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An Analysis of the True Rate of Return Project Evaluation Criterion

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The internal rate of return (IRR) is one of the most popular project evaluation criteria in the mineral industry, as well as in other industrial sectors. Analysis of its assumptions reveals, however, that this criterion does not function in the way it is supposed to. The search for a new criterion that would reflect the true rate of return (TRR) of an investment project or the true borrowing rate (TBR) of a borrowing project has led to the development of several approaches, yielding, for the same project, generally different values of the interest rate in question.

Two of those approaches have been selected as the most representative and practical. Each has been presented in two variants, and subsequently analysed. As a result, it is shown that the various approaches to the TRR/TBR are all consistent with the net present value (NPV) criterion, and that each can serve as an independent project evaluation instrument, provided its assumptions can be regarded in a given situation as realistic enough.

Introduction

A project evaluation criterion expressed in terms of the interest rate either yielded by a dollar invested (investment-type projects) or required for a dollar borrowed (borrowing-type projects) has always been a very attractive and convenient project evaluation criterion. It is therefore no wonder that the IRR criterion has for years been one of the most widely applied evaluation criteria in the mineral industry [1] and in other industries.[2]

Unfortunately, in recent years it has also become increasingly clear that the IRR, defined as a zero of the NPV treated as a function of the interest rate, does not represent the interest rate it has always been supposed to represent.[3] To fill the vacuum created by this realization, and to answer the question, what is then the TRR, or the TBR of a project, several approaches have been developed.[4-15] They differ in some of their assumptions and therefore provide, for the same project, generally different values of the interest rate in question. In the following, four variants of the two most representative approaches will be analysed and discussed.

The models for TRR/TBR calculation

Assume the condition of certainty and the environment of the perfect capital market with identical investment and borrowing discrete interest rates equal to m and being constant over the lifetime $[0,n]$ of a project. Let

$$\{b_j\} \ , \ j=0,1,\dots,n \ , \qquad [1]$$

where

$$b_0, b_n \neq 0 \ , \qquad [2]$$

be the project's cash flow pattern (CFP). Now, if $b_0<0$, the project is an investment project (IP), whereas $b_0>0$ indicates a borrowing project (BP). Further, if i is the effective discrete interest rate, constant over time, then the NPV function (NPVF) of CFP [1] is

$$NPV(i)=\sum_{j=0}^{n} b_j(1+i)^{-j} \ , \ i\in(-1,+\infty), \qquad [3]$$

and the IRR of the project is any real zero of the NPVF [3]. It is a widely represented view that the classical concept of the IRR includes an implicit assumption of the reinvestment (in case of an IP) or reborrowing (in case of a BP) rate being identical with any real IRR of the project. Since this assumption is entirely artificial and can only be incidentally true, it disqualifies the traditional concept of the IRR criterion and calls for a more realistic approach. There is a general consensus that the so-called reinvestment/reborrowing assumption should be the foundation of any such approach. This assumption states that in an IP all positive CFs are reinvested at the market investment rate until the end of the project's lifetime, and in a BP, all negative CFs are reborrowed from the market at the market borrowing rate until the project's termination.

At the same time, there are two different views on how the negative CFs of an IP, or the positive CFs of a BP should be treated. One approach to the TRR/TBR criterion (Approach 1) [4,5,8,10,11,15] assumes that the CFs mentioned are discounted at the market investment/borrowing rate on the project's start-up time 0, whereas the other approach (Approach 2) [6,9,12] leaves the CFs in question at their original locations.

In both approaches, the intermediate CFs generated by the project (the positive CFs of an IP, or the negative ones of a BP) can be tied to the market for the rest of the project's lifetime (Variant A), or they can interact with the project during this time by balancing the project's intermediate CFs of the opposite sign (Variant B). Combining the two approaches with the two variants of transferring the CFs generated by the project leads to four models, 1A, 1B, 2A and 2B (the figure identifies the approach and the letter indicates the variant), of the transformation of the project's original CFP.

To characterize the models, let us now denote by z the number of sign changes in CFP [1] and exclude from future considerations the trivial case $z=0$, in which there is no real IRR and, likewise, no real TRR/TBR to the project. Assuming then $z\geq 1$, let us further identify in CFP [1] the sub-sequences of CFs having the same sign. There will be $(z+1)$ such sub-sequences which will be called in the following the sub-sequences of constant sign (SCS). If we now define by u_r and v_r , $r=1,2,\dots,z+1$, the indices of the first and last CF in the r-th SCS respectively, so that $b_{u_r}, b_{v_r} \neq 0$, then, from [2], $u_1=0$ and $v_{z+1}=n$. For example, CFP {12,5,0,-18,0,-7,20,0,-15,-4} has $n=9$, $z=3$, $u_1=0$, $v_1=1$, $u_2=3$, $v_2=5$, $u_3=v_3=6$, $u_4=8$, $v_4=9$.

In this context, the four distinguished models can now be defined by the following algorithms outlining each model's transformation procedure:

Model 1A

Step 1. Calculate

$$b_0^* = \sum_{q=1}^{P} \sum_{u_{2q-1}}^{v_{2q-1}} b_j (1+m)^{-j} ,$$

where

$$P = \begin{cases} (z/2)+1 & \text{for an even } z \\ (z+1)/2 & \text{for an odd } z \end{cases}$$

Go to Step 2.

Step 2. Calculate

$$b_n^* = \sum_{q=1}^{R} \sum_{u_{2q}}^{v_{2q}} b_j (1+m)^{n-j} , \qquad [4]$$

where

$$R = \begin{cases} z/2 & \text{for an odd } z \\ (z+1)/2 & \text{for an even } z \end{cases} . \qquad [5]$$

Go to Step 3.

Step 3. Set $b_j^*=0$ for $j=1,2,\ldots,n-1$. Stop.

Model 1B

Step 1. Calculate

$$b_0^* = \sum_{j=0}^{v_1} b_j (1+m)^{-j} .$$

If $z=1$, go to Step 5. Otherwise, set $k=u_3$ and calculate

$$B_k = \sum_{j=u_2}^{v_2} b_j (1+m)^{k-j} + b_k . \qquad [6]$$

Go to Step 2.

Step 2. If $\mathrm{sgn}(b_0)*B_k>0$, go to Step 6. Otherwise, go to Step 3.

Step 3. If $k=n$, go to Step 7. Otherwise, go to Step 4.

Step 4. Set $k=k+1$ and calculate

$$B_k = B_{k-1}(1+m) + b_k . \qquad [7]$$

Go to Step 2.

Step 5. Calculate

$$b_n^* = \sum_{j=u_2}^{n} b_j (1+m)^{n-j} . \qquad [8]$$

Go to Step 8.

Step 6. Calculate $b_0^*=b_0^* + B_k(1+m)^{-k}$. Set $B_k=0$. Go to Step 3.

Step 7. Set $b_n^*=B_k$. Go to Step 8.

Step 8. Set $b_j^*=0$ for $j=1,2,\ldots,n-1$. Stop.

Model 2A

Step 1. Set $b_j^*=b_j$ for the following js: $j=u_r,u_r+1,\ldots,v_r$, $r=1,3,\ldots,z-1$, and $j=u_{z+1},u_{z+1}+1,\ldots,n-1$, for an even z; $j=u_r,u_r+1,\ldots,v_r$, $r=1,3,\ldots,z$, for an odd z. Go to Step 2.

Step 2. Calculate b_n^* using formulas [4] and [5]. If z is an even number, set $b_n^*=b_n^*+b_n$. Go to Step 3.

Step 3. Set $b_j^*=0$ for the following js: $j=u_r,u_r+1,\ldots,v_r$, $r=2,4,\ldots,z$, for an even z; $j=u_r,u_r+1,\ldots,v_r$, $r=2,4,\ldots,z-1$, and $j=u_{z+1},u_{z+1}+1,\ldots,n-1$, for an odd z. Stop.

Model 2B

Step 1. Set $b_j^*=b_j$ for $j=0,1,\ldots,v_1$.

If $z=1$, go to Step 5. Otherwise, set $k=u_3$ and calculate B_k using [6]. Go to Step 2.

Step 2. If $sgn(b_0)*B_k>0$, set $b_k^*=B_k$, set $B_k=0$, and go to Step 3. Otherwise, check if $k=n$. If true, set $b_n^*=B_n$ and stop. Otherwise, set $b_k^*=0$ and go to Step 4.

Step 3. If $k=n$, stop. Otherwise, go to Step 4.

Step 4. Set $k=k+1$ and calculate B_k applying [7]. Go to Step 2.

Step 5. Calculate b_n^* using [8] and set $b_j^*=0$ for $j=u_2,u_2+1,...,n-1$. Stop.

The TRR/TBR criterion

As a result of the transformations 'prescribed' by the four models, four transformed CFPs

$$\{b_j^*\},\ j=0,1,...,n,\quad b_0^*,b_n^*\neq 0, \qquad [9]$$

are obtained. CFPs [9] for Models 1A and 1B have only initial and terminal CFs $\neq 0$, whereas CFPs [9] for Models 2A and 2B will usually have, in addition, some, or all, intermediate CFs $b_j^*\neq 0$. These CFs will, by definition, satisfy condition $sgn(b_0^* * b_j^*)>0$.

The above means that, irrespective of the model considered, CFPs [9] can have a maximum of one sign change. If the number of sign changes is exactly one, which implies

$$sgn(b_0^* * b_n^*) = -1, \qquad [10]$$

then the TRR/TBR r of the project described by CFP [1] is defined as the only zero (IRR) of the NPVF

$$NPV^*(i)=\sum_{j=0}^{n} b_j^*(1+i)^{-j},\ i\in(-1,+\infty) \qquad [11]$$

of CFP [9]. Expressing it in mathematical terms gives

$$\sum_{j=0}^{n} b_j^*(1+r)^{-j}=0,\quad r\in(-1,+\infty). \qquad [12]$$

In the case of Models 1A and 1B and in those cases of Models 2A and 2B in which there are no intermediate CFs$\neq$0, the TRR/TBR r, resulting from [12], can be calculated directly as

$$r=(-b_n^*/b_0^*)^{1/n} - 1. \qquad [13]$$

In all other cases of Models 2A and 2B, r has to be either approximated or, if $n\leq 4$, it can also be calculated directly using the respective formulas for low order polynomials.

Note, that if the original CFP [1] would be composed of only two CFs$\neq$0, which, in view of $z\geq 1$, would require

$$u_2 - v_1 = n, \qquad [14]$$

then the TRR/TBRs r for all four models would be identical, and equal to the unique IRR i_0 of CFP [1], given by the formula $i_0 = r = (-b_n/b_0)^{1/n} - 1$. Because of its triviality, case [14] will be excluded from further considerations.

Now, if the interest rate r is to be regarded as the interest rate generated by (IP), or required from (BP) the project, the following conditions for absolute profitability should apply:

Investment project

(a) $r>m$, the project is profitable; [15]

(b) $r=m$, the project is marginally profitable; [16]

(c) $r<m$, the project is unprofitable. [17]

Borrowing project

(a) r<m, the project is profitable; [18]

(b) r=m, the project is marginally profitable; [19]

(c) r>m, the project is unprofitable. [20]

Since the introduced models use different CFP transformation procedures, it is obvious that, except for special cases, the CFPs [9] obtained for each model will, generally, differ from each other. They will, therefore, produce generally different NPVFs and TRR/TBRs for different models. There is, however, one feature that is common to all four models. Owing to the fact that all transformation procedures use the market interest rate m for the CFs' discounting and compounding, the NPVFs [11] of CFP [9] for i=m will be identical for all four models, and, at the same time, also identical with the NPVF [3] of the original CFP [1], calculated at the same interest rate m. This means:

$$NPV^*(m)=\sum_{j=0}^{n} b_j^*(1+m)^{-j} = \sum_{j=0}^{n} b_j(1+m)^{-j} = NPV(m) \quad [21]$$

for all four models.

Assume now, that CFPs [9] for all the models have one sign change each, which implies [10] and means that the NPVFs of these CFPs each have in $(-1,+\infty)$ a unique real and simple zero r = TRR/TBR.

This, in the context of [21], is now presented graphically in Figures 1 and 2. Figure 1 refers to an IP,

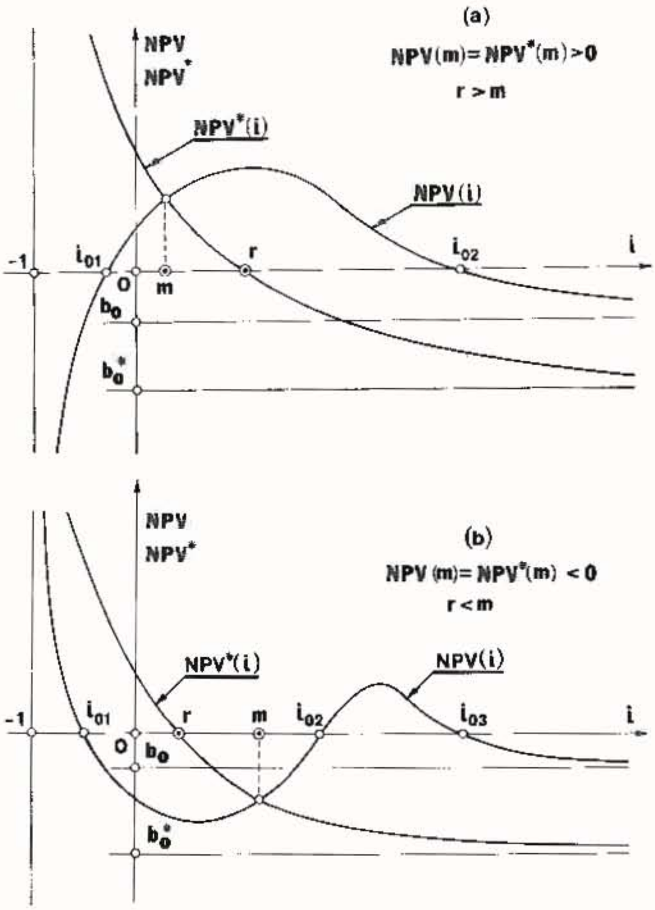

FIGURE 1. The TRR for an IP; CFP [9] has only two CFs≠0.

whereas Figure 2 refers to a BP. It is assumed, for demonstration purposes, that in Figure 1 CFP [9] has only two CFs≠0, while in Figure 2, it has at least three CFs≠0. Morerover, parts (a) of the two figures correspond to the situation when the identical NPVs of CFPs [1] and [9] for i=m are positive, whereas in the figures' parts (b), the two NPVs are negative.

Observe, further, that from the logic of transformation inherent in Models 2A and 2B, we always have for these two models

$$b_0^* = b_0, \quad [22]$$

which is reflected in Figure 2. For a change, the transformation logic of Models 1A and 1B implies that besides [22], also $|b_0^*| > |b_0|$ can be the

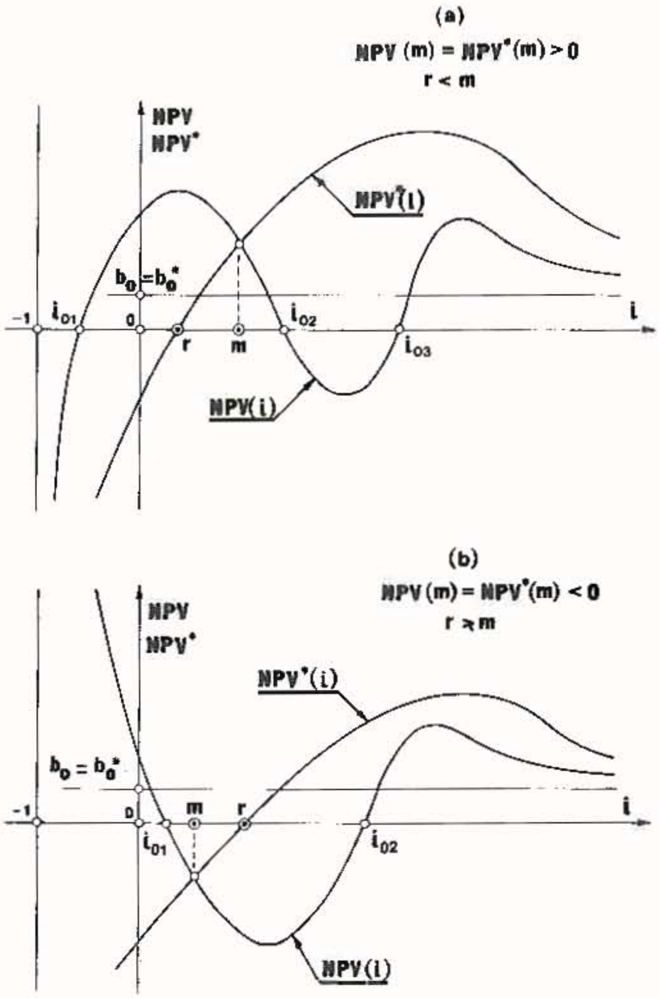

FIGURE 2. The TBR for a BP; CFP [9] has more than two CFs≠0.

case, which is presented in Figure 1. Note also, that $b_0^*=b_0^*(m)$ is a result of discounting, which implies that along with rising m the absolute value of $b_0^*(m)$ declines. In other words, moving with m towards $+\infty$ would cause in Models 1A and 1B the asymptote b_0^* to approach asymptote b_0 (Figure 1).

It can also be proven for all the models discussed that the profiles of NPVFs [3] and [11] intersect in $(-1,+\infty)$ only once, at a point with co-ordinates [m,NPV(m)]. This means that for an IP it is always $NPV^*(i) > NPV(i)$ if $i \in (-1,m)$, and $NPV^*(i) < NPV(i)$ if $i \in (m,+\infty)$, whereas for a BP the inequality signs are reversed.

Now, it should become clear that from Figures 1 and 2, from [10], [21] and the properties of function [11], the following implications result with respect to all the presented models:

Investment project

(a) r>m implies NPV(m)>0 ; [23]
(b) r=m implies NPV(m)=0 ; [24]
(c) r<m implies NPV(m)<0 . [25]

Borrowing project

(a) r<m implies NPV(m)>0 ; [26]
(b) r=m implies NPV(m)=0 ; [27]
(c) r>m implies NPV(m)<0 . [28]

Putting the above implications into the context of profitability conditions [15]-[20] proves that, <u>for all four models introduced, the TRR/TBR project evaluation criterion is entirely compatible with the NPV criterion</u> and, by the same token, with all the evaluation criteria that are NPV compatible, like Net Future Value, B/C-Ratio, Annual Worth and IRR, provided the latter is interpreted according to reference 3.

Practical application of the models

Since the TRR/TBR is generally different for each transformation model, the financial analyst must first select the model best fitting the financial strategy developed for the project. Then, the TRR/TBR corresponding to this model will be considered the best approximation of the true interest rate associated with the project.

Admitting, for example, Approach 1,

including Models 1A and 1B, would mean for an IP that the investor engages in the project an amount of b_0 at time 0, and, at the same time, invests in the market an amount of $(b_0^*-b_0)$, to balance, from the generated revenue, the intermediate outlays required by the project (the values and location of those outlays will depend on the reinvestment variant, A or B, combined with Approach 1). As a result of this operation, the investor's only investment, split between the project and the market, is b_0^* generating a single revenue b_n^* at the end of the project's lifetime.

The same Approach 1 to a BP would imply that the borrower gets from the project an amount of b_0 at time 0, and simultaneously borrows from the market an additional amount of $(b_0^*-b_0)$ which is then repaid to the market by the intermediate revenues generated by the project (the values and location of those revenues will depend on the reborrowing variant combined with Approach 1). As a result, the only amount borrowed by the borrower, partly from the project and partly from the market, is b_0^*, followed by a single outlay b_n^* at the end of the project.

Alternatively, assuming Approach 2, including Models 2A and 2B, means for the investor that, this time, he puts his money exclusively in the project, without interacting with the market. The values of the outlays and their location times will depend on the reinvestment variant, A or B, adopted in the particular model. As a result, the project, after the reinvestment of its intermediate revenues in the market, is typically characterized by a series of outlays, followed by a single revenue at time n.

A BP considered within the conceptual framework of Approach 2, implies, in turn, that the borrower gets his money entirely from the project, with no involvement from the market. The values of the project's revenues and their timing depend on the reborrowing variant,A or B, assumed in the model being examined. Consequently, the project's CFP, after the reborrowing of the intermediate payments to the project from the market, is generally composed of a series of revenues, followed by a single outlay.

From the above discussion, it appears that Approach 2 is undoubtedly more realistic then Approach 1. Indeed, it preserves the original input to (IP), or output from (BP), the project without distorting it by any influence of the market. Because of this, the resultant (transformed) CFP constitutes a basis for an objective evaluation of the project's input (IP), or output (BP), only.

Unlike Approach 2, Approach 1 divides the input to (IP), or the output from (BP), the project into two parts, of which one is associated with the project and the other with the market. This degenerates the transformed CFP of the project, with the result that this CFP no longer reflects the financial pattern of an exclusive investing into, or borrowing from the project, but rather the pattern of simultaneous investing into, or borrowing from both the market and the project.

Although the superiority of Approach 2 over Approach 1 seems rather evident, the impression is that in practical applications Approach 1 is still more popular than its competitor. One plausible explanation is that when applying Approach 1, the TRR/TBR can easily be calculated directly, using [13], whereas an application of Approach 2 frequently requires an approximation of the unique zero of the NPVF [11]. This approximation may, indeed, have been a (minor) problem some two or three decades ago; however, in today's highly computerized professional environment, it poses no difficulty at all. Formulas for the calculation of efficient bounds to the unique zero of function [11] already exist, [16] and all that one needs to do to approximate this zero in a few iterations and with sufficient accuracy, is to apply an algorithm as simple as 'Regula Falsi', or 'Pegasus'. [17]

It should be emphasized that if all that is wanted is to check the absolute profitability of a project, or to choose between two ME projects, the financial analyst will never fail by applying any of the four identified models, regardless of whether this model is realistic in the context of a given situation or not. The interest rate r obtained from the model will not necessarily reflect the true interest rate of the project, but it will still make possible, via [15]-[20], an evaluation that, through [23]-[28], will be fully consistent with the NPV criterion. However, exactly the same task can be accomplished by applying any evaluation criterion that is consistent with the NPV. The full advantage of the TRR/TBR criterion can only be achieved if the transformation model adopted for the project's evaluation corresponds reasonably well to the project's financial strategy.

In this context, it is clear that, apart from the approach applied, it is equally important which reinvestment/reborrowing variant is selected. Each of the two variants, A and B, can correspond to a practical situation, and the choice is, again, that of the analyst. It should be noted that variants A and B by no means reflect all the possible scenarios as to how the intermediate CFs generated by the project can interact with the market until the project's completion time. Instead, the two variants should be considered as representative of two contrasting scenarios, leaving room for other strategies (variants) that would include elements of the basic two.

Each such 'hybrid' reinvestment, or reborrowing variant, combined with the two approaches, 1 and 2, will increase the number of transformation models by two, thus increasing by two the total number of TRR/TBRs defined. In an identical way, as was done earlier with respect to the four transformation models, it can now be proven, for each additional new model, that the resultant TRR/TBR possesses exactly the same features as its predecessors. Hence, each new version of the TRR/TBR can serve as another independent, and NPV compatible, project evaluation criterion that, for a given project, also reflects the project's true rate of return (IP), or true borrowing rate (BP).

All this implies that the financial analyst in practice has a considerable flexibility in that, if not satisfied with any of the models already defined, he can define his own model by applying the reinvestment/reborrowing assumption in the way that best fits the financial strategy developed for the project.

The TRR/TBR as a function of the market interest rate *m*

According to the reinvestment/reborrowing assumption inherent in the concept of the TRR/TBR, and the participation of the market in investing (IP), or borrowing (BP), in the case of Approach 1, the TRR/TBR r is not an independent internal interest rate of the project, but a rate that is affected by the external market interest rate m. In other words, r is, generally, a function of m , and the only cases in which it is not constitute, for both approaches, projects with original CFPs satisfying [14], and, with respect to Approach 2, projects with CFPs having one change of sign and no intermediate CFs generated by the project. In all those exceptional cases, the TRR/TBR is identical with the unique IRR of the project. In the following, major properties of the TRR/TBR function (TRR/TBR-F) $r = r(m)$, will be discussed. For the sake of conciseness, the corresponding proofs will be omitted.

Beginning with the domain of function $r(m)$, note that this function exists for the values of m for which [10] holds. From the analysis of the algorithms defining the transformation models, it results that [10] is met for any $m \in (-1,+\infty)$ and for any CFP [1] only in the case of Model 1A. For the remaining three models, 1B, 2A and 2B, the entire interval $(-1,+\infty)$ constitutes the domain of the TRR/TBR-F only if CFP [1] has an odd number of sign changes z. If z is an even number, there will always be such an $m = m^* > -1$ that for $m \in (-1,m^*]$ [10] will not hold and, consequently, function $r(m)$ does not exist. In such cases, the domain of this function will be limited to the interval $(m^*,+\infty)$.

It is not difficult to find the values of m^* for the three models, although in many instances an approximation is necessary. The author has developed respective computer algorithms that, owing to space requirements, will be separately presented later. Note also, in this context, that m^* for Models 1B and 2B will be identical.

To illustrate, consider CFP {-20,80,-95,50,-20,30,-18} with two IRRs, -0.237019 and 1.432662, and for which $m^* = -0.464651$ for Models 1B and 2B, and $m^* = -0.570721$ for Model 2A.

Generalizing, we can then write, $r = r(m)$ for $m \in (M,+\infty)$, where

$M = -1$ for Model 1A and, if z is odd also for Models 1B, 2A and 2B; $M = m^*$ for Models 1B, 2A and 2B if z is even. [29]

If $m \in (-1,m^*)$, it means, in practical terms, that all the CFs of the transformed CFP are of the same sign. In such a case, it can be assumed, for the purpose of evaluation, that $r=-1$, which, using [17] and [18], means an unsatisfactory result for an IP, and a satisfactory one for a BP.

Further, it can be shown that, for all the models involved, the TRR/TBR-F is in $(M,+\infty)$ a continuous and monotonically increasing function, so that in this interval the following holds:

$$r(m_2) > r(m_1) \text{ for } m_2 > m_1. \qquad [30]$$

Moreover, it is $\lim_{m \to M} r(m) = -1$,

where M is given by [29], and

$$\lim_{m \to +\infty} r(m) = +\infty .$$

Now, assume that an IP is being considered and further suppose that the NPVF [3] for this project has $k \geq 1$ real zeros in $(M,+\infty)$. Let, for a given m be $NPV^*(m)=NPV(m)>0$ and let $i_0^*>m$ be the IRR nearest to m. Then, taking into account [15]-[20] and [30], it can be proven that $m<r(m)<i_0^*$. Similarly, if $NPV^*(m) = NPV(m) < 0$ and $i_0^+<m$ is the IRR closest to m, then $i_0^+<r(m)<m$. Applying the same reasoning to a BP and summarizing, we get $r_A < r(m) < r_B$, where r_A and r_B are defined as follows:

Investment project

$$\text{(a) } NPV^*(m) = NPV(m) > 0 \qquad [31]$$

$$r_A = m ,$$

$$r_B = \begin{cases} i_0^* & \text{if } i_0^* \text{ exists} \\ +\infty & \text{if } i_0^* \text{ does not exist} \end{cases} ,$$

$$\text{(b) } NPV^*(m) = NPV(m) < 0 \qquad [32]$$

$$r_A = \begin{cases} i_0^+ & \text{if } i_0^+ \text{ exists} \\ -1 & \text{if } i_0^+ \text{ does not exist} \end{cases}$$

$$r_B = m .$$

Borrowing project

For a BP, case (a), [31], is identical with case (b), [32], for an IP, and case (b), [32], is identical with case (a), [31], for an IP.

It can also be proven, that if CFP [1], for which $M=m^*$, has at least one real IRR in $(-1,+\infty)$, and if i_L is the smallest (leftmost) IRR, then it will always be $m^*<i_L$.

The above features of the TRR/TBR-F are illustrated graphically in Figures 3 and 4. For better exposure and analysis, in both figures profiles of function r(m) are presented together with profiles of the corresponding NPVFs in the background. Figure 3 refers to a BP with an even number of sign changes,

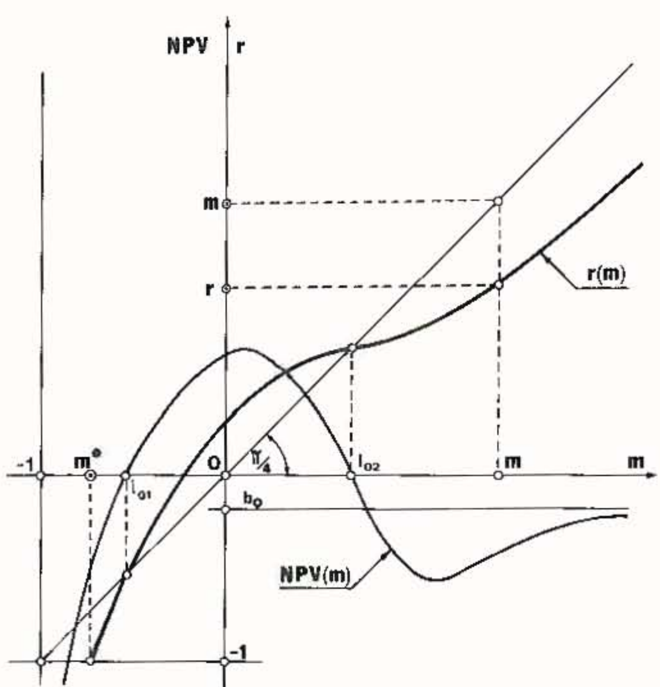

FIGURE 3. The TRR/TBR as a function of m for an even z (models 1B, 2A and 2B).

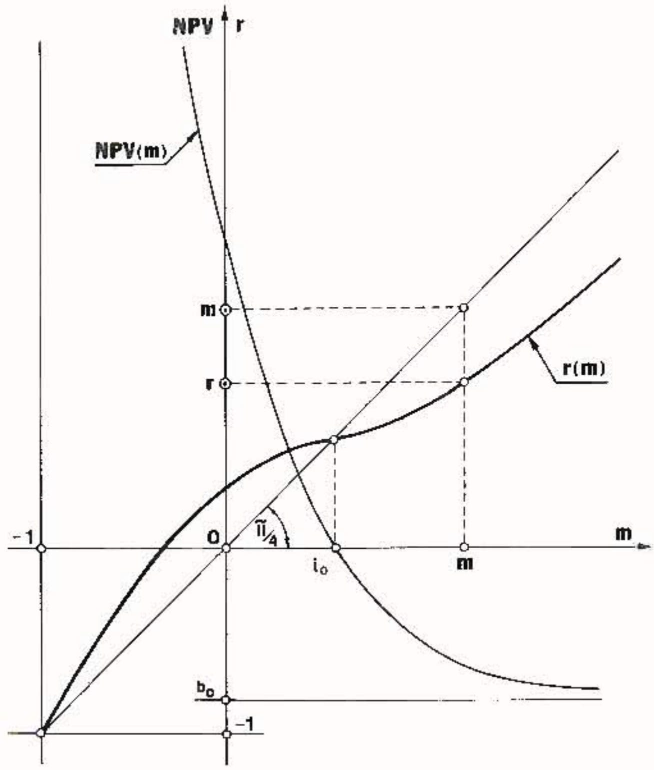

FIGURE 4. The TRR/TBR as a function of m for an odd z.

two simple IRRs and with $M = m^*$ (Models 1B, 2A or 2B), whereas Figure 4 relates to an IP with an odd number of sign changes and one IRR. Although the two figures do not cover all possible models and CFP cases, they do address well the characteristic features of the TRR/TBR-F.

A numerical complement to Figure 3 is presented in Table 1, which contains the values of function r(m), derived from CFP

{-50,-100,350,-200,650,-400, 520,-100} [33]

for all four transformation models. The NPVF of CFP [33] has two simple IRRs, -0.783830 and 0.979460, and since this CFP has an even number of sign changes (z=6), we have M=-1 for Model 1A, and $M=m^*$ for Models 1B, 2A and 2B, where m^*= -0.815665 for Model 2A, and m^*= -0.807692 for Models 1B and 2B.

Comparing mutually exclusive projects

A correct comparison of ME projects, or project alternatives, has, for

TABLE 1.
The TRR values for CFP [33]

m	-0.80	-0.60	-0.40	-0.20	0	0.20
Model 1A	-0.801769	-0.576440	-0.350711	-0.128595	0.086578	0.291125
Model 1B	-0.826945	-0.505370	-0.047922	0.135359	0.274644	0.416731
Model 2A	-0.848303	-0.433538	-0.196288	0.005244	0.189905	0.363807
Model 2B	-0.878718	-0.279227	0.050464	0.178415	0.305945	0.439095
m	0.40	0.60	0.80	1.00	1.20	1.40
Model 1A	0.483270	0.663553	0.833742	0.995777	1.151301	1.301568
Model 1B	0.560935	0.705904	0.850546	0.994146	1.136273	1.276694
Model 2A	0.529822	0.689631	0.844340	0.994723	1.141351	1.284659
Model 2B	0.576174	0.715195	0.854685	0.993695	1.131654	1.268233
m	1.60	1.80	2.00	2.20	2.40	2.60
Model 1A	1.447497	1.589763	1.728866	1.865185	1.999013	2.130585
Model 1B	1.415296	1.552041	1.686937	1.820020	1.951341	2.080958
Model 2A	1.424990	1.562623	1.697788	1.830681	1.961466	2.090289
Model 2B	1.403258	1.536648	1.668379	1.798462	1.926930	2.053826

some reason, always been a problem when applying the IRR criterion. There are still surprisingly many analysts and, even worse, authors who believe that a direct comparison of the IRRs of two ME alternatives is the proper way of ranking them. Since the TRR/TBR criterion represents the content which the IRR criterion has for decades been believed to express, it is likely that those who in the past have misinterpreted the IRR criterion will do now the same with respect to the TRR/TBR. Therefore, it has to be repeatedly stated that <u>in comparing ME alternatives the only correct way of evaluation is to apply the incremental analysis</u>. This principle, applied to the TRR/TBR criterion, will now be demonstrated in a few numerical examples.

Comparing alternatives with equal duration

Consider two ME projects, A and B, given by the following CFPs:

CFP A = {-10,-50,60,-30,40,190}, [34]

having one simple IRR = 0.586882, and

CFP B = {-44,20,45,-50,260,-35} [35]

with two simple IRRs, -0.706831 and 0.725709. The incremental analysis can now be done in two different ways. The incremental CFP (A-B) of the hypothetical incremental alternative (A-B) can be derived either directly from the original CFPs A and B (Incremental Approach 1 (IA1)), or from the transformed CFPs of the two competing projects (Incremental Approach 2 (IA2)). The two, generally different, CFPs of the incremental alternative will, for a given m, produce in either case the same NPV which, in turn, will lead to identical choices for both evaluation approaches. The fact that the TRR/TBRs for the two incremental CFPs will for the same transformation model, generally, differ from each other, does not have much significance, since an incremental alternative is an artificial one anyway. Therefore, in this case <u>it is not the value of the TRR/TBR which matters, but only how it relates to the market interest rate m.</u> It is obvious, that, owing to the compatibility of the TRR/TBR criterion with the NPV, the TRR/TBRs for the two alternative incremental CFPs will always relate to m in an identical way.

Assuming m = 0.10, the results of the evaluation of the ME projects A and B, given by CFPs [34] and [35], are presented for all four models and the two incremental approaches in Table 2. Note, that in the case of Models 1A and 2A, TRRs of Alternative A are higher then the corresponding rates of Alternative B, whereas in the case of Models 1B and 2B, the relationship is reversed. Yet, for all the models Alternative B is superior to A, which underscores once again how little value a direct comparison of the interest rates (however true they might be) of two ME projects has, if one attempts to determine the projects' relative profitability.

Although the two incremental approaches discussed lead always to identical choices, it seems that IA2 is, generally, less time consuming. It can also be credited with some

TABLE 2.

Evaluation of project alternatives A, [34], and B, [35], with equal duration.

Alt.	Model	Transformed CFP	TRR/TBR	Proj. type	NPV	Eval. results
A	1A	{-779.940,0,0,0,0,3138.600}	0.321098	IP	+1168.884	PROFIT.
	1B	{-554.545,0,0,0,0,2775.600}	0.380015	IP		
	2A	{-100,-500,0,-300,0,3138.600}	0.411018	IP		
	2B	{-100,-500,0,0,0,2775.600}	0.440802	IP		
B	1A	{-1032.980,0,0,0,0,3751.770}	0.294282	IP	+1296.574	PROFIT.
	1B	{-440,0,0,0,0,2796.770}	0.447574	IP		
	2A	{-440,0,0,-500,0,3401.770}	0.405578	IP		
	2B	{-440,0,0,0,0,2796.770}	0.447574	IP		
A-B (IA1)	1A	{2011.303,0,0,0,0,-3444.870}	0.113625	BP	-127.690	UNPROF.
	1B	{340,0,0,0,0,-753.220}	0.172434	BP		
	2A	{340,0,150,200,0,-1194.870}	0.154532	BP		
	2B	{340,0,0,0,0,-753.220}	0.172434	BP		
A-B (IA2)	1A	{253.040,0,0,0,0,-613.170}	0.193654	BP	-127.690	UNPROF.
	1B	{-114.545,0,0,0,0,-21.170}	-1.0	IP		
	2A	{340,0,0,200,0,-995.220}	0.163366	BP		
	2B	{340,0,0,0,0,-753.220}	0.172434	BP		

theoretical advantage, since it utilizes the final, transformed CFPs characterizing the two competing alternatives.

ME alternatives with unequal duration

There are several approaches documented in the literature to comparing ME projects with unequal duration. In the first step, using different assumptions, they propose different measures to transform the shorter project into a project with a lifetime identical to that of the longer project. The next step is then an evaluation identical to the evaluation of ME alternatives with an equal duration. In the following, two cases of ME projects with unequal duration will be distinguished, and, in each case, simple evaluation approaches will be presented and illustrated with numerical examples. For the sake of convenience and without loss of generality, let us assume in these considerations that of the two ME alternatives with unequal duration, A and B, alternative A will always be the shorter one.

Alternative A has a TRR/TBR > -1

This assumption means that the transformed CFP of the shorter alternative A has exactly one change of sign. The analyst should now answer the following question with regard to this alternative: is a project identical to project A likely to be realized after the completion of the latter ? If the answer to this question is 'yes', CF b_n^* generated by project A at the end of its lifetime should be reinvested (IP) in, or reborrowed (BP) from the market until the end of the lifetime of the longer alternative B. As a result, an expanded version of alternative A, let us call it alternative A^*, will be created, and characterized by CFPA* generated from the transformed CFP of alternative A. The lifetime of alternative A^* will now be identical with the lifetime of its competitor, alternative B.

Consider, as an example, CFP

{100,0,400,0,0,-600}, [36]

with n=5. Assume, CFP [36] is the transformed CFP of a borrowing project A and suppose the lifetime of alternative B is 8 discrete time units (n=7). Since the TBR of project A is r=0.054799, it will be used for the borrowing of $600 from identical projects until the end of period [0,7]. The transformed CFP of alternative A^* will now be {100,0,400,0,0,0,0,-667.5608}. The TBR of this project will, of course, remain the same as that of project A; however, the NPVs of the two projects will differ. If the market interest rate, for example, was m=0.15, we had NPV(A) = 104.1514 and NPV(A^*) = 70.5618.

If the answer to the earlier question is 'no', the only alternative for an extension of project A remains the market. Borrowing $600 from the market at m=0.15 until time 7 gives now a new alternative, let us call it A^+, with the following CFP

{100,0,400,0,0,0,0,-793.5}. [37]

Since this CFP is a result of an interaction of CFP [36] with the market, the two CFPs will produce identical NPVs; however, their TBRs will now be different. Indeed, the TBR for CFP [37] is r=0.088808.

Note, in this context, that any CFP reflecting a 'market project' will produce the NPV=0. Such a CFP, 'added' to CFP [36] and extending it to the period of full 8 time units, will, obviously, not change the NPV of CFP [36], which will still remain the same (104.1514). Consequently, if the market participates in the extension of project A, and if the NPV criterion is used for the evaluation, any CFP resulting from such an extension, whatever the complementary 'market project' might be, can serve for the comparison with alternative B. The results of the incremental evaluation, although expressed by different numbers, will always lead to identical decisions. Since the NPV and the TRR/TBR criteria are fully consistent, the same reasoning applies also to the latter criterion. In this case, each of CFPs A^+ generated from CFPs A and extended by different 'market projects', will produce different incremental alternatives with, generally, different TRR/TBRs; yet, they all will always lead to identical decisions.

Observe that a very special example of a CFP having the NPV=0 constitutes one with all CFs=0. Consequently, the extended alternative A of project

[36] could also be defined by CFP

{100,0,400,0,0,-600,0,0} [38]

having exactly the same chances for its competition with alternative B as in the case of CFP [37]. In fact, as shown earlier, a transformed CFP [38] would be identical with CFP [37].

Let us now consider, as an example, a comparison of two ME projects, A and B, with unequal lives and both having TRR/TBRs > -1. The original CFPs of the two projects are:

CFPA = {-200,-400,800,-350,1300} [39]

CFPB = {-300,-200,250,-400,1400, -100,2600,-200} [40]

and the market interest rate is m=0.20. In this comparison, because of space limitation, only transformation model 2B is applied and it is assumed that alternative A^+ is produced by reinvesting in the market the CF generated by project A at time 4. This reinvestment remaines in effect until time 7, which is the end of the lifetime of project B.

As a result, the transformed CFPs (TCFPs) of CFPs [39] and [40], along with their TRR/TBRs, are:

TCFPA = {-200,-400,0,0,2032} [41]

r_A = 0.435398

TCFPA* = {-200,-400,0,-350,0,0,0, 7010.554} [42]

r_{A^*} = 0.435398

TCFPA+ = {-200,-400,0,0,0,0,0, 3511.296} [43]

r_{A^+} = 0.319931

TCFPB = {-300,-200,0,-100,0,0,0, 5195.201} [44]

r_B = 0.406036.

These CFPs now produce the following two incremental alternatives, IA1 and IA2, with their TRR/TBRs: TCFP(A^*-B)= {100,0,0,0,0,0,0,424.491} with $r_{(A^*-B)}$ = -1, as IA1, and TCFP(A^+-B)= {100,0,0,0,0,0,0,-2073.741} with $r_{(A^+-B)}$ = 0.542083, as IA2. Note, that whereas CFPs [41]-[44] identify IPs, both incremental CFPs indicate BPs and reflect opposite judgments which are further confirmed by the NPV criterion. Indeed, we have: NPV(A)= NPV(A^+)= 446.605, NPV(A^*)= 1143.816, NPV(B)= 925.348, and, as a result, NPV(A^*-B)= 218.468 and NPV(A^+-B)= -478.743, which again emphasizes the importance of the analyst's choice as to the scenario for the extension of the lifetime of alternative A.

The TRR/TBR of Alternative A does not exist

In the preceding case, the TRR/TBR of project A has served as an instrument to extend this project beyond its lifetime while maintaining the project's profitability level expressed by the same TRR/TBR. In this case, the transformed CFP of alternative A does not have any sign change, hence its TRR/TBR does not exist. The equivalent extension of the project can therefore only be done by a direct reproduction of the project, as many times as required until the end of the lifetime of the competing project B is reached, or for the first time exceeded. If it is reached, the two projects are ready for comparison; if it is exceeded, two situations are possible: project B either has a TRR/TBR > -1, or it does not have any TRR/TBR at all.

If it does, the project should be extended, using either its TRR/TBR, or the market interest rate, until the end of the common evaluation period resulting from the multiplication of project A. To illustrate, consider two following transformed CFPs A and B:

$$CFPA = \{300,0,100\} \quad [45]$$

$$CFPB = \{500,50,0,-300\}. \quad [46]$$

One reproduction of project A produces alternative A^* with $CFPA^* = \{300,0,100,300,0,100\}$. Project B, with the TRR/TBR = - 0.188618, has now to be extended by borrowing $300 at the above interest rate until time 5. This will generate for alternative B^* the following $CFPB^* = \{500,50,0,0,0,-197.502\}$. Both alternatives, A^* and B^*, are now ready for the incremental analysis.

If the transformed CFP of project B does not produce any sign change, both projects, A and B, should be reproduced until they reach a time point that results from the least common denominator of their lifetimes.

If, for some reason, a reproduction of project A after its completion is not possible, an interaction with the market is necessary to make the lifetime of the project identical with that of project B. The simplest way to do it is to assign the additional CFs of CFP A with zero values, thus obtaining CFP A^+ ready for evaluation. In the case of the competing CFPs [45] and [46], we would get $CFPA^+ = \{300,0,100,0\}$. This approach has an additional advantage in that it can be applied irrespective of whether the CFP of project B produces a change of sign or not.

Summary and conclusions

1. In the presented paper, the concepts of the true rate of return, or the true borrowing rate are discussed. Their development and application in the recent years reflect two following tendencies: on the one hand, there is an apparent need on the part of the mineral industry to have a project evaluation criterion that would measure the project's profitability in terms of an interest rate; on the other hand, it has become increasingly clear that the IRR criterion, that has for many years been regarded as such a profitability measure, is not theoretically sound and, owing to its numerous shortcomings, does not serve its purpose.

2. Four representative models for the calculation of the TRR/TBR are presented and examined. It is shown that, in all four cases, the TRR/TBR criterion is fully consistent with the NPV criterion. Hence, all four models can be applied as profitability measures, provided their assumptions are close enough to a real-life situation. It is indicated, that, if necessary, the analyst can develop, for a particular project, his own model based on the reinvestment/reborrowing assumption, and that the resulting TRR/TBR will also be consistent

with the NPV.

3. The consequence of the reinvestment/reborrowing assumption, which is inherent in the concept of the TRR/TBR, is that, unlike the IRR, this rate is no longer a project's internal rate, but rather a hybrid interest rate that reflects the potential of both the project and the market. As a result, the TRR/TBR is a function of the market interest rate, and the main features of this function are discussed in the paper. They are also illustrated graphically against the background of the NPV function which underscores and visualizes the compatibility of the two project evaluation criteria.

4. A correct evaluation of ME alternatives when applying the IRR criterion has always created problems for both authors and analysts. To avoid a similar mishandling of the TRR/TBR criterion, it is shown for all possible CFP cases how ME projects should be evaluated if this criterion is applied.

5. The considerations presented in the paper prove that the TRR/TBR can serve as an independent project evaluation criterion. It is not only entirely compatible with the NPV but is also devised to express the projects own (although not internal) interest rate of return, or borrowing rate. This feature makes it both unique among all the NPV compatible evaluation criteria and, in many instances, superior to them. The TRR/TBR, however, does not challenge any of these criteria; rather, it complements them and should be used in concert with them.

6. The full advantage of the TRR/TBR criterion, as an indicator of a project's own interest rate, can be taken only when establishing the absolute profitability of the project. If the relative profitability of two ME projects is examined, the very value of the TRR/TBR of the incremental alternative, because of the artificial nature of this alternative, is meaningless. Therefore, when comparing ME projects, the NPV, as an evaluation instrument, may be, due to its computational simplicity, a wiser choice.

7. In the context of the above considerations, the crucial role of the financial analyst in the project evaluation process cannot be overemphasized. It is his knowledge, experience, common-sense and, last but not least, intuition that will help him decide what assumptions and anticipations to make. Without those qualities, any evaluation approach, however sophisticated, is doomed to failure.

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