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Improved Control of Phosphate Flotation by means of a Simple Mineralogical Regression Model

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A simple mineralogical regression model is described, which was devised in order to help the flotation plant cope with rapidly changing headgrades and feed mineralogy.

Minerals in the feed were determined by means of quantitative X-ray diffraction and compared with plant performance parameters and flotation reagent consumption rates, using regression analysis.

A mineralogical regression model for predicting recovery served to motivate flotation plant personnel and also set standards whereby flotation performance could be measured. The reagent consumption model indicated in which direction any or all of five flotation consumption rates should be altered.

All practically measurable parameters, even those showing poor or no correlation, were included in the model, provision being made for the continuous update of a batch of data of 30 lines.

Compared with the method of multiple linear regressions, the r^2 is poorer at 0,41 (versus 0,50), but the mean standard error of estimate for both methods is identical at ± 6,6 units.

The regression model is simpler than the method of multiple linear regressions, and although the accuracy of prediction is not high (41%), the mineralogical regression model proved to be successful in practice.

Flotation efficiency, as measured by the mean ratio of head to tails grade, for the 9 month period after the mineralogical regression model was implemented on an hourly basis, improved from 2,8 to 3,2, which is equivalent to an improvement in average flotation recovery of 5% relative.

Introduction

Foskor produces a phosphate concentrate, (fluor-apatite), by means of a froth flotation process, using various flotation reagents. These reagents are expensive and contributed more than 20% to production costs in 1985.

Nearly half of the phosphate flotation capacity is utilized in the processing of tailings, originating from another company. In other words, no prior planning or ore blending in order to ensure a constant or known headgrade to the flotation plant has been possible.

Although the yearly average headgrade is reasonably constant, average daily headgrades are found to vary between wide limits, and the hourly feed mineralogy is even more variable. A 30% increase or decrease in headgrade over a period of only 24 hours is not uncommon.

Statement of problem

The flotation of apatite in the presence of calcite is a difficult process, requiring a combination of four or five chemical reagents, whose reaction with

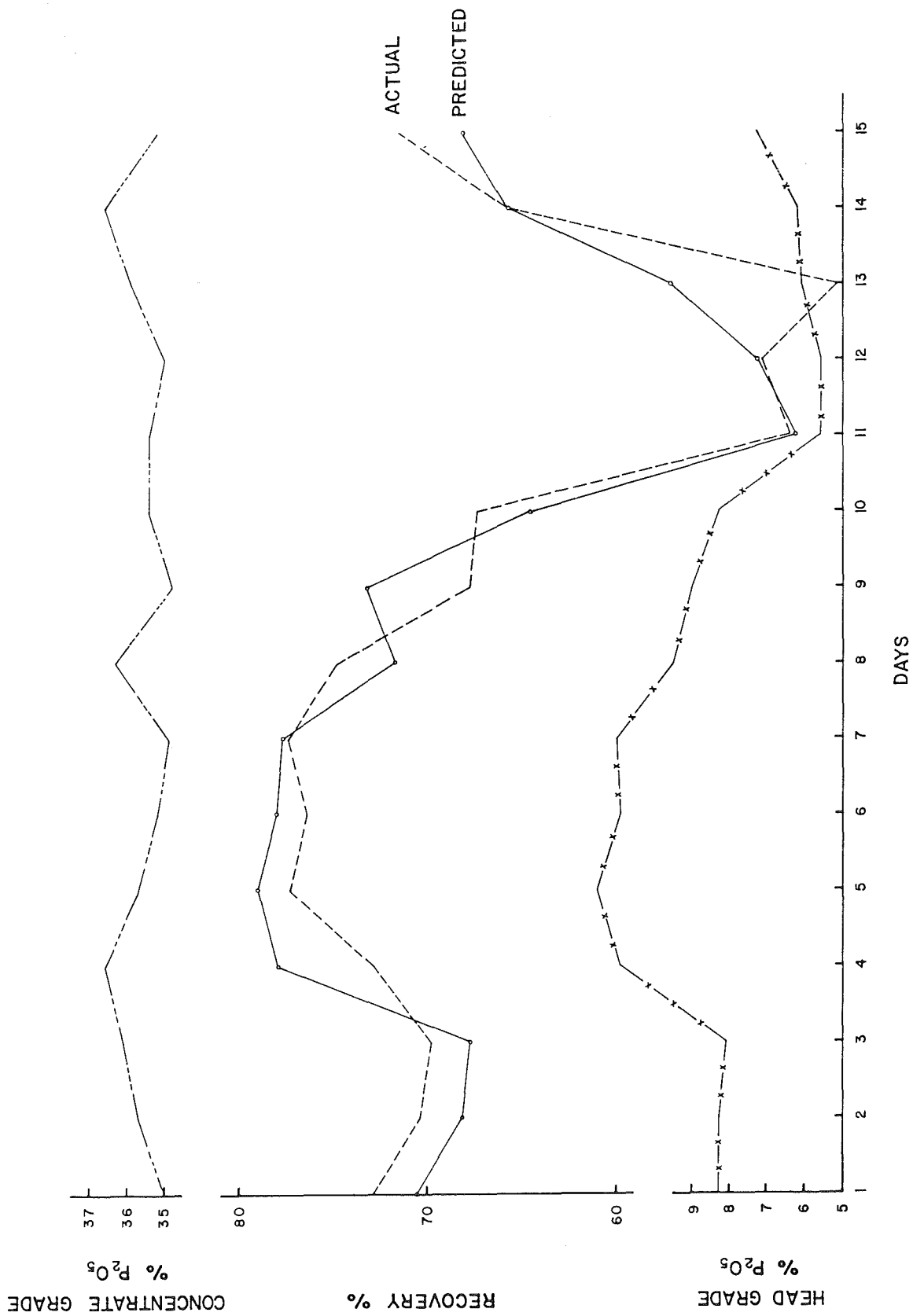

FIGURE 1. Typical flotation plant performance over a two-week period

one another and with the other gangue minerals is not yet fully understood[1,2]. Consequently, even during periods of relatively constant feed mineralogy, many minor adjustments have to be made to the reagent addition rate in order to optimize flotation efficiency, a process requiring a period of days, rather than hours, to effect.

The problem is compounded by the fact that chemical analyses of headgrade, combined tails and concentrate grades, are available only two hours after the pulp has been sampled, owing to the wet chemical method being used and logistic problems. Plant control is thus mainly in the hands of process controllers, who rely on the physical appearance and carrying power of the froth.

Figure 1 indicates daily headgrade, recovery and concentrate grade for a 2-week period plotted together with the recovery predicted from the feed mineralogy (treated in next section), normalized for the concentrate grade eventually produced.

On days 1 to 3, the headgrade remained relatively constant, and actual recoveries were better than the predicted recovery, but only on day 3 was the concentrate produced of an acceptable grade, i.e. it required 3 days to optimize flotation efficiency. On day 4 the headgrade suddenly improved by about 25%, reagent adjustments were not carried out quickly enough, 5% was lost on recovery and an unnecessarily high concentrate grade was produced. On days 5 to 7, the headgrade remained uniformly high, and the flotation plant managed to close the gap between predicted and actual recovery on day 7, also after 3 days of trial and error. Thereafter, headgrade deteriorated rather badly to day 12, but then slowly started to recover. The poor recovery on day 13 was a direct consequence of the plant superintendent's instruction to reduce reagent consumption rates, which were already unrealistically high on days 11 and 12, although the predicted and actual recoveries were then quite similar.

The problem is thus to narrow the gap between unpredictable changes in feed mineralogy and reaction by flotation plant, in adjusting reagent consumption rates. Clearly, flotation efficiency directly influences the Company's profitability.

Setting the standard — The mineralogical regression model for predicting flotation recovery

Obviously, flotation plant performance as depicted in Figure 1, without the line indicating potential or predicted recovery, is almost meaningless. Plant performance must be compared with a realistic standard.

Slurry feed to the flotation plant was sampled on an hourly basis, 24 hours per day and seven days per week. The minerals in the feed were determined by quantitative X-ray diffraction[3], using the matrix flushing method of Chung[4], where all the minerals are determined, so that no internal standard is necessary.

Average daily feed mineralogy was accumulated and linear regression analysis[5] was carried out, correlating the mineralogy with flotation recovery on the plant. Recoveries were normalized to a fixed concentrate grade by multiplying the actual recovery with the grade produced divided by 36,5. Some results in the form of a correlation matrix are given in Table 1.

As expected, recovery correlates best with the amount of apatite in the feed (r

TABLE 1. Correlation coefficient r for feed mineralogy and flotation recovery

	Phlogopite	Lizardite	Diopside	Calcite	Dolomite	Forsterite	Recovery
Apatite	-0,26	0,74	0,27	-0,70	-0,15	0,71	0,90
Phlogopite		-0,03	0,51	-0,36	-0,44	-0,34	-0,22
Lizardite			0,31	-0,63	-0,19	0,25	0,63
Diopside				-0,53	-0,44	-0,06	0,24
Calcite					0,09	-0,51	-0,68
Dolomite						0,12	-0,11
Forsterite							0,76

= 0,90 on a scale of ± 1 to 0, where 0 means no correlation). Also, certain relationships between the feed minerals are evident. Apatite, lizardite and forsterite correlate strongly with one another (r = 0,7), probably due to their natural association with one another in the Palabora Igneous Complex. Phlogopite and diopside are associated with each other, but do not have a significant effect on recovery. Calcite correlates negatively with apatite and all of the gangue minerals, except the other carbonate in the ore, namely dolomite. Significantly, calcite has the strongest negative influence on phosphate flotation recovery.

Figure 2 shows the results of plotting recovery against mineralogy, expressed as: log [Apatite - Total Carbonates/Total silicates] for a three-month period. The regression relationship is poor, with an r^2 of only 0,48 and standard error of estimate of ± 5,0 units. Regression constants a = 16,5 and b = 42,0 where y = a + bx. However, for the purpose of motivating the flotation plant to 'do better', it served its purpose. Daily

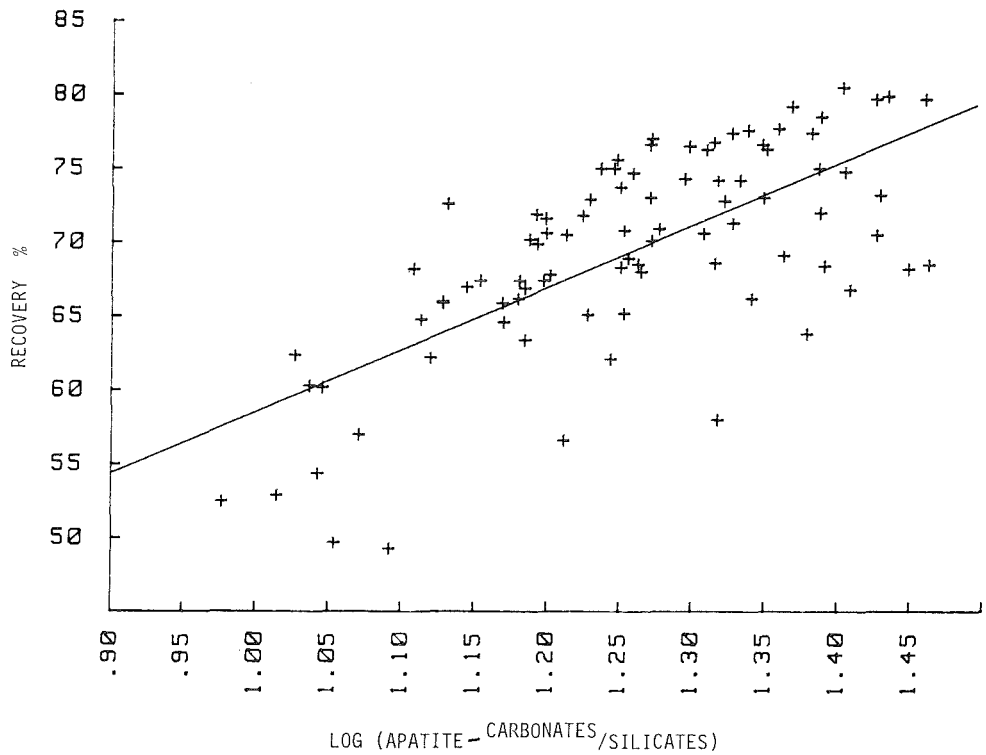

FIGURE 2. Relationship between flotation recovery and feed mineralogy

flotation recoveries were compared with the recovery predicted from the regression of the previous 3 months, which was later extended to 9 months, and 'gains' or 'losses' in concentrate tons were calculated. A cumulative tally of these 'tons' was kept.

Predicting flotation reagent consumption rates

Relationships between the feed parameters

Since a mineralogical regression model for reagent consumption can only be as effective as the data from which it is derived, some criteria were defined whereby optimal flotation efficiency could be recognized, and the daily average data were sifted accordingly.

In the first instance, all daily data where feed to the flotation circuits had been interrupted for whatever reason, i.e. the circulating and recirculating middlings fractions had been lost, were excluded. Any testing of non-standard flotation chemical reagents was excluded as well as the instances where the resultant concentrate grade fell outside certain limits. Finally, the average daily P_2O_5 content in the tailings was inspected and data excluded where these were outside acceptable limits. In this way, optimal flotation data and feed mineralogy for 30 days were accumulated, a process requiring a period of about 6 months. Linear regression analysis was then carried out.

Results of this programme are given in Tables 2 and 3. The mineralogy, grind (grain size), volume and pulp density of the incoming feed to the flotation plant are correlated with one another and with reagent consumption rates and recovery.

Referring to Table 2, certain trends are apparent:

(a) The positive correlation of the proportion of apatite (headgrade), with the volume of slurry received was rather surprising, since greater volumes passing through the flotation circuits usually reduce flotation retention times which should adversely affect flotation recovery. At the same time improved head grades lead to improved recovery.

(b) An increase in the amount of phlogopite in the feed is indicative of a coarser grind.

In Table 3, where the feed parameters are correlated with flotation reagent consumption rates and recovery, the positive correlation between the volume

TABLE 2. Correlation coefficient r for flotation feed parameters

	+425 micron	+300 micron	-212 micron	Volume	PD
Apatite	- 0,37	- 0,35	0,38	0,62	0,05
Phlogopite	0,53	0,44	- 0,26	0,02	- 0,13
Lizardite	- 0,25	- 0,24	0,27	0,44	0,05
Diopside	- 0,06	- 0,12	0,22	0,18	- 0,13
Calcite	0,00	0,05	- 0,17	- 0,48	0,05
Dolomite	0,00	- 0,00	- 0,06	- 0,22	0,07
Forsterite	- 0,22	- 0,15	0,11	0,44	0,12
+ 425 micron		0,93	- 0,81	- 0,13	0,35
+ 300 micron			- 0,85	- 0,12	0,36
- 212 micron				0,19	- 0,48
Volume					0,37

TABLE 3. Correlation coefficient r for feed parameters, flotation reagent consumption rates (cc/min) and recovery

	Waterglass	Caustic soda	Gum	Fatty acid	Polyglycol ether	Recovery
Apatite	0,57	- 0,18	0,40	0,53	- 0,76	0,90
Phlogopite	- 0,40	- 0,29	- 0,05	- 0,33	0,26	- 0,22
Lizardite	0,33	- 0,14	0,35	0,30	- 0,64	0,63
Diopside	- 0,01	- 0,05	0,06	- 0,14	- 0,08	0,24
Calcite	- 0,37	0,22	- 0,39	- 0,31	0,61	- 0,68
Dolomite	0,26	0,23	0,02	0,18	- 0,15	- 0,11
Forsterite	0,62	- 0,04	0,48	0,61	- 0,64	0,76
+ 425 micron	- 0,38	- 0,16	- 0,13	- 0,15	0,15	- 0,34
+ 300 micron	- 0,36	- 0,25	- 0,10	- 0,08	0,12	- 0,27
- 212 micron	0,20	0,05	- 0,02	0,01	- 0,08	0,22
Volume	0,27	0,12	0,33	0,15	- 0,45	0,56
Pulp density	0,18	0,38	0,35	0,26	- 0,27	0,16

of slurry received and flotation recovery must be due to the association of higher headgrades with larger volumes noted from Table 2.

The proportions of the various minerals in the feed also correlate to a greater or lesser degree with reagent consumption rates. For example, an increase in the proportion of apatite in the feed increases with the waterglass, gum and fatty acid consumption rates, whereas the consumption of polyglycol ether is significantly (r = -0,8) reduced, for optimal flotation efficiency. For the mineral calcite, the converse occurs with consumption of polyglycol ether tending to increase (r = 0,6) as the proportion of calcite increases in the feed.

The effect of grain size (grind) is not so marked, the minerals themselves probably masking its influence. However, coarser feed requires generally less flotation reagent due to reduced surface area, but recovery tends to decline.

The pulp density of the feed is measured at the receiving station and is a rough indication of tonnes of ore throughput. Reagent consumption rates tend to increase, except for polyglycol ether.

For the purpose of predicting flotation reagent consumption rates, it was decided to limit the feed parameters to mineralogy and pulp density, since only these data were available on an hourly basis.

The mineralogical reagent consumption regression model

In any mineral processing plant, particularly flotation, processes and procedures do not remain static for a long period of time, for example new flotation reagents may be used, the grind may change, ore throughput may change, or a desliming procedure may be introduced. It was thus thought essential to develop a regression model with an in-built continuous update. The model must also incorporate as many practically measurable parameters as possible, although some do not correlate particularly well in the present batch of data.

It is also vital to preserve credibility with the process controller, and a forecast of a recovery of 102%, or a negative reagent consumption rate, is not

TABLE 4. Linear regression constant b (in $\dot{y} \doteq a + bx$) for reagent consumption rates multiplied by pulp density versus minerals

	Pd x WG	Pd x CS	Pd x Gum	Pd x FA	Pd x PGE
Apatite	1,54	- 0,159	2,49	0,343	- 0,148
Phlogopite	- 0,823	- 0,199	- 0,289	- 0,165	0,035
Lizardite	1,78	- 0,242	4,30	0,384	- 0,247
Diopside	- 0,084	- 0,073	0,362	- 0,132	- 0,026
Calcite	- 0,437	0,092	- 1,04	- 0,089	0,054
Dolomite	0,777	0,220	0,157	0,131	- 0,030
Forsterite	2,41	- 0,049	4,34	0,565	- 0,173
Min.	16,3	- 1,53	27,5	3,33	- 1,37

easily forgiven or forgotten. The regression model tends towards conservative and average predictions rather than obviously incorrect forecasts.

The regression model was calculated as follows:

1. The determination of the daily average reagent consumption rates adjusted for variation in ore throughput was done by multiplying the daily average reagent consumption rate (in cc/min) by the pulp density. This figure of reagent consumption, adjusted for pulp density, is used in the rest of the calculation.
2. Linear regression constants were determined for each of the minerals versus each reagent consumption rate, separately.
3. Slopes, i.e. b values in $y = a + bx$, were divided by the value for apatite.

   The mineral regressions were combined for each reagent consumption rate.

5. A new regression with the combined minerals versus reagent consumption rate was determined, from which reagent consumption rates are then predicted.

Table 4 gives the linear regression constant b (in $y = a + bx$), for each of the minerals separately, and for the minerals combined as indicated above.

In Table 5 correlation coefficients are given for the regression relationships of Table 4. Comparing Tables 5 and 3, it can be seen that the inclusion of pulp density as a measure of incoming feed

TABLE 5. Correlation coefficient r for pulp density multiplied by reagent consumption rate versus minerals

	Pd x WG	Pd x CS	Pd x Gum	Pd x FA	Pd x PGE
Apatite	0,56	- 0,17	0,38	0,52	- 0,78
Phlogopite	- 0,40	- 0,29	- 0,06	- 0,33	0,24
Lisardite	0,33	- 0,13	0,33	0,29	- 0,65
Diopside	- 0,02	- 0,06	0,04	- 0,15	- 0,10
Calcite	- 0,36	0,23	- 0,36	- 0,30	0,63
Dolomite	0,26	0,22	0,02	0,18	- 0,14
Forsterite	0,62	- 0,04	0,47	0,60	- 0,64
Min.	0,74	- 0,35	0,56	0,61	- 0,83

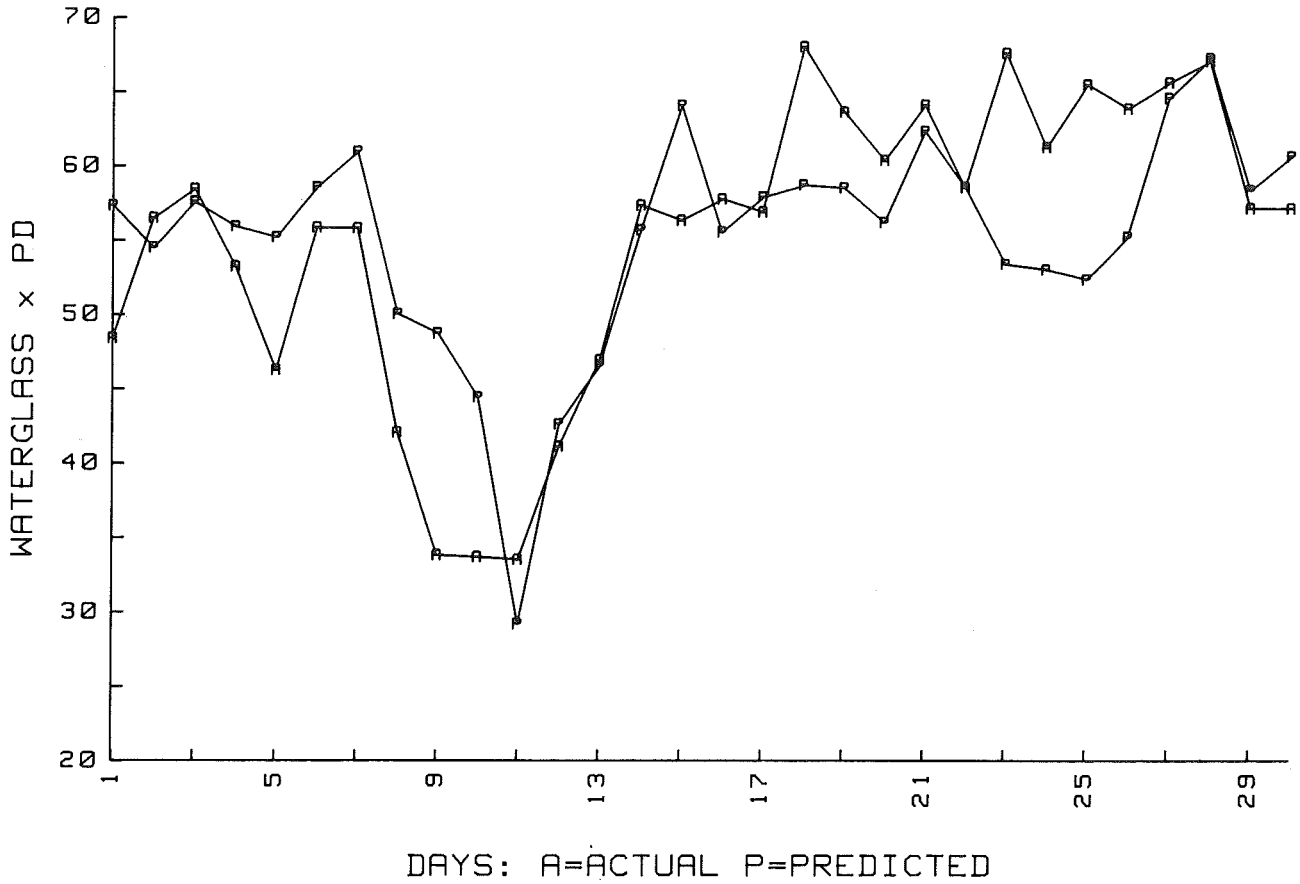

FIGURE 3. Actual vs predicted consumption of waterglass

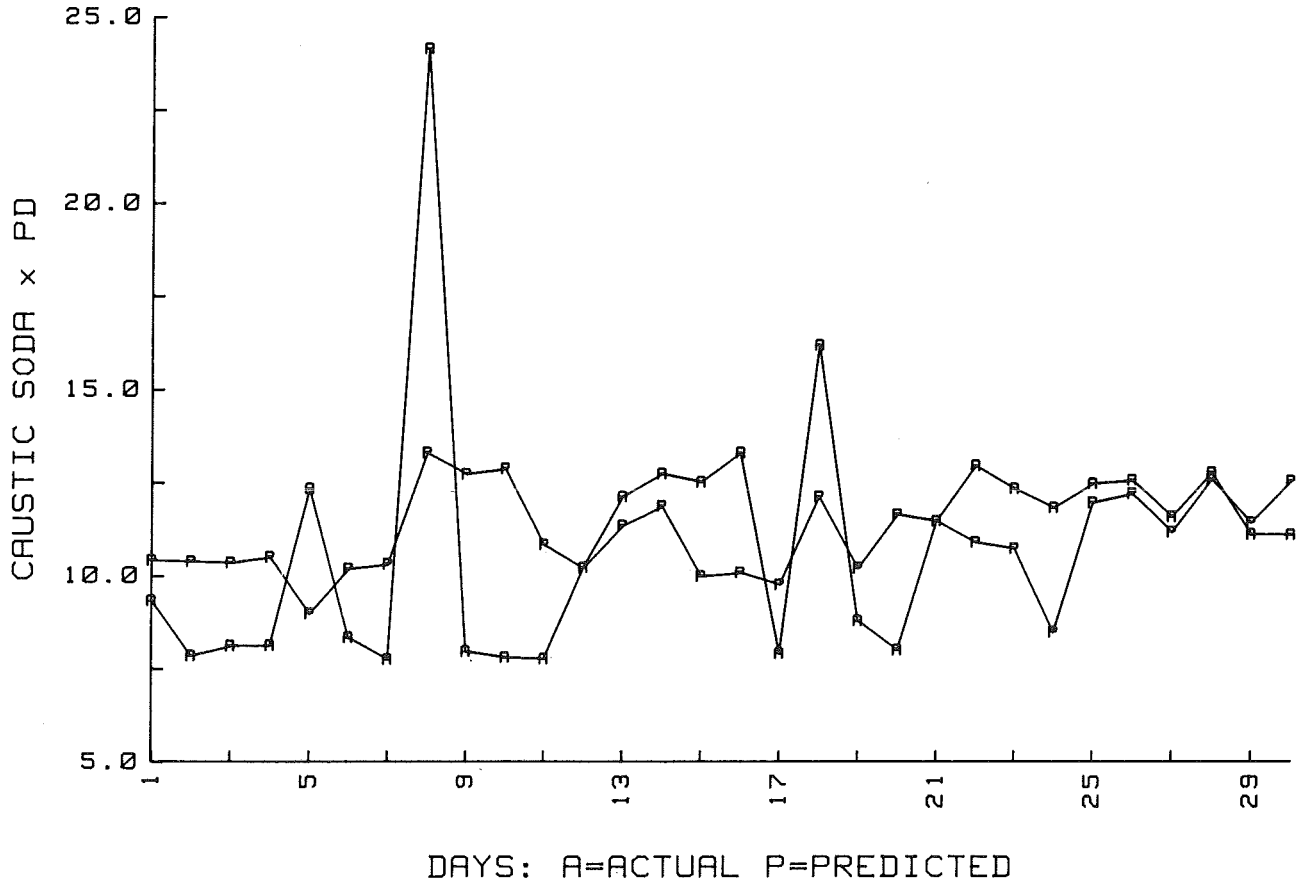

FIGURE 4. Actual vs predicted consumption of caustic soda

with the reagent consumption rate has not reduced the correlation coefficients appreciably, and in some cases has even improved them slightly. See, for example, the relationship between polyglycol ether and the amount of calcite in the feed: r = 0,61 without pulp density, and r = 0,63 with pulp density.

From Table 5, the inclusion of all the minerals in the model, even those showing no correlation, for example, diopside, where r varies between - 0,02 and - 0,15, has not weakened the overall regression relationship. The correlation between Min and reagent consumption rate multiplied by pulp density is in every case higher than for a single mineral on its own. For example, the polyglycol ether multiplied by pulp density consumption rate versus apatite in the feed has an r of - 0,78, but when all the minerals are combined, r is improved to - 0,83.

The equations for predicting reagent consumption rates reduce to the following form:

$$WG = [34,3 + 16,3\ A - 0,870\ P + 18,9\ L - 0,88\ Di - 4,63\ C + 8,22\ Do + 25,4\ F]\ PD^{-1}$$

$$CS = [11,5 - 1,53\ A - 1,91\ P - 2,33\ L - 0,702\ Di + 0,885\ C + 2,11\ Do - 0,471\ F]\ PD^{-1}$$

$$G = [141,8 + 27,5\ A - 3,19\ P + 47,6\ L + 3,99\ Di - 11,5\ C + 1,73\ Do + 47,8\ F]\ PD^{-1}$$

$$FA = [5,28 + 3,33\ A - 1,60\ P + 3,73\ L - 1,28\ Di - 0,866\ C + 1,27\ Do + 5,49\ F]\ PD^{-1}$$

$$PGE = [7,14 - 1,37\ A + 0,323\ P - 2,29\ L - 0,241\ Di + 0,500\ C - 0,278\ Do - 1,60\ F]PD^{-1}$$

where WG = Waterglass (sodium silicate) x 10^{-3} cc/min
CS = Caustic soda x 10^{-3} cc/min
G = Gum x 10^{-3} cc/min
FA = Fatty acid x 10^{-3} cc/min
PGE = Polyglycol ether x 10^{-3} cc/min
A = Fluor-apatite % in feed
P = Phlogopite % in feed
L = Lizardite % in feed
Di = Diopside % in feed
C = Calcite % in feed
Do = Dolomite % in feed
F = Forsterite % in feed
and PD is pulp density measured in g/ml.

In Figures 3 to 7, predicted and actual consumption rates for the five flotation reagents are shown for 30 observations.

Mineralogical regression model versus the method of multiple linear regressions

It is interesting to compare results between the mineralogical regression model with those obtained from the method of multiple linear regressions[5] on the same series of data. Using multiple linear regressions reagent consumption rates reduce to the following equations:

$$WG = [-908,7 + 9,97\ A + 9,19\ P + 9,31\ L + 10,05\ Di + 9,44\ C + 9,80\ Do + 11,1\ F]\ PD^{-1}$$

$$CS = [-41,2 - 0,20\ A + 0,23\ P + 1,16\ L + 1,21\ Di + 0,65\ C + 0,65\ Do + 1,12\ F]\ PD^{-1}$$

$$G = [-2\ 078 + 20,0\ A + 23,0\ P + 26,7\ L + 22,9\ Di + 22,4\ C + 21,7\ Do + 28,8\ F]\ PD^{-1}$$

$$FA = [-106,5 + 1,30\ A + 1,12\ P + 1,10\ L + 0,96\ Di + 1,10\ C + 1,15\ Do + 1,44\ F]\ PD^{-1}$$

$$PGE = [5,16 - 0,040\ A + 0,026\ P - 0,091\ L + 0,053\ Di + 0,042\ C - 0,017\ Do - 0,026\ F]\ PD^{-1}$$

where abbreviations have the same meaning as previously.

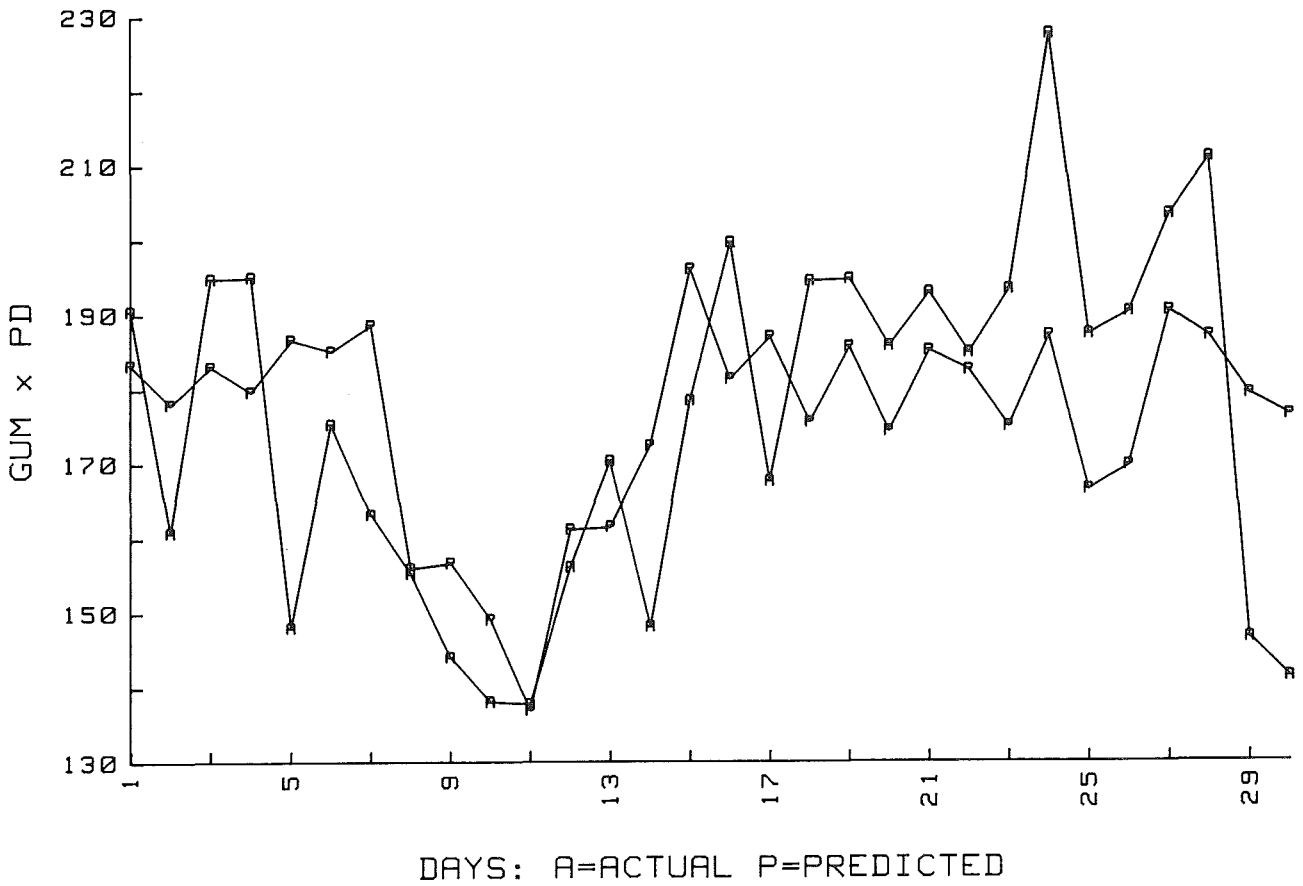

FIGURE 5. Actual vs predicted consumption of gum

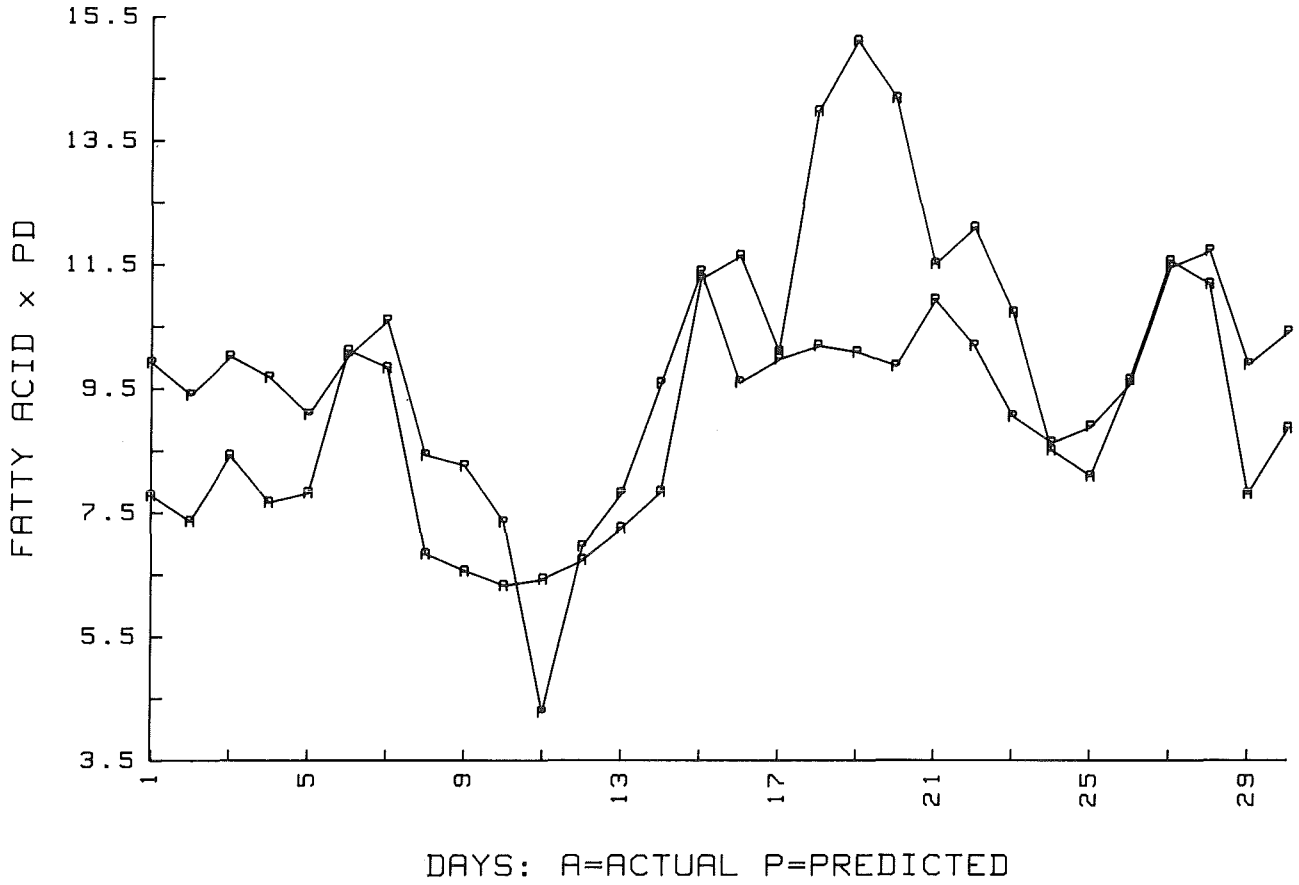

FIGURE 6. Actual vs predicted consumption of fatty acid

TABLE 6. Correlation coefficient r^2 for the two methods

	Min. regression model	Multiple linear regressions
Pd x WG	0,54	0,62
PD x CS	0,12	0,27
Pd x Gum	0,32	0,43
Pd x FA	0,37	0,46
Pd x PGE	0,69	0,72
Mean	0,41	0,50

TABLE 7. Standard error of estimate for predicted reagent consumption rates

	Min. regression model	Multiple linear regressions
Pd x WG	6,97	7,14
Pd x CS	3,22	3,30
Pd x Gum	20,2	20,0
Pd x FA	1,97	2,07
Pd x PGE	0,40	0,42
Mean	6,55	6,59

In Table 6 the fit of the correlation coefficient r^2 for the mineralogical regression model and the method of multiple linear regressions are compared. On average, the multinle linear regression method will give a prediction falling within one standard deviation of the actual reagent consumption rate, 50% of the time, compared with only 41% for the mineralogical regression model.

However, reagent consumption rates predicted by the mineralogical regression model are nearly identical to and marginally more accurate than the method of multiple linear regressions. Refer to Table 7, where the standard error of estimate for the two methods is given.

In Figures 8 to 12, the deviation between actual reagent consumption rates and the consumption rates predicted by the two methods are plotted. In most cases, the models deviate in the same

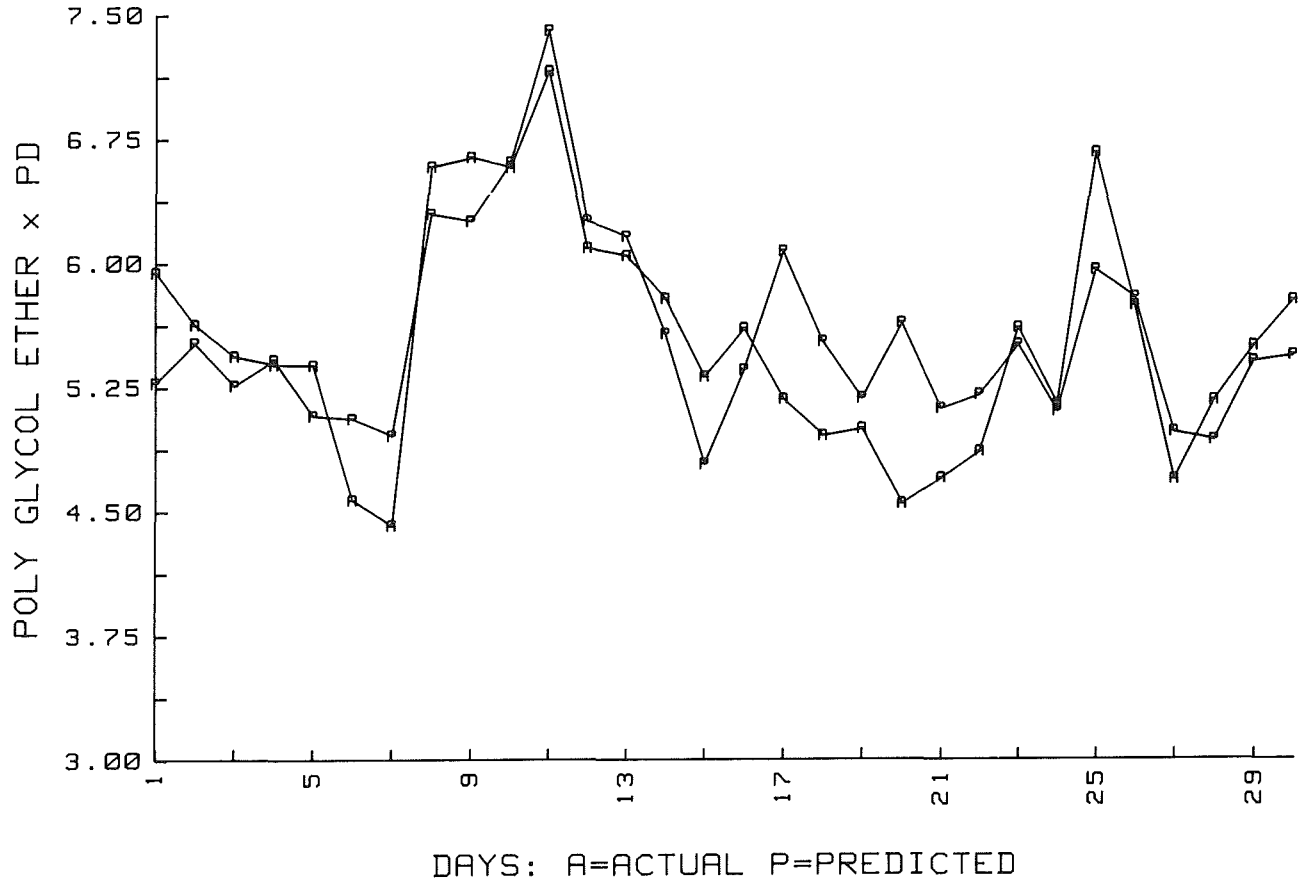

FIGURE 7. Actual vs predicted consumption of polyglycol ether

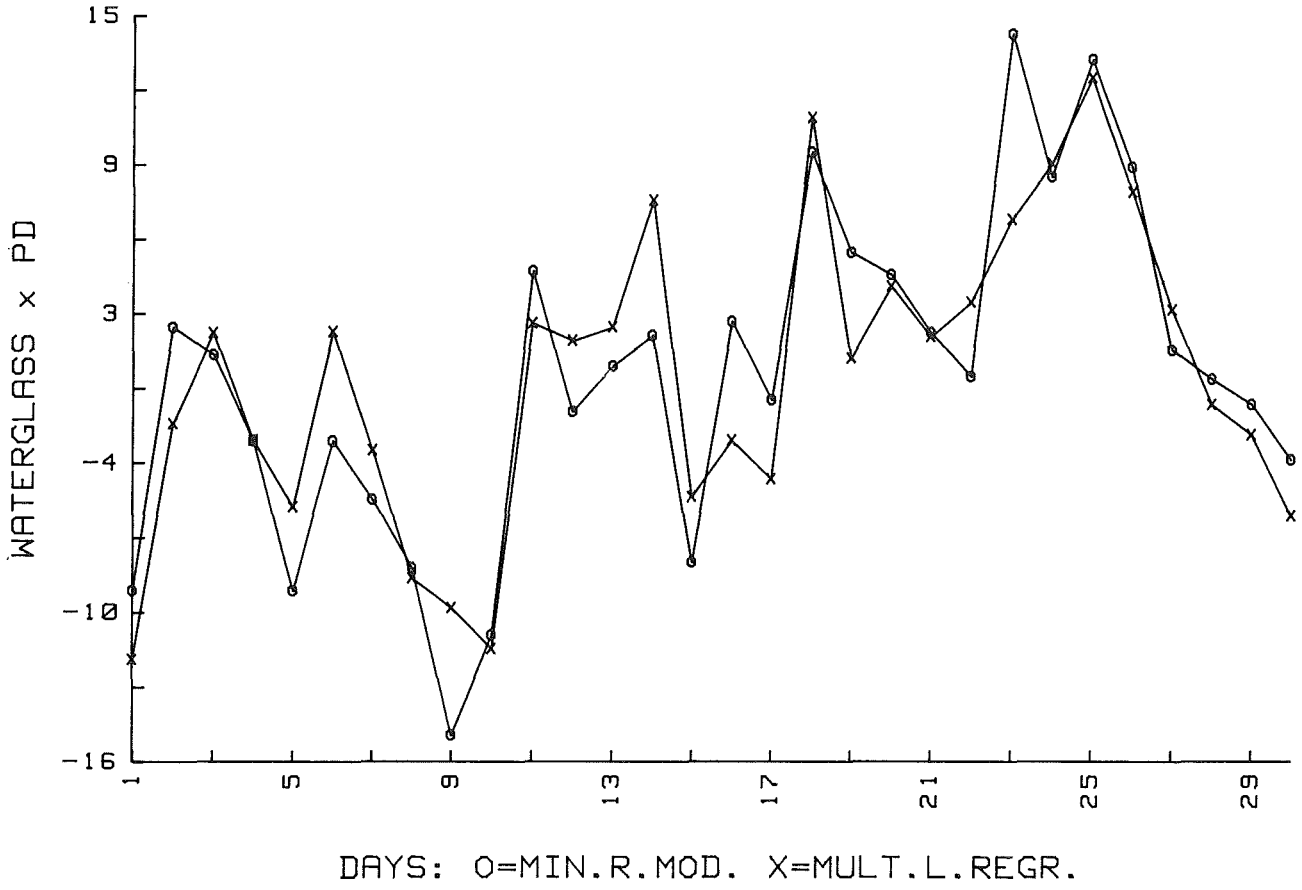

FIGURE 8 Deviation from actual dosage of waterglass

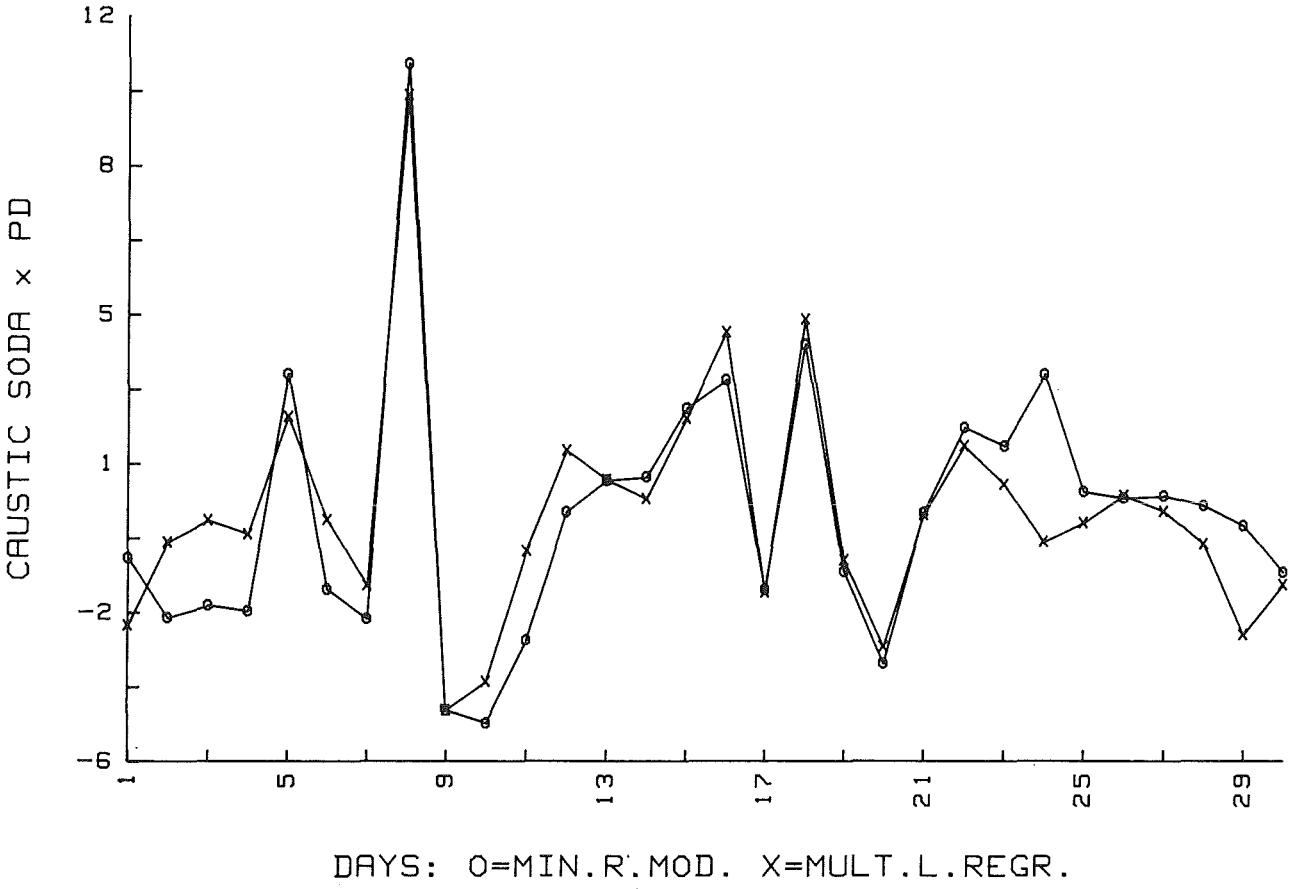

FIGURE 9. Deviation from actual dosage of caustic soda

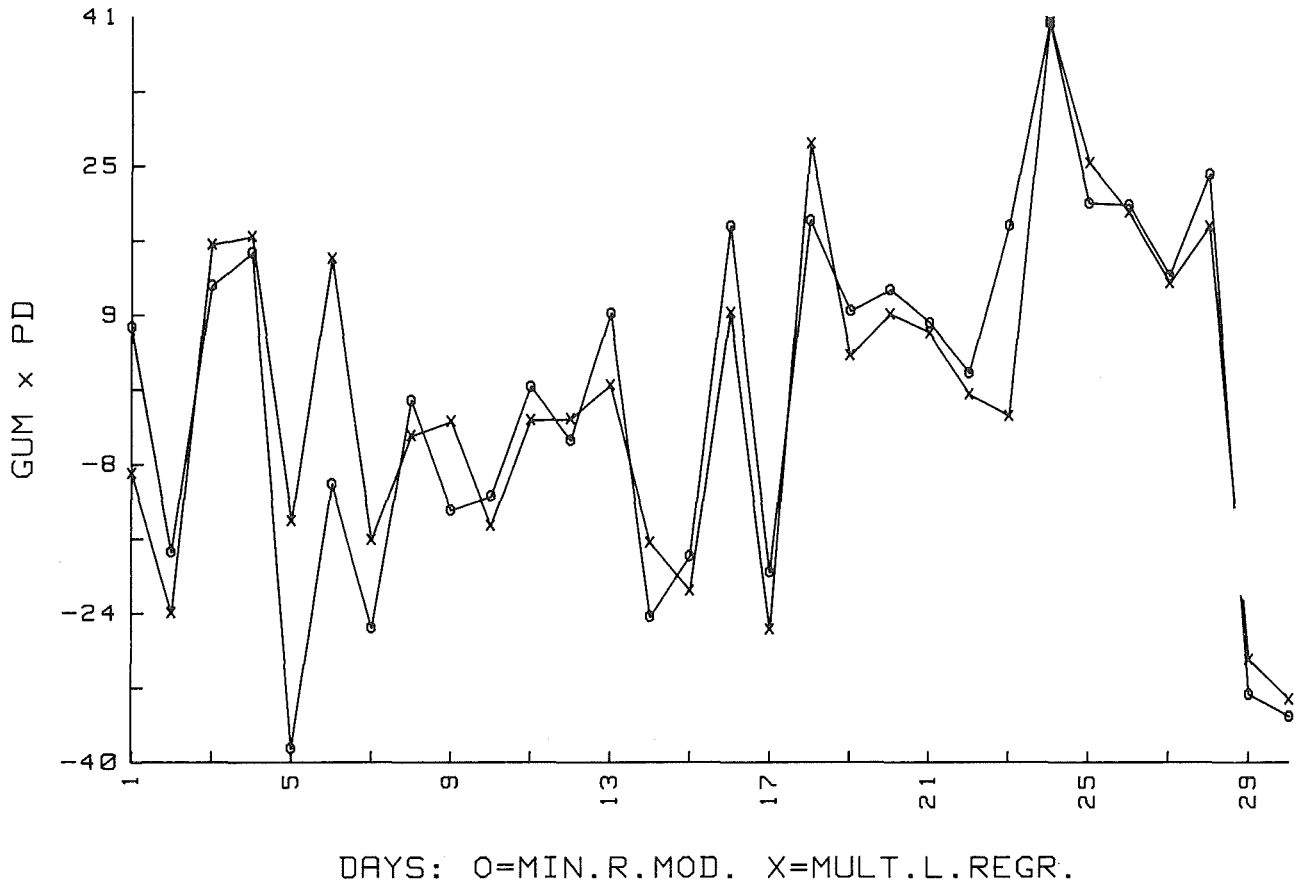

FIGURE 10. Deviation from actual dosage of gum

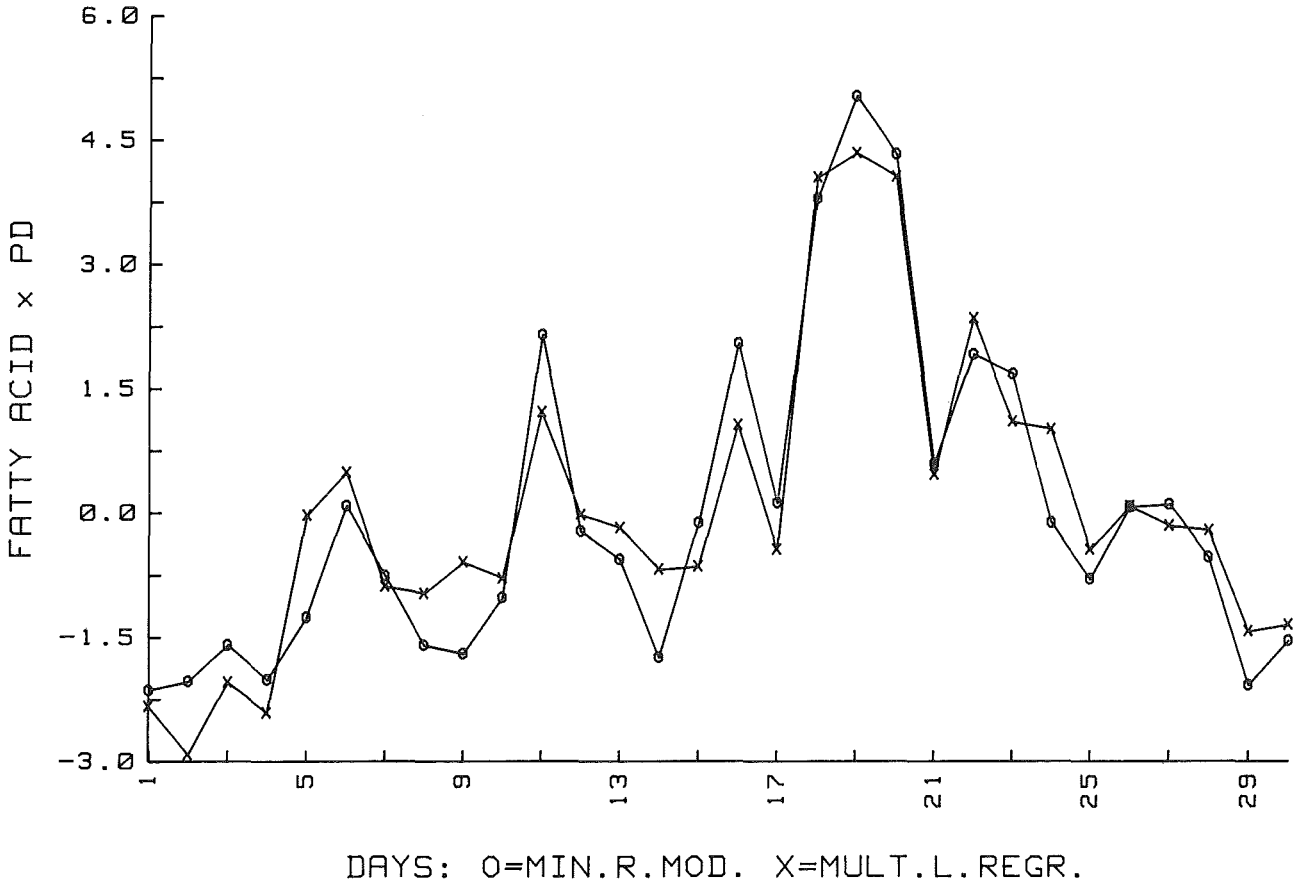

FIGURE 11. Deviation from actual dosage of fatty acid

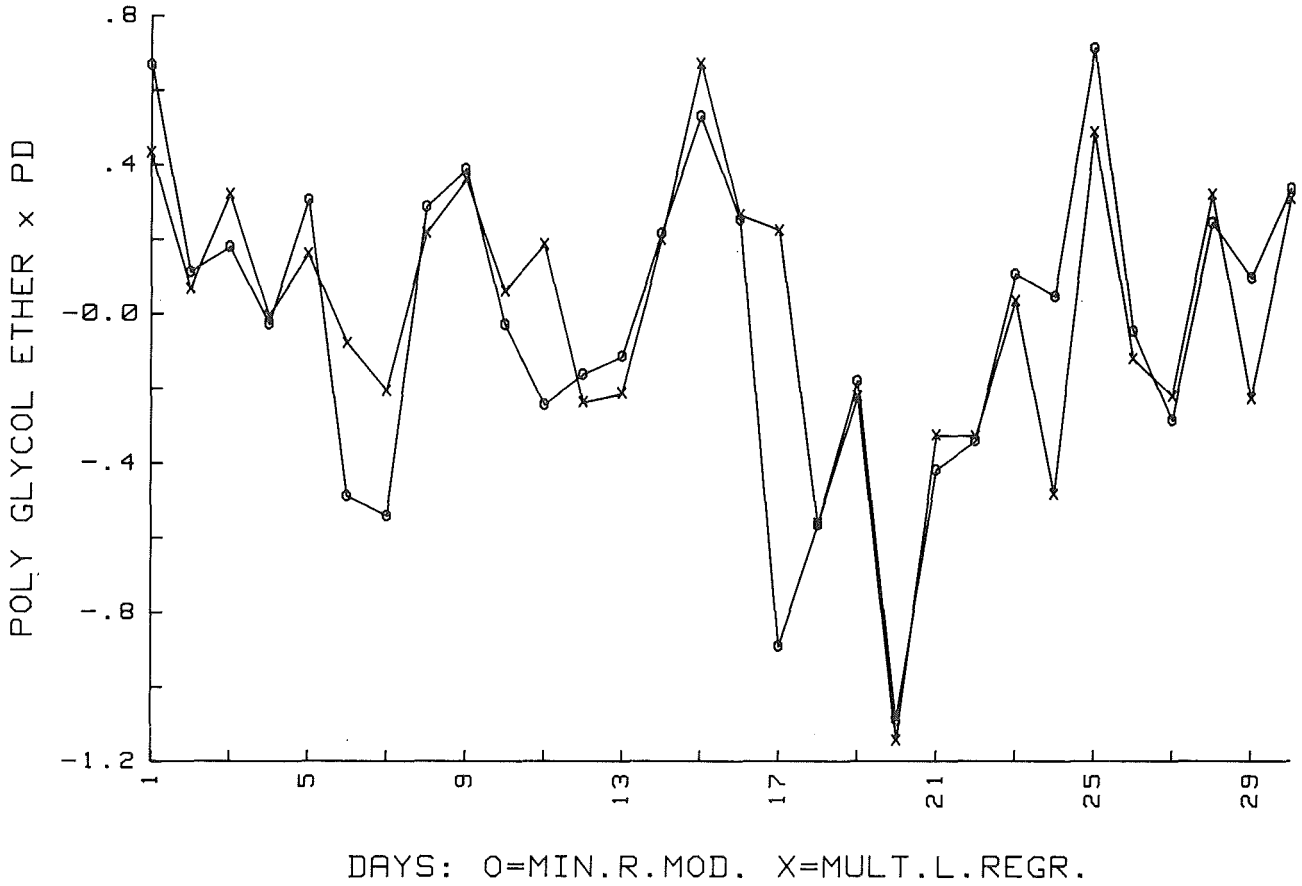

FIGURE 12. Deviation from actual dosage of polyglycol ether

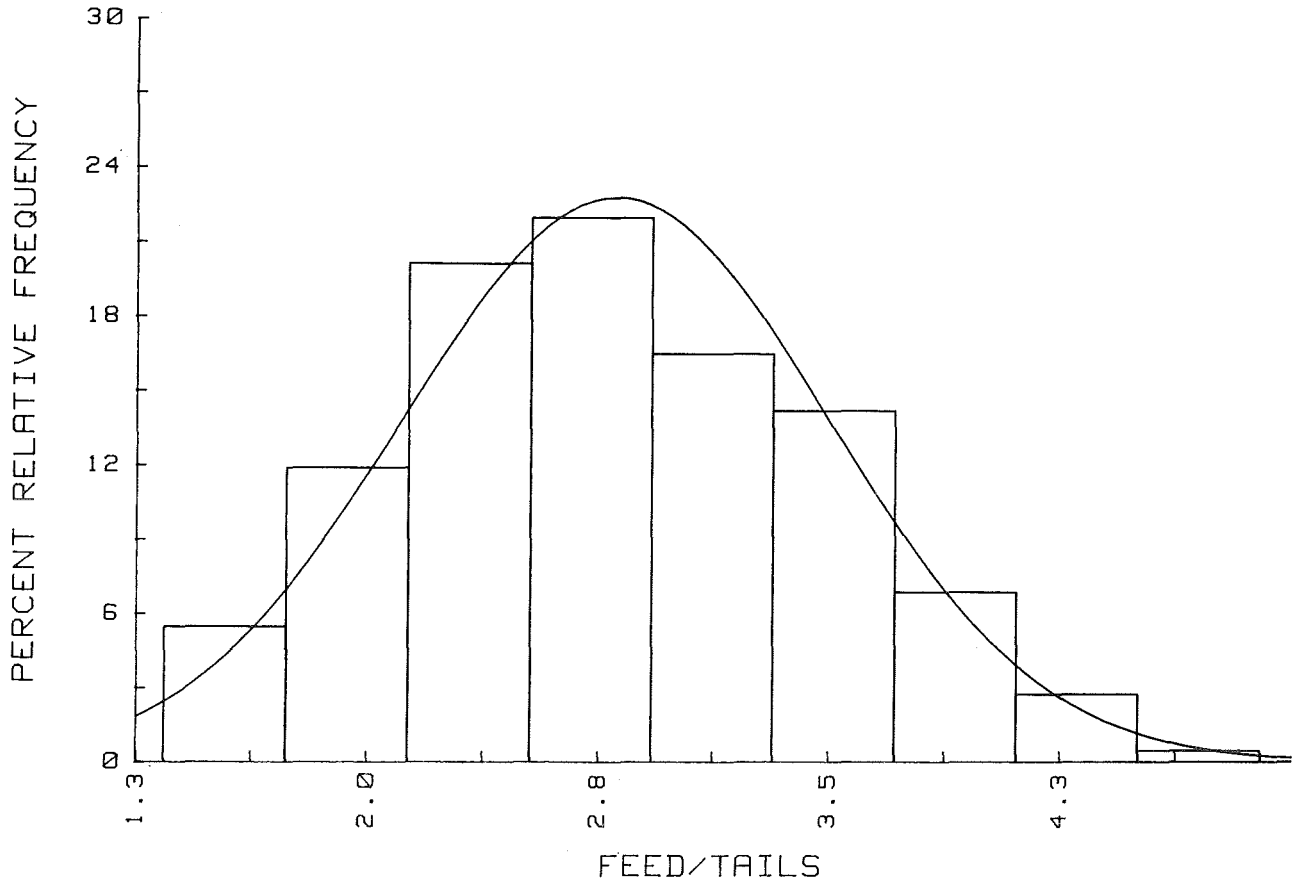

FIGURE 13. Before: Production increases P.R.F.

direction for the same observation, which indicates that factors other than feed mineralogy, as such, probably played a role.

Time has not been available to apply the recently published method of Hollaway[6] using residual regression analysis. It is also not clear how the automatic continuous update of the model should be carried out.

Results using the regression model

Although the mineralogical regression model as a whole was accurate only 41% of the time, this was still very much better than nothing at all, as is shown by results in the plant.

Figures 13 and 14 show the distribution of the ratios of head to tails grade for the period January to August 1985, i.e. before the mineralogical regression model was fully implemented, and for the 9-month period thereafter. Actual data for the plant were used and there is no doubt that flotation control has improved measurably. The mean head to tails grade ratio has improved from 2,83 with a variance of 0,47 to 3,18 with a variance of 0,28.

Assuming an average head grade of 9% P_2O_5, the improvement in relative recovery may be calculated.

$$\text{Recovery } \% = \frac{(v - u)k}{(k - u)v} \times 100$$

where v = head grade
u = tailings grade
k = concentrate grade

At head to tailings grade ratio of 2,83 - Recovery = 70,8%. At head to tailings grade ratio of 3,18 - Recovery = 74,3%

The difference is 3,5 or 5,0% relative.

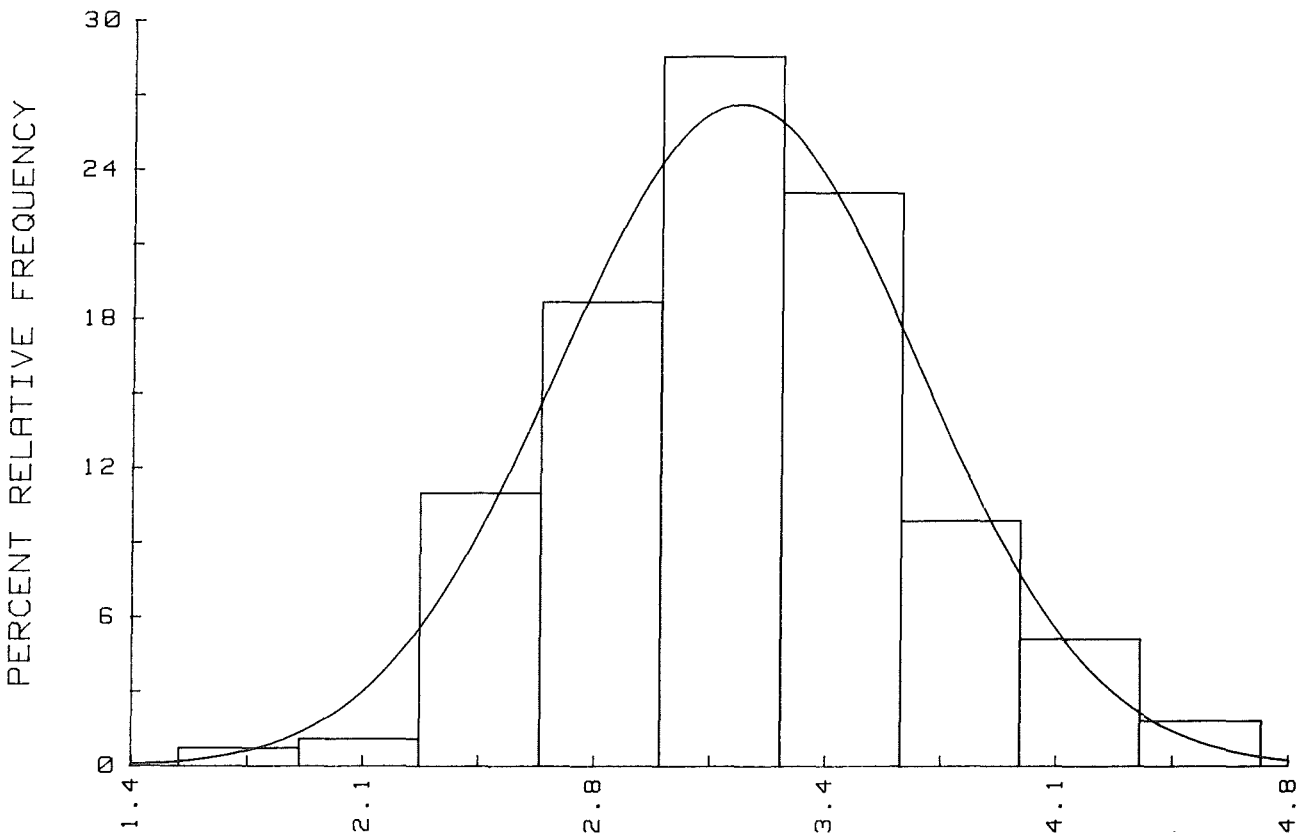

FIGURE 14. After: Production increase P.R.F.

Summary and conclusions

Foskor utilizes a difficult froth flotation process for the production of rock phosphate. Five expensive chemical reagents are used in the process.

By correlating the feed mineralogy with recovery, it could be shown that the flotation plant tends to lag behind in adjusting reagent consumption rates during periods of rapidly changing feed.

A simple mineralogical regression model was devised in which the proportion of each of the minerals in the feed was correlated with flotation reagent consumption rate. The minerals were then combined and a new linear regression calculated for the prediction of flotation reagent consumption rates.

Provision was made for the continuous update of the model by using the most recent acceptable 30 data lines.

Compared with the method of multiple linear regressions, which require more computing facilities, r^2 was poorer (0,41 versus 0,50), but the mean standard error of estimate for predicting flotation reagent consumption rates was identical at $\pm$ 6,6 units.

Over a 9-month period, the application of the mineralogical regression model lead to an improvement in flotation recovery of 3,5% absolute or 5% relative, compared with the previous 8 months.

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