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Computer-aided Use of a Screening Process Model

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The mathematical framework of a well-established model proposed earlier by the authors for the screening process is briefly recalled. Some numerical methods for the application of the model, concerning numerical solution of the equations and parameter estimation, are indicated.

Some applications of the model to improve screening operations, sizing of screen, simulation of screening and of integrated plant operations are discussed, and the computer software to be used for these purposes is described in detail.

Introduction

Mathematical models have been developed and accepted as powerful tools for mineral processing, plant design and optimization. In many cases, higher plant efficiency may be achieved, at a lower cost, through the optimization of the application of existing technology rather than through improving or refining unit operations. Therefore, process models and computer programs capable of assisting the process engineer in decision making are very important.

The screening model considered in this paper had been proposed by Ferrara and Preti ([1]) and subsequently reviewed by them and by other workers ([2,3,4]).

The model is chiefly aimed at giving a relationship between (A) variables which define the mineral fed to the screen (size distribution), (B) parameters which characterize the process, (C) dimensions of the screen and (D) results of the screening operations. These results can be either in the form of partition curves, here expressed as screen oversize efficiency, or in the form of size distribution curves for the products. From these results it is also possible to obtain synthetic parameters such as the efficiency formulae (efficiency of undersize removal or of undersize recovery) which define the screen performance.

Only steady-state continuous screening is considered in this paper. However, the model can also be applied to batch sieving by substituting time for length (from the feed point of the screen) and modifying the meaning and dimensions of the variables and the parameters accordingly.

The equations of the model and the calculation computer programs can be used in one of the following ways:

- (A), (B) and (C) being given, (D) is calculated (Simulation);
- (C) and (D) being given, (B) is calculated (Parameter estimation);
- (A), (B) and (D) being given, (C) is calculated (Design).

Problems have arisen in the solution of model equations and in the estimation of parameters; they were solved by appropriate mathematical methods that are presented in the next sections of this paper.

The model can be applied to dry screening

only; problems associated with wet screening, screening of moist and sticky materials and with the effects of blinding on the screening surfaces were not tackled.

Mathematical model of screening

Two different screening conditions, crowded and separate (or disperse), lead to two different rate processes ([1,5]). In the following, attention will be focused especially on crowded conditions, which generally occur in industrial screening where maximum throughput at a given level of efficiency is required. For the sake of clarity, for both of the conditions the model can be considered as composed of two parts:

(a) one expressing the kinetics of the process through the space (or time) dependent mass balance applied to populations of particles with different size;

(b) another expressing the relationship between the passage probabilities of particles, their size and the mesh aperture of the screening surface.

The model equations have been derived in both discretized ([1]) and continuous form ([6]). The latter, having a more general validity, is preferred for introducing the model in this paper.

Crowded screening

Crowded screening occurs when the flow rate exceeds a critical level, W_c, so that the material forms so thick a bed that only particles in immediate contact with the screen are capable of passage. As long as the upper layers are able to replenish the contact layer, the mass flow rate of passage remains constant.

(a) *Kinetics of the process*

If $w(X,l)dX$ is the mass flow rate per unit width for particles of size from X to X+dX entering the element dl, the mass balance

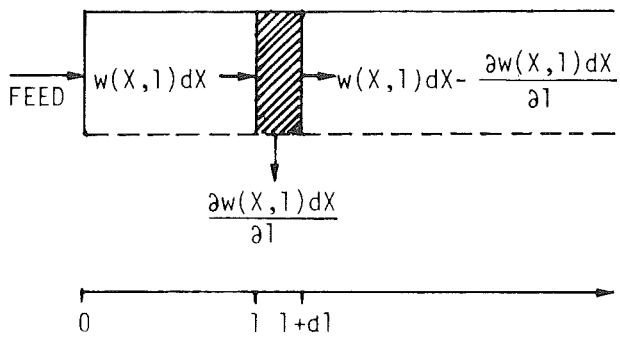

FIGURE 1. Schematic representation of the basic screening mass balance model

relative to this element for the same size class is as represented in Figure 1.

Since it has been shown experimentally that in crowded conditions the mass of passing particles of size X is proportional to a constant which depends on X and to the concentration of particles with size X, we have:

$$\partial w(X,l)/\partial l = -k(X)\, f(X,l) \qquad (1)$$

where f(X,l) is the concentration of particles of size X, which is given by the value of the density function of the size distribution of the material at the point l, f(x,l), for x = X:

$$f(X,l) = \frac{w(X,l)}{\int_0^\infty w(x,l)dx} = \frac{w(X,l)}{W(l)} \qquad (2)$$

Integrating between 0 and L and introducing the screen oversize efficiency (or partition number) for the size X

$$E(X,l) = w(X,l)/w(X,0) \qquad (3)$$

yields:

$$E(X,l) = \exp[-k(X)\int_0^L dl/W(l)] \qquad (4)$$

For this approach to be valid, the following assumptions are necessary: perfect mixing on the screen along the vertical and plug flow of the material along the screen. The first assumption has been discussed in an earlier paper by Ferrara and Preti ([1]).

Eq. (4) can be used to calculate the efficiency curve, once the function $\int dl/W(l)$ is known, or to fit experimental data. For using the model, however, it is preferable to write this equation in a different form, in which some important and known parameters are made explicit. If we consider the ratio of the logarithms of the screen oversize efficiencies [Eq.(4)] for two different values of the particle size, a new function can be defined:

$$\chi(x) = \ln E(x,l)/\ln E(X,l) = k(x)/k(X) \quad (5)$$

Substituting Eq. (2) into Eq. (1) and using the expressions (3) and (5) yields:

$$\int_0^\infty w(x,0)E(X,l)^{\chi(x)-1}\frac{\partial E(X,l)}{\partial l}\,dx=-k(X) \quad (6)$$

a differential equation which is equivalent to Eq.(1) but contains explicitly the variable X representing the size of the grains whose kinetics is being studied, as well as the variable x representing the generic grain size of the particle population over the screen, whose distribution affects the kinetics of the grains of size X.

If D is the screen aperture, then the conditions for x will be:

for $0 \leqslant x < D$ $\quad k(x) \neq 0 \quad \chi(x) \neq 0$

for $x \geqslant D$ $\quad k(x) = 0 \quad \chi(x) = 0$

Therefore, it is convenient to re-write Eq.(6) by dividing the integral into two parts:

$$\int_0^D w(x,0)E(X,l)^{\chi(x)-1}\frac{\partial E(X,l)}{\partial l}\,dx + \int_D^\infty w(x,0)\frac{\partial \ln E(X,l)}{\partial l}\,dx = -k(X) \quad (7)$$

Integrating the differential equation (7) between 0 and L and introducing the feed flow rate per unit width W_0 and the density function of the grain size distribution in the feed f(x,0) yields:

$$W_0\left\{\int_0^D f(x,0)\frac{1}{\chi(x)}\left[E(X,L)^{\chi(x)} - 1\right]dx + \int_D^\infty f(x,0)\ln E(X,L)\,dx\right\} = -k(X)L \quad (8)$$

Eq. (8) is valid for grains of size X between 0 and D; for $X \geqslant D$, it will be:

$$k(X) = 0 \qquad E(X,L) = 1$$

(b) *Probablity of passage*

Two functions appear in Eq. (8), $\chi(X)$ and k(X), which can be expressed by a model derived from the simplest of those proposed by Gaudin ([7]) : $p(x) = (D-x)^2/(D+B)^2$, where D is the mesh aperture (square) and B the wire diameter. Since ([1,5])

$$k(X) = W_c \; n \; p(X) \quad (9)$$

where W_c is the critical flow rate and n the number of particle presentations per unit length, both of them constant under given operating conditions, then the function $\chi(x)$ can be expressed as follows:

$$\chi(x) = k(x)/k(X) = p(x)/p(X) = [(D-x)/(D-X)]^\sigma \quad (10)$$

According to Gaudin's model, $\sigma = 2$ for square meshes and $\sigma = 1$ for wedge wire screens. Tests have shown that Eq. (10) fits very well the experimental data, and it was found that the σ values depend on the vibration conditions of the screen and on the type of screening surface.

For x/D = 0.5 in Eq. (10), one obtains:

$$k(X) = k_{50}\, 2^\sigma\, (1 - X/D)^\sigma \quad (11)$$

where k_{50} is the kinetic constant for $x/D = 0.5$.

Combining Eqs. (8) and (11) yields the complete equations describing the screening process in the crowded state:

$$W_0\left\{\int_0^D f(x,0)\,\frac{1}{\chi(x)}\left[E(X,L)^{\chi(x)} - 1\right]dx + \int_D^\infty f(x,0)\ln E(X,L)\,dx\right\} = -k_{50}\,2^{\sigma}\left(1-\frac{X}{D}\right)^{\sigma} L \qquad (12)$$

for $X<D$

$$E(X,L) = 1 \qquad \text{for } X \geqslant D \qquad (13)$$

The second term of the left-hand side of Eq. (12) can be written as follows:

$$\ln[E(X,L)]\int_\infty^D f(x,0)\,dx = \ln[E(X,L)]\ Yos$$

where Yos is the proportion of oversize >D in the feed.

Separate screening

Separate screening occurs when the particles on the screen do not interact with one another. The differential equation is:

$$\partial w(X,l)/\partial l = -\,s(X)\,w(X,l) \qquad (14)$$

where $s(X)$ is the kinetic constant for the size X.

Integrating Eq. (14) between 0 and L and introducing the screen oversize efficiency $E(X,L)$ yields the screening equation for the separate state:

$$E(X,L) = \exp[-s(X)\,L] \qquad (15)$$

for X between 0 and D, while for X>D it is always $s(X) = 0$ and $E(X,L) = 1$.

If $s(x)$ is written as a function of s_{50} and σ in a similar way to what was done in the case of crowded conditions, the following approximate equation governing the separate state is obtained:

$$E(X,L) = \exp[-s_{50}\,2^{\sigma}(1 - X/D)^{\sigma} L] \qquad (16)$$

Mixed conditions

If we designate with L_c the critical distance from the feed point for the transition from crowded to separate conditions and with W_c the corresponding flow rate, Eq. (12) with $L=L_c$ will provide the results for screening in the crowded state; the corresponding screen oversize efficiency will be $E(X,L_c)$. In the next portion of the screen, $(L-L_c)$, separate conditions will occur and the global screen oversize efficiency will be:

$$E(X,L) = E(X,L_c)\ E[X,(L-L_c)] \qquad (17)$$

Including Eqs.(4) and (16) in Eq.(17) and using the relationship $k_{50}=s_{50}\,W_c$ we obtain:

$$E(X,L) = \exp[-k_{50}\,n_d\,2^{\sigma}\,(1-X/D)^{\sigma}] \qquad (18)$$

where the term

$$n_d = \int_0^{L_c} dl/W(l) + (L-L_c)/\,W_c \qquad (19)$$

is unknown but constant under certain operating conditions.

Eq. (18) can be used to fit experimental curves of screen oversize efficiency by estimating its parameters ($n_d\,k_{50}$) and σ regardless of the screening conditions occurring on the screen (crowded, separate or mixed). It cannot be employed for design purposes because n_d varies with operating conditions in a way which is unknown.

In conclusion, it is possible to obtain a single equation for the screen oversize efficiency which is valid for crowded, separate or mixed conditions and therefore very useful for characterizing the screening pro-

cess with few parameters. The derivation had already been made by Hess ([8]) and reported in a earlier paper by the authors ([6]).

Significance of the parameters k_{50} and σ

The kinetic parameter k_{50} is the mass flow rate per unit area for particles of size X = 0.5 D, assuming the feed to be made up of grains all of the same size and the screening to occur in crowded condition. This parameter can be measured in g/sec cm^2 or in t/h m^2 (1 g/sec cm^2 = 36 t/h m^2).

The significance of k_{50} is similar to that of the so-called 'basic capacity' in the well-known formulae for screen sizing ([9,10]).

The parameter k_{50} depends chiefly on the mesh aperture, as does the capacity mentioned above. It also depends on parameters characterizing the screening surface (fraction open area, aperture shape, type of screening surface), on the vibration characteristics of the screen (frequency, amplitude, form of the oscillation) and on the inclination of the screen.

The parameter σ affects the ratio of passage probabilities for particles of different size (Eq. 10) and, once k_{50} is known, allows k(X) to be calculated for the generic size X by means of Eq. (11). To illustrate further the meaning of σ, Fig.2 gives a plot of k(X) versus X/D for k_{50}=1. As can be seen from the plot, for high σ values the k(X) values for the fine classes are high (X/D<0.5), while those for the near-submesh particles are low (X/D>0.5). Conversely, for low σ values the near-submesh particles remarkably increase their k(X) value.

The parameter σ, which is dimensionless, is independent of the mesh aperture, but depends on the type of screening surface, the vibration characteristics and the slope of the screen.

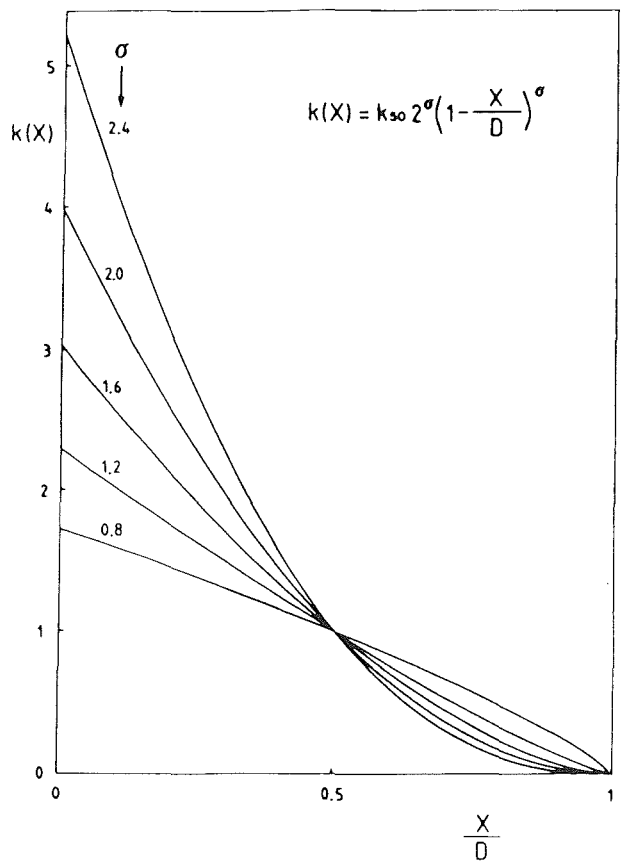

FIGURE 2. Plot illustrating the function of k(X) versus X/D for different σ values and K_{50} = 1

Numerical solution of the model equations

The use of continuous functions for the definition of the model, such as employed in the present study, improves the model accuracy but requires the knowledge of continuous functions defining the size composition of the feed.

In general, the size composition of the feed is known for discrete values of particle size; therefore, for practical applications, the model equations must be written in a discretized form.

The transformation of Eq.(12) in such a form leads to the following equation, already reported by the authors in previous papers:

$$W_0\left\{\sum_{j=1}^{n} y_{jo}\frac{1}{X_{ji}}\left[E(x_i,L)^{X_{ji}}-1\right]+\ln E(x_i,L)\sum_{j=n+1}^{m} y_{jo}\right\}=-k_{50}2^{\sigma}\left(1-\frac{x_i}{D}\right)^{\sigma} L$$

$$E(x_i,L) = 1 \quad \text{for } n+1 \leqslant i \leqslant m \qquad (20)$$

where the index i refers to the size class whose behaviour during screening is being studied, and j refers to the other size classes present in the feed which affect the behaviour of size class i; $\chi_{ji}=[(D-x_j)/(D-x_i)]^{\sigma}$. The classes from 1 to n are smaller than the mesh aperture of the screen, while the classes from n+1 to m are larger.

Calculation of screen oversize efficiency coefficients from the model equations

The screen oversize efficiency coefficients are defined by the values $E(x_i,L)$, where x_i is the average diameter of the i-th size class. They can be obtained solving Eq.(20) in which the kinetic constants and the screen length are given values. This equation, in order to obtain $E(x_i,L)$ for a given value of L, has to be solved for the n classes having $x_i<D$.

The numerical solution is made easier by the following variable transformation:

$$E(x_i,L) = \exp(\gamma_i) \qquad (21)$$

Therefore:

if $E(x_i,L) =1$ and $\exp(\gamma_i)= 1$,then $\gamma_i = 0$

if $E(x_i,L) =0$ and $\exp(\gamma_i)\rightarrow 0$,then $\gamma_i \rightarrow \infty$

Substituting Eq.(21) into Eq.(20) we obtain the following equation in the variable γ_i:

$$G(\gamma_i) = \sum_{j=1}^{n} \frac{y_{jo}}{\gamma_{ji}} \left[\exp(\gamma_i\chi_{ji}) - 1\right] + \sum_{j=n+1}^{m} y_{jo}\gamma_i + k_{50}2^{\sigma}(1-\frac{x_i}{D})^{\sigma}\frac{L}{W_0} \qquad (22)$$

since $G(\gamma_i)$ is derivable the Eq.(22) can be solved using the methods of numerical analysis, such as the Newton method. It is based on the iterative formula, which in our case can be written as:

$$\gamma_i^{(r)} = \gamma_i^{(r-1)} - G(\gamma_i)^{(r-1)}/G'(\gamma_i)^{(r-1)} \qquad (23)$$

where $G'(\gamma_i)$ is the derivative of $G(\gamma_i)$ and (r) is the index of iteration. The formula is updated by replacing $\gamma_i^{(r-1)}$ with $\gamma_i^{(r)}$. As a starting point, $\gamma_i=0.5$ is chosen, since this value has proved suitable for a fast convergence. The procedure stops at the (q)-th iteration when the convergence criterium

$$0< G(\gamma_i)^{(q)} < 10^{-4} \qquad (24)$$

is satisfied. The values of the screen oversize efficiency are obtained from Eq. (21) by substituting into it the roots of Eq. (22). The method has proved effective using the starting estimates suggested above.

Calculation of screening area to meet a given screening efficiency

Once the model parameters are known, the model can be used to size a screen requested to perform with a given overall recovery efficiency (ORCE) or with a given overall removal efficiency (ORME):

$$ORCE=\sum_{1}^{n} y_i\ (1-E_i)/\sum_{1}^{n} y_i$$

$$ORME=1-(\sum_{1}^{n} y_i\ E_i/\sum_{1}^{m} y_i\ E_i) \qquad (25)$$

where $E_i=E(x_i,L)$.

Since the right-hand side of Eq.(20) contains the screen length, L, the process can be simulated, the oversize efficiency curve determined and the selected type of overall efficiency calculated. The screen length, L, is changed iteratively according

to a searching method until the requested efficiency is met.

Calculation of model parameters

Given screening results, the model parameters can be calculated. The procedure is different for the cases of square and non-square mesh apertures of screening surface.

Up to now we have spoken in general terms of the mesh aperture D of the screening surface. In laboratory, for sieve analysis the ASTM series that employs square meshes is generally used. The model parameters are calculated on the basis of such analyses; consequently, for a correct use of the model equations the value of the mesh aperture D has to be referred to square apertures. Therefore, in the case of screening operations with square apertures, the model is completely defined by the two parameters k_{50} and σ.

If the model is employed to simulate screening operations with non-square apertures, the value of D cannot be selected a priori but has to be estimated from experimental sieve analysis data. A new parameter, the so-called equivalent aperture, De, has to be employed to replace D; thus, a third parameter has to be introduced into the model in addition to the kinetic constants.

Square screen surface aperture

A procedure is proposed which can be applied when the screening results are known for one screen section or, better still, for more screen sections, so as to permit a more accurate estimation of the parameters. The procedure is also applicable when the screening results for the whole screen are known, provided that on the whole screen crowded conditions occur. This procedure consists of two steps:

(a) Rough estimation of k_{50} and σ by means of Eq. (4) re-written in the following form for the generic class i-th and for the screen length from L_k to L_{k+1} :

$$\ln E_i = - k_i \int_{L}^{L_{k+1}} dl/W(l) \qquad (26)$$

When test results are available for screening sections (Fig.3) at L_k and L_{k+1}, then W_k and W_{k+1} are known and hence a rough estimate of the integral of Eq. (26) can be obtained by considering W(l) constant in the section k and equal to $(W_k+W_{k+1})/2$,

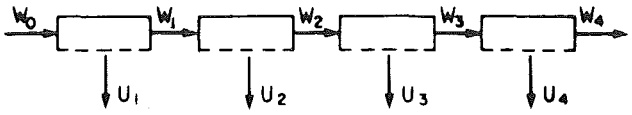

FIGURE 3. Schematic representation of a pilot plant screen with four sampling sections

the value for the integral being then $2(L_{k+1}-L_k)/(W_k+W_{k+1})$. In this way, k_i can be calculated from Eq. (26) for each class i, and k_{50} and σ can be obtained as the solution of an overdetermined system of equations. This is a system of n linear equations in two unknowns; n also represents the number of classes having the average size <D for which the calculation of k_i is possible. The generic i-th equation of the system obtained from Eq. (11) is:

$$\ln k_i = \ln k_{50} + \sigma \ln[2\,(1-x_i/D)] \qquad (27)$$

The algorithm we use for the solution of the system determines the couple k_{50} and σ that minimizes the sum of the absolute value of the residuals, $e(k_{50},\sigma)$:

$$e(k_{50},\sigma) = |\ln k_1 - \ln k_{50} - \sigma \ln[2(1-x_1/D)]|$$

$$+ \ldots\ldots\ldots \qquad (28)$$

$$+ |\ln k_n - \ln k_{50} - \sigma \ln[2(1-x_n/D)]|$$

The system is solved using Linear Programming (LP) methods. If the data contain some wild values (e.g. values that are very inaccurate compared to the overall accuracy of the data), it is advisable to calculate the least-first power (L1) approximation rather than the least-squares (L2) approximation or an Ln approximation. Indeed the data we are dealing with may contain some wild values of k_i due to the attachment of fine fraction particles to the coarser sizes; this phenomenon occurs because of the moisture very often present in the material.

Introducing the residual r_i, the generic i-th equation (27) can be rewritten as follows:

$$\ln k_i = \ln k_{50} + \sigma \ln[2(1-x_i/D)] - r_i \qquad (29)$$

Since the variables of an LP problem are conditioned to 'non-negativity' and since the residuals r_i may have any sign, the equation can be rewritten as follows:

$$\ln k_i = \ln k_{50} + \sigma \ln[2(1-x_i/D)] + z_{i*2-1} - z_{i*2} \qquad (30)$$

The change of variable allows the constraint of 'non-negativity' to be satisfied:

$$\text{if } z_{i*2-1}>0, \text{ then } z_{i*2}=0 \text{ and } r_i<0$$
$$\text{if } z_{i*2-1}=0, \text{ then } z_{i*2}>0 \text{ and } r_i>0 \qquad (31)$$

The objective linear function to be minimized, MIN(e), can be written as follows:

$$e(k_{50}, \sigma) = \Sigma |r_i| = \Sigma (z_{i*2-1} + z_{i*2}) \qquad (32)$$

and an improved version of the earlier simplex algorithm can be used to find the solution, min(e). An algorithm that passes through several neighbouring simplex vertices in a single iteration has been proposed by Barodale and Young ([11]). This algorithm appears to be superior computationally to any other known algorithm for this particular problem.

(b) Once a rough estimate of k_{50} and σ has been obtained, these values are used as guess values for a more accurate calculation method. This consists in:

- simulating the process by using Eq. (22) for each class i (1,..m) and calculating the values:

$$\Phi_i = |E_i(k_{50}, \sigma) - E_i^*| \, q_i \qquad (33)$$

  where E_i and E_i^* are the simulated and the experimental values of the screen oversize efficiency, respectively, and q_i (>0) are appropriate weight coefficients;
- searching the minimum value of an appropriate scalar objective multivariable Ψ function:

$$\Psi = \Psi [\Phi_i (k_{50}, \sigma)] \qquad (34)$$

Several methods can be used to obtain min(Ψ). The method we prefer to use is based on the general purpose sequential searching technique proposed by Box ([12]), which does not require any derivative calculation and tends to find the global minimum of the objective function because the initial set of points selected to calculate the function values is randomly scattered throughout the feasible region of the plane k_{50} and σ.

The kinetic constants of the screening process calculated as described in point (a) can be considered an appropriate solution if the screen section is short and therefore the assumption of the constancy of the feedrate W(l) is acceptable; in this parti-

cular case the second step of the procedure, (b), can be skipped.

Non-square screen surface apertures

In this case, the new parameter, the so-called equivalent aperture, De, has to be calculated, from the experimental data, in addition to the kinetic constants. In fact, considering elongated rectangular apertures having sides Ds and Dl (Ds<Dl) it cannot be De=Ds since some particles passing through the screen will not pass through the corresponding square sieve in the laboratory analysis. Therefore it has to be : Ds<De<Dl.

An empirical procedure, based on the observation of the efficiency curve, that allows to calculate De through the search of the minimum value of the size X for which E(X,L)=1, has been previously used by the authors.

In the present paper a more rigorous procedure is indicated that leads to the calculation of De as well as of the corresponding kinetic constants.

In this case the above-mentioned over-determined system becomes a non linear over-determined system in three unknowns, D=De, k_{50} and σ, and can be solved by:

$$\text{MIN } e(De, k_{50}, \sigma) \qquad (35)$$

or, decomposing the procedure, by

$$\text{MIN } \{\min e[De, \underline{k_{50}}(De), \underline{\sigma}(De)]\} \qquad (36)$$

where the underlined values are the kinetic constants that minimize the sum of the absolute value of the residuals (Eq.28) for the given value of D=De.

The method used requires searching for the value De in an appropriate range having as lower limit X_{v-1} and as higher limit X_v, v being the index of the coarser class in the undersize product; the range has to contain the abscissa X for which E(X,L)=1. The condition of minimum requires that:

$$d\,[\min e(De, \underline{k_{50}}, \underline{\sigma})]/d\,De = 0 \qquad (37)$$

The solution of this problem gives the min $e(De, k_{50}, \sigma)$ and the corresponding values of the independent variables De, k_{50} and σ necessary for the use of the screening model with non-square holes.

Influence of the sieve intervals on the model parameters estimation

The screen oversize efficiency is defined as a continuous function by Eq.(3) (which can also be written in the form of Eq.(38)), and as a discrete function for the class i-th (particle size from x_i to x_{i+1}) by Eq.(39) :

$$E(X,1) = W(1)\, f(X,1)/W_0\, f(X,0) \qquad (38)$$

$$E(x_i,1) = W(1)\, y_i/W_0\, y_{io} \qquad (39)$$

The efficiencies as defined from Eqs.(38) and (39) differ in values for wide sieve intervals: the former is the limit of the latter for sieve intervals approaching zero. Therefore, only Eq.(38) defines the true screen oversize efficiency, while Eq.(39) gives an 'averaged' value of the efficiency over a sieve interval which depends on the function (38), the sieve intervals and the size distribution of the feed.

In general, a continuous screen oversize efficiency function can be transformed to a discretized one as follows:

$$E(x_i,1) = \frac{\int_{x_i}^{x_{i+1}} f(X,0)\, E(X,1)\, dX}{\int_{x_i}^{x_{i+1}} f(X,0)\, dX} \qquad (40)$$

Frequently Eq.(40) cannot be integrated

analytically, either because E(X,1) or f(X,0) or both are unknown in the form of a continuous function.

Assuming the density function of size distribution to be constant within the sieve intervals, Eq.(40) can be approximated by the following expression:

$$E'(x_i,1) = \frac{1}{x_{i+1} - x_i} \int_{x_i}^{x_{i+1}} E(X,1)\, dX \qquad (41)$$

Methods of averaging the screen efficiency over sieve intervals according to Eq.(41) have been incorporated in the screen models of Whiten ([13]) and Batterham et al.([14]) (using analytical integration) and in that of King ([15]) (using numerical integration), which have been later reviewed by Hess ([8]).

As a consequence of the above and with reference to the application of the model proposed in this paper, a correct procedure would require the estimation of the true values of the model parameters k_{50} and σ (i.e. the values corresponding to the true screen oversize efficiency), which are independent of sieve intervals and feed size distribution. To this end, the experimental values of the screen oversize efficiency (related to wide sieve intervals) have to be compared with the simulated ones, calculated using narrow intervals and then averaged on the wide intervals corresponding to the experimental data by Eq.(40). Therefore, the same procedure described in the preceding section 'Calculation of Model Parameters' can be applied, E_i of Eq.(33) being averaged screen oversize efficiency values as explained before.

The software

On the basis of the above-described algorithms a multipurpose computer program called SEDOS (Simulation Estimation Design Of Screen) was developed. The software is structured in such a way that, at the highest level of interaction, the user needs only to initiate the system and answer by input figures the questions posed during the run of the program. No special skill is required to operate the software, which interactively allows the operator to select among the three main purposes it includes:

1) simulation of the screening process based on the knowledge of the model parameters;

2) calculation of the screening area needed to meet a required 'overall efficiency'; either recovery efficiency or removal efficiency can be specified;

3) estimation of model parameters:
   (a) square screen surface aperture: estimation of the kinetic constants k_{50} and σ, the parameter D (mesh aperture) being known a priori;
   (b) non-square apertures: simultaneous estimation of k_{50} and σ and of the equivalent aperture De.

At the present time the program is being run on the Control Data (CDC CYBER 170/730) mainframe of the University of Trieste Computer Centre (UTSCC). The 2400-statement program is written in Fortran 77 and requires a compilation time of about 16 seconds under the CDC NOS 2.1 operating system using the FTN5(OPT=0) compiler and about 54 K of CM for execution.

Execution time depends on the purpose of the use (1 or 2 or 3a/b) and on the complexity of the specific problem. By way of example, in the next main section the CP execution time will be reported for the application examples presented. The source of the program on appropriate support is available on request.

Application examples and discussion

Some examples of applications of the software which enables the use of the screening model are presented and discussed.

Estimation of parameters

On the basis of typical screen results data obtained from SKEGA AB, reported in Table 1, the model parameters were calculated. The data are referred to a pilot-plant constituted by a screen (1130 mm X 3600 mm) with a rubber deck and having facilities for sampling the undersize product of eight sections (8 X 450 mm = 3600 mm) and the oversize product. Operating conditions of the test are reported. The calculated values of the parameters are reported in Table 2 for different screen length.

TABLE 1. Percent passing on different screen sections

Hole size : 6x10 mm
Rubber deck open area : 26%
Stroke : 16 mm Speed : 905 rpm
Shape of stroke : Linear
Material : Gravel Bulk denisty : 1.9 gr/cm^3

SIZE mm	1.0	2.0	4.0	5.6	8.0	11.3	RATE t/h
FEED	6.7	16.1	28.6	39.1	61.1	87.0	110.6
U1-2	25.5	56.3	86.0	97.3	100.0	100.0	16.6
O2>3	3.5	9.1	18.5	28.9	54.4	84.8	94.1
U3	20.4	49.7	81.6	96.1	100.0	100.0	5.0
U1-3	24.3	54.8	85.0	97.0	100.0	100.0	21.6
O3>4	2.5	6.8	15.0	25.1	51.8	83.9	89.1
U4	19.7	48.4	82.1	97.4	100.0	100.0	5.6
U1-4	23.4	53.5	84.4	97.1	100.0	100.0	27.1
O4>5	1.3	4.1	10.5	20.3	48.6	82.9	83.5
U5	13.8	38.7	74.6	94.2	99.6	100.0	3.8
U1-5	22.2	51.7	83.2	96.7	99.9	100.0	31.0
O5>6	0.8	2.4	7.4	16.7	46.1	82.0	79.7
U6	9.9	29.1	63.6	89.5	99.3	100.0	2.7
U1-6	21.2	49.8	81.6	96.2	99.9	100.0	33.7
O6>7	0.4	1.5	5.4	14.2	44.3	81.4	77.0
U7	8.9	27.1	63.4	88.8	99.1	100.0	2.1
U1-7	20.5	48.5	80.5	95.7	99.8	100.0	35.8
O7>8	0.2	0.8	3.8	12.1	42.7	80.9	74.9
U8	5.3	19.6	55.8	86.3	98.3	98.9	1.1
U1-8	20.1	47.7	79.8	95.4	99.8	100.0	36.9
OVER	0.1	0.5	3.0	11.0	41.9	80.6	73.8

UN Undersize section N
U1-M Undersize of sections from 1 to M
ON>M Oversize of section N and feed of section M

TABLE 2. Model parameters calculated for different screen length

Sections	Length m	De mm	K_{50} t/h m^2	σ
1-2	0.90	7.82	21.53	2.19
1-3	1.35	7.85	19.94	2.21
1-4	1.80	7.87	20.92	2.37
1-5	2.25	7.98	20.47	2.52
1-6	2.70	8.01	20.51	2.53
1-7	3.15	8.27	18.76	2.84

It is to be remarked that an average value of the model parameters can be normally calculated having the size consist of the overall undersize product. However, when the undersize of different screen section are collected separately as in this example, it is possible to obtain a better estimation of the parameters and, at the same time, to check the quality of the experimental data. In this way, inaccuracy in parameter estimation occurring when the screen is not operating under crowded condition all over the surface can be avoided.

Table 2 shows that, excluding the influence of the sections near to the discharge end where the screening process is almost exhausted and only the critical particles remain, no significant variations of the parameters calculated for different screen length were observed. This fact validate the test under consideration and, when verified for a large number of tests as we did, is an indirect validation of the model. In case of Table 2, the arithmetic mean of the value, rounded to the first decimal digit (k_{50}=20.6, σ =2.4 and De=7.9) is a good estimate of the parameters.

Simulation

The simulation of integrated plant operations is becoming increasingly widespread in mineral processing for feasibility studies, prediction of results,

TABLE 3. Determination of screening partition numbers and efficiency

INPUT DATA:
Capacity per Unit Area = 28 t/h m^2
Equivalent Screen Aperture = 7.9 mm
Number of Class in Feed = 19
Kinetic Constants : K_{50} = 20.6 σ = 2.4

N	SIZE CLASS	FEED % wt	PART. NUMB.	OVER % wt	UNDER % wt
1	0.0 - 1.0	5.3	0.9864	0.07	5.23
2	1.0 - 1.2	1.3	0.9704	0.04	1.26
3	1.2 - 1.4	1.7	0.9622	0.06	1.64
4	1.4 - 1.7	2.5	0.9490	0.13	2.37
5	1.7 - 2.0	2.6	0.9293	0.18	2.42
6	2.0 - 2.4	3.2	0.9013	0.32	2.88
7	2.4 - 2.8	4.0	0.8571	0.57	3.43
8	2.8 - 3.4	5.0	0.7854	1.07	3.93
9	3.4 - 4.0	5.9	0.6734	1.93	3.97
10	4.0 - 4.8	6.7	0.5154	3.25	3.45
11	4.8 - 5.6	7.3	0.3234	4.94	2.36
12	5.6 - 6.7	8.7	0.1262	7.60	1.10
13	6.7 - 7.9	8.3	0.0103	8.21	0.09
14	7.9 - 8.0	0.7	0.0000	0.70	0.00
15	8.0 - 9.5	8.8	0.0000	8.80	0.00
16	9.5 -11.2	7.9	0.0000	7.90	0.00
17	11.2 -13.2	6.8	0.0000	6.80	0.00
18	13.2 -16.0	6.2	0.0000	6.20	0.00
19	16.0 -19.0	7.1	0.0000	7.10	0.00
		100.0		65.88	34.12

Recovery Efficiency = 0.54585
Removal Efficiency = 0.55930

3.677 CP seconds Execution Time

selection of alternative processes and as an aid in designing. The development of computer simulation programs requires the knowledge of the models for each of the process units included in the circuits.

The screening model considered in the present paper is perfectly suitable for this use and has already been included in an integrated plant simulation program developed by us. It has also been included by Herbst et al. ([16]) in the modular computer simulation package UTAH-MODSIM.

Table 3 gives an example of simulation of screening: input data are k_{50}, σ and De (20.6, 2.4 and 7.9 respectively); feed rate (140 t/h), screen area (5 m^2), hence capacity per unit area 28 t/h m^2; feed size composition. Output data are: partition numbers, size composition of the products, recovery and removal efficiencies.

Design

It is to be noted that the classical design methods for screens, even if conservative, do not always allow one to avoid possible failures. Difficult screening conditions exist (e.g. with circulating load containing a high proportion of near-submesh particles), for which calculation methods may be inadequate. Especially when such conditions are to be expected, the use of simulation procedures to verify the plant efficiency is advisable. Moreover, the classical design methods allow the sizing referred to a value of the screen efficiency but cannot give the size composition of the products.

Tables 4 and 5 show possible applications of the model in the designing stage. Given the model parameters (in this case the values of the previous example), the screen capacity per unit area that allows different efficiencies to be met can be calculated (Table 4). Table 5 shows as the model, for one of the cases of Table 4, can predict the size composition of the products.

By operating in this way, more accurate sizing can be obtained than by using classical design methods.

The use of the model for simulating the results can be very helpful in the checking of the dimensions adopted for the machines,

TABLE 4. Screen capacity per unit area for different values of efficiency

Recovery Efficiency	Removal Efficiency	Specific Capacity t/m^2
0.40	0.50	50.24
0.50	0.54	33.78
0.60	0.60	22.04
0.65	0.63	17.26
0.70	0.67	13.07
0.80	0.75	6.20

TABLE 5. Determination of screening capacity to meet a given efficiency

Required Recovery Efficiency = 0.650

INPUT DATA:
Equivalent Screen Aperture = 7.9 mm
Number of Class in Feed = 19
Kinetic Constants : K_{50} = 20.6 σ = 2.4

N	SIZE CLASS	FEED % wt	PART. COEF.	OVER % wt	UNDER % wt
1	0.0 - 1.0	5.3	0.9995	.00	5.30
2	1.0 - 1.2	1.3	0.9980	.00	1.30
3	1.2 - 1.4	1.7	0.9969	0.01	1.69
4	1.4 - 1.7	2.5	0.9948	0.01	2.49
5	1.7 - 2.0	2.6	0.9907	0.02	2.58
6	2.0 - 2.4	3.2	0.9833	0.05	3.15
7	2.4 - 2.8	4.0	0.9679	0.13	3.87
8	2.8 - 3.4	5.0	0.9341	0.33	4.67
9	3.4 - 4.0	5.9	0.8616	0.82	5.08
10	4.0 - 4.8	6.7	0.7220	1.86	4.84
11	4.8 - 5.6	7.3	0.4985	3.66	3.64
12	5.6 - 6.7	8.7	0.2121	6.85	1.85
13	6.7 - 7.9	8.3	0.0181	8.15	0.15
14	7.9 - 8.0	0.7	0.0000	0.70	0.00
15	8.0 - 9.5	8.8	0.0000	8.80	0.00
16	9.5 -11.2	7.9	0.0000	7.90	0.00
17	11.2 -13.2	6.8	0.0000	6.80	0.00
18	13.2 -16.0	6.2	0.0000	6.20	0.00
19	16.0 -19.0	7.1	0.0000	7.10	0.00
		100.0		59.40	40.60

Recovery Efficiency = 0.649
Removal Efficiency = 0.631
Capacity per Unit Area = 17.26 t/h m^2

23.259 CP seconds Execution Time

whatever calculation method is applied in sizing the screen. The effect of the various external factors on the screen performance (e.g. variations in the feed ore characteristics, or in the behaviour of other machines included in the circuits) can be investigated accurately by sensitivity and risk analyses as already reported for other problems in a previous paper by Ferrara et al.([17]).

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Symbols

x, X	particle size	mm
D	screen aperture	mm
De	equivalent screen aperture	mm
l	distance from feed point	cm
L	considered length of the screen	cm
L_c	critical length	cm
W(x,l)	mass flow rate per unit width of particles of size from 0 to x at distance l	g/sec cm
W(l)	mass flow rate per unit width of all particles over the screen at distance l	g/sec cm
W_0	value of W(l) at feed point (feed rate)	g/sec cm
W_c	value of W(l) at critical point (transition from crowded to separate conditions)	g/sec cm
f(x,l)	density function of size distribution at point l	dimensionless
f(x,0)	density function of size distribution at feed point	dimensionless
w(x,l)	= W(l) f(x,l) = W(x,l)/ x	g/sec cm
k(x),k_i	kinetic constants in the crowded region	g/sec cm^2
k_{50}	k(x) for x = D/2	g/sec cm^2
s(x),s_i	kinetic constants in the separate region	cm^{-1}
s_{50}	s(x) for x = D/2	cm^{-1}
n	number of attempts at passage of a particle per unit length of screen	cm^{-1}
p(x)	single event probability of passage	dimensionless
y_i	weight fraction of the particles of class i in the bed of material at distance L from the feed point	dimensionless
y_{i0}	weight fraction of the particles	

of class i in the feed dimensionless

E(X,L) screen oversize efficiency referred to particles of size X and to a length L of the screen dimensionless

$\chi(x)$ see Eqs.(5) and (10) dimensionless

χ_{ji} $= k_j/k_i$ dimensionless

σ see Eqs.(10) and (11) dimensionless

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