Optimizing Design Capacity and Field Dimensions of Underground Coal Mines

Z. LI and E. TOPUZ

Department of Mining and Minerals Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA

This paper presents a quantitative approach to the problem of determining design capacity and field dimensions of underground coal mines using room-and-pillar mining systems. In evaluating these design variables, relationships among mining costs, number of production sections in mines, location of central shafts, mine field dimensions, section production, mine output, and cost of production losses due to underground man-traveling are analyzed. Thereby the unit cost of coal is expressed as a function of the design variables as well as input parameters such as seam angle, seam thickness, seam depth, underground traveling speed of men, mine recovery and plant recovery. The problem is then formulated as a nonlinear optimization model in terms of minimizing the unit cost of coal subject to a set of constraints and solved analytically for flat seams and numerically for inclined seams. Sensitivity analysis of the variables is also included in the paper.

Introduction

The determination of mine design capacity and mine field dimensions under various mining conditions is of primary importance to the economics of developing and subsequently operating an underground coal mine. For example, an overrated design capacity of the mine can immobilize a large amount of capital; oversized dimensions of the mine field can lead to decreased availability of working time at faces and increased costs for mine ventilation, underground materials handling, and roadway maintenance. Undersized mine field dimensions, on the other hand, bring about frequent new mine developments which are of course costly, especially considering that minable coal seams become deeper and deeper as the resource depletes. An improper selection of location for the production shaft can result in increased capital and operating costs. Decisions related to the problem of determining mine shaft location, mine field dimensions, and mine design capacity have a long-term effect on the overall economics of underground coal mine operations, and a well-planned approach to the problem can yield substantial monetary savings.

This paper presents a quantitative approach to the problem of determining mine design capacity and mine field dimensions.

Formulation of the problem

Cost-size relationship

In order to formulate the problem, the relationship between mining cost and mine size needs to be quantified.

Data

To derive the quantitative relationship between mining cost and mine size, use was
made of the cost estimates for hypothetical drift mines with the number of sections in a mine ranging from 2 to 24, extracting seams of 1.83 meters, lying 229 meters below the surface as accounted in the literature. Through a statistical analysis of the data, capital cost \( (B_c) \) and operating cost \( (B_o) \) in 1977 U.S. dollars for the base-case drift mines are expressed as functions of the number of production sections in mines \( (S) \):

\[
B_c = 9394200 \cdot S^{0.9}, \quad [1a]
\]

and

\[
B_o = 2628200 \cdot S^{1.0}. \quad [1b]
\]

It is noted that the data, though quite old, does serve the purpose of analyzing the problem. For a more accurate modeling, however, the data may need to be adjusted to current price levels using historical inflation data or preferably current data need to be used.

**Effect of seam thickness on mining costs**

Seam thickness influences mining costs. In a thin seam, more extensive workings have to be developed and a greater area of ground has to be mined in order to achieve the same mine output as that from a thicker seam. As a result, those costs related to the extent of extraction are increased. For example, costs for conveyor belts, rail track, water and power lines, haulage of coal, rocks, and supplies, ventilation, entry construction, roadway maintenance, spillage cleaning and rock dusting become greater in thinner coal seams. To take the effect of seam thickness into account, adjusting capital cost \( (T_c) \) and operating cost \( (T_o) \) are estimated as

\[
T_c = 19685000(1.83 - m), \quad [2a]
\]

and

\[
T_o = 574000(1.83 - m), \quad [2b]
\]

where \( m \) is seam thickness. These adjustments will be added to the base-case costs.

**Effect of seam depth**

The effect of seam depth on mining costs results primarily from the variation in the length of access openings. For deeper coal seams, the capital cost for shaft sinking and hoist facilities becomes greater. In addition, coal hoisting, men and supplies transporting, and mine ventilation become more expensive. Deep seams may also require heavier, thus more costly supports than do shallow seams.

The cost for shaft sinking varies depending on geological conditions and geotechnical properties of the strata through which the shaft is sunk, sinking methods used, lining requirements, length of shaft, and sectional area of the shaft. Because the sectional area of shaft is dependent on the mine capacity, so also is the shaft sinking cost.

The shaft sinking cost \( (S_c) \) is estimated to be

\[
S_c = -9650000 + 49213(h + b \tan \alpha), \quad [3a]
\]

where \( h \) is the depth of seam at the shallower boundary of the property, \( b \) is the horizontal distance of the production shaft from the shallower boundary of mine field along the dip direction, and \( \alpha \) is the inclination of seam (shown in Figure 1).

The operating cost for coal hoisting is formulated as

\[
S_o = c_oA(h + b \tan \alpha), \quad [3b]
\]

where \( c_o \) is the cost for hoisting a unit of coal along a unit length of shaft and \( A \) is the annual mine output.

**Costs for cleaning and loading facilities**

Coal cleaning costs depend on the degree of cleaning and the tonnage of raw coal feed. Katell\(^2\) has suggested a cost to size factor of 0.96 for the capital cost \( (P_c) \) and 0.85 for the operating cost \( (P_o) \), that is,

\[
P_c = K_cA^{0.96}, \quad [4a]
\]

and

\[
P_o = K_oA^{0.85}, \quad [4b]
\]

where \( K_c \) and \( K_o \) are constants for a specified degree of cleaning.
In addition, according to Katzell, loading facilities add 2% to the capital cost and 1% to the operating cost.

Cost-size relationship

In summary, the cost-size relationships are derived through a statistical analysis of the U.S. Bureau of Mines and the Energy Information Administration cost estimates for hypothetical drift mines. Cost adjustments are made for shaft mines extracting seams with various thicknesses and depths and having surface loading and cleaning facilities. The cost-size relationship for the capital cost \( C_c \) is

\[
C_c = (1 + 0.02)(B_c + T_c + S_c + P_c) \quad [5a]
\]

and for the operating cost \( C_o \) is

\[
C_o = (1 + 0.01)(B_o + T_o + S_o + P_o) \quad [5b]
\]

Assuming a mine life of \( T \) years and an interest rate of 1 and summing up the annual capital and operating costs, one obtains the annual cost for the extraction and processing of coal as

\[
C_t = i_T C_c + C_o
\]

where \( i_T \) is the equal-payment-series capital recovery factor.

Locating a production shaft

The problem of locating the central shafts and surface facilities within the mine field serves as the basis upon which the problem of determining the mine field dimensions and design capacity is approached.

Many factors may affect the location of production shaft and central surface facilities. Among them are costs for underground haulage of coal and rocks, transportation of men and supplies, ventilation, and roadway maintenance. Other factors include inclination of the seam, distribution of coal over the seam, geology and hydrology of the surrounding strata, surface topography, and closeness of roads and water and power supplies.

In the analysis, it is assumed that the thickness of seam and the angle of inclination are constant, the geology and hydrology of overlying strata are uniform over the mine field, and the influence of distances to accessible rail and roads and water and power supplies is negligible.

It is also assumed that the shape of mine field is rectangular, the boundaries of mine field are parallel or perpendicular to the strike or dip direction of the seam, and all underground haulage roadways are developed either along or across the strike. In other words, coal, men, and supplies are transported along orthogonal sets of openings and the transportation distances are rectilinear.

Under these conditions, the location of production shaft and central surface facilities can be determined by analyzing costs for transportation of coal, men, and supplies. As a result, the optimal shaft loca-
tion in terms of minimizing the cost for underground transportation can be found to be at
\[ a = \frac{x}{2} \]  
and
\[ b = a_1 y - a_2 x, \]  
where \( x \) is the mine field dimension along the strike, \( y \) is that across it, and \( a_1 \) and \( a_2 \) are constants. 3

It may be inferred that the inclusion of costs for mine ventilation and roadway maintenance would change the optimal location of shaft as determined above. However, the deviation is usually insignificant because these costs are relatively small compared to the cost for transportation. This is true especially when the inclination of seam is small and the production shaft is located approximately at the center of the mine field because it can be shown that for flat seams, a centrally-located shaft is the most favorable in terms of minimizing the costs for mine ventilation and roadway maintenance. 3

**Number of production sections in mines**

Let the production rate (\( p \)) be defined as the production per unit of productive time. Then the section production per shift can be evaluated as
\[ p_s = p_T p \frac{d}{d} (T_g - T_L - T_t - T_d) \]  
where \( p \) is an efficiency factor and \( T_p \) is the productive shift time which equals the total shift time \( (T_g) \) less the lunch break time \( (T_L) \), the time spent on traveling to and from the face \( (T_t) \), and the total shift delay time \( (T_d) \).

It is important to note that \( T_t \) is a function of mine field dimensions. The traveling time can be evaluated as
\[ T_t = 2(D/v_t + T_w) \]  
where \( v_t \) is the traveling speed of men, \( T_w \) is the waiting time during transporting men into or out of the mine, and \( D \) is the to-

Cost of production losses

In a previous section, mining costs were dealt with. To be specific, the tangible costs, that is, equipment investment, labor cost, and supplies and power costs were estimated. For a more detailed analysis, however, it is necessary to include intangi-
Formulation of the problem

Adding the cost of production losses to the cost for equipment, labor, and power and supplies results in the total annual cost

\[ C_T = T_r C_o + C_o + C_L. \]  

[15]

Dividing the annual salable coal, which is equal to \( r_pA \), into the total annual cost \( (C_T) \), the total cost for a unit of salable coal \( (C_u) \) can be expressed as a function of \( x \), \( y \), and \( A \), denoted as

\[ C_u = f(x,y,A). \]  

[16]

The problem of determining the field dimensions and design capacity of a mine is then formulated in terms of finding \( x \), \( y \), and \( A \) such that the cost for a unit of salable coal is minimized under the condition that the annual mine output equals the tonnage of recoverable coal in the property divided by the life of mine. Mathematically, the problem is

\[ \min \ C_u = f(x,y,A) \]  

[17a]

\[ \text{s.t. } A = r_m y x / (T \cos \alpha) \]  

[17b]

and \( x, y > 0 \).  

[17c]

Solution of the problem

Analytical solution for a special case

The problem can be solved analytically under the condition that the inclination angle of coal seam is zero. An analytical solution of this special case is important in that it demonstrates how the objective function behaves, provides guidance in selecting a method for solving the general problem in which the dip angle of seam is not zero, and serves as a test problem for the programmed algorithm which is used to solve the general problem.

The following input values are used in solving the problem:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>1.83 m</td>
</tr>
<tr>
<td>( b )</td>
<td>229 m</td>
</tr>
<tr>
<td>( Q_s )</td>
<td>50,001.085 t/m</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.90</td>
</tr>
<tr>
<td>( t )</td>
<td>20 yr</td>
</tr>
<tr>
<td>( d )</td>
<td>220 days/yr</td>
</tr>
<tr>
<td>( T_s )</td>
<td>270 s</td>
</tr>
<tr>
<td>( s )</td>
<td>2 shifts/day</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>10%</td>
</tr>
<tr>
<td>( \nu )</td>
<td>3.31/t</td>
</tr>
<tr>
<td>( \eta )</td>
<td>50%</td>
</tr>
<tr>
<td>( T_p )</td>
<td>85%</td>
</tr>
<tr>
<td>( T_f )</td>
<td>60%</td>
</tr>
<tr>
<td>( T_f )</td>
<td>28800 s</td>
</tr>
<tr>
<td>( T_f )</td>
<td>10%</td>
</tr>
<tr>
<td>( T_f )</td>
<td>1800 s</td>
</tr>
<tr>
<td>( T_f )</td>
<td>6600 s</td>
</tr>
</tbody>
</table>

It can be analytically found that with a processing plant, the optimal mine field dimensions, mine design capacity, and unit cost of salable coal, respectively, are

\[ x = y = 6,053 \text{ m}, \]  

\[ A = 2,253,834 \text{ t/yr}, \]  

and \( C_u = \$22.75/t \); and without a processing plant,

\[ x = y = 4,432 \text{ m}, \]  

\[ A = 1,208,350 \text{ t/yr}, \]  

and \( C_u = \$14.50/t \).

Numerical solution of the problem

To solve the general problem in which the inclination angle of seam does not equal
zero, numerical methods must be used and three additional pieces of information are required, namely, the unit costs ($/t/m) for transporting along horizontal haulage ways, upward and downward along inclined haulage ways, denoted by $c_h$, $c_u$, and $c_d$, respectively.

The problem was solved on an IBM personal computer using the Newton method and the Davidson-Fletcher-Powell multidimensional search algorithm. Given that $\alpha = 5^\circ$, $c_h = 0.001085$, $c_u = 0.001121$, and $c_d = 0.001049$, it was found that with a processing plant, the optimal mine field dimensions along the strike and the dip, mine design capacity, and unit cost of coal, respectively, were

$x = 7,654$ m and $y = 4,678$ m,

$A = 2,202,523$ t/yr,

and

$Cu = $23.58/t;

and without a processing plant,

$x = 5,896$ m and $y = 3,052$ m,

$A = 1,106,925$ t/yr,

and

$Cu = $15.19/t.

**Discussion of results**

**Analysis of sensitivity**

To study the effects of input parameters on mine field dimensions, mine design capacity, and unit cost of coal, an analysis of sensitivity was conducted by solving the problem under various mining conditions. This was made possible by repeating the execution of the problem on an IBM personal computer with one parameter varied in a range of possible values and the others kept fixed at a value commonly found in practice. Those fixed values are the same as those used in the special case except that instead of 229 m, a value of 457 m is used as the depth of seam in the sensitivity analysis.

The input parameters that were individually analyzed include seam angle, seam thickness, underground travel speed of men, mine recovery, and plant recovery. The results from analyzing the effects of these input parameters on the optimal mine field dimensions, mine design capacity, and unit cost of coal are given in Table 1 to 6.

**Discussion of results**

Some observations can be made on the individual effects of the input parameters on mine field dimensions, design capacity, and unit cost of coal.

**Seam angle**

The inclination angle of seam affects the optimal shape of the mine field. When the seam is flat, a square shape is most favorable. As the inclination angle is increased from 0 to 6 degrees, the ratio of the optimal field dimension along the strike to that across it changes from 1 to 1.6 and the minimum unit cost increases from $23.62/t to $24.59/t.

**TABLE 1.**

<table>
<thead>
<tr>
<th>$\alpha$ (°)</th>
<th>$x$ (m)</th>
<th>$y$ (m)</th>
<th>$A$ (t/yr)</th>
<th>$Cu$ ($$/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7,156</td>
<td>7,156</td>
<td>3,149,325</td>
<td>23.62</td>
</tr>
<tr>
<td>2</td>
<td>7,541</td>
<td>6,508</td>
<td>3,178,604</td>
<td>24.01</td>
</tr>
<tr>
<td>4</td>
<td>8,699</td>
<td>5,990</td>
<td>3,129,000</td>
<td>24.33</td>
</tr>
<tr>
<td>6</td>
<td>8,862</td>
<td>5,553</td>
<td>3,026,352</td>
<td>24.59</td>
</tr>
</tbody>
</table>

**Seam thickness**

As the thickness of seam is increased from 1.22 to 2.44m, the optimal mine field dimensions decrease from 8,019 to 5,745m at an increasing rate of change and the minimum cost decreases from $30.30/t to $18.58/t at a decreasing rate of change.

**TABLE 2.**

<table>
<thead>
<tr>
<th>$m$ (m)</th>
<th>$x$=y (m)</th>
<th>$A$ (t/yr)</th>
<th>$Cu$ ($$/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.22</td>
<td>8,019</td>
<td>2,636,906</td>
<td>33.30</td>
</tr>
<tr>
<td>1.52</td>
<td>7,621</td>
<td>2,976,891</td>
<td>27.52</td>
</tr>
<tr>
<td>1.83</td>
<td>7,156</td>
<td>3,149,325</td>
<td>22.62</td>
</tr>
<tr>
<td>2.13</td>
<td>6,577</td>
<td>3,103,632</td>
<td>20.78</td>
</tr>
<tr>
<td>2.44</td>
<td>5,745</td>
<td>2,707,000</td>
<td>18.58</td>
</tr>
</tbody>
</table>
Seam depth

As the depth of seam is increased from 229 to 838 m, the optimal mine field dimensions, mine design capacity, and the minimum cost of coal increase from 6,053 to 8,269 m, from 2.25 to 4.21 million t/yr, and from $22.75/t to $24.83/t at a decreasing rate of change, respectively.

<table>
<thead>
<tr>
<th>Table 3. Effects of seam depth (h) on the optima</th>
</tr>
</thead>
<tbody>
<tr>
<td>h (m)</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>229</td>
</tr>
<tr>
<td>381</td>
</tr>
<tr>
<td>533</td>
</tr>
<tr>
<td>686</td>
</tr>
<tr>
<td>838</td>
</tr>
</tbody>
</table>

Traveling speed

As the underground traveling speed of men is increased from 1.5240 to 1.9034 m/s, the optimal mine field dimensions and design capacity increase from 6,639 to 7,632 m and from 2.71 to 3.58 million t/yr at an approximately constant rate of change, respectively, but the minimum cost of coal decreases from $24.11/t to $23.23/t.

<table>
<thead>
<tr>
<th>Table 4. Effects of travel speed (v_t) on the optima</th>
</tr>
</thead>
<tbody>
<tr>
<td>v_t (m/s)</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>1.5240</td>
</tr>
<tr>
<td>1.6256</td>
</tr>
<tr>
<td>1.7272</td>
</tr>
<tr>
<td>1.8288</td>
</tr>
<tr>
<td>1.9304</td>
</tr>
</tbody>
</table>

Mine recovery

As the recovery of coal is increased from 50% to 90%, the optimal mine field dimensions and the minimum cost of coal decrease from 7,990 to 5,992 m, from 3.93 to 2.14 million t/yr, and from $28.96/t to $15.56/t at a decreasing rate of change, respectively.

<table>
<thead>
<tr>
<th>Table 5. Effects of mine recovery (r_m) on the optima</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_m (%)</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>70</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>90</td>
</tr>
</tbody>
</table>

Plant recovery

As the recovery of plant is increased from 80% to 100%, the optimal field dimensions, mine design capacity, and unit cost of coal decrease from 7,990 to 5,992 m, from 3.93 to 2.14 million t/yr, and from $28.96/t to $15.56/t at a decreasing rate of change, respectively.

<table>
<thead>
<tr>
<th>Table 6. Effects of plant recovery (r_p) on the optima</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_p (%)</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>85</td>
</tr>
<tr>
<td>90</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

Conclusions

The study indicates that the optimal mine field dimensions increase with seam depth and traveling speed of men and decrease with seam thickness, mine recovery, and plant recovery; the minimum unit cost of coal increases with seam depth and seam angle and decreases with seam thickness, traveling speed, mine recovery, and plant recovery; and the optimal mine design capacity increases with seam depth, traveling speed, and mine recovery and decreases with plant recovery.

When the seam is flat, a square shape of mine field is most favorable and as the seam angle increases the ratio of the optimal field dimension along the strike to that across it becomes larger. It is interesting to notice that the optimal design capacity
does not vary monotonically but concavely with seam thickness or seam angle.

Also, it appears that the plant recovery and the seam thickness have a significant effect on the minimum unit cost of salable coal even though the unit cost of coal is insensitive to the field dimensions and design capacity.

Finally, by building such a quantitative model, the optimal mine life can be determined and the effects of other input parameters such as the rate of interest on the mine design capacity, mine field dimensions, mine service life, and the unit cost of coal can be analyzed.

It should be mentioned that the optimal solution from the quantitative analysis is not intended to be the terminal decision because some qualitative factors affecting the evaluation of mine field dimensions and design capacity were not incorporated into the analytical formulation. Instead, the quantitative solution is expected to serve as a useful aid in reaching the right decision concerning the design of underground coal mines. The methodology presented here can be an important step toward increased applications of quantitative methods in designing underground coal mines.

References