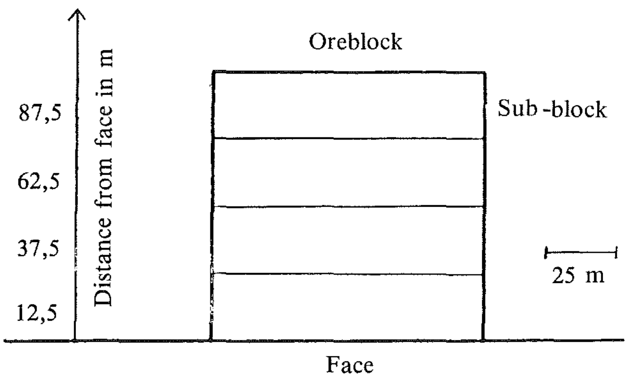


FIGURE 10. Orientation of (25 m × 100 m) sub-blocks in (100 m × 100 m oreblock.

TABLE 5. Log. estimation variance of (100 m × 25 m) blocks, estimated from face samples.

| | S1.1 | | S1.2 | | S2.1 | | S2.2 | | S2.3 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Distance from face | Var. | % | Var. | % | Var. | % | Var. | % | Var. | % |
| 12,5 m | 0,122 | 53 | 0,112 | 36 | 0,051 | 36 | 0,034 | 34 | 0,043 | 46 |
| 37,5 m | 0,221 | 95 | 0,259 | 84 | 0,100 | 71 | 0,070 | 70 | 0,746 | 81 |
| 62,5 m | 0,230 | 99 | 0,297 | 97 | 0,123 | 88 | 0,088 | 88 | 0,086 | 93 |
| 87,5 m | 0,231 | 100 | 0,305 | 99 | 0,134 | 95 | 0,095 | 95 | 0,091 | 98 |
| True block variance: | 0,231 | 100 | 0,307 | 100 | 0,140 | 100 | 0,100 | 100 | 0,092 | 100 |

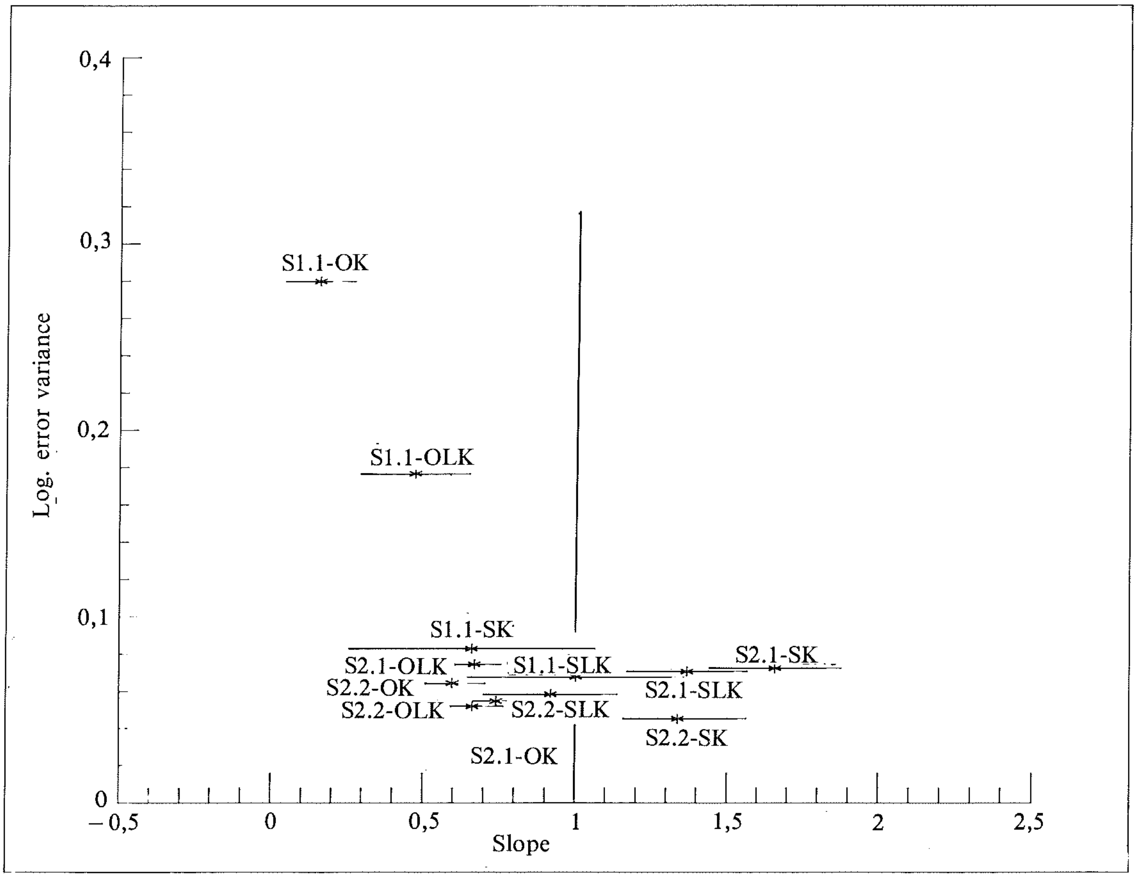


FIGURE 11. Plot of log. error variance vs slope, Local semivariograms (200 m × 50 m) oreblocks

## Conclusion

On the basis of this study, the following conclusions can be drawn for longwall mining in the Carbon Leader reef:

- Subdivision into geostatistically homogeneous subareas improves the overall unbiasedness of the estimates in the individual subareas.
- Conditional unbiasedness does not seem to be affected by subdivision.
- Subdivision also yields acceptable kriging errors, indicating that the local semivariograms are stationary throughout the subarea.
- Simple kriging gives conditionally unbiased estimates.
- Logtransformed and untransformed kriging give similar results.
- The extrapolation problems associated with longwall mining make it advisable to extend the oreblock size perpendicular to the face to not more than a third of the range of influence of the semivariogram. At the same time, the blocksize parallel to the face should be extended to the maximum allowed by mining constraints.


## Acknowledgements

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