

A Filtration Model for Optimization of a Gold Extraction Plant

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A theoretical model for prediction of dissolved gold loss from a rotary vacuum filter was developed as part of a program to implement optimizing control on gold plants

The filter was modelled using a modification of the Hagen-Poiseuille equation and the Choudhury and Dahlstrom washing equation. The model has three parameters which must be determined from accumulated data before predictions can be made. These are the cake and cloth resistances and the filter washing efficiency.

The model predicts with a fair degree of accuracy the response of the filter to changes in the independent variables and will therefore be a useful tool in the plant optimization study.

Introduction

Gold Fields of South Africa operates several gold mines in the Transvaal. Attention has recently been given to the implementation of optimizing control on these plants.

In optimizing control, the best combination of the control variables will be chosen by the optimization method to yield the maximum profit. The setpoints of the control loops will then be driven to their optimum values. It is foreseen that the implementation of optimizing control will result in four main benefits:

- 1) improvements in recovery and profit,
- 2) less manual adjustment being necessary on the plant, resulting in less operating error,
- 3) less supervision being necessary which would result in the operating staff being able to devote more attention to other areas,
- 4) increased process stability yielding more consistent metallurgical behaviour.

Methodology

In order for the optimization to proceed, valid mathematical models of the process must be formulated.

When the functional form of a model $f(a_1, a_2, \dots, a_m, S_1, X_2, \dots, X_n)$ is derived from fundamental principles, it is called a theoretical model. An empirical model, on the other hand, is fitted to actual data.

Theoretical models are valid over the entire operating range of the process, provide an understanding of basic mechanisms and are the most reliable route towards process modelling. They have the disadvantage, however, of utilizing large numbers of parameters and may therefore

be difficult to obtain.

Empirical models do not normally provide an understanding of basic mechanisms and are dangerous when extrapolated outside the limits of the data. However, the development of empirical models is generally less time-consuming and therefore less expensive.

Modelling of an entire plant might well involve both theoretical and empirical models, the method(s) chosen being dependent on the complexities of each individual unit or sub-unit. The project team decided that the two approaches should be conducted in parallel: empirical modelling being used to stabilize the plant and increase profits in the short term, while further examining the requirements and implications of theoretical models, where possible, in order to refine and improve on the optimization in the long term.

The aim initially is to stabilize each unit in the plant, after which optimization can be extended to the plant as a whole. A 'unit' in the broad sense refers to a section of the plant, such as the filter plant or the mill circuit, each of which comprises a number of sub-units, e.g. a bank of filters or even an individual filter. In some cases, the unit might equate to a black box; in others there would be a need to monitor and/or control parameters on individual sub-units.

The project has proceeded with the modelling of the filter section of the plant. Experimentation will be performed to determine whether empirical/theoretical modelling of the entire filter section for the purpose of the optimization would require extension to the modelling of a filter bank or even an individual filter.

A theoretical model has been developed for a single filter. This could be adapted for use on a filter bank or the entire filter plant as determined by the above-mentioned experimentation. The model has been tested on a limited amount of data captured on a single filter at West Driefontein.

The development of this model is summarized. This is followed by a description of the computer program written to implement the model and test the predictions of the model against the experimentally acquired results.

The model

Description of a rotary vacuum filter

Although gold can be extracted from ore with more than one type of process, the standard practice in the South African gold mining industry at present consists of milling the ore down to a predetermined fineness and dissolving the gold from the ground aggregate with a cyanide solution in 'Browns' tanks. The slurry from the Browns tanks, consisting of solid particles (i.e. rock) and dissolved gold solution (called the pregnant solution), is routed to the filter plant where the solid and liquid phases are separated. The solid particles are repulped and discarded on the slimes dam while the pregnant solution (called filtrate) is treated further to extract the gold. Rotary vacuum filters are currently in use on the company's gold mines.

With reference to Figure 1 the operating principle of the rotary vacuum filter is as follows: Slurry from the Browns tanks is fed into the slurry tank of the filter. A slurry agitator is used to minimize the separation into

layers of different particle sizes in the slurry.

The circumference of the filter is divided into segments which are individually connected with pipes to a central suction/blow-off port valve. The adjustment of the port valve is such that vacuum is applied to every segment immediately after entry into the slurry. During submergence pregnant solution is extracted from the slurry, and solid particles are deposited on the filter cloth as a cake.

Subsequent to emergence from the slurry washwater is sprayed on to the cake to wash out the pregnant solution trapped in the voids between the solid particles. Depending on the flowrate of the washwater, the cake washing angle may be quite small, with relatively dry areas of cake in the vicinity of the slurry pan. With large flow rates of the washwater ('flood washing') the cake washing angle may extend from the point where the drum emerges from the slurry up to the position where the cake is discarded. The excess washwater flows into the slurry pan and, presumably, the cake discard pan.

Just before the leading edge of any segment passes the scraper, pressure instead of vacuum is applied to the segment to assist in discarding the cake. The discarded cake is repulped with the addition of washwater and pumped to the slimes dam.

Subsequent to a small 'dead angle' as the segment passes the scraper and re-enters the slurry, vacuum is once again applied and the process is repeated.

The Choudhury equation

Choudhury and Dahlstrom accumulated extensive data on several filter applications.¹ Plots of the logarithm of the fraction of original solute remaining in the cake after washing versus the wash ratio showed a strong linear trend. The equation of the best-fitting straight line was found to be

$$R_s = \left[1 - \frac{E}{100} \right]^n \quad [1]$$

where R_s = mass fraction original solute remaining in cake after washing

n = wash ratio

= volume of washwater going through cake per unit volume of original liquor in the unwashed cake

E = washing efficiency

= a measure of how successful the water permeating the cake is in displacing the original liquor; $0 \leq E \leq 100$. It is equal to the percentage of original solute which would be washed out of the cake at $n = 1,0$.

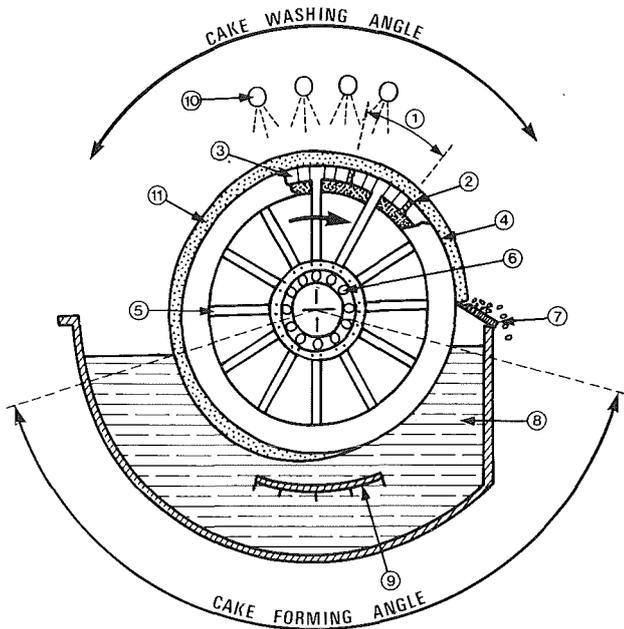
Derivation of equation for the dissolved gold concentration in the cake

Koudstaal defined R_s such that when $E = 0\%$ there is no wash at all and when $E = 100\%$ there is perfect wash.²

$$R_s = \frac{DG_c - DG_{ww}}{DG_p - DG_{ww}} \quad [2]$$

where DG_{ww} = dissolved gold concentration in washwater (g/m^3)

DG_c = dissolved gold concentration in cake (g/m^3)



- | | |
|--------------------------------|-------------------|
| 1 Filter segment | 7 Scraper |
| 2 Segment division strips | 8 Slurry in tank |
| 3 Filter cloth supporting grid | 9 Slurry agitator |
| 4 Filter cloth | 10 Water spray |
| 5 Suction piping | 11 Cake |
| 6 Rotating valve parts | |

FIGURE 1. Schematic illustration of a rotary vacuum filter²

DG_p = diluted gold concentration in pan (g/m^3)

Substituting [2] into [1] yields

$$\frac{DG_c - DG_{ww}}{DG_p - DG_{ww}} = \left[1 - \frac{E}{100} \right]^n \quad [3]$$

When $DG_c = DG_p$, no wash has occurred and $E = 0\%$ and

where $DG_c = DG_{ww}$, perfect wash has occurred and $E = 100\%$.

After manipulation of [3] an expression for DG_c is developed:

$$DG_c = DG_{ww} + \{DG_p - DG_{ww}\} \cdot \left[1 - \frac{E}{100} \right]^n \quad [4]$$

Expressions for DG_p and n are developed in Appendices 1 and 2 respectively, and are given below:

$$DG_p = \frac{DG_r Q_s (2690 - \sigma_s) + 1690(1-f)Q_{ww} DG_{ww}}{Q_s (2690 - \sigma_s) + 1690(1-f)Q_{ww}} \quad [16]$$

$$n = f Q_{ww} \left[\left(\frac{Q_s (\sigma_s - 1000)}{1690} \cdot \frac{\epsilon}{(1-\epsilon)} \right) \right]^{-1} \quad [21]$$

where DG_r is the dissolved gold concentration in the feed (g/m^3)

σ_s is the incoming slurry density (kg/m^3)

Q_s is the incoming slurry flowrate (m^3/s)

Q_{ww} is the total flowrate of washwater (m^3/s)

f is the fraction of washwater which passes through the cake.

It is clear that one can only predict DG_2 once E , the filter efficiency, and f , the fraction of washwater passing through the cake, are known.

The estimation of the filter efficiency, E

E is estimated using 'measured' values for DG_c . The dissolved gold concentration cannot easily be measured directly as it would have to be measured at several posi-

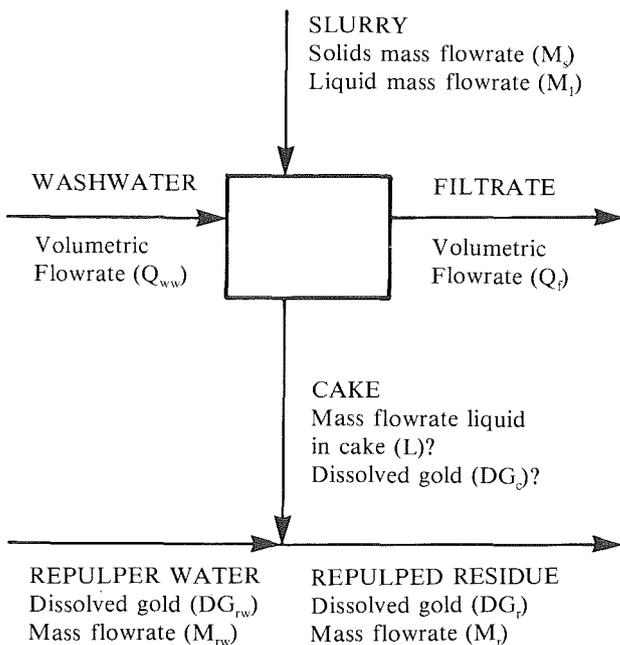


FIGURE 2. Filter mass balance

tions along the drum to get a representative sample. It is therefore calculated using mass balances.

With reference to Figure 2, the mass flowrate of liquid in the cake (L) can be calculated in one of two ways:

- (A) Mass flow liquid in cake
 = Mass flow repulped residue
 - Mass flow solids in cake
 - Mass flow repulper water

$$L = M_r - M_s - M_{rw} \text{ kg/s} \quad [5]$$

or

- Mass flow liquid in cake (b)
 = Mass flow liquid in incoming slurry
 + Mass flow washwater
 - Mass flow filtrate

$$L = M_l + (Q_{ww} - Q_f) 1000 \text{ kg/s} \quad [6]$$

The value of DG_c can now be calculated in a second mass balance:

- Gold in cake
 = Gold in repulped residue
 - Gold in repulper water

$$DG_c = \frac{(L + M_{rw}) DG_r - M_{rw} DG_{rw}}{L} \text{ g/m}^3 \quad [7]$$

The estimation of the cake and cloth resistances

As f , the fraction of washwater passing through the cake, is a function of the cake and cloth resistances these have to be estimated before f can be calculated.

The estimation is using a mathematical model developed from the Hagen-Poiseuille equation.¹

$$\frac{1}{A} \frac{dV}{dt} = \frac{\Delta P}{\eta(rwV/A + R)} \quad [8]$$

- where η = viscosity (pa.s)
 A = surface area of the drum (m^2)
 w = mass of dry cake deposited per unit volume of filtrate (kg/m^3)
 = specific cake resistance (m/kg)
 R = filter cloth resistance (l/m)
 dV/dt = volumetric flowrate of filtrate (m^3/s)
 ΔP = pressure difference over the bed (Pa)
 V = volume of filtrate (m^3).

Now ΔP changes during a cycle of the drum. One cycle consists of periods of cake formation t_f , cake washing t_w and cake discharge and dead time t_d . During discharge time the vacuum is switched off and positive pressure applied to the pipes. No filtrate is collected at this stage, nor during dead time. The total volume of filtrate from each segment per cycle is therefore equal to the sum of the filtrate volumes collected during cake formation and cake washing.

Integrating over the drum surface area yields the total volume of filtrate per cycle. Multiplication of this result by v , the drum speed in revolution per second, yields the volume of filtrate from the filter per second, W_f :

$$Q_f = \frac{-vAR}{wr}$$

$$\begin{aligned}
& + \frac{vA\sqrt{R^2\eta^2 + 2P_{av}t_fwr\eta}}{wr\eta} \\
& + \frac{t_w v A P_{aw}}{\sqrt{(R^2\eta^2 + 2P_{av}t_fwr\eta)}} \text{ m}^3/\text{s} \quad [9]
\end{aligned}$$

in which t_f = time for cake formation(s)
 t_w = time for cake washing(s)
 v = drum speed (rev/s)
 P_{av} = average ΔP during cake formation (Pa)
 P_{aw} = average ΔP during cake washing (Pa)

Expressions for t_f , t_w , P_{av} , P_{aw} are derived in Reference 2. The formula for w is derived in Appendix 4.

The first two terms represent the volumetric flowrate of filtrate during cake formation while the last term represents the volumetric flowrate of filtrate during washing. Given the operating point and a value for r and R , Q_f can be calculated. Values for r and R are chosen which minimize

$$\sum_i (Q_{f, \text{calculated}_i} - Q_{f, \text{measured}_i})^2$$

The fraction of washwater which passes through the cake, f , can now be calculated:

$$\begin{aligned}
f &= \frac{\text{flowrate washwater through cake}}{\text{total flowrate of washwater}} \\
&= \frac{t_w v A P_{aw}}{Q_{ww} \sqrt{(R^2\eta^2 + 2P_{av}t_fwr\eta)}} \quad [10]
\end{aligned}$$

If

$$Q_{ww} < \frac{t_w v A P_{aw}}{\sqrt{(R^2\eta^2 + 2P_{av}t_fwr\eta)}} \quad [11]$$

then all the washwater goes through the cake and $f = 1$.

The computer program

A simplified flowchart of the program is shown in Figure 3. The program reads in data and then calls subroutine STEADY to determine when steady-state conditions exist in the filter. This is necessary as the model is only valid at steady-state. Subroutine E045DF is then called to iterate for the cake and cloth resistances, r and R , by minimization of the sum of squares of the differences between the actual and predicted filtrate flowrates. These differences are calculated by subroutine FSFUNI for each combination of values of R and r being tested. Subroutine LSFUNI calls subroutine DILUTION to iterate for the diluted slurry density in the pan, which is needed in order to calculate w and P_{av} . This routine can therefore be considered to be iteration within the iteration of the cake and cloth resistances. Once the iterations are completed, the values of the cake and cloth resistances are returned to the main program.

The dissolved gold concentration in the cake can now be predicted at steady-state conditions in the filter. Subroutine STEADY is therefore called again to determine where steady-state conditions exist. Predictions with the existing parameters continue until the residuals become unacceptable. When this happens, the parameters are re-estimated.

Results and discussion

A plot of actual and predicted dissolved gold concentrations in the cake versus data block number is shown in Figure 4. The units on the x-axis are not time-based as data capture was erratic, long time periods elapsing between the acquisition of some data blocks. This was partly due to the periodic acid treatment/replacement of the filter cloth.

As can be seen from the figure, the predicted values generally followed the trend of the actual values. The model predicts the response of the filter to changes in the control parameters with a fair degree of accuracy and is therefore expected to be a useful tool in the optimization study.

Values of the cake and cloth resistances were re-

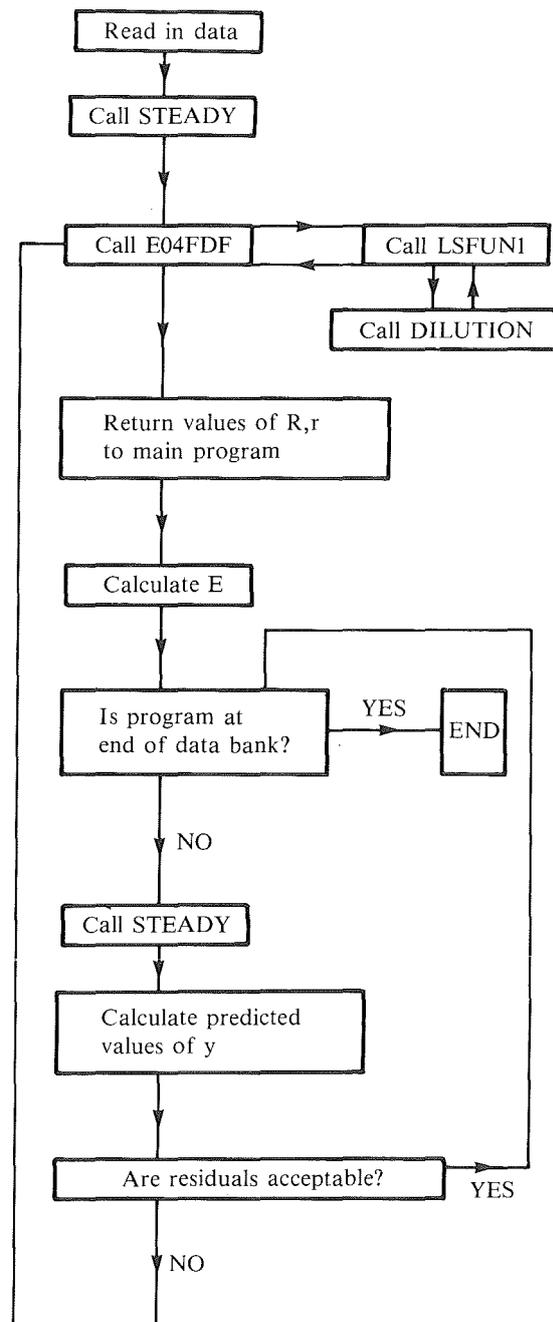


FIGURE 3. Program flowchart

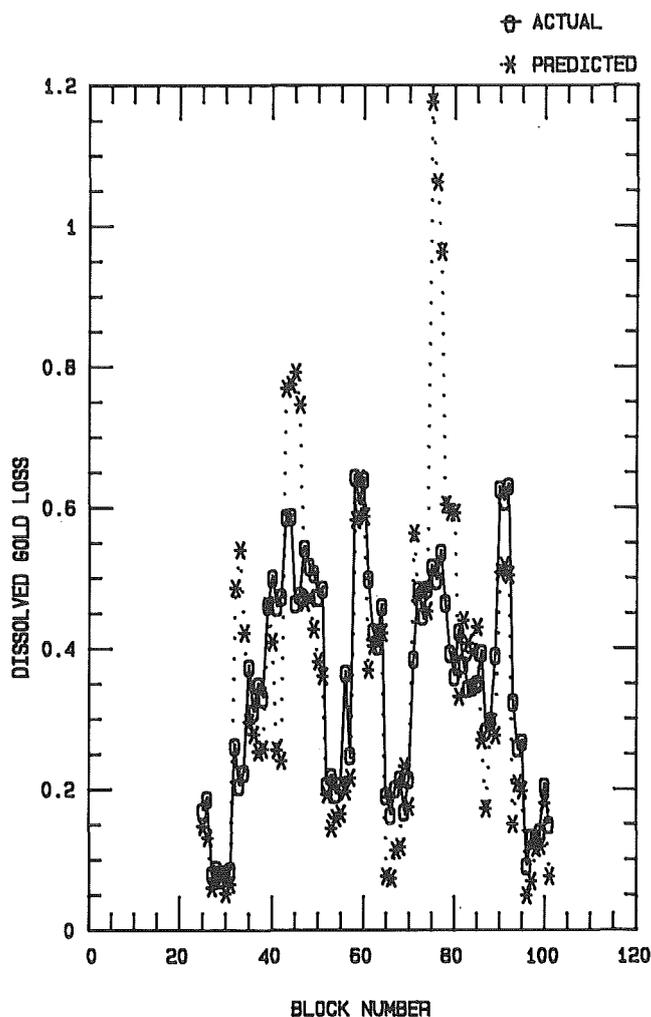


FIGURE 4. Actual and predicted dissolved gold losses

estimated when the values of the residuals became too high. In the majority of cases, this re-estimation led to an improvement in the accuracy of prediction. The elapsed time period between data blocks where re-estimations were made varied between 8 hours and 8 days. No conclusions can be drawn from this, however, because of the inconsistency of data capture. On average, the results for the estimation of the cake and cloth resistances were:

$$r = 28,7.10^9 \pm 23,2.10^9 \text{ (m/kg)}$$

$$\text{and } R = 231,2.10^9 \pm 205,47.10^9 \text{ (l/m)}$$

With reference to the specific cake resistance, r , the relatively large standard deviation of $\pm 23,2.10^9$ was surprising. Bearing in mind that the fineness of solids in the slurry and the porosity of the cake are fairly constant, a smaller standard deviation was expected.

The calculated value for the filter cloth resistance, R , varied considerably with time. This effect can most probably be attributed to the periodic blinding and cleaning of the cloth. During experimentation it was cleaned roughly every four days with diluted hydrochloric acid.

The main application of this model will be in the prediction of the dissolved gold losses from the filter at different values of control variables such as drum speed, submergence level, washwater flowrate and incoming slurry density.

It is intended that an empirical model be developed shortly and the two compared with respect to accuracy, robustness and ease of manipulation.

In the case of the filter section, whichever proves to be the superior of the two in terms of the above three factors will be used for the purpose of optimization.

Two possible optimization strategies for the filter plant are given below:

Each filter bank is modelled as a unit

In this case, individual filters are not modelled separately. If used, the theoretical model would be adapted for use on a filter tank.

An expression for the dissolved gold loss from the filter plant is derived in Appendix 3 and is given below:

$$\begin{aligned} \text{OVERALLDG} &= \frac{\text{g dissolved gold out of plant}}{\text{t solids out of plant}} \\ &= \frac{DG_1(L/S)_1Z_1 + DG_2(L/S)_2Z_2 + \dots + DG_N(L/S)_NZ_N}{Z_T} \end{aligned} \quad [24]$$

where $DG_1, DG_2 \dots DG_N$; $(L/S)_1, (L/S)_2 \dots (L/S)_N$ and $Z_1, Z_2 \dots Z_N$ represent the dissolved gold concentrations from, liquid to solid ratios from and throughputs to filter banks 1, 2 ... N , respectively. Z_T represents the total throughput to the filter plant.

As there are no facilities for storing slurry in a filter plant, the slurry pumped in must be processed immediately. Therefore, the total throughput to the filter plant is a constraint on the system which must not be violated. Now for each bank, Z is likely to be a function of washwater flowrate, vacuum, drum speed, slurry temperature, density of incoming slurry and possibly submergence level. Equations relating the throughput to each filter bank to these variables will be formulated by means of a regression using plant data.

Values of the control variables will be chosen for each bank so that the overall dissolved gold loss is minimized and the total throughput constraint is satisfied, i.e. at present the optimum washwater flowrates and slurry densities to each of the three banks would be determined, as well as the optimum drum speeds and submergence levels (these would be the same for each filter in a bank).

Each filter is modelled separately

The equation for overall dissolved gold loss is derived in the same way:

$$\begin{aligned} \text{OVERALLDG} &= \frac{DG_A(L/S)_AZ_A + DG_B(L/S)_BZ_B + \dots + DG_N(L/S)_NZ_N}{Z_T} \end{aligned} \quad [12]$$

where $DG_A, DG_B \dots DG_N$; $(L/S)_A, (L/S)_B \dots (L/S)_N$; $Z_A, Z_B \dots Z_N$ represent the dissolved gold concentrations from, liquid to solid ratios from and solids throughputs to filters A, B ... N , respectively.

The optimum washwater flowrates and slurry densities to each filter would be determined, as well as the optimum

drum speeds and submergence levels (these could be different for each filter).

It is clear that the modelling of each filter as opposed to each bank would require extra equipment in the form of washwater flowmeters, slurry flowmeters and equipment to measure repulped residue densities, etc. It would therefore have to be proved that the modelling of each filter separately would lead to sufficient savings in dissolved gold to make this worthwhile.

In both cases, other control variables which affect upstream sections of the plant as well as the performance of the filters, e.g. particle size, will only be manipulated once models to describe their effect on these upstream processes have been formulated. Until then, these variables will be considered to be uncontrollable. Empirical models will be obtained initially for the remaining units as these are usually obtained faster. Theoretical models will be developed where possible as these are likely to provide a better understanding of the mechanisms of the process concerned.

The described theoretical filter model is not dynamic in nature, although it is well known that mineral processing plants are never completely at steady-state because of fluctuations in process conditions. After a study of the process, it was discovered that most of the variables which have an influence on dissolved gold loss are eventually controllable. Variations in uncontrollable variables such as ambient temperature and concentration of dissolved gold in the feed which are sufficiently high to cause a change in the optimum values of the control variables

are not expected over small time periods. Before the optimization process is implemented on the mines, a sensitivity analysis will be completed. This will establish what change(s) in the uncontrollable variables will necessitate a rerun of the optimization program and subsequent adjustment to the setpoints of the control loops.

References

1. CHOUDHURY, A.P.R. and DAHLSTROM, D.A. Prediction of cake-washing results with continuous filtration equipment. *A.I. Ch. E. Journal*, vol. 3, no. 4, 1957. pp. 433 – 438.
2. KOUDSTAAL, J. A study of the control of a rotary vacuum filter in a gold reduction plant. PhD thesis, University of Natal (Durban Campus), 1976.
3. FRITZ, J.T.D. A feasibility study on improving the throughput and efficiency of rotary vacuum filters used in the gold mining industry. *CSIR Contract Report* (private communication), January 1984.
4. MULAR, A.L., BULL, W.R., PUTTMAN, R.E.J. and BLACK, K. Training Manual for the SAIMM Summer School. *Mineral Processes: Their Analysis, Optimization and Control*. Mular, A.L. and Bull, W.R. Johannesburg; 1973.
5. ENGELBRECHT, P.C., NORTON, A.E. and BECK, R.D. The process monitoring and control of a circuit for the concentration of copper, lead and zinc. *Proceedings, 12th CMMI Congress*. Glen, H.W. Johannesburg, 1982. pp. 809 – 816.

Appendix 1

Diluted gold concentration in pan DG_p

The incoming slurry flowrate = Q_s (m^3/s) and its density = σ_s (kg/m^3). The liquid density will be taken equal to 1000 kg/m^3 and the density of the solids as 2690 kg/m^3 .

Volumetric flowrate of liquid in incoming slurry

$$= \frac{2690 Q_s - Q_s \sigma_s}{1690} m^3/s \quad [13]$$

Total flowrate of liquid into pan

= Flowrate of liquid in incoming slurry
+ Flowrate washwater into pan

$$= \frac{2690 Q_s - Q_s \sigma_s}{1690} + (1 - f)Q_{ww} m^3/s \quad [14]$$

where f is the fraction of washwater which passes through the cake,

Q_{ww} is the total flowrate of washwater (m^3/s).

The grams of dissolved gold flowing into the pan per second

$$= \left(\frac{2690 Q_s - Q_s \sigma_s}{1690} \right) DG_f + (1 - f)Q_{ww}DG_{ww} \quad [15]$$

where DG_f and DG_{ww} are the dissolved gold concentrations in the feed and washwater respectively.

$\therefore DG_p$

= Concentration of dissolved gold in the pan

$$= \frac{DG_f Q_s (2690 - \sigma_s) + 1690(1 - f)Q_{ww}DG_{ww}}{Q_s(2690 - \sigma_s) + 1690(1 - f)Q_{ww}} g/m^3 \quad [16]$$

Appendix 2

Wash ratio n

$$\epsilon = \frac{V_{void}}{V_{cake}} \quad [17]$$

$$\frac{\epsilon}{1 - \epsilon} = \frac{\text{Void volume of cake}}{\text{Volume of solids in cake}} \quad [18]$$

It is assumed that the entire void volume of the unwashed cake is filled with liquid.

The volumetric flowrate of solids in the cake

$$= \frac{Q_s (\sigma_s - 1000)}{1690} m^3/s \quad [19]$$

where Q_s and σ_s are the incoming slurry flowrate and density respectively.

\therefore The volumetric flowrate of liquid in the unwashed cake

$$= \frac{Q_s (\sigma_s - 1000)}{1690} \cdot \frac{\epsilon}{1 - \epsilon} m^3/s \quad [20]$$

$n = \frac{\text{volume washwater going through cake}}{\text{volume liquor in unwashed cake}}$

$$= \frac{fQ_{ww}}{Q_s (\sigma_s - 1000)} \cdot \frac{\epsilon}{1 - \epsilon} \quad [21]$$

Appendix 3

Total dissolved gold loss OVERALLDG

For each filter tank

$$DG = f(S_1, X_2, X_3 \dots X_n) g/m^3$$

where $X_1, X_2 \dots X_n$ are the independent variables found to be significant in predicting DG.

$$\begin{aligned} \text{Now, DGLOSS} &= \frac{\text{g gold}}{\text{kg solids in cake}} \\ &= \frac{DG(g/m^3) \times L/S (kg/kg)}{1000 (kg/m^3)} \quad [22] \end{aligned}$$

where L/S = liquid to solid ratio in cake

OVERALLDG

$$\begin{aligned} &= \frac{\text{g dissolved gold out of filter plant}}{\text{t solids out of filter plant}} \\ &= \frac{DGLOSS_1 Z_1 + DGLOSS_2 Z_2 + \dots DGLOSS_N Z_N}{Z_T} \\ &= \frac{g/kg \times 1000 (kg/t)}{\quad} \quad [23] \end{aligned}$$

$$= \frac{DG_1(L/S)_1 Z_1 + DG_2(L/S)_2 Z_2 + \dots DG_N(L/S)_N Z_N}{Z_T} \quad [24]$$

where $DG_1, DG_2 \dots DG_N; (L/S)_1, (L/S)_2 \dots (L/S)_N$ and $Z_1, Z_2 \dots Z_N$ represent the dissolved gold concentrations from, liquid to solid ratios from and throughputs to filter banks 1, 2 ... N respectively.

Z_T = total throughput to filter plant (kg/s)

$$= Z_1 + Z_2 + \dots Z_N.$$

Appendix 4

Derivation of w , the mass of dry cake deposited per unit volume of filtrate

$$w = \frac{\text{mass of dry cake deposited}}{\text{unit volume of filtrate}} (kg/m^3)$$

The mass flowrate of incoming solids

$$= \frac{2690(Q_s \sigma_s - 1000Q_s)}{1690} kg/s \quad [25]$$

Volumetric flowrate of filtrate

= Volumetric flowrate of liquid in slurry

- Volumetric flowrate of pregnant solution trapped in voids of cake

$$= \frac{2690Q_s - Q_s \sigma_s}{1690} - \frac{Q_s(\sigma_s - 1000)}{1690} \cdot \frac{\epsilon}{1 - \epsilon} m^3/s \quad [26]$$

$$w = \frac{2690(\sigma_s - 1000)(1 - \epsilon)}{1690 - 1690 \epsilon - \sigma_s} kg/m^3 \quad [27]$$

As washwater may spill over into the slurry pan, it follows that σ_s represents the diluted slurry density in the pan.