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The relationship between certainty and value in mineral asset management.

Johnny van den Berg, Mining Executive, MineRP

Mineral assets migrate through a logical lifecycle from exploration target to final product. It is critical that mines harvest maximum economic value at each stage during this lifecycle in order to demonstrate the optimum value of the company to the investor.

Calculating Nett Worth

Investors use a variety of instruments to determine the value of natural resource companies—but it all boils down to \[ \text{Nett Worth} = \text{Total Assets} - \text{Total Liabilities} \].

In this sense mines are particularly interested in how the Mineral Asset component of the Total Assets is derived. The rules for publication of the Mineral Inventory are highly regulated and prescribed by legislation or reporting codes. Published inventories however are open to interpretation by analysts and investors, leaving the question of the intrinsic value of the asset open to speculation. Intrinsic value (of course) lies in the eye of the beholder! The mine owner’s valuation often differs vastly from that of a potential acquiring company or a trade union negotiator trying to get maximum benefits for its members.

The fact that mineral inventories only gets published annually leads to further value speculation— speculation that could easily be avoided through the implantation of accurate, transparent and auditable systems for on-demand reporting and analysis of mineral resources—irrespective of their current state.

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MineRP’s GeoInventory and GeoFinance Solutions allow for the cradle to grave management of the mineral asset. This is done through standards-based translation of the impact of mining activities (managed in mining technical systems) into mineral inventory transactions carried in enterprise resource planning (ERP) systems such as SAP. ERP systems track business resources—

including raw material—and other business fundamentals such as production, supply chain, human resources, assets and others.

Impact on balance sheet

MineRP’s GeoInventory solutions potentially impact the balance sheet by increasing the availability, consistency, accuracy, transparency and auditability of mineral asset reports and valuation at every stage of the mineral asset lifecycle.

The diagram above demonstrates the intersection points at which the investor or analyst adds value to mineral assets. Traditionally, these intersections mark the stages at which investors recognise added resource value. The reality is however that mining companies incur the most cost between the reserve and the sold intersections (For some companies this can last as long as 40 months). If the mine can show an ability to consistently perform well in bringing its mineral inventory through the value chain, investors may add additional valuation intercepts, which in turn will lead to a positive re-rating of the company.

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The initiative for this volume came from the Southern African Institute of Mining and Metallurgy with the aim of commemorating Professor Danie Krige’s work, his contribution to geostatistics, and the creation of new knowledge that he has stimulated. At the outset I would like to sincerely thank the authors for their contributions and for the swift and diligent responses to the call for papers. Since most of the work fell to the authors of the papers, compiling this volume has been a pleasure. The first invitation to contribute to the volume went out on 10 May 2013 and as the volume stands, it will contain 37 papers from 67 authors from around the globe, all completed in a period of nine months. The readiness and willingness of geostatisticians to contribute their research to this volume signals their appreciation for the work and person of Danie Krige. Something worth noting is that most authors found the writing of articles in this volume an enjoyable experience. In fact one author commented that he hoped I would enjoy reading the paper as much as he enjoyed writing it.

I thought that a summary of the early development of geostatistics might be in order. Early publications by Danie Krige were translated into French and Russian and the effects of his new insights into mineral resource evaluation spread to mining centres around the world. From this interest the APCOM meetings arose, and as the need for a forum to discuss such issues in the South African and Australian mining industries became evident, the Geostatistical Association of South Africa and the Geostatistical Association of Australia were formed. Early development of the subject material (before the term geostatistics was coined) in the literature of the 1945 to 1965 period was couched in phrases such as ‘statistical approach’ or ‘statistical analysis’ and ‘mine valuation’. At that stage geostatistics was unknown as a subject, but terms such as lognormal, correlation, regression, trend surface, moving average, and interpolation procedures began to appear in this literature.

It is interesting to use the number of publications in the field of geostatistics as a proxy for the way the discipline of geostatistics has evolved over the years since its first developments in the early 1950s. In 1988 Patricia Sheahan produced a bibliography of geostatistics for the Economic Geology Research Unit at the University of the Witwatersrand, in which she lists 315 publications on geostatistics in the period 1951 to 1988. Her definition for geostatistics from the Glossary of Geology, published in 1987 by the American Geological Institute: (a) statistics as applied to geology; the application of statistical methods or the application of statistical data for use in geology; (b) statistical techniques developed for mine valuation by the French school of Matheron’, guided her choice of what to include, noting that her bibliography refers to practical aspects of geostatistics with an emphasis on literature since the mid-1970s, and that it is by no means comprehensive. However it is the best record that we have for that period of geostatistical development, so the data for the number of publications related to the subject for each year between 1951 and 1988 (Figure 1) is source from Sheahan (1988).

The period between 1950 and 1987, when the number of publications on geostatistics reached 133, is of particular interest. Figure 1 shows a moderate growth in the number of publications after 1964, but an exponential rise in 1976. Publications output in the 1978 to 1981 period was relatively subdued, but from 1982 growth in interest and output was rapid. I have found no bibliography of works relating to geostatistics after 1988 that meaningfully captures publications output. Webster’s Timeline History 1962-2007 of Geostatistics (Parker, 2009) fails to capture many publications that were produced after 1987, and probably before that time as well. For interest, the numbers of publications by Danie Krige and Georges Matheron are also shown on the diagram.

It is probably dangerous to go down this road for risk of offending people by either getting the numbers wrong or by missing someone, but anyone with more than five publications in Sheahan’s bibliography for this period is listed below. Some who have made huge contributions to the advancement of geostatistics, for example Ed Isaacs and Mohan Srivastava, besides many others (M. David, Ch. J. Huijbregts, P. Delfiner, and J.P. Chiles), do not feature in the list. So it should be said that the number of publications does not necessarily reflect the importance of the contributions to

Figure 1—The number of publications produced in each year from 1945 to 1987; publications by D.G. Krige until 2002 are shown, as are publications by G. Matheron (data sourced from: Sheahan, P. 1988. A bibliography of geostatistics. Economic Geology Research Unit, Information Circular No. 202. University of the Witwatersrand. 41 pp.)
the development of geostatistics. Number of publications is not the only metric by which we should measure the development of a scientific discipline, but it really is interesting to see who was publishing geostatistical papers during the period (listed in the Table below).

The number and timeline of publications indicate two important features of Danie Krige as an outstanding researcher and academic – firstly the consistency, and secondly the number of publications produced during his lifetime.

Matheron and Kleingeld (1987), in their consideration of the evolution of geostatistics (APCOM 87, Geostatistics, vol. 3, pp. 9-12), note that the term ‘random function’ was not in use when Danie Krige and Herbert Sichel introduced statistics into ore evaluation. The term ‘random variable’ has been used fairly extensively in the past. For example, Paul Lévy (Ecole Polytechnique) wrote an article entitled ‘Wiener’s random function, and other Laplacian random functions’ (pp. 171-187 in Proceedings, Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, 1951), and in 1962 A.M. Yaglom wrote the book ‘An Introduction to the Theory of Stationary Random Functions’ (published by Prentice-Hall). This term was first introduced by B. Matérn in his original 1948 publication on forestry in Sweden and was also used in the 1960 translation (with updates) from the Swedish version. The term ‘géostatistique’ was introduced by Matheron in 1962, as was the term ‘kriging’, although the latter appeared as a French word in a footnote. Matérn and Matheron also note that the concern around the adoption and acceptance of geostatistics was not related primarily to the mathematical complexities, but rather to the poor communication about the ‘… conceptual and psychological interpretation of the given phenomena …’ between methodical mathematicians, practical mining engineers, and intuitive geologists.

They also identified three periods in the development of geostatistics, the earliest being 1945–1965 when linear geostatistics was developed. Application of the lognormal distribution to gold mining by Herbert Sichel was followed by Danie Krige’s application of regression analysis between sampling and mining blocks for resource estimation. These works allowed Georges Matheron to fully develop the linear geostatistics of his doctoral thesis in 1965.

The second period, between 1966 and 1974, saw the development of nonlinear geostatistics on location at Centre de Geostatistique in Fontainebleau. M. David and G. Matheron made important contributions, but A.G. Journel, Ch. J. Huijbregts, P. Delfiner, P. Chauvet, and J.P. Chiles are said to have developed nonlinear geostatistics during this period. It is interesting to note how the number of publications escalated after the end of this period (Figure 1), but I leave the reader to interpret this observation. The third period between, 1974 and 1987, dealt with complex problems including ore/waste selection, change of support, transfer functions, parameterization of reserves, disjunctive kriging, and indicator kriging (Matheron and Kleingeld, 1987). Since that time, the interest and growth in geostatistics has shaped the extraction of minerals and metals from ore deposits around the world.

This volume is about geostatistics, its development, growth, and applications in the earth and natural sciences, but its importance lies in both the quality of the papers and the number and stature of the contributors in the geostatistical fraternity. This will indeed be a memorable volume. Of particular importance is the fact that the research presented here is at the forefront of the thrust in geostatistical research, and the collection of papers taken together indicates the forward direction for the frontiers of geostatistical research. I asked an eminent geostatistician where he thought the new frontiers of geostatistical research lay, and I believe that, collectively, the papers in this volume answer that question.

It is hoped that you will enjoy reading this volume as it commemorates the work of Professor Danie Krige and reflects the latest in global thinking about geostatistics.

R.C.A. Minnitt

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<th>Name</th>
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IN MEMORY OF DANIE KRIGE
A SA LEGEND OF INTERNATIONAL ACCLAIM FOR HIS DISTINGUISHED CONTRIBUTIONS TO THE MINING INDUSTRY
1919 ~ 2013

Professor Danie Krige was a world renowned pioneer in mineral resource evaluation. Nearly all the methods used in modern mining practice to evaluate the grade and tonnage of mineral deposits were either introduced by Danie Krige, or found their origins in the research and development that he initiated. Almost all of the growth and development in geostatistic disciplines today stem from Danie's early work on regressing sampling data and mining blocks, to estimate the amount of metal that could be recovered from mineral resources. Once international mining institutions became aware of Danie's work, a large number of researchers took up the newly developed techniques of spatial analysis, which led to the introduction of the term 'géostatistique' and later termed 'Kriging' (in recognition of his distinguished pioneering work), for the mathematical process of assigning grade to individual points or mining blocks. These methods improved the quality of ore evaluation, reduced the financial risk facing investors, allowing mineral resources to be quantified in terms of uncertainty associated with the estimates.

After joining the office of the Government Mining Engineer in 1943, Danie was exposed to the problem of limited borehole valuations associated with gold mines, including mining to greater depths. Applying his own statistical insights and those of Prof. Herbert Sichel in valuing new gold mines, Danie developed a statistically based procedure to calculate ore reserves in existing operations, which was submitted as a Masters degree thesis at the School of Mining Engineering at Wits University. Danie's thesis provided a statistical explanation of the problem of conditional bias in ore block valuations. The application of Danie's geostatistical 'Kriging' procedure was successfully implemented at Anglovaal's two large gold mines in the early 1960's. The outcome of his pioneering work led to the establishment of 'Le Centre de Geostatistique de l’École des Mines de Paris', a now famous geostatistics research centre in Fontainebleau, France.

For his outstanding work, Danie was awarded honorary degrees from the Moscow State Mining University, the University of the Witwatersrand, the Pretoria University and the University of South Africa, as well as a Doctorate of Science (Engineering) degree from his alma mater. Danie was elected as a Foreign Associate of the United States National Academy of Engineers (NAE) in 2010 for his distinguished contributions to Earth Resource Engineering and Technologies, the first South African ever to receive this award. Other awards include those from the SA Institute of Mining and Metallurgy, the US Society for Mining, Metallurgy & Exploration, the South African Academy for Science and Arts, the International Association for Mathematical Geology, the International APACOM Council (Application of Computers and Operations Research in the Mineral Industry) and the University of Antofagasta in Chile. He also received the Order of Meritorious Service Class 1 gold medal (1989) and the Order of the Baobab silver award (2012) from the South African State Presidents for his exceptional and distinguished achievements.

In 1981, Danie took up the post of Professor of Mineral Economics at the School of Mining Engineering at Wits University, supervising many MSc and PhD qualifications, as well as presenting courses in geostatistics, mineral economics and decision making for mining investments, until 1991. He continued his research and consulting activities well beyond his retirement from academia, giving lectures and providing valuable consulting services to local and international mining companies.

GASA remembers the substantial impact Danie made in the global mining industry.
This issue of the Journal commemorates Danie Krige and his pioneering work in geostatistics, which has set the global standards in this discipline. His work is one example of many innovations emanating from the South African mining industry, which because of its diversity and complexity, requires continual innovation in order to survive and grow.

The South African gold mining industry started with mines at relatively shallow depths and with high head grades, making them extremely profitable. Initially the mines were located in the Witwatersrand area but innovative geological exploration led to the discovery of the Carletonville, Klerksdorp, Free State, and Evander goldfields. It is hard to imagine now that back in 1970 South Africa’s gold production was a staggering one thousand tons, about 35 million ounces. As mining proceeded deeper, the development of new mining methods was instituted. We are now heading for depths in excess of 4 km, well beyond any other deep-level mining. With the decrease in gold head grades, metallurgists had to look at more cost-effective gold processing routes. This is where the carbon-in-pulp /carbon-in-leach (CIP / CIL) technology took over from the traditional cyanide leaching and cementation technology. Although the CIP / CIL technology was not pioneered in South Africa, the development of carbon elution, carbon regeneration, and gold electrowinning was mainly a result of South African innovation.

The South African platinum mining industry started with the mining and processing of the Merensky Reef. Initially the UG2 chromitite was not considered because of the metallurgical challenges. However, as Merensky reserves started to dwindle, the mining and processing of the UG2 became a necessity. Again, South African metallurgists rose to the challenge and developed innovative approaches to concentration and smelting. Today more UG2 is processed than Merensky.

When the global consumption of stainless steel and speciality steel increased significantly after the Second World War, the opportunity for producing ferroalloys in South Africa was identified. The design of submerged arc electric furnaces for the production of ferromanganese and ferrochromium was developed in South Africa. Later, the application of DC electric furnaces for the production of ferrochromium from fine chromite, and then for the smelting of ilmenite to produce pig iron and titanium slag, was pioneered in South Africa. This demonstrates the creativity of South Africa’s pyrometallurgists.

There are also significant achievements to highlight in the field of hydrometallurgy. The Rand Refinery is the world’s largest gold refinery and the technologies employed are efficient and innovative. In the platinum industry there are three precious metals refineries with capacities in excess of any other facility globally. Over the years these refineries have introduced significant technology changes, again mainly locally conceived.

I have used the above examples to illustrate that innovation is alive and well in the South African mining industry. The SAIMM promotes innovation mainly via the convening of conferences and also through the Journal, as well as access to papers via its website. Danie Krige was a shining example of innovation, and may his legacy inspire further innovation well into the future.

M. Dworzansowski
President, SAIMM
IN MEMORIAM

Professor D.G. Krige FRSSAf

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Daniel Gerhardus ("Danie") Krige, whose name is world-renowned in the field of mineral resource estimation and evaluation, passed away peacefully on Sunday morning, 23 March 2013; he was 93 years old. The funeral at Constantia Kloof Dutch Reformed Church on 28 March was attended by more than 200 family members, friends and colleagues from industry and universities, with eulogies by Oscar Steffen, Richard Minnitt and Winfred Assibey-Bonsu. Such was the renown of Daniel Krige that his death is recorded together with that of Margaret Thatcher in Wikipedia (Deaths in April, 2013). Born in Bothaville, Free State, on the 26 August 1919, Daniel G. Krige grew up in Krugersdorp, matriculating from Monument High School in Krugersdorp in 1934 at the age of 15. He graduated with a BSc(Eng) degree in mining engineering from the University of the Witwatersrand at the end of 1936 (aged 19). In 1938 he joined Anglo Transvaal where he worked on a number of gold mines until 1943, gaining a wide range of valuable practical experience in surveying, sampling and ore valuation. He then joined the Government Mining Engineer's Department where he worked for eight years before returning to industry as Group Financial Engineer of the Anglovaal Group. He held this post until 1981, after which he spent 10 years as Professor of Mineral Economics at the Witwatersrand University. He remained a registered Professional Engineer although his activity as a consultant naturally diminished.

HONOURS AND AWARDS
His contributions were recognised by the Witwatersrand University through the award of the DSc(Eng) degree in 1963 and a DIng(HC) degree in 1981 by the University of Pretoria. He received further honorary doctorates from the University of South Africa in April 1996 and from the Moscow State Mining University in September 1997. He received many merit awards from the SA Institute of Mining and Metallurgy, including two gold medals in 1966 and 1980 and two silver medals in 1979 and 1993. In 1984 he received the Institute's highest award, the Brigadier Stokes platinum medal. He was awarded the William Krumbein medal from the International Association of Mathematical Geology in 1984, the Gold Medal for Scientific and Technical Achievements from the Suid Afrikaanse Akademie vir Wetenskap en Kuns in 1982, the Distinguished Achievement Award from the APCOM International Council in 1989 and in the same year the Percy Fox Foundation Award in South Africa. In 1987 he received from the American Society of Mining Engineers one of its highest awards, the Daniel Jackling Award, and in 1988 he was made a 'Distinguished Member'; in both cases he was the first, and to date, the only South African to receive these honours. In 1992 the University of Antofagasta in Chile also honoured him with a special award. The South African State President awarded him with the Order for Meritorious Service Class I, Gold in 1989. In 1998 The Royal Society of South Africa awarded him the John F. Herschel Medal for outstanding contributions to science in South Africa.

In February 2010 his distinguished contributions to engineering were acknowledged by the United States National Academy of Engineering (NAE) and he was elected a Foreign Associate, Section 11, Earth Resources engineering. Danie was the first South African to ever receive this award from the NAE. In December 2010 he received an Honorary Doctorate from the University of the Witwatersrand, and in 2011 he was awarded the Order of the Baobab (Kre'met'art Silwer), by President Jacob Zuma.

CAREER ACCOMPLISHMENTS
During his period in government service Danie Krige handled several of the post-war lease applications in the Free State and Klerksdorp goldfields. The fact that decisions on new gold mines of critical importance to the State and the economy as a whole were being taken on a limited number of drillholes, without any scientific analysis of the risks of failure, stimulated him to start basic research into ore evaluation. His approach was based on the application of mathematical statistics to these problems, an approach of which very little was known worldwide at that stage but which had already
been initiated in South Africa by Herbert Sichel via the lognormal frequency distribution model. In Krige’s 1951 paper, published in the Journal of the Chemical, Metallurgical and Mining Society of South Africa, he covered the statistical explanation of the conditional biases in ore block valuations and stimulated the use by several gold mines of regression corrections for routine ore reserve valuations, a technique which, in effect, was the first use on an elementary basis of what is now known as kriging. This paper introduced, inter alia, the basic geostatistical concepts of “support,” “spatial structure,” “selective mining units” and “grade-tonnage curves.”

As the Anglovaal Group’s Financial Engineer, he was responsible for the Group’s ore evaluation, mine surveying, financial analyses of mining projects and negotiations, share valuations and technical computing facilities. During the early 1960s he implemented geostatistical kriging procedures on the two large gold mines of the Group. This was the first routine application of the kriging of ore reserves in the world. Apart from ore evaluation, his career led to significant contributions in the fields of investment and financial analysis and mining taxation. This is evidenced by his contributions to the establishments of the original South African uranium contracts, and by a substantial number of local and overseas publications in his field. These include the publication in 1955, in Afrikaans, of what was probably one of the first papers on risk analysis for new mining investments.

As Professor of Mineral Economics in the Mining Engineering Department of the University of the Witwatersrand he was responsible mainly for postgraduate courses in geostatistics and mining economics and supervised many masters and doctoral theses. After retirement from the university he continued his research, lecturing and publication activities and remained active as a consultant in the valuation of resources and reserves of mineral deposits and financial analysis for several of the Mining Houses and various local international mining and consulting companies. He presented courses in geostatistics and/or lectured at local universities (Pretoria, UNISA, RAU and Rhodes) and overseas (Australia, Germany, Taiwan, Chile, Russia and China). He has participated in, and contributed to, many international mining congresses in South Africa, the USA, Canada, Germany, Spain, Chile, Colombia, Slovenia, Australia, the UK, Russia, France and China; in several cases as the keynote speaker.

OUTCOMES
Krige’s work led directly, or contributed largely, to the following:
1. His recognition worldwide in mining circles as the principal pioneer in modern statistical methods of ore evaluation, or geostatistics as it is now called.
2. Since the early 1960s his surname has been used to describe the geostatistical techniques of ‘kriging.’ The term was coined by Georges Matheron and is now applied world-wide mainly in the fields of exploration and ore evaluation, but the environmental, petroleum, hydrology, agriculture and other disciplines.
3. The teaching of geostatistics in graduate and postgraduate mining engineering and other courses at universities worldwide.

PUBLICATIONS
Danie Krige published some 90 technical papers both locally and overseas, including Russia. His early research papers that had stimulated interest in several mining circles overseas were republished in French in 1955, resulting in a major research effort by French mining engineers in this field. A 1951 paper, based on his MSc(Eng.) thesis submitted to the Department of Mining Engineering to the University of the Witwatersrand, expounded his pioneering work in geostatistics in more detail. His 1978 publication was the first Monograph (Geostatistics) in the monograph series of the SA Institute of Mining and Metallurgy.

A complete record of all Krige’s publications is available on a CD disc from the SA Institute of Mining and Metallurgy. They are presented under the following headings:
1. Original basic concepts and developments
2. Routine block kriging on mines
3. Geostatistical techniques, Simple kriging versus Ordinary kriging, Conditional biases
4. Bayesian approach
5. Valuation of new mines from drillholes
6. Reference works
7. Reviews
8. Economic and Financial.
OTHER PROFESSIONAL CONTRIBUTIONS

As a Professional Engineer, Danie Krige served for many years on the mining committee of the Engineering Council of South Africa. For an extensive period he was honorary treasurer on the Council of the SA Institute of Mining and Metallurgy and became an honorary life member. He was also a mining engineering member of the Income Tax Special Court, a founder member of the International Association for Mathematical Geology and of the Geostatistical Association of Southern Africa, a founder-member and honorary life Fellow of the Statistical Association, an honorary life member of the Institute of Mine Surveyors of South Africa and a Fellow of the Royal Society of South Africa. He also served as a director of several mining companies, as well as for the South African Development Trust, the Lebowa Development Corporation, and the Lebowa Mineral Trust.

Danie served on the sub-committee of the Prime Minister’s Economic Advisory Council which investigated State Aid for marginal gold mines in 1967/8. He designed the State Aid formula which assisted a large number of gold mines to survive the period of low gold prices. This scheme significantly contributed to the stabilisation and growth of the gold mining industry and the economy as a whole during a difficult time. He also served for many years on various committees of the Chamber of Mines. In 1974 he was a Chamber-nominated member of the Government-Chamber mining mission to Iran which investigated aspects of a closer co-operation on mining matters. More recently, he served as a member of the Marais Committee on mining taxation and on the Melamet Commission of Enquiry into further State aid for the ERPM gold mine; he was also an observer for the State on this mine’s Management Committee and Board of Directors until early 1994. He was also a member of the SAMREC Working Committee which developed the South African Code for reporting of Mineral Resources and Reserves as published in 2000.

He was South Africa’s representative on the International APCOM Council from its inception and initiated the arrangements for the Symposia held in South Africa in 1972 and in 1987, and he assisted in the preparation of this Symposium in Cape Town in 2003. He served as Chairman of the International Council, the first non-USA member to be elected to this position, from 1990 to 1993.

The outstanding feature of Danie Krige’s contributions was his focus on, and dedication to, the basic tenets of geostatistics and the use, wherever practical, of large databases to undertake practical follow-up studies. This style of approach to statistical and geostatistical research became apparent in his initial 1950/2 work and consistently underpinned his research. His rigorous practice of verifying new geostatistical techniques using large data sets, allowed him to test and audit their applicability and interrogate alternative approaches. His high standards of research contributed significantly to the advancement of the science of geostatistics and provided many fruitful avenues for future research. His lasting contributions are a tribute to a lifetime of dedication and he was a worthy leader and an example to all who practice and research in the field of geostatistics.

DANIE KRIGE’S ACKNOWLEDGEMENT OF THE GRACE GIVEN TO HIM

Thoughts from an interview he gave to Richard Minnitt during 2012

With his weight of achievement and a life of distinguished contributions to science and engineering behind him, Professor Danie Krige was a devout Christian who also recognised and acknowledged that he had been the recipient of gifts of grace from the Creator. He drew attention to six specific areas in which he could identify the grace of the Almighty at work in his life and career. The first was a tribute to his parents for the practical application of a godly lifestyle, the establishment of a firm foundation, and a life philosophy that was modelled by them in every area of life. An example of this was that even with the limited resources of a pastor, his parents saw to it that seven of the nine siblings received a tertiary education.

The second of the gifts of grace that Danie acknowledged was the support he had received from his two spouses. He was happily married for 45 years to his first wife (until her death), and for 20 years to Anjie, his second wife.

The third gift of grace was the way in which his career developed, and the various turns in direction that it took as his research unfolded. Having graduated from Wits he was employed in the sampling and survey departments of the Anglovaal Group. In the interview Danie stated that the most important event of his career occurred when an extensive prospecting programme of deep drilling was launched to determine the extent and value of the extensions of the Witwatersrand gold deposits westwards to Klerksdorp and the Orange Free State. Mining companies involved in exploration applied to the Government Mining Engineer for mining leases which led to the opening up of the new goldfields. The GME enrolled additional Mining Engineers to handle this work and he was one of the lucky ones to be chosen. Danie was involved full time in collecting, analysing, and using statistical modelling of the data to determine the underlying patterns of the gold distribution in the widening fields. These patterns proved to be adaptable to modern statistical programmes and these were in turn developed to yield improved grade estimation procedures. This work was successfully submitted for a Master’s thesis at the University of the Witwatersrand and a technical paper that was published here and overseas, raised interest. A French team under Professor Georges Matheron in Paris translated the publication and republished it in French. Professor Matheron insisted for the worldwide acceptance of the term KRIGING for this new valuation method, a term for a practice that is now accepted internationally.

Danie and Anjie at Danie’s 90th birthday party.

The fourth gift of grace was that when Danie returned to work at Anglovaal, they began to apply these advanced methods of valuation on their mines. Comparisons with later
follow-up values demonstrated the advantages of the new methods. Following the clear demonstration of the validity of the new methods, Anglovaal fully supported the application of Danie’s methods on their mines, as well as the publication of these findings, both locally and internationally, particularly at international conferences. In addition, Wits University awarded Danie a DSc(Eng) degree in recognition of this work. This led to the award of two Honorary Doctorate degrees in South Africa, from the University of Pretoria and the University of South Africa (UNISA), and a third from the Moscow State University, in addition to numerous other awards both locally and overseas.

The fifth gift of grace Danie acknowledged was that on retirement from Anglovaal at the age of 60, he accepted the unexpected opportunity of taking up the chair of Professor of Mine Economics at Wits University, which he occupied for the next 10 years. This enabled him to teach and undertake extensive consulting work for mining companies both locally and internationally, and was, in his opinion, a great blessing. The final gift of grace that Danie recognised was that after his retirement from Wits University he was able to undertake extensive national and international consulting work which he believed kept him occupied and young for the following 20 years. Danie also acknowledged, with deep gratitude, that while the opportunities presented themselves to him, his was the responsibility to make good use of them, and that without these gifts of grace his life’s work would not have been possible.

Acknowledgement

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Criteria for the Annual Danie Krige Medal Award

Daniel Gerhardus Krige (26 August 1919 – 3 March 2013), one of South Africa’s most influential mining engineers and geostatistician of international repute, passed away last year. Danie was a recipient of the Brigadier Stokes award in 1984 – this is the Institute’s highest recognition of contribution to the minerals industry.

Following discussions at Office Bearers and Council during 2013 it was agreed to honour his memory and contribution to the mineral industry through three activities:

• The publication of a Danie Krige Commemorative Volume of the Journal. This is planned for March 2014 with a number of papers (37) having been submitted to the publications committee to date
• An annual Danie Krige Memorial Lecture to be facilitated by the School of Mining Engineering at the University of the Witwatersrand
• The annual award of a Danie Krige medal for a qualifying geostatistics paper published by the SAIMM in the previous year.

Selection criteria

The Danie Krige Medal will be awarded annually to the author (or co-authors) of the best geostatistical paper published in the previous calendar year. Accordingly, SAIMM members would be invited to nominate and/or submit papers for consideration on an annual basis.

The following criteria will govern the award:

i. Papers on theoretical or applied geostatistics are eligible
ii. The papers must have been published in the Journal of the SAIMM in the preceding calendar year
iii. Nominations for the award may be made by a member of the SAIMM (who is not an author) or submissions may be made by the author(s)
iv. Nominations and submissions must be submitted by email in pdf format to the SAIMM for attention of the Chairperson of the Danie Krige Medal Committee;
v. An individual may only submit one paper (or be nominated, based on one paper) for the award in any year
vi. No award will be made if none of the papers in a given year meet the minimum standards of the Danie Krige Medal Committee. In evaluating papers, the committee will use the following criteria and apply their professional judgement:
   a. The impact and contribution to knowledge of the paper in its specific field
   b. How innovative are the ideas or techniques described in the paper
   c. The relevance of the problem being addressed
   d. How well the paper is written (language, structure, supporting figure etc.)
vii. Only one paper, or one series of papers on a topic by the same author, per year will qualify for the award
viii. The decision of the Danie Krige Medal Committee on the award of the medal will be final
ix. Award of a Danie Krige Medal excludes the winning paper from consideration for any other SAIMM publications awards i.e. the SAIMM Gold and Silver medals for Journal papers.

The Danie Krige medal will comprise a 38 mm diameter medal in 9 carat gold in an engraved rosewood case and carry an impression of Danie Krige on one side and the SAIMM logo on the other.

G.L. Smith
Immediate Past President, SAIMM
Memories of Danie Krige
Geostatistician Extraordinaire

Let me start the story in 1969, when I was a final-year mining engineering student at the University of Chile. At that time, Andre Journel, a young man from the Paris School of Mines (Fontainebleau), came to Chile to teach a two or three months-long seminar on geostatistics. If I am not mistaken, that was the very start of geostatistics in Chile. This effort was continued in the early 70’s by Andre’s colleague Alain Merechal. In those days, geostatistics was highly mathematical and theoretical, as commercial software did not exist and enthusiastic users had to write their own code. It is not surprising that geostatistics in Chile had a distinct French flavor and Kriging was referred to as ‘Krigeage’. In my ignorance, I grew up thinking that Daniel Krige was a very clever Frenchman.

I had no further contact with geostatistics until 1975. At that time, I was working for Anglo American Corporation in Welkom, South Africa, in the Management Sciences Department, developing mixed integer programming models for mine planning. Of course, the gold grades assigned to the different mining areas had a large impact on the sequencing, as well as on the expected profits. My esteemed friends, the geologists, had the almost impossible task of predicting the gold and uranium grades of large blocks, based on perhaps half a dozen boreholes covering an area as large as 50 km2. On the other hand, closely spaced underground sampling showed large variability and intricate grade patterns. Grade predictions based on a very limited number of drill hole results seemed almost impossible. This is what started my interest in the subject.

At that time, publications were few - many of them in French, with at least 50 integrals per page. It was not easy! I admit I was battling to get to grips with the subject. One fine day, my boss said ‘Why don’t you go and see Danie Krige, perhaps he can give you a hand’. Immediately I replied ‘Great, I’m off to France’. ‘I’m afraid not, Danie works for Anglovaal in Johannesburg, so just get in your car’, came the reply.

I visited Danie every three or four months. He introduced me to his co-worker, Jean-Michel Rendu. They were extremely helpful and shared their knowledge, papers and lots of advice. During one visit, Danie explained that he was rather sad since Jean-Michel had accepted and offer as Professor at a prestigious University in the USA. Before returning to Welkom that evening, I walked into Danie’s office and said to him ‘What about the job, then?’ I thought to myself that I was being rather cheeky. To my great surprise Danie said ‘If you want the job, it is all yours’. I was delighted and could not wait to get home and tell my wife Patricia the great news.

I joined Anglovaal in 1976 and worked with Danie for 10 years. Danie was a patient boss and explained the famous log-normal regression model – the first steps towards kriging – on more than one occasion.

In the early 80’s, we gave four one-week courses at the University of Clausthal-Zellerfeld in Germany. On one of these occasions, our wives Ansie and Patricia accompanied us. We stayed in the silver-mining medieval town of Goslar, in Lower Saxony, close to the beautiful Harz Mountains. The hotel was close to the central square, housed in a thousand year-old building. In the evenings, after classes, we would sit in a quaint little coffee shop on the square, enjoying coffee with wonderful pastries and listening to the Town Hall ‘glockenspiel’.

For a weekend, we visited Berlin and took a bus tour to East Berlin. We crossed the wall at Check Point Charlie. A uniformed lady, who looked and acted as if she were a member of the Gestapo, boarded the bus, checked all the passports and returned them to each passenger, except Danie’s and mine. She took our passports into an office and stayed there for about 20 minutes. The other passengers were all muttering in German and looking at us. We had a quiet chat with Danie: ‘We are going to be famous; you coming from the land of apartheid and me from General Pinochet’s dictatorship. We might be here for a long, long time’. In the end the police woman came back, gave us our passports with a dour look and the tour continued normally.
Memories of Danie Krige

Unfortunately, Ansie suffered from severe asthma and passed away in 1989. We remember her as a very kind and loving person. Some years later, Danie married a wonderful lady, co-incidentally also named Ansie.

South Africa has produced a number of great statisticians. On one occasion, we had lunch at the Johannesburg Club with Danie, D.M. Hawkins, H.S. Sichel and Peter Brooker from the UK. In my modest opinion, having had the honour and the pleasure of working with some of these great minds, they all seemed to have something in common: they are all unassuming, approachable, helpful and always willing to share their knowledge. It seems to be the mark of truly great people.

I remember those 10 years I worked at Anglovaal with Danie as very happy, friendly and productive. The company and Danie were very special and encouraged research, conferences, publications and teaching part time at the Mining Engineering Department of the University of the Witwatersrand. We developed a fine working relationship with my favorite clients and lifelong friends, the geologists; in particular with Jan Mostert.

In 1986, due mainly to family pressures, we moved back to Chile, where I have been working as a part time Professor at the University of Chile and as a mining consultant in geostatistics and sampling ever since. Over the years in Chile, our working relationship with Danie continued and we presented some short courses together, met at conferences and seminars and completed ore resource calculation reviews for several large Chilean copper mines. On one occasion, Danie was accompanied by Ansie and we went on holiday to the beautiful Chilean Lake District, some 900 km south of Santiago. On the way south, I got a speeding fine. A few kilometers further, another policeman stopped me for yet another traffic offence. Ansie could not bear it, got out of the car and gave the policeman a dressing-down in English. To our great surprise, he let us go, remarking that my driving was not at all brilliant that day.

Some years ago, I was teaching the undergraduate course on linear geostatistics to the mining engineering students. We had covered the estimation variance material and had still to continue with Kriging. By coincidence, Danie was in Chile and I asked him if he would be so kind as to give the Kriging lecture himself. Typical of Danie, he immediately accepted. As we walked into the class, there was silence. I told the students that the next topic was Kriging and the best person to present the subject was none other than Professor Krige himself. You should have seen the look on their faces …….. That evening, the students came to our home to socialize and enjoy a few drinks with Danie. They took many photos and had a wonderful time with Ansie and Danie.

In 2007, I chaired APCOM in Chile. Danie very kindly delivered the opening keynote address with Clayton Deutsch on ‘The Origins and Future of Geostatistics’. The conference went very well and many people almost queued to have their pictures taken with Danie. As a keepsake, I still have a bottle of good Chilean wine with Danie’s signature on the label.

In July 2007, we went on safari to Tanzania with our dear friends Jeanne and Jan Mostert. On our return to Johannesburg, Ansie and Danie invited us for dinner at their home. We had a very pleasant, friendly evening. That was the last time we saw Danie.

Through the years, I lost track of the number of well-deserved Honorary Doctorates and distinctions that Danie received. A concept that was started off by comparing estimated block values against follow up mining values (true block values) in the South African Gold Mines has developed into a global industry with perhaps thousands of users in mining and other applications, by software developers and researchers all around the world. Danie’s seed idea was picked up by research centers such as Fontainebleau, Stanford, Alberta, at various centers in South Africa and more recently in Chile, and numerous new applications are continually being proposed and developed. An aspect perhaps less known was Danie’s extensive knowledge and ability in subjects such as taxation and financial evaluation of mining projects. In fact, his company designation was that of Financial Engineer.

While travelling through Patagonia in early March, 2013, we received a message from Oskar Steffen saying that after an illness, Danie had passed away on Saturday March 2. It was a long, sad day.

Danie was my friend, my mentor. We remember him with great fondness and respect.

E. Magri
At Implats we believe that people are the foundation on which exceptional performance is built. We are therefore committed to ensuring their well-being and our vision of ‘zero harm’ remains a cornerstone of our approach to ensuring their safety. Alongside world-class safety management standards, we also place an equally high priority on first-class health care and medical facilities.

The development of our people and the empowerment of our local communities is also a key priority. Over the last five years we have invested in excess of R3.1 billion in social investment projects benefitting more than 100,000 people in South Africa and over 12,000 in Zimbabwe, including 28,000 in local communities.

We also believe in taking responsibility for the ecosystems in the areas in which we operate and consistently apply world-class environmental practices in order to mitigate the impact of our business on these environments.
Cokriging for optimal mineral resource estimates in mining operations

by R.C.A. Minnitt* and C.V. Deutsch†

Synopsis
Cokriging uses a sparsely sampled, but accurate and precise primary data-set, together with a more abundant secondary data-set, for example grades in a polymetallic orebody, containing both error and bias, to provide improved results compared to estimation with the primary data alone, as well as filtering the error and mitigating the effects of conditional bias. The method described here may also be applied in polymetallic orebodies and in other cases where the primary and secondary data could be collocated, and one of the data-sets need not be biased, unreliable, etc. An artificially created reference data-set of 512 lognormally distributed precious metal grades sampled at 25x25 m intervals constitutes the primary data-set. A secondary data-set on a 10x10 m grid comprising 3200 samples drawn from the reference data-set includes 30 per cent error and 1.5 multiplicative bias on each measurement. The primary and secondary non-collocated data-sets are statistically described and compared to the reference data-set. Variograms based on the primary data-set are modelled and used in the kriging of 10x10 m blocks using the 25x25 m and 50x50 m data grids for comparison against the results of the cokriged estimation. A linear model of coregionalization (LMC) is established using the primary and secondary data-sets and cokriging using both data-sets is shown to be a significant improvement over kriging with the primary data-set alone. The effects of the error and bias are filtered and removed during the cokriging estimation procedure. Thus cokriging using the more abundant secondary data, even though it contains error and bias, significantly improves the estimation of recoverable reserves.

Keywords
Cokriging, primary data-set, secondary data-set, linear model of coregionalization (LMC), ordinary kriging, optimal resource estimates.

Introduction
Cokriging is a widely proclaimed, but infrequently applied estimation technique using two or more sets of collocated or non-collocated data of quite different type and support, over the same domain. The method is applicable in polymetallic orebodies where one metal is well-sampled and the other less so, but may also be applied in orebodies where the primary and secondary data could be collocated, and one of the data-sets need not be biased, unreliable, etc. The method produces better estimates of recoverable reserves than ordinary kriging of either of the two data-sets individually. The application of cokriging demonstrated in this paper involves a primary random function \(Z(u), \ u \in A\) of sparsely sampled, good quality measurements \(z(u_\alpha), \alpha = 1, \ldots, n\) evenly distributed across a domain of interest for which grade and recoverable reserves are to be estimated. Over the same domain an often more densely distributed, but poorer quality, non-collocated secondary random function \(Y(u), \ u \in A\) whose measurements \(y(u_\beta), \beta = 1, \ldots, r\) include sampling errors and bias, is also available for use in the estimation procedure. Secondary data is often easier, quicker, or cheaper to collect than primary data, but its information content is suspect because of the concern that including poorer quality data with sampling error and bias would compromise the estimate.

Examples include open pit mining operations where reverse circulation drilling is augmented by blast-hole sampling data, projects where both RC and diamond drill-hole data are available, tabular orebodies where regional exploration diamond drilling is supported by numerous face or chip samples in adjacent underground workings, or in exploration targets where a primary set of gold grades from diamond drilling is augmented by an abundance of secondary geochemical or geophysical information such as ground magnetic readings. Examples of cokriging two highly correlated variables sampled at different locations within a domain have been described in various applications by Meyers (1982), Wackernagel (2010), Goovaerts (1997), Chiles and Delfiner (2012), and Isaaks and Srivastava (1989).

Cokriging
Generally, data-sets will contain more than one variable of interest, and these are usually spatially cross-correlated as expressed in the cross-variogram. Cokriging is a method that takes advantage of the information embedded in it by combining the information from multiple data-sets. It is especially useful in situations where one data-set is sparse but accurate, and another is abundant but may contain errors or biases. By using both data-sets, cokriging can provide more accurate estimates of recoverable reserves than ordinary kriging using either data-set alone.
Cokriging for optimal mineral resource estimates in mining operations

in the cross-correlation of a second variable in order to minimize the variance of the estimation error (David, 1977; Journel and Huijbregts, 1978). In this paper cokriging is considered in the context of simulated gold grades that have been sampled in two different campaigns, which are then used to estimate the recoverable reserves. Cokriging is the preferred method of estimation where there is an undersampled primary data-set and a closely sampled secondary data-set containing sampling error and bias. The advantage of cokriging with equally sampled data is unclear, but will be investigated in the future.

Isaaks and Srivastava (1989) indicate that in some situations cokriging will not improve an ordinary kriging estimate. Such situations arise when the primary and secondary variables are collocated, meaning that there is not one variable that is undersampled with respect to another, and that the auto- and the cross-variograms are proportional to the same basic model. In such cases the cokriging estimates will be identical to the ordinary kriging estimates. Isaaks and Srivastava conclude that, ‘Thus if all the variogram models are “quite similar” in shape and the primary variable is not noticeably undersampled, cokriging will not improve things very much’ (p. 405). Cokriging therefore is meant specifically for different, but correlated, non-collocated variables of different quality that are sampled at different densities across a domain. Deutsch and Journel (1998) note that variograms and cross-variograms can be used in the cokriging systems only provided that the requisite constraints on cokriging weights are met. Provided this is so, sample cross-variograms can be transformed into a corresponding cross-variogram C_{ZY}(h). The cross-variogram can then be modelled with a linear model of coregionalization to be used in the cokriging system. Where a single secondary variable (Y) is considered, the ordinary cokriging (OCK) estimator of a primary variable is:

\[ Z_{OCK}(u) = \sum_{\alpha=1}^{n} \lambda_{\alpha}(u)Z(u_{\alpha}) + \sum_{\beta=1}^{r} \lambda'_{\beta}(u)Y(u_{\beta}) \]  

where \( \lambda_{\alpha} \) represents weights applied to the \( n \) primary data \( (z) \), \( \lambda'_{\beta} \) represents weights applied to the \( r \) secondary data \( (y) \), and the bold \( u \) identifies a vector.

Deutsch and Journel (1998) show that while kriging requires a model for the Z covariance, cokriging requires a joint model for the matrix of covariance functions including the Z covariance \( C_Z(h) \), the Y covariance \( C_Y(h) \), and the cross \( Z-Y \) covariance \( C_{ZY}(h) = C_Z(Z(u), Y(u+h)) \). In traditional ordinary kriging the sum of the weights applied to the primary variable is unity, while the sum of weights applied to the secondary variable is zero. Provided these constraints are met (Equation [2]) the estimator is unbiased:

\[ \sum_{\alpha=1}^{n} \lambda_{\alpha}(u) = 1, \quad \text{and} \quad \sum_{\beta=1}^{r} \lambda'_{\beta}(u) = 0 \]  

The second constraint is that the sum of the secondary weights must be zero, which implies that the secondary data makes no net contribution to the estimate, a severe constraint on ordinary cokriging according to Deutsch and Journel (1998). The error variance is minimized under the constraints in Equation [1] as follows:

\[ \sum_{\alpha=1}^{n} \lambda_{\alpha}(u)C_{zz}(u_{\alpha} - u_{\beta}) + \sum_{\beta=1}^{r} \lambda'_{\beta}(u)C_{zy}(u_{\alpha} - u_{\beta}) + \mu_{\alpha}(u_{\beta}) = C_{Zz}(u_{\alpha} - u_{\beta}) \]

\[ \mu_{\alpha}(u_{\beta}) = \gamma_{Zz}(u_{\alpha} - u_{\beta}) - \sum_{\alpha=1}^{n} \lambda_{\alpha}(u)C_{Zz}(u_{\alpha} - u_{\beta}) - \sum_{\beta=1}^{r} \lambda'_{\beta}(u)C_{zy}(u_{\alpha} - u_{\beta}) \]

where the Lagrangian multipliers \( \mu_{\alpha} \) and \( \mu_{\beta} \) account for the two unbiasedness constraints. The covariances are classically obtained from variograms as follows:

\[ C_{Zz}(h) = C_{Zz}(0) - \gamma_{Zz}(h) \]

\[ C_{ZY}(h) = C_{ZY}(0) - \gamma_{ZY}(h) \]

\[ C_{ZY}(0) \] are the variances of the \( Z \) and \( Y \) variables, respectively. \( \gamma_{Zz}(h) \) is the estimated cross-covariance of collocated \( Z \) and \( Y \) data. The direct and cross-variograms are fitted with the well-known linear model of coregionalization (LMC).

\[ \gamma_{Zz}(h) = \sum_{l=0}^{nst} C_{Zl}I_l(h) \]

\[ \gamma_{ZY}(h) = \sum_{l=0}^{nst} C_{Yl}I_l(h) \]

where \( nst \) is the number of structures and \( I_l(h), l = 0, \ldots, l \) is the \( l \)th nested structure defined by a shape, three angles, and three ranges; and \( C \) is the contribution of the \( l \)th structure to the variogram model, \( ZZ, ZY, \) or \( YY \) as appropriate.

In order for the model to be positive definite the following determinant must also be positive definite for all values of \( l \) and the cross-product of these coefficients must be greater than zero.

\[ \det \left[ \begin{array}{cc} C_{Zl} & C_{Yl} \\ C_{Zl} & C_{Yl} \end{array} \right] \geq 0 \quad \text{for all} / \text{that is} \quad C_{Zl} \geq 0 \]

and \( C_{Zl}C_{Yl} - C_{Yl}C_{Zl} \geq 0 \)

where the 0 identifies a scalar. Deutsch and Journel (1998) suggest that cokriging has not been extensively used in practice firstly because of the tedious joint modelling required by \( R^2 \) variograms when \( R \) variables are used, and secondly because of the screen effect of the better correlation among \( z(u_{\alpha}) \) data compared with the weaker correlation between the \( z(u_{\alpha}) \) and \( y(u_{\alpha}) \) data. Furthermore, they describe two other cokriging techniques that are applied, namely simple cokriging and standardized ordinary cokriging.

**Simple cokriging (SCK)** with the prior assumption of a known mean is mathematically the best estimation method in that there are no constraints on the weights, it minimizes the estimation variance, and it produces estimates that are unbiased. The simple cokriging estimator at location \( (u_0) \) of a primary variable \( z \) using a secondary variable \( y \) is:

\[ \sum_{\alpha=1}^{n} \lambda_{\alpha}(u_0)C_{zz}(u_{\alpha} - u_{0}) + \sum_{\beta=1}^{r} \lambda'_{\beta}(u_0)C_{zy}(u_{\alpha} - u_{0}) + \mu_{\alpha}(u_{0}) = C_{Zz}(u_{\alpha} - u_{0}) \]

\[ \mu_{\alpha}(u_{0}) = \gamma_{Zz}(u_{\alpha} - u_{0}) - \sum_{\alpha=1}^{n} \lambda_{\alpha}(u_0)C_{Zz}(u_{\alpha} - u_{0}) - \sum_{\beta=1}^{r} \lambda'_{\beta}(u_0)C_{zy}(u_{\alpha} - u_{0}) \]
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\[ Z^*_\text{SCK}(u_0) - m_z = \sum_{\alpha=1}^{n} \lambda_\alpha [Z(u_\alpha) - m_z] \]

\[ - m_z] + \sum_{\beta=1}^{r} \lambda'_\beta [Y(u'_\beta) - m_y] \]

where \( \lambda_\alpha \) and \( \lambda'_\beta \) are the weights attributed to the primary and secondary data, and \( m_z \) and \( m_y \) are the primary and secondary means in a stationary environment. The error variance at \( (u_0) \) is given by:

\[ \sigma^2_Z(u_0) = \text{Var}(Z(u_0) - Z^*(u_0)) \]

This is minimized by solving the following system of linear equations:

\[ \sum_{\alpha=1}^{n} \lambda_\alpha (u_\alpha) C_{xz} (u_\alpha - u_\beta) + \]

\[ \sum_{\beta=1}^{r} \lambda'_\beta (u'_\beta) C_{xz} (u'_\beta) = \]

\[ C_{xz} (u_\alpha - u_\beta) \quad \alpha = 1, \ldots, n \]

\[ \sum_{\beta=1}^{r} \lambda'_\beta (u'_\beta) C_{xy} (u'_\beta - u_\beta) + \]

\[ \sum_{\beta=1}^{r} \lambda'_\beta (u'_\beta) C_{xy} (u'_\beta) = \]

\[ C_{xy} (u'_\beta - u_\beta) \quad \alpha = 1, \ldots, r \]

As with simple kriging, this version of cokriging employs the residuals from the data, or requires the standardization of the means such that they are zero. Although it is free of major constraints affecting other methods, the major concern in regard to application of this technique is the strong assumption of stationarity across the domain.

The standardized ordinary cokriging estimator (SOCK) uses a secondary variable that has been standardized such that the means for the primary and secondary variables are the same, and all the weights are constrained to add up to one. In this case Equation [1] can be written:

\[ Z^*_\text{SOCK}(u_0) - m_y = \sum_{\alpha=1}^{n} \lambda_\alpha [Z(u_\alpha) - m_z] \]

\[ \sum_{\beta=1}^{r} \lambda'_\beta [Y(u'_\beta) - m_y] \]

\[ \frac{1}{\sigma_z} \sum_{\alpha=1}^{n} \lambda_\alpha [Z(u_\alpha) - m_z] \]

\[ \frac{1}{\sigma_y} \sum_{\beta=1}^{r} \lambda'_\beta [Y(u'_\beta) - m_y] \]

where \( \lambda_\alpha \) and \( \lambda'_\beta \) are cokriging weights obtained by solving the ordinary cokriging system of equations (Equation [3]) expressed in terms of correlograms (Goovaerts, 1998), with the single condition that:

\[ \sum_{\alpha=1}^{n} \lambda_\alpha + \sum_{\beta=1}^{r} \lambda'_\beta = 1 \]

where \( m_z = \text{E}(Z(u)) \) and \( m_y = \text{E}(Y(u)) \) are the stationary means of \( Z \) and \( Y \) (Deutsch and Journel, 1998).

In this case both the SOCK and SCK weights are linearly related. Goovaerts (1998) has shown that estimation with standardized or non-standardized data results in the same cokriging estimate. The non-standardized estimate is a rearrangement of Equation [4]:

\[ Z^*_\text{SCK} = Z + \sigma \]

With these constraints in mind, the use of cokriging and the necessity of a linear model of coregionalization (LMC) are now examined.

Linear model of coregionalization (LMC)

The linear model of coregionalization (LMC) informs the cokriging process as to how much of the secondary data to use. The cross-variogram carries the information content for the calibration that filters the error from the secondary data. The cross-variogram is constructed using the cross-covariance since the cross-variogram requires collocated data, but the cross-covariance does not; unequally sampled data is used directly. For a cross-covariance that is flat at zero distance it is possible to model an LMC, but all the secondary data would be assigned a weight of zero during the cokriging procedure. If the cross-covariance looks exactly like the direct variogram, it is again possible to model the LMC, but the primary and secondary data would be treated in exactly the same way, i.e. they would be assigned equal weights during the cokriging procedure.

The covariance matrix for jointly distributed primary and secondary variables must be positive definite. A simple way of constructing a valid cross-covariance is through the use of an LMC. While each variable has its own direct variogram and each pair of variables have their own cross-variogram, the LMC uses the same variogram structures and the same ranges for the direct variograms of \( Z \) and \( Y \) (although the contributions can vary), and for the cross-variograms of \( Z \) with \( Y \) (see Equation [3]). This ensures that the variance of any possible linear combination of these variables is always positive (Isaaks and Srivastava, 1989).

The model for each of the single variograms may consist of one or more components of the basic models, giving rise to so-called nested structures, which is acceptable since 'any linear combination of positive definite variogram models with positive coefficients is also a positive definite model' (Isaaks and Srivastava, 1989, p. 375). These matrices must be positive definite, therefore the following relations must be true:

\[ C(i)_{XX} - \sum_{l=0}^{nst} C(i)_{YY} \geq 0 \]

\[ \det \left[ \begin{array}{c} C(i)_{XX} - \sum_{l=0}^{nst} C(i)_{YY} \end{array} \right] \geq 0 \]

where \( l \) refers to the number of structures (nst), the 0 identifies a scalar, and \( C(i) \) is the contribution of the \( i \) th structure to the variogram model \( ZZ, ZY, \) or \( YY \) as appropriate.

Isaaks and Srivastava (1989) state it is sufficient that all the eigenvalues of the matrices of \( b \) coefficients are positive, but generally the conditions shown in Equation [5] are more widely applied. They also warn that these constraints make the modelling of coregionalization difficult and that one of the single or cross-variograms may not fit the sample variogram well. They suggest that each individual model can be considered as being a small part of a total model and the overall fit judged accordingly. They also provide the helpful suggestion that when compiling a model of coregionalization, a basic model in the single variogram does not necessarily
have to appear in the cross-variogram model. However, any basic model in the cross-variogram must be included in all the single variogram models.

**Practical application of cokriging**

In the progressing of a mineral prospect to a going mining concern it is common to accumulate primary data such as metal grade, as well as other types of data from a variety of studies. The latter may have different support and are termed secondary data. Directly or indirectly, the goal of such studies would be related to efforts to derive the best estimate of grades and their distribution across the orebody, the primary data source for such estimates being a limited number of carefully drilled diamond drill-holes. Although generally abundant, secondary data may be suspected of being in error or biased. Rather than applying ad-hoc calibration or correction factors to improve the quality of the secondary data, or simply rejecting secondary data as not usable, cokriging is an attractive alternative that allows secondary data to be used in a theoretically acceptable manner. However, the challenges with respect to the practical implementation of cokriging can be formidable, and it is the aim of this study to provide an understanding of and to document these challenges through a study of synthetic primary and secondary data-sets.

In a real-life mining situation it is essential to assemble the required modelling parameters for the primary data alone, considering issues such as stationarity, managing outliers, declustering, and variography. Attention to such details ensures that the appropriate parameters are available for investigating volume-variance relationships and the kriging of the data. Considerations about stationarity would answer questions such as whether the data belongs together, or does it need an adjustment to account for a trend. The way in which outliers are dealt with is always a contentious point, but where it is clear that outliers need attention, ‘following local practice’ or conforming to the mining personnel standard is a prudent position to take. When required, declustering would also be considered in order to produce an appropriately weighted distribution that, having taken account of sample redundancy and clustering, is summarized by a mean and a variance. Understanding and extracting the underlying variogram is essential to testing the sensitivity of the deposit to SMU size and volume-variance relationships, and in this case the kriging will require a variogram in original units. One might use normal scores, median indicators, correlograms, relative variograms, or any other tools available to extract information about the underlying spatial structures and to overcome the challenge of limited or skewed data.

In this particular exercise, the use of a simulated data-set and regularly spaced data means that none of the issues described has to be considered. The primary data is therefore suitable for incorporation, can be considered to be stationary, with no outliers, and the distribution within a regular grid obviates the issues that would normally have to be addressed in regard to declustering.

**Reference data-set**

A 400 m × 800 m Gaussian point data-set was simulated at a 1×1 m resolution to give 320 000 points. A lognormal distribution of fictitious metal grades with a mean of 0.70 g/t and a standard deviation of 1.08 g/t was created by transforming the normal scores data. The simulated lognormal reference data-set was sampled such that three highly correlated, non-collocated data-sets all having units of grams per ton (g/t) were created on regular square grids at 10×10 m, 25×25 m and 50×50 m, hereafter referred to as 10 m, 25 m, and 50 m grids. The relative position of these three grids in the first 100×100 m block is shown in Figure 1a.

At a point scale (1×1×1 m), assuming a reef thickness of 1 m and a density of 3 t/m³, the project area contains 960 000 t at a grade of 0.70 g/t with a total metal content of 672 kg. True values for tons, grade, and contained metal were averaged for panels at 10×10 m, 25×25 m, and 50×50 m grid size from the simulation reference data-set. Recoverable reserves were calculated at a cut-off of 1 g/t. The tons, average grade, and recoverable metal at point support, 10 m, 25 m, and 50 m panels are listed in Table I.

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**Figure 1**—(a) Layout of the samples on 10×10 m (3200 data), 25×25 m (512 data), and 50×50 m (128 data) grids, (b) location of non-collocated primary and secondary data within a 10 m radius of one another

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In proceeding from 1 m points to 50 m panels the grade of the recoverable reserves above 1 g/t decreases rapidly with a marginal increase in tons, due to the inclusion of increasing amounts of waste dilution in the SMUs. As a consequence the recoverable metal decreases from 418 kg to 324 kg as the size of the SMUs increases, the balance of the total metal in the deposit being lost in ore sent to waste and waste rock itself.

**Sampling the reference data-set**

Two primary data-sets (Z) sampled on 25×25 m and 50×50 m square grid spacings represent the 512 and 128 good quality drill intersections of a flat tabular orebody, carefully drilled, logged, and assayed diamond borehole cores, respectively. The more abundant secondary data (Y) was sampled at much closer 10×10 m square grid spacing representing 3200 measurements of more easily collected and probably cheaper geochemical or geophysical data. A 30 per cent random error and multiplicative bias of 1.5 were introduced to this secondary data during the sampling procedure. Descriptive statistics for each data-set as well as experimental, direct, and cross-variograms were calculated and modelled (Table II). Data on the 25 m and 50 m grids are cokriged independently with data from the 10 m grid for comparison with the true recoverable reserves.

The effect of introducing sampling errors and bias, compared to the true grades for the10 m grid data is shown in a scattergram in Figure 2. The material lying below the 1 g/t cut-off in the 3rd and 4th quadrants represents 10.9 per cent dilution (waste sent to the mill) and 3.1 per cent lost ore (ore sent to the waste dump), a total of 14.0 per cent.

The recoverable reserves in the true tons, grade, and recoverable metal for different panel sizes listed in Table I are compared with those of the sampled values listed in Table III.

Recoverable reserves of the sampled 10 m data are estimated to contain nearly 1.4 times more tonnage, and 1.85 times more contained metal, than the true block values. The reason for this is the sampled 10 m panels with introduced error and bias have 35 per cent higher grade, and 36 per cent higher tonnage, than the true values. For the 25 m panels (Table III), the tonnage is lower, but the higher grade means a 16 per cent higher metal content relative to the true values. For 50 m panels the much lower sampled ton is compensated by a much higher sampled grade to give equivalent amounts of metal.

**Statistics and suitability of the secondary data**

Assessing the suitability of the secondary data relative to the primary data for use in cokriging is essential. Cokriging depends heavily on two assumptions about the primary and secondary data: firstly, that data within the domain is stationary, and secondly, that it is sufficiently correlated for the data to be used together. Valid application of cokriging means there should also be a relationship between pairs of the primary and nearby secondary data, as reflected in their correlation coefficient. Where secondary data displays no

<table>
<thead>
<tr>
<th>Table I</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>True recoverable reserves above 1 g/t for points, and for 10 m, 25 m, and 50 m panels averaged from the point reference data-set</strong></td>
</tr>
<tr>
<td>Scale</td>
</tr>
<tr>
<td>Point (1x1 m)</td>
</tr>
<tr>
<td>10×10 m</td>
</tr>
<tr>
<td>25×25 m</td>
</tr>
<tr>
<td>50×50 m</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Data-set</th>
<th>Secondary 10×10 m: poorer quality, abundant data</th>
<th>Primary 25×25 m: good quality, limited borehole data</th>
<th>Primary 50×50 m: good quality, limited borehole data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid size</td>
<td>10×10 m</td>
<td>25×25 m</td>
<td>50×50 m</td>
</tr>
<tr>
<td>Number of samples</td>
<td>3200</td>
<td>512</td>
<td>128</td>
</tr>
<tr>
<td>Error and bias</td>
<td>30% error and 1.5 times multiplicative bias introduced</td>
<td>No error or bias</td>
<td>No error or bias</td>
</tr>
<tr>
<td>Mean</td>
<td>1.02 (1.46 times bias)</td>
<td>0.706</td>
<td>0.695</td>
</tr>
<tr>
<td>Variance</td>
<td>1.48 (26.6% error)</td>
<td>1.377</td>
<td>1.019</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.321</td>
<td>1.175</td>
<td>1.009</td>
</tr>
<tr>
<td>Min.</td>
<td>0.01</td>
<td>0.011</td>
<td>0.102</td>
</tr>
<tr>
<td>Max.</td>
<td>21.91</td>
<td>11.867</td>
<td>9.643</td>
</tr>
</tbody>
</table>

Figure 2—Scatterplot of the true 10×10 m panel values versus the data with the 30% error and 1.5 times bias introduced; the 1 g/t cut-off lines define areas of waste, dilution, ore, and lost ore.
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Evidence of correlation with the primary data, it is unlikely to contain suitable information and would have to be excluded from the analysis.

Cokriging also depends on primary and secondary data being near to, but not collocated with, one another. Although there are 3200 points on the 10 m grid with error and bias, only 512 secondary paired data lying within a 10 m radius of the primary data were extracted as shown in Figure 1b. The descriptive statistics for these two sets of data are shown in Table IV.

The paired data were plotted in a scattergram (Figure 3) to determine the correlation coefficient, examining the effect of the error and bias on the relationship between the data. The higher mean and variance of the secondary data relative to the primary data indicate a 30.6 per cent error in the mean, and 1.27 times bias in the variance. The scatterplot of the primary and secondary data is shown in Figure 3 and indicates a correlation coefficient of 0.689, which is considered significant at the 95 per cent level of confidence (we would reject the null hypothesis that there is no correlation between the primary and secondary data at \( r(P_{95}) = 0.1449 \), since \( r = 0.689 \), and accept the alternative hypothesis: there is a correlation between the variables at 95 per cent significance).

In this case the difference in the mean of the primary and secondary data will give the bias associated with the secondary data, while the difference in variances provides an indication of the error associated with the secondary data which is due to the sampling error. The significant correlation (0.7) suggests that the secondary data is indeed suitable for incorporation in the estimation of recoverable reserves as part of a cokriging exercise. Permissible limits on the correlation coefficient are unclear, except to say that at or above 0.7 the method suggested here works very well, below 0.2 it does not work at all, and between 0.2 and 0.7 the results may be questionable.

Ordinary kriging with the primary data (25 m and 50 m)

Compiling an ordinary kriging model of the primary data-sets, i.e. the drill-hole data on the 25 m and the 50 m grids, is the best evaluation available to miners and mine planners if the benefits of cokriging are not applied. A model produced from ordinary kriging of the primary 25 m and 50 m data to be compared with the results from the cokriging is the only way in which the results of this exercise and purpose of this paper can be demonstrated. The basis of this comparison is to confirm that cokriging using the secondary data results not only in a meaningful improvement in the estimation of recoverable reserves, but also removes the bias and reduces the error in the final estimates. In a real-life analysis using cokriging, the number of samples, \( n \), would be reasonably large, and should be subject to all stationarity considerations.

The variograms shown in Figure 4a and 4b for use in the ordinary kriging estimation were modelled on the experimental variogram of the sampled 25 m and 50 m grid data respectively (see Figure 1a), the best option in a real-life situation where the only data available is what has been sampled. A nugget effect of 0.4 is used in both variograms following the \( C_0 \) value of 0.4 in the point support simulation model. The variogram for the true regularized 25 m data and 50 m data does not provide any information at distances shorter than 25 m and 50 m respectively, but this data must be used since it is the only information available for these data-sets.

---

**Table III**

Recoverable reserves in the true and sampled primary (25 m and 50 m) and secondary (10 m) datasets

<table>
<thead>
<tr>
<th>Scale</th>
<th>Tons (t)</th>
<th>Grade (Z, g/t)</th>
<th>Metal (Q, kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True</td>
<td>Sampled</td>
<td>True</td>
</tr>
<tr>
<td>Point (1×1 m)</td>
<td>189 000</td>
<td>2.22</td>
<td>418</td>
</tr>
<tr>
<td>10×10 m</td>
<td>206 000</td>
<td>1.88</td>
<td>388</td>
</tr>
<tr>
<td>25×25 m</td>
<td>219 000</td>
<td>1.69</td>
<td>371</td>
</tr>
<tr>
<td>50×50 m</td>
<td>216 000</td>
<td>1.49</td>
<td>324</td>
</tr>
</tbody>
</table>

---

**Table IV**

Descriptive statistics for the paired primary and secondary data-sets

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Primary data</th>
<th>Secondary data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid spacing</td>
<td>25×25 m</td>
<td>10×10 m</td>
</tr>
<tr>
<td>Error (%)</td>
<td>Nil</td>
<td>30</td>
</tr>
<tr>
<td>Bias (multiplicative)</td>
<td>Nil</td>
<td>1.5</td>
</tr>
<tr>
<td>Mean</td>
<td>0.707</td>
<td>1.019</td>
</tr>
<tr>
<td>Variance</td>
<td>1.377</td>
<td>2.19</td>
</tr>
<tr>
<td>Std deviation</td>
<td>1.175</td>
<td>1.48</td>
</tr>
</tbody>
</table>
The distribution of recoverable reserves shown in Figures 5a and 5b is the complete kriged model for the project area using 25 m and 50 m data kriged into 10 m blocks, respectively. The grade, tons, and metal content of the true, sampled, and kriged models are listed for comparison in Table V.

The recoverable reserves estimated by ordinary kriging in the 10 m panels shown in Figure 5b and 5c are only grades above 1 g/t cut-off. The recoverable reserves for the true 10 m panels shown in Figure 5a are quite erratic, but there is a reasonable correspondence in the location of highs and lows in the true and kriged recoverable reserves for the 25 m data. The differences between the 10 m true and 50 m kriged data are quite marked (Figure 5a and c).

As the size of the panels increases from 10 m to 50 m the effect of smoothing due to the kriging procedure is more evident; the tonnage and grade estimates change predictably given the cut-off grade and distribution.

**LMC for the case study**

The aim of this study is to compare the benefits of cokriging against ordinary kriging using models of coregionalization at different data spacing. The first LMC is for a combination of 3200 secondary data on a 10 m grid with the 512 primary data on the 25 m grid. The second LMC uses a combination of the 3200 secondary data on the 10 m grid with the 128 primary data on the 50 m grid, as shown in Table VI.

The variograms for the primary (25 m and 50 m) and secondary (10 m) data are required. The variogram for the primary data is not very informative because there are only 512 data for the 25 m grid, and only 128 for the 50 m grid, but especially because at short distances there are no pairs closer than 25 m or 50 m (Figures 8a and 9a). The variograms of the secondary data for the 25 m LMC and the 50 m LMC are identical (Figures 8c and 9c), and have better continuity because of the abundance of data at distances of 10 m. Although the direct variograms do not appear very informative, we know that any structure in the cross-variogram must also be present in the direct variograms. This means that the structure seen in the cross-variogram is also present in the direct variograms, but this was not revealed.
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Table V
Recoverable reserves in the true, sampled, and ordinary kriged (OK) (25 m and 50 m) data-sets

<table>
<thead>
<tr>
<th>Scale</th>
<th>Tons (t)</th>
<th>Grade (Z, g/t)</th>
<th>Metal (Q, kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True</td>
<td>Sampled</td>
<td>OK</td>
</tr>
<tr>
<td>10×10 m</td>
<td>206 000</td>
<td>281 000</td>
<td>1.88</td>
</tr>
<tr>
<td>25×25 m</td>
<td>219 000</td>
<td>184 000</td>
<td>1.69</td>
</tr>
<tr>
<td>50×50 m</td>
<td>218 000</td>
<td>145 000</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Figure 6—Compilation of LMC for 25×x25 m data. (a) Extrapolation of the covariance back to the y-axis (1.25) gives the axis of rotation for the variogram, and (b) the method used in Excel to invert the covariance

Table VI
Combinations of primary and secondary data for two separate models of coregionalization

<table>
<thead>
<tr>
<th>Models of coregionalization</th>
<th>First model</th>
<th>Second model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary data</td>
<td>512 on 25×25 m grid</td>
<td>128 on 50×50 m grid</td>
</tr>
<tr>
<td>Secondary data</td>
<td>3200 on 10×10 m grid</td>
<td>3200 on 10×10 m grid</td>
</tr>
</tbody>
</table>

before the cross-variogram was compiled; thus, the secondary data improves estimation of the variograms as well as local estimates.

Plotting the direct variograms for the primary and secondary data and the covariance for the combined primary-secondary data provides a route to the LMCs. The cross-variogram between the primary and secondary variables is plotted as red dots in Figure 6a and 7a. At short lag distances the cross-variogram is positive and decreases to zero with increasing lag distance. Once this structure is inverted (flipped over) using an Excel® spreadsheet to manipulate the data, it takes on the form of a variogram. The nugget effect on the cross-variogram does not appear because there is no collocated Z-Y values.

It is necessary to fit an LMC which provides the quantitative calibration of the primary to the secondary data and permits the filtering of the error during the cokriging. Fitting of the LMC is done manually, although it is possible to fit the LMC automatically using the appropriate software, but this is usually done only if the LMC is fitted to more than two variables. The first step is to fit the cross-variogram as simply as possible in all directions. The sill is equal to the constant used to invert the covariance, and the nugget effect is zero. By applying the structure used in the cross-variogram it is now necessary to fit the direct primary and direct secondary variograms. There may be nested structures or completely different models in each of the direct primary and direct secondary variograms that do not appear in the cross-variogram. However, the constraint for the method is that any structures that appear in the cross-variogram must also appear in the direct primary and secondary variograms. This is the reason for starting with the cross-variogram, followed by the direct variograms.

Examples of this procedure for compiling the LMCs for the 25 m and the 50 m data are illustrated in Figures 6 and 7. In these figures the experimental points of the covariance are extrapolated back to intersect the y-axis at 1.25 for the 25 m data, and 0.93 for the 50 m data, when the distance is zero. These then become the points about which the rest of the covariances are rotated to give lines that look like variograms (Figures 8 and 9).
Cokriging for optimal mineral resource estimates in mining operations

First LMC: The first model, using the 25 m primary data and the 10 m secondary data, requires the variograms for the primary and secondary data, and the covariance for the primary-secondary combination. The model variogram is used to identify the nugget effect and the sill for the direct primary and secondary data, as well as for the cross-variogram between primary and secondary data.

The equations for the variogram models shown in Figure 8 are as follows:

\[ Z : \gamma_z(h) = 0.4 \text{Nug}(h) + 1.08 \text{Sph}_{160}(h) \]
\[ Y : \gamma_y(h) = 0.75 \text{Nug}(h) + 1.45 \text{Sph}_{160}(h) \]
\[ ZY : \gamma_{ZY}(h) = 0.0 \text{Nug}(h) + 1.25 \text{Sph}_{160}(h) \]

\[ \text{Nug}(h) = \begin{cases} 0, & \text{at } h = 0 \\ 1, & \text{otherwise} \end{cases} \]

\[ \text{Sph}_a(h) = \frac{3h^2}{2a} - \frac{1}{2} \left( \frac{h}{a} \right)^3, \text{ for } h \leq a \]

For a valid LMC it is also essential that the matrix be positive definite, meaning that the difference between the products of the primary and secondary direct variograms and cross-variogram (Equation [6]) must be positive (Goulard, 1989). This implies that all the direct variograms must be positive. Also, if there is some spatial structure in the cross-variogram between two variables it must be evident in the corresponding direct variograms as well, which implies that there is correlation between the two variables. On the diagonal of the kriging matrix the nugget of the primary and the nugget of the secondary are used, but the nugget effect of the cross-variogram is never used. The cross-variogram is also positive definite because there could be nugget effects on both direct variograms, but there is no nugget effect on the cross-variogram; only the structured portion of the covariance has a nugget effect. The positive definiteness condition for the first LMC is satisfied as:

\[ W_0 = \begin{bmatrix} w_{Z0} & w_{Zy0} \\ w_{yZ0} & w_{y0} \end{bmatrix} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.75 \end{bmatrix} \]

has \(0.4 > 0, 0.75 > 0, 0.4 \times 0.75 > 0.0^2\)

\[ W_1 = \begin{bmatrix} w_{Z1} & w_{Zy1} \\ w_{yZ1} & w_{y1} \end{bmatrix} = \begin{bmatrix} 1.08 & 1.25 \\ 1.25 & 1.45 \end{bmatrix} \]

has \(1.08 > 0, 1.45 > 0, 1.08 \times 1.45 > 1.25^2\)

The important implication here is that the cross-variogram will usually lie between the two direct variograms, so that in actually fitting the variograms it is convenient to fit the cross-variogram first so that the cross-variogram structures are defined, and then fit the direct variograms, because evidence of spatial structure in the cross-variograms...
Cokriging for optimal mineral resource estimates in mining operations

constrains the direct variograms and it must also be evident in the direct variograms (Goulard and Vlotz, 1992). The fitting process is iterative but it is important to keep the range of the individual structures constant for all variograms.

**Second LMC:** The second model, using the 25×25 m primary data and the 10 m secondary data, also requires the variograms for the primary and secondary data, and the covariance for the primary-secondary combination. The model variogram is used in the same way as the first LMC to identify the nugget effect and the sill for the direct primary and secondary data, as well as for the cross-variogram between primary and secondary data.

The models for the direct variograms for primary and secondary data, and the cross-variogram for the combined primary and secondary data shown in Figure 9, are very acceptable fits of the equally acceptable experimental variograms. The equations for the variogram models shown in Figure 9 are as follows:

\[ Z : \gamma_{Z} (h) = 0.3 \text{Nug} (h) + 0.88 \text{Sph}_{160} (h) \]

\[ Y : \gamma_{Y} (h) = 0.86 \text{Nug} (h) + 1.45 \text{Sph}_{160} (h) \]

\[ ZY : \gamma_{ZY} (h) = 0.0 \text{Nug} (h) + 0.93 \text{Sph}_{160} (h) \]

The positive definiteness condition for the second model is also satisfied as:

\[
W_{0} = \begin{bmatrix}
Z_{0} & Z_{Y} \\
Y_{0} & Y_{Y}
\end{bmatrix} = \begin{bmatrix}
0.3 & 0 \\
0 & 0.86
\end{bmatrix}
\]

has \(0.3 > 0, 0.86 > 0, 0.3 \times 0.86 > 0.02\)

\[
W_{1} = \begin{bmatrix}
Z_{X1} & Z_{Y1} \\
Y_{X1} & Y_{Y1}
\end{bmatrix} = \begin{bmatrix}
0.88 & 0.93 \\
0.93 & 1.45
\end{bmatrix}
\]

has \(0.88 > 0, 1.45 > 0, 0.88 \times 1.45 > 0.93^{2}\)

Having established that the cross-variograms are positive definite, it is now possible to perform the cokriging.

**Comparing cokriging and ordinary kriging**

The final step in the procedure is to create a block model that correctly accounts for all of the available data and at the same time demonstrates improvements in local and global estimates, by filtering the error and without transferring a bias. Firstly a single data file containing the primary and secondary data for use in the cokriging software routine is compiled. The compilation of the final data file is generally software-dependent, and many packages simply omit the cokriging capability because of the difficulties associated with the user interface to calculate the LMC; a further reason that cokriging is not used as often as it might be. Apart from additional implementation details, the cokriging and kriging algorithms are the same, and all the parameters for the cokriging routine are virtually identical to the kriging parameters. Usually the abundance of secondary over primary data in the cokriging routine means that a decision must be made about how much of the secondary data to use without overwhelming the process and disadvantaging the primary data. The main reason for limiting the amount of secondary data used is to avoid smoothing. If the model is being developed for a long-term resource estimate, over-smoothing may mean that the recoverable reserves will be estimated too low. A second reason to limit secondary data is that secondary data may be closer to the point being estimated than primary data, but the latter is more relevant than the secondary data. For this reason the cokriging software allows one to specify how many of each data type, usually a maximum of 12 each, will be used in the cokriging process.

The following analysis of the recoverable reserves above 1 g/t gold, derived from the kriging and cokriging of the 25 m and 50 m data-sets into 10 m panels, is compared against the known true data within the project area. The tonnage, grade, and recoverable metal are compared against the true data for the 10 m panels, in order to demonstrate that cokriging improves local and global estimation, that the bias has not been transferred, and that the error has been filtered out of the final result. Generally, ordinary kriging with the primary data will be unbiased, but the different varieties of cokriging with the primary and secondary data are also going to be unbiased. The caveat is that the mean of the primary, and particularly the secondary, data must be known, which is easy enough through the data.

The bias is dealt with through the means for the primary and secondary data, whereas the error is managed by essentially depressing the covariance between the primary and secondary variables \(C_{ZY}(h)\).

\[
C_{ZY}(h) \leq \begin{bmatrix}
C_{Z}(h) \\
C_{Y}(h)
\end{bmatrix}
\]

The secondary variable has error content, and for this reason we would not want the variogram of the secondary data to have too much weight (Figure 10). The control of the weight attributed to the secondary variable is by reducing the sill. The amount of error decreases as the sill moves upwards and increases as the sill moves downwards. At the two
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extremes, i.e. when $C_{zy}(h)$ equals the variance of the primary variable, there is no error at all, and when $C_{zy}(h)$ is flat at zero lag there is only error and the data is not weighted at all. In effect this is a continuum that must be calibrated. The difference of the mean grades, (indicating bias), and the difference between variances, (indicating error), for the cokriging results using data on the 25 m and 50 m grid are closer to the true values than the ordinary kriged results. Again, these outcomes indicate that the cokriging procedure is highly efficient at removing the bias and filtering the error that were shown to be embedded in the secondary data.

Kriging and cokriging data on 25 m grid
Sample data on the 25 m grid has been kriged (OK) and cokriged into 10 m blocks (Figures 11b and 11c), and visually compared with the known true values of 10 m blocks (Figures 11a).

The tons, grade, and recoverable metal for the 25 m kriged and cokriged panels are summarized in Table V.

Kriging and cokriging data on 50 m grid
Sampled data on a 50 m grid has been kriged (OK) and cokriged into 10 m blocks (Figures 12b and 12c, respectively), and compared with the known true distribution of 10 m blocks (Figures 12a) in the project area.

Again, it is evident that the co-kriging output is visually more similar to the known true distribution than the ordinary kriging outcome, shown in Figure 12.

Summary
A summary of the cokriging and ordinary kriging estimates using the data-sets on 25 m and 50 m grids to give tons, grade, and recoverable metal in the 25 m and 50 m panels is presented in Table VII. The reference data-set for the points and the 10 m, 25 m, and 50 m panels is the true recoverable reserves above 1 g/t in the deposit and represents the base case against which the sampled, ordinary kriged, and cokriged data is compared. The sampled data-set for the 10 m grid size is the poorer quality secondary data, while that for the 25 m, and 50 m grid sizes represents the higher quality primary data for use in cokriging.

Differences between the reference and the sampled data-sets at 10 m, 25 m, and at 50 m are a reflection of random selection at that grid spacing. It is noteworthy at each grid size that the sampled data-set marginally overstates the grade and contained metal.
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The results of the kriging and cokriging into 10 m panels using sample data on the 25 m grid indicate that the tons predicted from ordinary kriging (195 000 t) are 11.0 per cent less than the known true tonnage (219 000 t) while cokriging predicts (221 400 t), 1.1 per cent more than the known tonnage. The metal predicted in the ordinary kriged panels (315 kg) is 15.1 per cent lower than the true metal content (371 kg), while that for the cokriged panels (425 kg) is 14.6 per cent higher. The difference between the true known grade (1.69 g/t) and the ordinary kriged grade (1.61 g/t) is 4.7 per cent, but for the cokriged grade (1.92 g/t) it is 13.6 per cent higher. Cokriging provides tonnages and grade that are increased relative to the true values, representing an increase over the true metal content of 371 kg, but this is 0.5 per cent closer to the real content than the result obtained from ordinary kriging.

In tons and recoverable metal the cokriging estimates are closer, if only marginally in the case of recoverable metal, to the known true values than the ordinary kriged estimates. The much better estimate of tonnage using cokriging relative to ordinary kriging is offset by the higher value for cokriged grade, but there is an overall marginal improvement in estimation of contained metal. These benefits due to cokriging have been further resolved by comparing the tons, grade, and recoverable metal for designations referred to as lost ore, dilution, ore, and waste estimated by ordinary and cokriging, and are given in Table VIII for the 25 m grid samples and 50 m grid samples. These figures, while useful in their own right, are better displayed as percentages in the relevant areas of Figure 13. The benefits of cokriging over ordinary kriging, provided the underlying assumptions are valid, are illustrated in the way that mine production will be...

Figure 12. Recoverable reserves above 1 g/t cut-off in 10 m panels for (a) known true ore, (b) the ordinary kriged 50 m data, and (c) the cokriged 50 m data

<table>
<thead>
<tr>
<th>Table VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recoverable reserves above 1 g/t in the true, sampled, ordinary kriged, and cokriged (25 and 50 m) datasets</td>
</tr>
<tr>
<td></td>
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<tr>
<td>-----------</td>
</tr>
<tr>
<td>1 m points</td>
</tr>
<tr>
<td>10 m grid</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>25 m grid</td>
</tr>
<tr>
<td></td>
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<tr>
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<tr>
<td>5 0m grid</td>
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</table>
allocated as lost ore, dilution, ore, and waste rock in a cross-plot of the kriged and cokriged estimates against the known true grade of materials in the designated areas shown in Figure 13.

By inspection of Figure 13 it is evident that for each classification category (lost ore, dilution, ore, and waste), standardized cokriging (SOCK) improves the estimation outcomes over ordinary kriging (OK). There are considerably fewer tons of lost ore for both the 25 m (29 400 t as against 67 500 t) and 50 m (47 100 t as against 130 500 t) sampled data, as well as fewer lost kilograms of metal (23.3 kg versus 47.8 kg) in the 25 m grid and in the 50 m grid (61.2 kg versus 230.8 kg). The same is true for dilution, in that there is 18 600 t and 36 900 t less dilution, and 32.9 kg and 62.7 kg improved metal recovery in the 25 m and 50 m sampled grids respectively, due to cokriging. In the case of ore, cokriging yields more tons, better grade, and more metal compared to ordinary kriging at the 25 m and 50 m sampling size. For the 25 m grid size there are more tons, lower grade, and less metal sent to waste, and the same is true for the 50 m grid size except that 8 kg of metal are lost to waste. The improvements due to cokriging over ordinary in the four categories lost ore, dilution, ore, and waste are shown as percentages in Figure 13.

The benefits of cokriging are particularly noticeable in the area representing lost ore, where cokriging gives fewer lost tons (3.1 per cent and 4.9 per cent) and less recoverable metal (3.4 per cent and 8.7 per cent) compared to ordinary kriging (7.0 per cent and 13.6 per cent) for tons (6.7 per cent and 29.0 per cent) and for metal in the 25 m and 50 m sampling grids (Figure 13). Lost ore is estimated to be below the cut-off grade, but it is truly economic in that it could make a positive contribution to the mining operation. However the company does incur a lost opportunity cost. This ore will never be accounted for in the balance sheet, nor will it add any value to the mine, except perhaps towards the end of the mine life when plant superintendents attempt to feed the plant from the low-grade stockpile.

Cokriging also yields less dilution in terms of tons (4.7 per cent versus 6.6 per cent ) and recoverable metal (8.5 per cent versus 12.6 per cent ) compared to ordinary kriging for the 50 m grid. The same is true for the 50 m grid, where tons (3.9 per cent versus 7.8 per cent ) and recovered metal (6.5 per cent versus 13.7 per cent ) are improved by cokriging (Figure 12). Dilution is mine production that is estimated to be above the cut-off, but is truly uneconomic in that its contribution is far outweighed by the costs incurred during milling and processing. Such material does show up in the balance sheet in that it raises milling and processing costs, and constitutes an opportunity cost in that it occupies milling capacity that could be better utilized by truly high-grade ore.

### Table VIII

<table>
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<tr>
<th>Predicted</th>
<th>Lost ore</th>
<th>Tons</th>
<th>Grade</th>
<th>Metal</th>
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<td>25 m grid</td>
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<td>67 500</td>
<td>0.71</td>
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</tr>
<tr>
<td></td>
<td>SOCK</td>
<td>29 400</td>
<td>0.79</td>
<td>23.3</td>
</tr>
<tr>
<td>50 m grid</td>
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<td>130 500</td>
<td>1.77</td>
<td>230.8</td>
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<tr>
<td></td>
<td>SOCK</td>
<td>47 100</td>
<td>1.29</td>
<td>61.2</td>
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</table>

<table>
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<th>Grade</th>
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<td>SOCK</td>
<td>44 700</td>
<td>1.28</td>
<td>57.3</td>
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<td>1.23</td>
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<table>
<thead>
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<th>Tons</th>
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<tbody>
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<td></td>
<td>SOCK</td>
<td>176 700</td>
<td>2.08</td>
<td>368.1</td>
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<td>143.9</td>
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<tr>
<td></td>
<td>SOCK</td>
<td>159 000</td>
<td>1.75</td>
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</table>

<table>
<thead>
<tr>
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<th>Waste</th>
<th>Tons</th>
<th>Grade</th>
<th>Metal</th>
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<tbody>
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<td></td>
<td>SOCK</td>
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</tr>
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<td>50 m grid</td>
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<td>0.46</td>
<td>311.3</td>
</tr>
<tr>
<td></td>
<td>SOCK</td>
<td>716 400</td>
<td>0.45</td>
<td>319.8</td>
</tr>
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Cokriging for optimal mineral resource estimates in mining operations

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Table VIII

<table>
<thead>
<tr>
<th>Tons, grade, and contained metal for kriged and cokriged 25 m and 50 m data in terms of lost ore, dilution, and correctly classified ore and waste</th>
</tr>
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<tbody>
<tr>
<td>Predicted</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>25 m grid</td>
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<tr>
<td></td>
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</table>

Figure 13—Comparison of the percentage improvements in estimation of cokriging over ordinary kriging for the categories lost ore, dilution, ore, and waste, (a) for the 25 m grid sample data, and (b) for the 50m grid sample data
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The work flow in practice

Practical application of cokriging in mining operations would broadly follow a four-stage work flow involving the making of assumptions, exploratory data analysis, variography, and estimation by cokriging. In addition, the underlying assumptions that must be established before cokriging is applied are summarized here.

Assumptions

Some of the assumptions would preclude the use of this technique, but where there are opportunities to apply it the benefits over ordinary kriging are substantial. The chief weakness of the cokriging method is non-stationarity, for example if the bias changes across the domain. The \( Z \) variable may be accompanied by another variable, \( Y \), that is biased high and not perfectly correlated, but the method works well here because the data is strongly correlated, and in particular the bias is stationary. The bias may be as much as 30 per cent but provided it has the same average (uniform) across the whole domain the method is still applicable. A difficult case occurs if the data is biased in a non-stationary way. It is possible that data in the domain may on average have a bias that is close to zero, but if the bias is non-stationary application of this method will be flawed. The main assumption therefore is that of stationarity across the domain under consideration.

Exploratory data analysis

Evaluation of the primary data: this would involve a detailed evaluation of the primary data in terms of understanding the distribution of the variables using diagrammatic representations of the data in histograms, cumulative and probability distribution functions (cdf and pdf), as well as compiling the basic descriptive statistics, mean, variance, and coefficient of variation. The data location and spacing will indicate the necessity of declustering of the data to ensure that representative statistics are obtained. Plotting the declustered mean against the cell size will give an indication of the optimal cell size to be used in the analysis. Alternatively, the cell size can be estimated from the drill-hole spacing in sparsely sampled areas.

Evaluation of the secondary data: secondary data is subject to the same analysis as primary data except that there should be an evaluation of the bias and error content of the secondary data.

Confirmation of the stationarity of the populations: The strongest and most important assumption underlying the use of cokriging is that of stationarity within the domain. Where there is a trend it would need to be modelled. Another assumption concerns the nature of the data in that there should be two sets of non-collocated data. This method will work where the data is collocated. The primary data-set generally consists of few, high quality data while the secondary data-set comprises abundant, but poorer quality data that may contain sampling errors and sampling bias.

The method also assumes that there is sufficient primary data to give a meaningful variogram, and usually where there is abundant secondary data there is no restriction on obtaining a stable variogram.

Variography

Establishing the experimental variogram for the primary data: this is usually not as easy to achieve as might be expected. Generally the primary data will have been drilled on a widely spaced grid, which means that the first point on the variogram occurs at a long lag. In addition there is very little to inform the short-range variability of the data and in particular the nugget effect is hard to estimate; in reality it is usually less than might be expected from an inspection of the experimental variogram.

Establishing the experimental variogram for the secondary data: this is generally easier than for the primary data, simply because there is more data and consequently the experimental variogram for the secondary data is better-behaved than that for the primary data.

Establishing the experimental variogram for the cross-variogram: this is done by firstly calculating the covariance for the primary and secondary data. This is a decreasing function with increasing lag, but its importance lies in the smooth extrapolation through points on the covariance curve to their point of intersection on the y-axis. This is used to invert the covariance in order to produce the cross-variogram.

Fitting the linear model of coregionalization: the main requirement of the LMC is that it be positive definite. This can be achieved by following the advice presented above.

Kriging estimation

Ordinary kriging: the only reason that ordinary kriging is performed is to provide a basis for comparison with the cokriging results. It might even be necessary to cross-validate the output to confirm that the ordinary kriging produces acceptable results.

Standardized cokriging: the means for the primary and secondary data must be used in the cokriging procedure. In addition it is important that the primary and secondary data be combined in a specific way (depending on the software) during the cokriging process. An important step in the analysis of the output is a comparison between the efficiencies of ordinary and cokriging. This is probably best done by comparing the improvements in a scatterplot of the four categories of production materials: lost ore, dilution, ore, and waste.

Conclusions

This paper demonstrates a little-used geostatistical technique that combines different sets of data, from different sources, with different qualities, in a cokriging procedure. The data needs only to be weakly correlated and not necessarily be collocated either. Cokriging provides better estimates of recoverable resources at local and global scales, compared to ordinary kriging of the best available data, and is marked by the highly significant benefit that errors and bias due to poor sampling in the secondary set of data are not transferred to the resulting estimates.

Acknowledgements

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Cokriging for optimal mineral resource estimates in mining operations

Africa in August 2011. The comments, discussion, and inputs arising from the other 15 attendees, some of which are incorporated here, are acknowledged. The authors would like to acknowledge Dr Chris Prins for editing various drafts of this paper.

References


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Localized uniform conditioning (LUC): method and application case studies

by M.Z. Abzalov*

Synopsis

A new method, localized uniform conditioning (LUC), was proposed in 2006 for modelling grades of small blocks of ore when data spacing is too broad for their accurate modelling by the linear regression based techniques, such as kriging (Abzalov, 2006). It represents a modified uniform conditioning (UC) technique that calculates the grade distribution functions for the large panels. LUC uses partitioning of the panels onto the small blocks and then ranks them in increasing order of grade. Based on the block ranks, a single grade value can be deduced for each block from the UC model of the grade-tonnage relationships of the corresponding panel.

After being first presented in 2006, the LUC method has been implemented in ISATIS© (commercial software) and became one of the common approaches for grade estimation when data spacing is broad in comparison with the estimated block size. Several years of study on the LUC method and its application to different geological environments, have allowed identification of the strengths and weaknesses of the method, which are as follows:

➤ The method produces accurate grade-tonnage functions, which are in a good accordance with a volume-variance relationship principles
➤ An initial ranking of the selective mining unit (SMU) blocks can be made by direct kriging from the sparse data grid. Therefore, the LUC method can be particularly useful at the early stages of exploration and mining project evaluations when sparsely distributed data is often the only available information
➤ Accuracy of the local estimation depends on the SMU ranking techniques. When ranking performed by direct kriging of the SMU blocks their spatial distribution is approximate. When the variogram of the studied variable is characterized by a large nugget effect, the block ranks produced by kriging can significantly differ from their 'true' distribution
➤ Block ranking can be improved using auxiliary data, either geophysical or geochemical. This allows use of the LUC method for integrating different data sets. In particular, LUC can be used for grade control in open pits by integrating resource definition data (e.g. drill-hole assays) and blast-hole assays. The latter are used for the block ranking.

Keywords
geostatistics, localized uniform conditioning, LUC, resource modelling.

Introduction

It is well known in the geostatistical community that techniques based on linear regression are unsuitable for modelling grades of small blocks when the data spacing is too broad in comparison with the estimated block sizes (Armstrong and Champigny, 1989; Ravenscroft and Armstrong, 1990; Pan, 1998). To overcome this problem, a new method, localized uniform conditioning (LUC), was proposed (Abzalov, 2006). The LUC method represents a modified uniform conditioning (UC) technique. It calculates the grade distribution functions for the large panels by a conventional UC method and then uses partitioning of the panels onto the small blocks and ranks them in increasing order of grade. Based on the block ranks and using the calculated grade-tonnage relationships of the panels as a guide, a single grade value is deduced for each block. In other words, the proposed method localizes the UC model results; it is therefore called localized uniform conditioning (LUC).

After first presentation in 2006 the LUC method has been implemented in ISATIS© (Bleines et al., 2001) and became one of the common approaches for grade estimation when data spacing is too broad in comparison with the estimated blocks size. Several years of continuing studies of the LUC method, applying it in different geological environments, have allowed an assessment of the strengths and weaknesses of the method, which are presented in this paper.

Method

Uniform conditioning (UC)

Uniform conditioning (UC) is a nonlinear geostatistical technique for calculating tonnage ($T_v$) and mean grade ($M_v$) of recoverable resources distributed in a large panel ($V$) as the small blocks of size ($v$) representing a partitioning of this panel (Figure1). In geostatistical terms the UC technique (Rivoirard, 1994; Chiles and Delfiner, 1999; Wackernagel, 2002) consists of calculating a conditional expectation of a nonlinear function

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The localized uniform conditioning (LUC) method was developed with an intention to overcome the limitations of conventional UC (Abzalov, 2006). It calculates the grade distribution functions for the large panels by a conventional UC method and then localizes the UC model results.

**Partitioning the panel on the small blocks**

The first step is to split (partition) the panel on sub-cells whose size are equal to that chosen for estimation of small blocks (Figure 2a). Usually these are the blocks whose size matches the proposed selectivity of the mining method and therefore they are referred as selectively mineable units (SMU).

**Ranking SMU blocks in increasing grade order**

The SMU blocks distributed in each panel should be ranked in order of increasing grade. This is the underlying concept of the LUC method. It is obvious that an accurate ranking would require high-density information, such as high-resolution geophysics. However, reasonably accurate rankings of the SMU blocks in the panels can be deducted from the spatial distribution patterns of the grade values, such as zoning or grade trends. The latter approach is particularly relevant for continuous mineralization, characterized by a low nugget effect, such as occurs in stratiform base metal sulphide or iron oxide deposits. Spatial grade distribution patterns are easily recognized by geoscientists in many stratiform deposits at the early stages of exploration, when drill spacing is still too broad for direct accurate modelling of grades of small block, but sufficient for identification of the major distribution trends.

The global distribution features of the grade variables exhibiting a strong continuity can be reconstructed by interpolating available data nodes using any conventional linear interpolator, such as ordinary kriging (OK). In other words, it is suggested that direct kriging of the small blocks can be used to rank them approximately in increasing order.
of grade in the panels, even when the drill spacing is too wide for non-biased SMU grade estimation. The proposed approach to ranking of the SMU blocks in the panels using a linear estimator is schematically shown in Figure 2b.

The validity of the obtained grade ranks depends on the complexity of the grade distribution patterns. It is obvious that further studies are required to quantify the limitations of application of linear estimators (e.g. OK) for ranking the SMU size blocks. At this stage it is assumed that the above assumption is applicable to grade variables whose spatial distribution satisfies a border effect condition and which are also characterized by a low nugget effect and exhibit a good continuity at their variogram origins.

The OK-based ranking of SMU blocks can be further enhanced using suitable high-resolution geophysical techniques. The precision of geophysical methods is usually insufficient for a quantitative interpretation of the geophysical responses; however, it can be adequate for a relative ranking of the SMU blocks in the panels. These reconstructed distribution patterns are finally used for the definition of the grade relationships between SMU blocks.

**Defining the grade classes and estimating their mean grades**

The next step is discretization of the UC model on the grade classes and estimation of the mean grade of every grade class. The grade classes are defined for each panel using the relationships between the cut-off grade and the tonnage of recoverable mineralization at the different cutoff values (Equation [6]).

\[ GC_i \subset \{ T_i(Z_i), T_{i+1}(Z_{i+1}) \} \text{ and } GC_i \subset \{ Z_i, Z_{i+1} \} \]

where \( T_i \) is recoverable tonnage at cut-off \( Z_i \) and \( T_{i+1} \) is recoverable tonnage at cut-off \( Z_{i+1} \).

The mean grade of each grade class \( \langle MGC_i \rangle \) is deduced from the UC model estimating grade of recoverable mineralization at the different cutoff values (Figure 3b).

**Assigning grade to SMU blocks according to their rank**

The mean grades of the grade classes can be assigned to the SMU blocks by matching their ranks with the grade classes. To do so it is necessary to convert the SMU ranks to the grade classes (Figure 3c). This is deduced from the relationships between the SMU rank and the proportions of the panel (Equation [7]).

\[ SMU_{RANK} \subset (T_{RANK}, T_{RANK+1}) \]

where \( T_{RANK} \) is the proportion of the panel distributed in SMU blocks whose rank is equal or lower than \( RANK \), and \( T_{RANK+1} \) is proportion of the panel distributed in SMU blocks having higher rank \( RANK+1 \).

The grade class are determined for each \( SMU_{RANK} \) by comparing its \( (T_{RANK}, T_{RANK+1}) \) intervals with the intervals of the grade classes \( \{ T_i(Z_i), T_{i+1}(Z_{i+1}) \} \) (Figure 3c). \( SMU_{RANK} \) will be assigned grade class \( \langle TGC_i \rangle \) if the \( T_{RANK} - T_{RANK+1} \) intervals falls within the limits \( (T_i - T_{i+1}) \) of the grade class \( \langle GC \rangle \) (Figure 3c).

Finally, a mean grade \( \langle MGC_i \rangle \) of each class is transferred to the \( SMU_{RANK} \) blocks by matching their grade class indexes \( \langle MGC_i \rangle \) and \( \langle TGC_i \rangle \) (Figure 3d).
Localized uniform conditioning (LUC); method and application case studies

Implementation of the LUC methodology

The procedure of localizing the UC model results and assigning a single value to the SMU blocks (Figure 5) assumes an exact match between grade class intervals \( [T_i, T_{i+1}] \) and intervals of SMU blocks \( [T_{\text{RANK}}, T_{\text{RANK}+1}] \), which is readily achieved in practice.

Researchers designing the computerized scripts for implementation of the LUC approach need to consider the cases when the range of SMU \( [T_{\text{RANK}}, T_{\text{RANK}+1}] \) does not precisely matches that of the grade classes \( [T_i, T_{i+1}] \). The problem can be partially overcome by using a large number of grade classes. Personal experience shows that a good match between grade-tonnage relationships estimated by the conventional UC method and by the LUC approach is achieved when 50 grade classes are used. Further improvement can be achieved if the mean SMU grade is estimated by weighting grades of the classes to their proportions of the SMU. This approach was used by the author in the case studies described in the following sections.

Case studies

Iron ore deposit

The LUC method was tested on pisolitic iron ore mineralization in the eastern Pilbara, Western Australia (Hall and Kneeshaw, 1990; Abzalov et al., 2010). The resources of the deposit were defined by drilling using the grids as follows:

- Measured: 100 x 50 m
- Indicated: 200 x 100 m
- Inferred: 300 x 200 m.

However, it has been recognized that use of large blocks, such as 100 x 50 x 10 m, for definition of Measured Resources and Proved Reserves can lead to a substantial underestimation of the actual variability of the orebody, which is mined with a selectivity of approximately 25 x 25 x 10 m (Abzalov et al., 2010). As a consequence, using the large blocks for the reserve model can cause incorrect estimation of the recoverable mineralization. For example, if <2.6% Al₂O₃ is a metallurgically acceptable impurity threshold, then modelling grade distribution as 100 x 50 x 10 m blocks would overestimate recoverable tonnage by 5.7% in comparison with the model estimated using 25 x 25 x 10 m blocks, which matches the mining selectivity (Figure 4).

Direct estimation of the small blocks by kriging is not feasible because of the large distances between the drill-holes. Therefore, in order to obtain more accurate estimation of the recoverable resources it was decided to test the LUC method.

The exercise was based on a detailed study area that was drilled at 50 x 50 m centres and contained 8121 samples. The drill data was sampled in order to create a more sparsely distributed subset, with the drill-holes distributed at 100 x 50 m centres, which matches the grid used for definition of Measured Resources. The subset, containing 4801 samples, was used to generate block models through application of the LUC technique to estimate the Al₂O₃ grade distributed as blocks of 25 x 25 x 10 m in size (Figure 5a). For comparison, Al₂O₃ grades of the same blocks were estimated by OK applied to the same subset of the data, distributed as 100 x 50 m centres (Figure 5b).

The LUC model exhibits significantly higher resolution than OK model constructed using the same data (Figure 5). The resolution of the LUC method matches the mining selectivity and therefore is suited for detailed production planning at this project.

The model was validated by averaging sample and the block grades into the large panels and plotting both grades against the centres of the panels (Figure 6). In the current study the grades of the LUC model blocks have been averaged into 100 m wide panels drawn across the entire deposit. The average block grades are compared with the average grades of the all drill-hole samples contained in the same 100 m wide panel. Distribution of the average grades on the spidergram shows that LUC model reconciles well when compared with the drill-hole samples when data are averaged by the large panels (Figure 6).

![Figure 4](image-url)  
**Figure 4**—Maps showing spatial location of the mineralization at the different cut-offs. (a) Modelled as 100 x 50 x 10 m blocks; (b) modelled as 25 x 25 x 10 m blocks

![Figure 5](image-url)  
**Figure 5**—Examples of the block models constructed using subsets of the drill-holes distributed at 100 x 50 m centres. (a) OK model; (b) LUC model

![Figure 6](image-url)  
**Figure 6**—Spidergram showing distribution of the Al₂O₃ grades along the strike of mineralization. Grade values are averaged by 100 m wide slices drawn across the entire orebody
Comparison of the local estimates shows that correlation of the block grades estimated by LUC method with sample grades is 0.66, which is lower than the 0.85 correlation obtained for OK blocks (Figure 7). However, the LUC model is constructed using only 4801 samples out of the 8121 samples that were used for the OK model (Figure 7), therefore a lower precision in the local grade estimation is not unexpected. The error level is possibly acceptable considering that SMU grades are estimated from the drill-holes centred at 100 x 50 m (Figure 7).

Bauxite deposit

The study was undertaken as part of the long-term mine planning at the Weipa bauxite operation in Queensland, Australia. Evaluation of the project expansion required creation of a 3D model of the project area (Abzalov and Bower, 2009). The bauxite seam had to be represented as 0.5 m high slices, which represents the mining selectivity at the operation. However, direct kriging of the 3D blocks was impossible because most of the drill-holes (1419 holes) had been drilled in the 1970s, and at that time the holes were sampled in a 2D format where a single sample is taken for entire thickness of the seam. 3D data was available only from 117 holes which had been sampled as continuous strings of 0.2 m samples (Figure 8).

The 2D model was converted into a 3D model using the LUC method. The bauxite model was discretized to panels of 500 x 500 x thickness (m) and then a grade-tonnage relationship was estimated for each panel using the UC method. The 500 x 500 x 0.5 m blocks were ranked using the 3D samples of the 117 holes (Figure 8) and their grades deduced from the UC model using the LUC technique (Figure 9).

Application of the LUC methodology allowed us to reconstruct a vertical profile of the bauxite seam maintaining the total sum of the contained metal (Figure 9) and, at the same time, adhering to the principals of the volume-variance relationships.

Discussion and conclusions

Since it was first presented in 2006 the LUC method has been implemented in ISATIS© (commercial software) and has become one of the common approaches for grade estimation.
Localized uniform conditioning (LUC); method and application case studies

when data spacing is too broad in comparison with the estimated block size. It is one of the geostatistical methods used for recoverable resource estimation. In comparison with other methods, such as disjunctive kriging, service variables, and residual or multiple indicator kriging, LUC is relatively simple and benefits from its simplicity.

The key feature of the LUC approach is the ability to partition the panel into the small blocks (SMU) and estimate their grades maintaining the volume-variance relationship. The procedure (Abzalov, 2006) is not directly attached to uniform conditioning and can be applied like a post-processing algorithm to any recoverable resource estimate. The same approach was recently applied for localization of recoverable resources estimated by indicator kriging (i.e. localized indicator kriging). In order to choose between Gaussian-based algorithms, such as LUC, and indicator-based algorithms (e.g. LIK), the border effect needs to be checked and tested by estimating the ratios of indicators (Abzalov and Humphreys, 2002). If the ratio of indicators cross-variogram changes regularly with distance, the Gaussian-based models are applicable.

The accuracy of the local estimation depends on the SMU ranking techniques. When ranking is performed by direct kriging of the SMU blocks their spatial distribution is approximate. Accuracy of localization of the SMU grades decreases when the variogram of the studied variable is characterized by a large nugget effect.

Block ranking can be improved using auxiliary data, either geophysical or geochemical. This allows the use of the LUC method for integrating different data-sets, which will enhance the practicality of the LUC technique. However, this requires further investigation in order to obtain a better understanding of the strengths and limitations of the technique when it is applied in the multivariate environment.

Acknowledgements

This paper is dedicated to Professor D. Krige, who was a reviewer of the paper (Abzalov, 2006) in which the method was first published. His comments and suggestions have helped to improve paper and made it more legible. The author expresses his sincere gratitude to J. Bower and T. Riggs for permission to publish the case study results. I am also grateful to B. Sommerville and T. Riggs for reviewing the paper and for many useful comments.

References


Notations

\[ Z(v) \] variable of interest (metal) which is estimated as the blocks of size \((v)\)

\[ Y(v) \] Gaussian variable with mean 0 and variance 1

\( \phi_\varepsilon \) Gaussian anamorphosis

\( H_k \) Hermite polynomial

\( k \) coefficient of a Hermite polynomial expansion

\( r \) point-to-block correlation coefficient

\( T_r(z_C) \) tonnage of mineralization recovered at the cut off \((z_C)\) at the support \((v)\)

\( Q_r(z_C) \) quantity of contained metal \((z)\) recovered at the cut off \((z_C)\) and at the support \((v)\)

\( M_r(z_C) \) mean grade of mineralization recovered at the cut off \((z_C)\) and at the support \((v)\)

Appendix

Discrete Gaussian point-block model

The distribution of the SMU \((v)\) grades can be expressed using a Hermite polynomial expansion

\[ Z(v) = \phi_\varepsilon Y(v) = \sum_{k=0}^{\infty} \frac{Q_k}{k!} r^k H_k(Y(v)) \]

where \(k\) are coefficients established in the normal score transformation (Gaussian anamorphosis), \(Y(v)\) is the Gaussian variable with mean 0 and variance 1, and \(r\) is the point-to-block correction coefficient.
The underlying assumption of the above equality is that pairs of Gaussian transformed values $Y(x)$ (point anamorphosis) and $Y(v)$ (block anamorphosis) are bi-Gaussian linearly correlated values with a correlation coefficient $r$. This coefficient is unknown and needs to be calculated. The procedure for calculating the point-to-block correction coefficient $r$ is as follows.

The first step is to calculate a point anamorphosis (i.e., normal score transformations) $Z(x) = \phi(Y(x))$.

The next step is to calculate an empirical point variogram $g(h)$ using the available data $Z(x)$ (i.e., samples) and fit the variogram model.

From the point variogram $g(h)$ of $Z(x)$, the point-to-block correction coefficient $r$ of the block ($v$) anamorphosis $Z_v(x) = \phi_v(Y(v))$ can be calculated using the following geostatistical relationship between the variance of $Z(v)$ and block anamorphosis function:

$$\text{Var}(Z(v)) = \text{Var}(\phi_v(Y(v))) = \sum_{k=1}^{\infty} \frac{\sigma^2_k}{k!} \gamma^{2k}$$

At the same time the variance of $Z(v)$ is equal to a block covariance $C(v,v)$, which can be easily calculated from the variogram model:

$$\text{Var}(v) = C(v,v) = \gamma(\infty) - \gamma(v,v)$$

Therefore, using the above relationships the final equation for calculating the point-to-block correction coefficient $r$ can be expressed as follows:

$$(\gamma(\infty) - \gamma(v,v)) = \sum_{k=1}^{\infty} \frac{\sigma^2_k}{k!} \gamma^{2k}$$

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**Localized uniform conditioning (LUC): method and application case studies**

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Following the challenging years brought about by the International Financial Crisis, the local and international diamond industry is steadily redressing itself. Diamond mining continues to be active in the Northern Cape and adjacent areas and large diamonds of exceptional quality are regularly recovered from alluvial deposits associated with the Vaal, Orange, and Riet River drainages. Reprocessing of old tailings dumps in and around Kimberley has opened a new phase of exploitation in this historic area, and ongoing technological advances are helping to sustain an industry which has been active since the 1860’s.

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Communicating confidence in Mineral Resources and Mineral Reserves
by M.A. Noppé*

Synopsis
Mining is an inherently risky business; from the technical, environmental, social, and economic uncertainties associated with advancing an exploration prospect to a viable project to the operating, market, and safety risks and uncertainties attached to a developed mine. Since we cannot totally escape the risk and uncertainty related to resource projects, as an industry we should improve our presentation of the upside and downside risks in the context of the project’s development path and maturity. More transparent, consistent, and balanced views of technical confidence will better inform both internal and external stakeholders about the expected risk in the project.

International reporting codes set out the minimum standards, recommendations, and guidelines for public reporting of Exploration Results, Mineral Resources, and Mineral Reserves. However, the reporting code principles of transparency and materiality are largely subject to the interpretation of the competent person(s), which may introduce a degree of subjectivity in reporting, particularly the level of disclosure regarding supporting information. It is fundamental that Mineral Resources, Mineral Reserves, and study outcomes are reported so as to unambiguously present the level of inherent technical uncertainty (or confidence) in a project, while conveying a balanced view of the opportunities a project presents. Reporting needs to consider various stakeholders who may rely on this information, and present the data in the context of the changing risk profile associated with project development paths and project maturity.

This paper discusses the interdependence of resource-to-reserve conversion, the consideration of various technical-economic study types, and the level of confidence conveyed to stakeholders relying on these technical reports and other company public announcements.

Keywords
mineral resources, mineral reserves, ore reserves confidence, accuracy, reporting, classification, JORC Code, SAMREC Code, risk, uncertainty, reporting codes, scoping study, pre-feasibility, feasibility study.

Introduction
Despite the definitions and guidance on the reporting of Mineral Resource and Mineral Reserve categories provided by the current and previous versions of the SAMREC Code and the JORC Code, the categories and technical-economic study outcomes are not necessarily reported with a consistent or common expectation of confidence in the estimates. Inconsistencies in clearly relaying the expected accuracy, precision and confidence in the estimates as they advance from Mineral Resources to Mineral Reserves and the reporting of associated project economics may result in misleading or incorrect interpretations of the project risk by those relying on this information.

The Competent Person (CP) should strive to improve the presentation of the technical risk and uncertainty associated with resource projects to provide a more consistent, and balanced, view of confidence, risk, and opportunities for both internal and external stakeholders relying on this information. Greater consistency is required across the resources industry to better convey the accuracy, precision, and confidence when assigning and reporting Mineral Resource and Mineral Reserve categories and the outcomes of technical studies in the context of the project’s development path and maturity.

Reporting codes such as the SAMREC Code and the JORC Code (‘the Codes’) set out minimum standards, recommendations, and guidelines for Public Reporting of Exploration Results, Mineral Resources and Mineral Reserves (SAMREC, 2009; JORC, 2004; JORC, 2012). The Codes have been adopted by and included in the listing rules of the relevant securities exchanges (ASX, NZX, and JSE), and impose specific requirements on exploration and mining companies reporting to these exchanges. Furthermore, the Codes have been adopted by the relevant professional bodies, associations, and councils and are binding on members of those organizations.

While there is an expectation that all stakeholders involved in interpreting or relying on information reported under the Codes are familiar with their contents, this is not always the case. Despite the Codes being relatively brief documents, stakeholders relying on the...
Communicating confidence in Mineral Resources and Mineral Reserves

reported results tend to skim the contents of the Codes and focus on parts deemed most applicable in the circumstance. This paper quotes extensively from the 2012 edition of the JORC Code (JORC, 2012) and the 2007 edition of the SAMREC Code as amended in July 2009 (SAMREC, 2009) to draw attention to parts of the Codes providing support and guidance for classifying and reporting Mineral Resources and Mineral Reserves, and particularly their accuracy, precision, and confidence.

Consider, for example, the requirement that a Mineral Resource must have ‘reasonable and realistic prospects for eventual economic extraction’ (SAMREC, 2009). While some resource practitioners debate what this actually means, the JORC and SAMREC Codes state this ‘implies an assessment (albeit preliminary) by the Competent Person in respect of all matters likely to influence the prospect of economic extraction including the approximate mining parameters.’

Some argue that the consideration of ‘approximate mining parameters’ is too restricting or conservative for defining a Mineral Resource and is akin to the consideration and application of modifying factors required for defining a Mineral Reserve. Others have compromised the intent of the Codes and selectively applied this economic assessment when defining Indicated and Measured Resources but not when defining Inferred Resources. There are then cases where the interpretation of ‘eventual economic extraction’ has been stretched to speculate on mining methods, metallurgical extraction, or land access that may in the future be possible, but are as yet not demonstrated, available, or viable. These extremes, or even more subtle variations, in interpreting the Mineral Resource definition and confidence criteria can result in materially different reported Mineral Resource categories, and even different quantities (tonnages and grades of total Mineral Resources) for the same deposit by different CPs. While the Codes do not prescribe how CPs should carry out their assessments, the Codes provide definitions and guidance to facilitate consistency and transparency, particularly through their checklists of the factors to be considered and reported, in order to avoid, or at least explain, such obvious differences in interpretation.

Consider another general example, namely the definition in the Codes that Mineral Reserves are derived from the Indicated and Measured portions of a Mineral Resource through the consideration and application of modifying factors assessed at the level of at least a pre-feasibility study, including a mine plan and production schedule. This does not necessarily mean that all the Measured Resources will automatically convert to Proved Reserves (or the Indicated Resources to Probable Reserves). The study must demonstrate a technically achievable and economically viable mine plan and schedule for the reported Mineral Reserves, and furthermore, the level of confidence in the relevant modifying factors must be sufficient to support the category of Mineral Reserve. It is therefore possible that only a portion of the Mineral Resource will convert to a Mineral Reserve, thus reflecting the uncertainty in the process. It is also possible that a lower level of confidence in even one key modifying factor may mean the relevant portion of the Measured Resource may be better classified as a Probable Reserve rather than a Proved Reserve, and indeed, the Indicated Resource may not convert to a Mineral Reserve at all. This downgrade in reported confidence recognizes and reflects the material impact of that one factor on the technical or economic viability of the project at the time of reporting, and has been allowed for in the 2012 JORC Code and the 2009 SAMREC Code, as illustrated in Figure 1.

Without transparent reporting and support for the key Mineral Resource and Mineral Reserve assumptions, those relying on the publicly reported Mineral Resource and Mineral Reserve category and study outcomes at face value may not be fully aware of the risks or opportunities inherent in this data, and thus may not be in a position to make an informed decision on the reported values.

The JORC and SAMREC Codes provide extensive guidance on the relative hierarchy of accuracy, precision, or confidence in reporting Mineral Resource and Mineral Reserve categories and technical studies. While the Codes do not insist that the relative accuracy, precision, and confidence level of estimates are described, they strongly encourage CPs to discuss this and, where possible, provide a statement of the relative accuracy and confidence level, or at least a qualitative discussion of the relevant uncertainties. Indeed, the Codes highlight the importance of the CP’s assessment of confidence in reporting through the use of various terms, including ‘accuracy’, ‘uncertainty’, ‘reliability’, ‘confidence’, ‘confidence level’, ‘quality of data’, ‘quality of information’, and ‘quality of reported results’.

The Codes rely on the CPs to provide their own interpretation of what is meant by confidence and accuracy levels in the context of their project. In the author’s opinion, this requires better disclosure of the expected accuracy, precision, and confidence in reported Inferred, Indicated and Measured Resources, Probable and Proved reserves, and indeed the outcomes of scoping, pre-feasibility, and feasibility studies.
Communicating confidence in Mineral Resources and Mineral Reserves

Project development stages

It is important to remember that the purpose of advancing prospects and projects and developing mines is to achieve a profitable business outcome. It is therefore equally important to understand and present a project’s maturity when interpreting reported Mineral Resource and Mineral Reserve statements. The Australasian VALMIN Code (VALMIN, 2005) and South African SAMVAL Code (SAMVAL, 2009) classify mineral assets according to their maturity in the following project development stages: exploration, advanced exploration, pre-development/resource, development, and operating/producing. These development stages are outlined in Table I.

Logically, as a prospect or project advances along the development stages outlined in Table I, the understanding of the risks and opportunities improves with more and better-quality technical data collected and assessed through increasing levels of rigour and detail in technical and economic studies. The increasing level of project maturity reflects the increasing level of certainty in the estimated project outcomes, and it is reasonable to expect the value of the project to increase with this increasing confidence. The interrelationship of increasing certainty and project value with advancing development stages, including the definition of Mineral Resource and Mineral Reserve and the level of technical study, is presented schematically in Figure 2.

Technical study types

The resources industry, like other industries, seeks to convey confidence (accuracy, precision, and risk) in study outcomes through the level of detail of the particular technical and economic study. The levels of study reported in both private and public announcements, and subsequently the expectation of certainty in the study outcomes, is generally conveyed simply by the study names, without providing the study definitions or expected levels of accuracy, precision, and confidence. Sometimes there is an inconsistent or even incorrect use of study terminology in our industry, for example referring to a feasibility study, when the study is actually at the level of a pre-feasibility study or perhaps even a scoping study. One way to consider the intended level of study is to consider the stage at which a project advances from an ‘aspirational’ project to one that is considered to be strictly ‘data-driven’. In the latter case, the project is supported by extensive and good-quality data, technical studies, and engineering design at particular levels of detail.

For example, a scoping study may be considered largely ‘aspirational’ since, although it is generally partly data-driven (it may be based on Inferred Resources or better), it may be effectively conceptual as regards its technical and economic assumptions. A pre-feasibility or feasibility study, on the other hand, may be considered ‘data-driven’ as these are generally based mostly on Indicated and/or Measured resources and sufficiently detailed assessments of the modifying factors to enable a Mineral Reserve to be determined. This is why, even though under the JORC Code definitions a scoping study can be carried out on Indicated or Measured resources, the level of confidence in the modifying factors is not considered sufficient to determine a mine plan and production schedule that is technically achievable and economically viable, and from which the Mineral Reserves can be derived and reported.

Note that the term ‘scoping study’ is not defined or used in the SAMREC Code, which is to be expected since a scoping level study does not necessarily result in the delineation and reporting of Mineral Resources or Mineral Reserves.

It is not unusual to find quite different interpretations of Mineral Resource categories between different CPs, ranging from conservative to the highly optimistic. Indeed, it is not

\[
\text{Table I} \\
\text{Mineral asset development stages (VALMIN, 2005; SAMVAL, 2009)}
\]

<table>
<thead>
<tr>
<th>Project development stage</th>
<th>Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration areas</td>
<td>Mineralization may or may not be defined, but where a Mineral Resource has not been identified.</td>
</tr>
<tr>
<td>Advanced exploration areas</td>
<td>Considerable exploration has been undertaken and specific targets identified. Sufficient work has been completed on at least one prospect to provide a good geological understanding and encouragement that further work is likely to result in the determination of a Mineral Resource.</td>
</tr>
<tr>
<td>Pre-development / resource</td>
<td>Mineral Resources and/or Mineral Reserves were identified and estimated. A positive development decision has not been made. This includes properties where a development decision has been negative and properties are either on care and maintenance or held on retention titles.</td>
</tr>
<tr>
<td>Development</td>
<td>Committed to production but not yet commissioned or not initially operating at design levels.</td>
</tr>
<tr>
<td>Operating</td>
<td>Mineral properties, in particular mines and processing plants, which were fully commissioned and are in production.</td>
</tr>
</tbody>
</table>

Figure 2—The interrelationship of increasing certainty and project value with advancing development stages and the level of technical study (after Lilford, 2011)
Communicating confidence in Mineral Resources and Mineral Reserves

unusual to see quite different implications applied to the meaning of a ‘feasibility study’, again ranging from a scoping study level to that of a final feasibility study. Unless the statement or report accompanying these results provides sufficient clarity or transparency of the material assumptions supporting the public announcement, the recipients of such data could indeed be misled by the information. For example, one CP’s feasibility study, may be another’s scoping study, one CP’s Measured Resource may be another’s Indicated Resource. Indeed, if the requirement for ‘reasonable prospects for eventual economic extraction’ for reporting any category of Mineral Resource are not considered by the CP and the assumptions made transparent, then one CP may report very different Mineral Resource quantities to another, even using the same basic data and a similar geological model. Clearly, stakeholders using this public information, for example investors or valuers, should be able to rely on this information or be able to drill down into the detail behind the announcements to establish the correct context of this information.

The definitions of study types are repeated here.

➤ A Scoping Study (JORC, 2012) is an order-of-magnitude technical and economic study of the potential viability of Mineral Resources. It includes appropriate assessments of realistically assumed modifying factors together with any other relevant operational factors that are necessary to demonstrate at the time of reporting that progress to a pre-feasibility study can be reasonably justified.

➤ A Pre-feasibility Study (SAMREC, 2009) is a comprehensive study of the viability of a range of options for a mineral project that has advanced to a stage at which the preferred mining method in the case of underground mining or the pit configuration in the case of an open pit has been established and an effective method of mineral processing has been determined. It includes a financial analysis based on realistic assumptions of technical, engineering, operating, and economic factors and the evaluation of other relevant factors that are sufficient for a CP, acting reasonably, to determine if all or part of the Mineral Resource may be classified as a Mineral Reserve. The overall confidence of the study should be stated. A pre-feasibility study is at a lower confidence level than a feasibility study.

➤ A Feasibility Study (SAMREC, 2009) is a comprehensive design and costing study of the selected option for the development of a mineral project in which appropriate assessments have been made of realistically assumed geological, mining, metallurgical, economic, marketing, legal, environmental, social, governmental, engineering, operational, and all other modifying factors, which are considered in sufficient detail to demonstrate at the time of reporting that extraction is reasonably justified (economically mineable) and the factors reasonably serve as the basis for a final decision by a proponent or financial institution to proceed with, or finance, the development of the project. The overall confidence of the study should be stated.

When conducting technical studies, from scoping studies through to final feasibility studies, it is considered crucial that such studies are suitably matched not only to the accuracy and precision of the cost estimates, but also to the level of confidence in the underlying asset, namely the Mineral Resource and Mineral Reserve base. The level of technical study needs to convey the appropriate risk and opportunity profile of the project to the stakeholders. For example, it is completely misleading to report a resource project at a final feasibility study level if there are insufficient Measured and Indicated Mineral Resources and Proved and Probable Mineral Reserves defined to support the minimum economic mine life, regardless of what level of detail is reportedly available on other inputs, such as equipment costs or processing plant.

Over recent years the need to maintain rigour in conducting robust technical and economic assessments has been under pressure by the ‘need’ of many developers to fast-track studies to keep timelines and costs down and to take advantage of commodity demand and price cycles. The short timelines applied to technical studies often result in various investigations running in parallel rather than in series, as would traditionally have been the case. This invariably results in some redundancies in the process, and can also lead to misleading interim results from incomplete study phases. So in effect, some ‘final’ feasibility studies resulting from a fast-track process may effectively be at the level of confidence that many major engineering and mining companies would consider to be only at a pre-feasibility study level. Stakeholders should be made aware if higher levels of uncertainty are associated with the outcomes of some of these fast-tracked studies to allow them to properly assess the associated project risks. In some instances, the fast-tracking approach may mean alternative scenarios, normally identified during scoping phases and pursued during pre-feasibility assessments, are not fully considered before advancing to the so-called final feasibility study, and stakeholders deserve to be made aware if this has been the case.

Does the completion of a final feasibility study mean there is no more technical work to be done? Is no further definition and resolution of the Mineral Resources and Mineral Reserves required once a final feasibility study has been completed? This is certainly not the case, since for most deposits, a Measured Resource and Proved Reserve do not provide sufficient detail for short-term mining control. Furthermore, after completion of a final feasibility study, a project still requires additional detail in terms of final engineering design during its development stage to improve the accuracy and precision of the results for planning, contracting, and construction purposes. In the case of the Mineral Resource and Mineral Reserve, final definition drilling and sampling and the development of a prototype short-term grade control model is typically required for the start-up mining areas, followed by ongoing grade-control activities. Clearly, a Measured Resource and Proved Reserve are not necessarily at the ultimate level of accuracy, precision, and confidence required for reliable short-term mine planning and scheduling.
Communicating confidence in Mineral Resources and Mineral Reserves

The international Mineral Resource and Mineral Reserve reporting codes, including the JORC and SAMREC Codes, do not quantify the level of accuracy, precision, or associated uncertainty/risk expected to be conveyed by the various Mineral Resource and Mineral Reserve categories and technical study types. However, some rules of thumb for the levels of accuracy, expressed as confidence intervals, expected from the three main levels of study are presented in Table II. The levels of accuracy expressed as confidence intervals in Table II do not include the expected confidence levels, where for example a ±15 per cent accuracy interval at 90 per cent confidence limits would mean there is a 1 in 20 chance for the result to be less than 85 per cent of the estimate, and a 1 in 20 chance it may be 15 per cent higher than the estimate.

Approximate mining parameters versus modifying factors

➤ The 2012 JORC Code notes that with respect to a Mineral Resource—‘... in discussing ‘reasonable prospects for eventual economic extraction’ in Clause 20, the Code requires an assessment (albeit preliminary) in respect of all matters likely to influence the prospect of economic extraction including the approximate mining parameters by the Competent Person. While a Scoping Study may provide the basis for that assessment, the Code does not require a Scoping Study to have been completed to report a Mineral Resource’.

➤ The JORC Code goes on the clarify that—‘In other words, a Mineral Resource is not an inventory of all mineralisation drilled or sampled, regardless of cut-off grade, likely mining dimensions location or continuity. It is a realistic inventory of mineralisation which, under assumed and justifiable technical, economic and development conditions, might, in whole or in part, become economically extractable’.

➤ The SAMREC Code clarifies similarly that—‘The term ‘reasonable and realistic prospects for eventual economic extraction’ implies a judgement (albeit preliminary) by the Competent Person in respect of technical and economic factors likely to influence the prospect of economic extraction, including the approximate mining parameters. In other words, a Mineral Resource is not an inventory of all mineralization drilled or sampled, regardless of cut-off grades, likely mining dimensions, location or continuity. It is a realistic inventory of mineralization that, at the time of reporting and under assumed and justifiable technical and economic conditions, might become economically extractable. Portions of a mineral deposit that do not have reasonable and realistic prospects for eventual economic extraction must not be included in a Mineral Resource.’

Some CPs argue that if these ‘approximate mining parameters’ are applied when estimating and reporting a Mineral Resource, then this is effectively the same as reporting a Mineral Reserve. However, as the following extracts from the 2009 SAMREC Code and the 2012 JORC Code show, the modifying factors for the reporting of Mineral Reserves are more stringently determined than the approximated mining parameters.

➤ The SAMREC Code states—‘The term ‘economically mineable’ implies that extraction of the Mineral Reserve has been demonstrated as viable and justifiable under a defined set of realistically assumed modifying factors. What constitutes the term “realistically assumed” will vary with the type of deposit, level of study that has been carried out, and financial criteria of the reporting entity. Deriving a Mineral Reserve without a mine design or mine plan through a process of factoring of the Mineral Resource is unacceptable.”

➤ The JORC Code states—‘Confidence in the Measured Resource estimate is sufficient to allow application of Modifying Factors within a technical and economic study as defined in Clauses 37 to 40. Depending upon the level of confidence in the various Modifying Factors it may be converted to a Proved Mineral Reserve (high confidence in Modifying Factors), Probable Mineral Reserve (some uncertainty in Modifying Factors) or may not be converted at all (low or no confidence in

<table>
<thead>
<tr>
<th>Measure/item</th>
<th>Scoping study</th>
<th>Pre-feasibility study</th>
<th>Final feasibility study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost accuracy</td>
<td>±25%–50%</td>
<td>±15-25%</td>
<td>±10-15%</td>
</tr>
<tr>
<td>Cost contingency</td>
<td>30-50%</td>
<td>15-30%</td>
<td>&lt;15%</td>
</tr>
<tr>
<td>Proportion of engineering complete</td>
<td>&lt;5%</td>
<td>&lt;20%</td>
<td>&lt;50%</td>
</tr>
<tr>
<td>Resource categories</td>
<td>Mostly Inferred</td>
<td>Mostly Indicated</td>
<td>Measured and Indicated</td>
</tr>
<tr>
<td>Reserve categories</td>
<td>None</td>
<td>Mostly Probable</td>
<td>Proved and Probable</td>
</tr>
<tr>
<td>Mining method</td>
<td>Assumed</td>
<td>General</td>
<td>Optimized</td>
</tr>
<tr>
<td>Mine design</td>
<td>None or high-level conceptual</td>
<td>Preliminary mine plan and schedule</td>
<td>Detailed mine plan and schedule</td>
</tr>
<tr>
<td>Scheduling</td>
<td>Annual approximation</td>
<td>3-monthly to annual</td>
<td>Monthly for much of payback period</td>
</tr>
<tr>
<td>Risk tolerance</td>
<td>High</td>
<td>Medium</td>
<td>Low</td>
</tr>
</tbody>
</table>

Sources: (Parsons, 1999; McCarthy, 2003; Pincock, 2004; Barton, pers. comm., 2005; Macfarlane, 2007; Hatch, 2010; Bullock, 2011; AACE International, 2012)
Communicating confidence in Mineral Resources and Mineral Reserves

some of the Modifying Factors; or no plan to mine, e.g. pillars in an underground mine or outside economic pit limits)'.

Both the SAMREC Code and JORC Code provide extensive guidelines on the meaning and consideration of modifying factors in determining Mineral Reserves. The relevant portions of the JORC Code that discuss the application of modifying factors for the reporting of Mineral (Ore) Reserves are therefore reproduced here with little need for commentary:

The words “ore” and “reserves” must not be used in describing Mineral Resource estimates as the terms imply technical feasibility and economic viability and are only appropriate when all relevant Modifying Factors have been considered. Reports and statements should continue to refer to the appropriate category or categories of Mineral Resources until technical feasibility and economic viability have been established. If re-evaluation indicates that the Ore Reserves are no longer viable, the Ore Reserves must be reclassified as Mineral Resources or removed from Mineral Resource/Ore Reserve statements.

‘An ‘Ore Reserve’ is the economically mineable part of a Measured and/or Indicated Mineral Resource. It includes diluting materials and allowances for losses, which may occur when the material is mined or extracted and is defined by studies at Pre-Feasibility or Feasibility level as appropriate that include application of Modifying Factors. Such studies demonstrate that, at the time of reporting, extraction could reasonably be justified.

‘In order to achieve the required level of confidence in the Modifying Factors, appropriate Feasibility or Pre-Feasibility level studies will have been carried out prior to determination of the Ore Reserves. The studies will have determined a mine plan and production schedule that is technically achievable and economically viable and from which the Ore Reserves can be derived. The term ‘Ore Reserves’ need not necessarily signify that extraction facilities are in place or operative, or that all necessary approvals or sales contracts have been received. It does signify that there are reasonable grounds to expect that such approvals or contracts will eventuate within the anticipated time frame required by the mine plans. There must be reasonable grounds to expect that all necessary Government approvals will be received. The Competent Person should highlight and discuss any material unresolved matter that is dependent on a third party on which extraction is contingent’.

Transparency in reporting requires the assumptions on which the reasonable grounds are based for expecting approvals, sales contracts, transport infrastructure etc., to be summarized and the risks regarding any ‘material unresolved matter’ provided by the CP. The other important point made above is that a Mineral Reserve must be re-classified if a sustained change in technical or economic parameters indicates that the Mineral Reserve is no longer viable. Indeed, the Mineral Resource may even need to be removed from future statements if the criteria for eventual economic extraction are no longer valid. This is not meant to include short-term commodity price movements or immediate demand constraints, but rather longer term price trends, market conditions, political, environmental, social or infrastructure issues, legislation or approvals, or funding issues that may prevent the intended project being developed in the time frame considered in the relevant technical study. If the driving factors are no longer appropriate and the revised factors are such that the project is no longer economically viable, then the outcomes of the original study, including the estimated Mineral Reserves, may not be current or meaningful.

Accuracy, confidence, and quality

The terms and definitions for Mineral Resources and Mineral Reserves provided in the SAMREC Code, and summarized in Figure 1, are intended to provide a consistent meaning for the stakeholders assessing company and technical statements, reports, and announcements. However, do all CPs really mean the same thing when they use the defined terms?

The intent of SAMREC and JORC code-compliant reporting of Mineral Resources and Mineral Reserves is to ensure that the CP has considered the requirements of the Codes, followed the guidelines, and provided supporting information in terms of at least the Codes’ Table 1 checklists. But what is the intended and expected outcome of such reporting? Ultimately, the reported values must be placed into the correct context with respect to the reliability and intended, but not necessarily quantified, accuracy or certainty of the reported results.

The Codes place as much reliance on the quality of the supporting data or information, including the quality and level of detail of the technical-economic study, as on the quantity of such data, when defining a Mineral Resource and/or a Mineral Reserve. The vertical axis in Figure 1 clearly highlights the consideration of the ‘quality’ of data when considering both Mineral Resource and Mineral Reserve classification. The importance of the quality or confidence in the data for classifying the Mineral Resources is generally well-accepted by CPs and the Codes re-emphasize the quality versus quantity relationship by commenting that the choice of the appropriate category of Mineral Resource (or Mineral Reserve) depends upon the nature, quantity, distribution, and quality of data available and the level of confidence that attaches to that data.

It is important to reiterate the application of the principle of confidence in the quality of data, information, and/or study type when considering the appropriate Mineral Resource or Mineral Reserve category. The Codes provide the following guidance (JORC, 2012):

‘Measured Mineral Resources may be converted to either Proved Ore Reserves or Probable Ore Reserves. The Competent Person may convert Measured Mineral Resources to Probable Ore Reserves because of uncertainties associated with some or all of the Modifying Factors which are taken into account in the conversion from Mineral Resources to Ore Reserves’ and that ‘Depending upon the level of confidence in the various Modifying Factors it may be converted to a Proved Ore Reserve (high confidence in Modifying Factors), Probable Ore Reserve (some uncertainty in Modifying Factors) or may not be converted at all (low or no confidence in some of the Modifying Factors; or no plan to mine, e.g. pillars in an underground mine or outside economic pit limits).’
Communicating confidence in Mineral Resources and Mineral Reserves

Clearly, the same consideration applies to the possible conversion of Indicated Mineral Resources to Probable Mineral Reserves: namely, depending on the confidence in the modifying factors, the Mineral Resource may not be converted to Probable Mineral Reserves at all.

It is worth noting that while a Probable Reserve has a lower level of confidence than a Proved Reserve, it is still of sufficient quality to serve as the basis for a decision on the development of the deposit. This is why some project financiers rely on the total Mineral Reserve rather than necessarily the proportion of Proven or Probable reserves in their assessments of project risk.

Further guidance within the 2012 JORC Code regarding reliability or confidence in reporting includes the following:

‘Where there are as yet unresolved issues potentially impacting the reliability of, or confidence in, a statement of Ore Reserves (for example, limited geotechnical information, complex orebody metallurgy, uncertainty in the permitting process, etc.) those unresolved issues should also be reported’, and that ‘If there is doubt about what should be reported, it is better to err on the side of providing too much information rather than too little’.

In the preamble to Table 1 in the 2009 SAMREC Code, the principle of materiality of the information with respect to reliability, uncertainty or confidence in reporting is reiterated: ‘... as always, relevance and materiality are overriding-principles that determine what information should be publicly reported’. The 2012 JORC Code goes on to say ‘the Competent Person must provide sufficient comment on all matters that might materially affect a reader’s understanding or interpretation of the results or estimates being reported. This is particularly important where inadequate or uncertain data affect the reliability of, or confidence in, a statement of Exploration Results or an estimate of Mineral Resources or Ore Reserves’.

The topic of reliability, accuracy, or confidence in the final reporting and level of study supporting the Mineral Resources and Mineral Reserves is further addressed in the Codes, and in particular the JORC Code more so than the SAMREC Code.

The preamble to Table 1 in the SAMREC Code (SAMREC, 2009) calls on the CPs to ‘Discuss whether account has been taken of all relevant factors, i.e. relative confidence in tonnage / grade computations, density, quality, value and distribution of primary data and information, confidence in continuity of the geological and mineralization models’.

The 2012 JORC Code expands on the requirement to discuss accuracy, precision and confidence as follows:

‘Where appropriate a statement of the relative accuracy and confidence level in the Ore Reserve estimate using an approach or procedure deemed appropriate by the Competent Person. For example, the application of statistical or geostatistical procedures to quantify the relative accuracy of the reserve within stated confidence limits, or, if such an approach is not deemed appropriate, a qualitative discussion of the factors which could affect the relative accuracy and confidence of the estimate, and

‘Accuracy and confidence discussions should extend to specific discussions of any applied Modifying Factors that may have a material impact on Ore Reserve viability, or for which there are remaining areas of uncertainty at the current study stage’.

‘Competent Persons are encouraged, where appropriate, to discuss the relative accuracy and confidence level of the Mineral Resource [or Ore Reserve] estimates with consideration of at least sampling, analytical and estimation errors. The statement should specify whether it relates to global or local estimates, and, if local, state the relevant tonnage. Where a statement of the relative accuracy and confidence level is not possible, a qualitative discussion of the uncertainties should be provided in its place’.

The ultimate outcome of any Mineral Resource and Mineral Reserve process and feasibility study path is to secure funding to develop a technically and economically viable mine. The final feasibility study is referred to by some as a ‘bankable’ study because it can be used to support project financing by commercial bank loan facilities, or other forms of financing. So what accuracy, precision, and level of confidence do the financiers require? Parker (pers. comm., 1997) has noted that ‘the first rule is, there are no rigid standards, only guidelines ... some lenders are very liberal, and others conservative’. Parker has also commented on the accuracy of feasibility study outcomes to say that ‘although the relative width of the confidence interval is defined, the confidence level is not often mentioned or accurately quantifiable, because no two projects are exactly alike’. Parker noted that banks in general require sufficient confidence in the planned production to cover the loan service.
Communicating confidence in Mineral Resources and Mineral Reserves

period plus a contingency period of 50 to 100 per cent of the payback period, depending on the perceived risks in the project. Typically this translates to a required amount of total Mineral Reserves over the payback plus contingency period, and some banks may require a certain proportion of Proved to Probable reserves in evaluating feasibility study outcomes.

Valuations of mineral assets using market-based or comparable transaction methods tend to rely ultimately on the quantities of reported Mineral Resources and Mineral Reserves. Proved and/or Probable reserves obviously command a premium value per unit, but where these are not defined, the valuations will tend to rely on the quantities of Measured and Indicated resources, generally as a total Measured plus Indicated Resource, to guide the valuation (McKibben, pers. comm., 2013; VALMIN, 2005). Inferred Resources may provide further support depending on the potential to convert these to Indicated or Measured resources within a ‘reasonable’ timeframe (Lawrence, 2012).

The above examples of how Mineral Resource and Mineral Reserve confidence is recognized by project financiers and mineral asset valuers illustrate why it is so important to provide transparent and consistent reporting of Mineral Resources, Mineral Reserves, and study outcomes, and to provide a discussion on the expected accuracy, precision, and confidence levels of the estimates. It is considered highly desirable, when discussing and reporting accuracy and confidence levels, that the measures and terminology consider the practical use and interpretation of these results for the various stakeholders. In general, stakeholders wish to appreciate the expected accuracy, precision, and confidence levels over a particular period of time. For example, the period of time may relate to a mine production scale, such as the confidence for a one-week, one month, one quarter, one year, or life-of-mine period; or whether the confidence relates to a resource block, selective mining unit, mining panel or strip, or the entire resource domain.

The following descriptions of expected accuracy, precision, and confidence limits provide examples that would be meaningful to various stakeholders, from mine operators and management to investors and financiers:

- Measured Resource (or Proved Reserve): ±10 to 15 per cent (at 90 per cent confidence limits) for three-monthly production parcels
- Indicated Resource (or Probable Reserve): ±10 to 15 per cent (at 90 per cent confidence limits) for annual production parcels.

In other words, these metrics imply that 1 out of 20 periods are expected to be less than 90 to 85 per cent of the estimate, and 1 out of 20 periods may be 10 to 15 per cent higher than the estimate.

Alternatively, the measures could be reported over the same period or scale, but with different precision ranges:

- Measured Resource (or Proved Reserve): ±5 to 10 per cent (at 90 per cent confidence limits) for annual production parcels
- Indicated Resource (or Probable Reserve): ±10 to 15 per cent (at 90 per cent confidence limits) for annual production parcels,

or

- Measured Resource (or Proved Reserve): ±10 to 15 per cent (at 90 per cent confidence limits) for three-monthly production scale parcels
- Indicated Resource (or Probable Reserve): ±15-25 per cent (at 90 per cent confidence limits) for three-monthly production parcels.

Clearly, the accuracy, precision, and level of confidence that can be attained using this form of relative measurement depends very much on the nature of the deposit under consideration. For example, Mineral Resource estimates for a nuggety gold deposit may never attain the same measure of relative confidence as a stratiform-style copper deposit. However, that is exactly the point: no two deposits are identical, and that is why further discussion and reporting of the CP’s meaning when reporting a particular Mineral Resource or Mineral Reserve category is required. For example, it may be possible to express the same intended accuracy, say ±10 to 15 per cent confidence interval for Measured Resources, for various deposit styles but for different scales or periods of production; so it may be possible to achieve this accuracy and precision over three-month production areas for a stratiform base metal deposit, but this would need to be over six-month or one-year periods for a nickel sulphide deposit, or over the full life-of-mine for a nuggety gold deposit.

Examples of poor project outcomes and reporting

The importance of completing a study to the required level of detail and placing the outcomes in the correct context of technical risk and confidence is highlighted by following anecdotes (cited in a presentation by Peter McCarthy to the Melbourne branch of the AusIMM in 2013, titled ‘Why pre-feasibility studies fail’; McCarthy, 2003):

- In the 1980s, a study of 35 Australian gold mines found that 68 per cent failed to deliver the planned head grade (Burmeister, 1988)
- A review of nearly 50 North American projects showed that only 10 per cent achieved their commercial aims, with 38 per cent failing within about one year (Harquail, 1991)
- A study of the start-up performance of nine Australian underground base metal mines found that only 50 per cent achieved design throughput by year 5 and 25 per cent never achieved it at all (McCarthy and Ward, 1999)
- A US study comparing the final feasibility study production rate with the average sustained production rate from 60 steeply-dipping tabular deposits found that 35 per cent of the mines did not achieve their planned production rate (Tatman, 2001).

There have been a few recent company public announcements where the Mineral Resource, Mineral Reserve, and study level have presented the project in the incorrect context of the expected risk and level of maturity of the project.

In one example, a joint venture partner reported double the coal resource tonnage of the other JV partner, each using...
Communicating confidence in Mineral Resources and Mineral Reserves

the same drill hole data. The simple reason for the difference was that the partner with double the resource had not applied the likely mining parameters for the expected underground mining scenario. The CP had therefore essentially reported all the in situ coal without adequately justifying how this coal would be economically and realistically extracted. Clearly, a stakeholder relying on the higher reported coal resource without being made aware of the implications of these assumptions for the accuracy and confidence in the reported tonnage and quality would be expected to derive a significantly higher valuation for the project.

In another example, where a company reported the technical and financial outcomes of a ‘definitive feasibility study’, the financial project outcomes were questioned by the Australian Securities and Investments Commission. The company subsequently downgraded the level of study to a ‘feasibility study’, but one for which no Mineral Reserves were reported and where it was acknowledged that the Mineral Resources were still to be updated before the study could be completed and Mineral Reserves actually defined. In other words, the company’s original public statement reporting a definitive study supporting a project net present value of over A$2 billion was misleading and in effect, the original and revised company statement seem more correctly to represent the results of a scoping study carried out on preliminary Mineral Resource estimates. This context would have been abundantly clear if the announcement had used the JORC Code terminology and classifications correctly, and also contained a proximal cautionary statement to the effect: ‘The Scoping Study referred to in this report is based on low-level technical and economic assessments, and is insufficient to support estimation of Ore Reserves or to provide assurance of an economic development case at this stage, or to provide certainty that the conclusions of the Scoping Study will be realised’ (JORC, 2012). Further examples of the ‘use and abuse’ of feasibility studies can be found in Mackenzie and Cusworth (2007).

Conclusions

Resource development and mining is an inherently risky business. Mineral Reserve estimation is not simply a measure of maximum net present value or return on investment, but involves a thorough appreciation of the underlying Mineral Resource assessments of the relevant modifying factors at least to a Pre-feasibility level, to satisfy a range of business objectives, both quantitative and qualitative. Uncertainty and errors in Mineral Resource and Mineral Reserve estimates remain a major reason for the economic failure of mining projects. The appreciation and consideration of this uncertainty is critical for realistic project reporting, planning, and risk aversion.

It must be recognized that two deposits can have the same drill hole data. The simple reason for the difference was that the partner with double the resource had not applied the likely mining parameters for the expected underground mining scenario. The CP had therefore essentially reported all the in situ coal without adequately justifying how this coal would be economically and realistically extracted. Clearly, a stakeholder relying on the higher reported coal resource without being made aware of the implications of these assumptions for the accuracy and confidence in the reported tonnage and quality would be expected to derive a significantly higher valuation for the project.

In another example, where a company reported the technical and financial outcomes of a ‘definitive feasibility study’, the financial project outcomes were questioned by the Australian Securities and Investments Commission. The company subsequently downgraded the level of study to a ‘feasibility study’, but one for which no Mineral Reserves were reported and where it was acknowledged that the Mineral Resources were still to be updated before the study could be completed and Mineral Reserves actually defined. In other words, the company’s original public statement reporting a definitive study supporting a project net present value of over A$2 billion was misleading and in effect, the original and revised company statement seem more correctly to represent the results of a scoping study carried out on preliminary Mineral Resource estimates. This context would have been abundantly clear if the announcement had used the JORC Code terminology and classifications correctly, and also contained a proximal cautionary statement to the effect: ‘The Scoping Study referred to in this report is based on low-level technical and economic assessments, and is insufficient to support estimation of Ore Reserves or to provide assurance of an economic development case at this stage, or to provide certainty that the conclusions of the Scoping Study will be realised’ (JORC, 2012). Further examples of the ‘use and abuse’ of feasibility studies can be found in Mackenzie and Cusworth (2007).

Conclusions

Resource development and mining is an inherently risky business. Mineral Reserve estimation is not simply a measure of maximum net present value or return on investment, but involves a thorough appreciation of the underlying Mineral Resource assessments of the relevant modifying factors at least to a Pre-feasibility level, to satisfy a range of business objectives, both quantitative and qualitative. Uncertainty and errors in Mineral Resource and Mineral Reserve estimates remain a major reason for the economic failure of mining projects. The appreciation and consideration of this uncertainty is critical for realistic project reporting, planning, and risk aversion.

It must be recognized that two deposits can have the same reported Mineral Reserves and the same expected mining costs but have a very different financial attractiveness, solely as a result of different degrees of certainty inherent in their Mineral Resource and Mineral Reserve estimation. It should be standard practice to carry out a full risk analysis, including a detailed assessment of all sources of error, as an integral part of reporting any Mineral Resource or Mineral Reserve. For Mineral Resource reports, this will include all technical sources of error, while for Mineral Reserves this will also include economic, social, environmental, and infrastructure modifying factors as an integral part of the risk analysis. The aim is to provide a degree of quantification of the risk in the reported estimate to allow for better decision making by resource project stakeholders such as planners, operators, and investors.

Competent Persons should strive for better presentation of the technical risk and uncertainty associated with resource projects in the context of project maturity to provide more consistent and balanced views of confidence, risk, and opportunities for both internal and external stakeholders relying on this reported information. The international reporting codes, such as the SAMREC Code and the JORC Code, provide extensive guidance on the topics to be assessed. However, Competent Persons needs to step up and deliver the increased transparency that those relying on the reported data require.

In summary, together with due consideration of the assessment and reporting criteria outlined in the SAMREC and JORC Code checklists, the Competent Person and stakeholders relying on the Competent Person reports, need to challenge the reporting by asking themselves the following key questions:

➤ Would the vast majority of my Competent Person peers agree with my logic in defining, classifying, and reporting the Mineral Resources and Mineral Reserves?
➤ Does the stage of project development and level of confidence in the associated data and technical-economic studies support the reporting, and is this clearly and correctly presented?
➤ Would my peers and informed stakeholders be able to appreciate the assumptions, factors, and process followed in the reporting from the way in which the results are reported and supported?
➤ Are my assumptions for eventual economic extraction reasonable, realistic, and transparent, and have I adequately applied approximate mining parameters for reporting the Mineral Resources?
➤ Have I considered and used all representative data, and if I have excluded data have I adequately considered the advantages and risks in doing so?
➤ Have I applied realistic and justifiable mining factors in determining the mine plan and schedule for reporting Mineral Reserves, in particular geotechnical considerations, ore loss, dilution, mining extraction rates, ore sizing/fragmentation, blending requirements, and practical metallurgical recovery?
➤ Whether or not I have applied any quantitative assessment of accuracy and precision in my Mineral Resource and Mineral Reserve classification, have I considered the confidence I expect empirically over different length mining periods from the various categories, and are these consistent with what my peers and stakeholders would expect?
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- Have I included a statement of the relative accuracy and confidence level in the estimates and provided a similar discussion regarding any of the input data and factors that may have a material impact on Mineral Resource or Mineral Reserve viability?
- Have I adequately presented any remaining areas of uncertainty at the current study stage, and how these will be addressed in future work and studies?

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References


McKibben, J.J. 2013. Personal communication.


Checks and measures of performance for kriging estimates

by J.L. Deutsch*, J. Szymanski*, and C.V. Deutsch*

Synopsis

Block model estimates are commonly calculated by the well-established technique of kriging. The mathematics are well established, but implementation details require site-specific considerations. Checking and measuring the performance of the estimates is important to ensure the block model is suitable for its intended purpose. There are different criteria for long-term planning and short-term planning. The implementation of kriging should be checked with cross-validation and assessed for conditional bias and departure from theoretical optimality with the calculation of kriging efficiency. A new expression of kriging efficiency, which compares the kriging variance with the theoretically optimal kriging variance, is developed to aid in this assessment. The assessment tools presented here can be applied in a number of situations; however, ordinary kriging with a reasonably large search performs well in most cases.

Keywords

block model, kriging, variance.

Introduction

The challenge of spatial prediction is common in the earth science and other disciplines including meteorology and hydrology. Estimating spatially related geological variables is difficult; estimates must be made given samples taken from only one billionth of the deposit volume or less. Early computerized kriging techniques were developed by Krig (1951), Sichel (1973) and others in South Africa. Matheron (1962, 1963) formalized the methods for mining resource and reserve estimation and named them kriging after the South African pioneer Danie Krig. There were parallel developments in forestry (Matérn, 1960), bathymetry (Switzer et al., 1963), meteorology (Gandin, 1965), and mathematics (Kolmogorov et al., 1962). Further discussion of the historical development of kriging is given by Cressie (1990).

The properties of kriging are well known; it is a best, linear, unbiased estimator that minimizes the variance of estimation errors; often referred to as the estimation variance or kriging variance (Journel and Huijbregts, 1978). In practice, many decisions are required to make a kriging estimate such as kriging type, search parameters, and data selection. The resulting estimates should then be checked and assessed. In this paper, we examine kriging and how necessary decisions should be made and assessed with metrics and measures of performance. There are three types of estimates, each requiring a different strategy and criteria for assessing results: (1) estimates for visualization and geological understanding, (2) estimates for interim planning, and (3) final estimates for reserve classification.

Review of kriging framework

Consider a regionalized random variable $Z$ (a realization is referred to as the ‘data’). The data is subset into reasonable geologic rock types or domains called stationary populations in geostatistics. The data is assembled:

\[ \{ Z(\mathbf{u}_i) = z_{i\alpha}, \alpha = 1, \ldots, N \} \]

The notation $\mathbf{u}_i$ denotes a particular sample location within the domain. Estimates at unsampled locations are required. The data is usually at a nominal point scale, whereas the unsampled locations are volumes relevant for mining reserves and resources assessment. That is, they are blocks of a much larger size. The block estimate $\hat{Z}$ at an unsampled location $\mathbf{u}$ is computed by a linear combination of $n$ nearby data:

\[ \hat{Z}_i(\mathbf{u}) = m + \sum_{i=1}^{n} \lambda_i (z_i - m) \]

The notation $V$ denotes the larger block volume. The weights applied to each of the nearby data are denoted by $\lambda_i$. The stationary mean is $m$, and the usual assumption is that
this applies throughout the domain. The challenge is to choose the nearest \( n \) data to combine and to choose the weights. One evident choice is to retain 4 to 20 data and assign weights by inverse distance interpolation to the unsampled location. This can work well; but there are nuances of spatial structure and anisotropy that can be exploited for even better estimates.

Consider a separation vector \( h \) defined by an anisotropic distance and direction (two angles, usually azimuth and inclination from horizontal). The spatial variability for \( h \) is represented by the semivariogram \( \gamma(h) \). The semivariogram (commonly referred to as the variogram) is defined as the half expected squared difference for the random variable at data points separated by \( h \):

\[
\gamma(h) = \frac{1}{2} \mathbb{E} \{ (Z(u) - Z(u + h))^2 \}
\]

The variogram is experimentally estimated using the \( N(h) \) data separated by some defined tolerance:

\[
\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (Z(u_i) - Z(u_i + h))^2
\]

The experimental variogram is fit with a parametric variogram model. This permits calculation of the closely related covariance function \( C(h) = \sigma^2 - \gamma(h) \) where \( \sigma^2 \) is the variance of the data for the domain. The local weights for the estimate are optimized to minimize the variance of the errors of estimation (estimation variance):

\[
\min(\sigma^2_0) = \min(\mathbb{E}(\{Z'(u) - \bar{Z}(u)\}^2))
\]

\[
= \min \left( \sum_{i=1}^{n} \lambda_i C(u_i, u_i) - 2 \sum_{i=1}^{n} \lambda_i C(u_i, u_j) + C(V, V) \right)
\]

Where \( C(u_i, u_j) \) is the covariance between two data locations \( u_i \) and \( u_j \); \( C(u_i, u_i) \) is the average covariance between the block centered on \( u_i \) and the data at \( u_i \), and \( C(V, V) \) is the average covariance value between any two points within the block variance (Journel and Huijbregts, 1978). Minimizing this function by taking the partial derivatives and setting them equal to zero results in the simple kriging (named by Matheron) equations which are solved to determine the optimal weights:

\[
\sum_{i=1}^{n} \lambda_i C(u_i, u_j) = \bar{C}(u_i, u_j), \quad i = 1, \ldots, n
\]

It should be noted that for simple kriging, there is no constraint that the sum of the weights equals unity and that the simple kriging equation can be expressed differently to include the weight given to the mean; this expression can be found in any standard geostatistical text (Chiles and Delfiner, 2012; David, 1977; Journel and Huijbregts, 1978). The corresponding minimized estimation variance (kriging variance) can then be calculated:

\[
\sigma^2_{sk} = \bar{C}(V, V) - \sum_{i=1}^{n} \lambda_i \bar{C}(u_i, u_i)
\]

Choosing to constrain the weights to sum to unity leads to the ordinary kriging equations (so named by Matheron). The error variance is minimized subject to the unbiasedness constraint that the sum of the weights is equal to unity (Journel and Huijbregts, 1978). This relaxes the assumption of global stationarity since the mean is effectively estimated using the local search data. Other variants of kriging such as universal kriging or intrinsic random function kriging can also be considered for specific cases. Complete development of these kriging methods can be found in many geostatistical texts (Chiles and Delfiner, 2012; David, 1977; Journel and Huijbregts, 1978). For the estimation of mining variables in practice, ordinary kriging is used more often than simple kriging or alternative formulations, although integrated simulation approaches are also being adopted (Godoy and Dimitrakopoulos, 2011), and simple kriging is used in areas of wide data spacing.

### Decisions for kriging and assessment of results

When kriging is used for estimation, there are a number of significant decisions to be made. The type of kriging, generally either simple or ordinary kriging, must be decided. A restricted search is often considered to reduce the computational burden of computing and inverting large covariance matrices and, in the case of ordinary kriging, to reduce reliance on the hypothesis of a stationary mean (Rivoirard, 1987). A restricted search for simple kriging may also be used to eliminate the presence of negative weights and therefore decrease the weight applied to the mean (Boyle, 2010). As such, the next step is data selection and the definition of search parameters. Search parameters commonly include a maximum range around the location being estimated to search for local data and a maximum number of local data to consider. Other search parameters could include a maximum number of data to use from the same drill-hole and a maximum number of data to use from each octant of the local neighbourhood (Deutsch and Journel, 1998). This data pooled together composes the kriging neighbourhood (Rivoirard, 1987; Vann et al., 2003).

Rivoirard (1987) suggests the use of the weight given to the mean in simple kriging and the slope of regression for ordinary kriging as diagnostics on the search parameters. If the weight given to the mean in simple kriging with a limited neighbourhood is large, then a larger neighbourhood (less restrictive search) can be used. Conversely, if the weight applied to the mean in simple kriging is small, then the local neighbourhood has a strong influence. From Rivoirard’s analysis, it is almost always necessary to use a larger neighbourhood for ordinary kriging compared to simple kriging to reduce conditional bias. The slope of regression of the true values against the estimated values is used as a diagnostic for conditional bias. Ideally, the slope of this regression would be as close to unity as possible. This diagnostic depends heavily on the stationarity of the domain, as many of the diagnostics presented here do.

The large number of decisions required for kriging requires careful assessment of the resulting estimates. The most widely used method for assessing kriging results is cross-validation. Data locations are estimated with local data, excluding data from the same drill-hole. The estimated values are compared to the true values. Ideally, the estimates would correlate very strongly with the truth. In practice, the variance of the estimates is lower, so on average, high values are underestimated and low values are overestimated. A more robust cross-validation method is jackknifing, where a subset of the data is omitted from the beginning of modelling, and all calculations (estimation of the mean, variance, and variogram) are performed without this subset of the data.
Cross-validation and jackknifing are excellent assessment methods, but can be used only for locations where we know the true value. For the majority of locations where we do not know the true value, kriging metrics such as the slope of regression and kriging efficiency can be considered. The slope of regression provides a measure of the conditional bias in the estimates. The kriging efficiency as defined by Krige (1997) is akin to a goodness of fit. In this paper, a novel definition for kriging efficiency estimates the departure from the theoretical minimum squared estimation error. The results of cross-validation and these metrics can be used to guide decisions regarding data selection and kriging type.

**Data selection**

**Search anisotropy and ties**

The determination of search anisotropy for kriging is not trivial. The decision on search anisotropy parameters is guided by the variogram and data spacing. The variogram provides information on the spatial variability law of the domain, and the data spacing is used to choose a practical range. The search anisotropy should align with the variogram anisotropy, but be modified by the data spacing. As each of the variogram structures can have a different orientation, an effective anisotropy across all nested variogram structures should be considered. A reasonable distance for the search anisotropy should then be used, such as two data spacings. Other considerations can include restricting the search to a maximum number per octant or maximum number per drill-hole. Imposing additional restrictions may be necessary for very complicated deposits or deposits with long-range geologic continuity.

Care should be taken in the case of evenly spaced drill-hole data when a limited number of data are used for the kriging estimate. In this case the data used in the estimation (if they are equidistant) will depend on the kriging program and original order in the file. This can lead to inconsistencies between estimates if the data order in the file changes. This is a concern in mining, where establishing an audit trail and the reproducibility of estimates are critically important. One solution is to pre-sort the data along an arbitrary vector. The positions of data at tied distances are then chosen in order of the pre-sort vector. Alternatively, equidistant data could always be included if the situation arises.

A related issue when using only a subset of the data for kriging is the generation of discontinuous kriging surfaces. For some applications such as hill shading, hydrological modelling, and visualization, this is undesired (Gribov and Krivoruchko, 2004; Rivoirard and Romary, 2011). Gribov and Krivoruchko propose the use of a smoothing kernel which is applied to the weights of distance data. Rivoirard and Romary use a penalizing function to drive distance data weights to zero. These approaches are useful when the kriging application calls for it, but the resulting histogram of estimates should be checked.

**Number of search data**

The choice of the number of local search data to use for the kriging estimate is very important. As discussed earlier, the number of search data is commonly limited to reduce computational time and reliance on a global stationary mean. With increasing computer speed, this first concern is relatively minor. These kriging decisions were evaluated by the authors using a series of case studies similar to the evaluation of nonlinear kriging methods by Moyeed and Papritz (2002). A series of case studies were carried out to determine the effect of the number of search data on the mean squared error of the estimates. For each test case, kriging was carried out in cross-validation mode considering both simple and ordinary kriging with between 5 and 100 local data used to make each estimate. The mean squared error between the estimates and true value were then compared. In all cases, data from the same drill-hole as the estimate being made was excluded as is typical in cross-validation studies.

The first case study was a low-grade porphyry copper deposit. This data set includes copper data from 134 drill-holes with an average spacing of 100 m. The vertical copper data was composited into 3 m sections. The copper assays are moderately skewed with a mean of 0.25% copper and standard deviation of 0.27%. The variograms for this deposit are approximately isotropic and well behaved with a nugget effect of 20% of the sill. Figure 1 plots the effect of the number of search data on simple and ordinary kriging results using this copper data. For a low number of search data, simple kriging performed better than ordinary kriging, but with a large number of search data, ordinary kriging was the better estimator.

The second case study is a set of bitumen data from the Athabasca oil sands. The percentage bitumen was sampled at 280 drill-holes with the vertical data composited into 3 m sections. Due to the stratified geology of the oil sands, this deposit displays very strong vertical to horizontal anisotropy of approximately 150:1. The bitumen histogram is approximately uniform with a mean of 7.7% bitumen. As with the copper case study, ordinary kriging performed better than simple kriging for a large number of search data (Figure 2).

The third and final case study considered was a zinc deposit with zinc assays from 367 drill-holes. For this deposit, the metal assays were skewed, and there was a moderate amount (approximately 3:1) of anisotropy between the horizontal and vertical directions observed in the variograms. For this deposit, the mean squared errors for ordinary or simple kriging with large numbers of search data were comparable (Figure 3).

**Figure 1—Effect of the number of search data on simple and ordinary kriging results with low-grade copper porphyra data**
Checks and measures of performance for kriging estimates

Metrics

Given the large number of choices that must be made by the practitioner, we are motivated to consider measures and metrics of how kriging performs. Clearly, the number one measure is the minimized mean squared error variance (kriging variance); however, if search parameters are not used there will be less emphasis on local stationarity. In such cases other measures related to conditional bias and departure from theoretical optimality should be considered.

Slope of regression

Consider cross-plotting the unknown true value of a random variable $Z$ with estimation volume $V$ against the known estimate of the random variable $Z'$ on the same volume $V$. The regression of the true values given the estimate is an indication of the conditional bias in the estimate (Rivoirard, 1987). Specifically, the slope of regression, $b$, approximates the conditional bias of the kriging estimate:

$$E(Z' \mid Z' = z) \approx a + bz$$  \[8\]

If the bivariate relationship between the truth and estimate was Gaussian, then the linear regression would be exactly the conditional expectation. Even in non-Gaussian settings, the slope of regression is a reasonable approximation of the conditional bias. The slope of regression is calculated using the estimation weights:

$$b = \frac{\sum_{i=1}^{n} \lambda_i \bar{C}(v_i, V)}{\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j C(v_i, v_j)}$$  \[9\]

For simple kriging, the slope of regression is exactly 1. In the case of ordinary kriging where the Lagrange multiplier is used, the slope of regression is generally less than 1. Given this, we could say that ordinary kriging is conditionally biased unless the slope of regression is equal to 1.

Kriging efficiency

Kriging efficiency was first introduced by Krige (1997) as a measure of the efficiency of block estimates. The kriging efficiency is expressed as the kriging variance ($\sigma^2_K$) normalized by the variance of the true blocks ($\sigma^2$) as a percentage (Equation [10]). For block kriging, we consider the estimates to be made on some volume $V$, and the corresponding variance of the blocks is equal to the average covariance between points within the blocks: $C(V, V)$. We express Krige's kriging efficiency, $KE_{DK}$, subscripted by Krige's initials:

$$KE_{DK} = \frac{\sigma^2 - \sigma^2_K}{\sigma^2} \times 100\%$$  \[10\]

A high efficiency means that the kriging variance is low, and the variance of the block estimates is approximately equal to the variance of the true block values. A low efficiency implies a high kriging variance relative to the block variance. The kriging variance varies from block to block, so the kriging efficiency will vary as well. There are a number of limiting cases discussed by Krige (1997). For perfect valuations, the efficiency is 100%:

$$\sigma^2_k = 0, \sigma^2_{z'} = \sigma^2 \text{ and } KE_{DK} = \frac{\sigma^2 - 0}{\sigma^2} = 100\%$$  \[11\]

Where all blocks are valued at the global mean (global estimate of all blocks is the only estimate made):

$$\sigma^2_{z'} = 0, \sigma^2 = \sigma^2 \text{ and } KE_{DK} = \frac{\sigma^2 - \sigma^2}{\sigma^2} = 0\%$$  \[12\]

With no conditional bias for imperfect valuations:

$$\sigma^2_{z'} = \sigma^2 - \sigma^2_K \text{ and } KE_{DK} = \frac{\sigma^2 - \sigma^2}{\sigma^2} = \frac{\sigma^2_{z'}}{\sigma^2}$$  \[13\]

With a conditional bias for imperfect valuations:

$$\sigma^2_{z'} > \sigma^2 - \sigma^2_K \text{ and } KE_{DK} = \frac{\sigma^2 - \sigma^2}{\sigma^2} < \frac{\sigma^2_{z'}}{\sigma^2}$$  \[14\]

Krige also notes that the efficiency can be negative if the kriging variance is greater than the true block variance. When the estimation variance exceeds the block variance, Krige deems this a kriging anomaly and states that valuing the block with the mean would be more efficient assuming we know the mean accurately.
Checks and measures of performance for kriging estimates

Statistical efficiency

Efficiency is a measure of the relative amount of effort to accomplish a task. If a different process can accomplish the same task with less effort, then it is more efficient. In statistics, the efficiency of a statistical quantity is defined differently depending on the property. The three most commonly considered are the efficiencies of (1) an estimator, (2) an experimental design matrix, and (3) a hypothesis test. For an unbiased estimator, the efficiency is defined as the minimum possible estimation variance divided by the variance of the estimator (Fisher, 1922). The minimum possible estimation variance is determined by the sample size; the minimum possible estimation variance will change depending on whether we have 10, 100, or 1 million samples. This minimum possible estimation variance is given by the Cramér-Rao bound, which states that the minimum possible variance of the estimator is the inverse of the Fisher information matrix (Rao, 1945). The most efficient estimator (if one exists) will have an efficiency of 1. Less efficient estimators will have efficiencies in [0,1).

Kriging's definition of kriging efficiency differs from the classical definition in that the kriging efficiency is a measure of the variance in the true value that is not represented in the kriged estimate. This is more akin to a goodness-of-fit or proportion of variance explained. The absolute minimum variance would be the global simple kriging variance. If the definition of efficiency proposed by Krige were to be reworked as a statistical measure of efficiency, it should incorporate the global simple kriging variance which is the absolute minimum kriging variance. If the kriging variance is equal to the global simple kriging variance, then the estimator is efficient and will have an efficiency of 1. If the kriging variance is larger than the global simple kriging variance, then the efficiency will be less than 1. This could also be expressed as a percentage. The efficiency will vary depending on the block being estimated and data available. Although global simple kriging is the lowest variance linear estimator, there may exist lower variance nonlinear estimators. Restricting the definition of efficiency to linear, unbiased estimators, a new definition for the kriging efficiency is:

\[ KE(u) = \frac{\sigma^2_{\text{OK}}(u)}{\sigma^2(u)} \]  \[15\]

Global simple kriging is the most statistically efficient unbiased linear estimator. It does, however, force the model to rely heavily on a strong global stationarity assumption, which is unrealistic in many cases. For example, issues with an increase in the mean squared error and decrease in the actual slope of regression were encountered by Boyle (2010) when a very large search area was used. The reason for using ordinary kriging is to obtain a local estimate of the mean at the expense of introducing some conditional bias by limiting the search, and therefore reduce reliance on the assumption of a global stationarity mean throughout the whole domain.

Constraining the estimator will introduce a conditional bias. Constrained estimators are sometimes used in statistics because it is possible that an estimator with a small bias will have a smaller mean squared error (the mean squared error is equal to the variance plus the square of the bias). To compare efficiencies, the ratio of efficiencies is taken. This gives a measure of how much more efficient one estimator is compared to a second estimator.

For example, kriging with a more restrictive search radius will decrease the relative efficiency compared to kriging with a large search radius. The ratio of efficiencies would give a measure of how much variance is being incurred to decrease reliance on the global stationarity assumption. Ultimately, knowledge of how much the decision of stationarity could be relied upon would be necessary to determine an acceptable efficiency level (McLennan and Deutsch, 2004).

Application of kriging metrics to case studies

Recall the three case studies introduced earlier: (1) a low-grade copper porphyry deposit, (2) an oil sands deposit, and (3) a zinc deposit. We can consider calculating the kriging metrics discussed for the same range of search data. For simplicity, the global simple kriging variance is approximated using 100 search data. This approximation was found to be very reasonable with the data-sets.

Kriging metrics were calculated for the copper porphyry case study (Figure 4). The mean slope of regression, Krige’s efficiency, and kriging efficiency are each given as the mean of the calculated theoretical values. The mean weight given to the simple kriging is also plotted for reference. The shape of Krige’s efficiency, the theoretical slope of regression, and kriging efficiency are all similar, with the exception of the simple kriging slope. Recall that for simple kriging, the slope of regression will always be 1. A key aspect of this plot is that as the number of search data increases, the mean kriging efficiency (using the measure introduced in this paper) for both ordinary and simple kriging asymptotically approaches 1. This is because the estimator variances are approaching that of the theoretically best linear estimator: global simple kriging. This contrasts with what is observed for Krige’s efficiency, which plateaus around a value of 0.3. This is because the kriging variance is not being compared to the best possible case but to the block variance instead.

For the oil sands case study, the kriging metrics follow a similar shape (Figure 5), although with a distinct elbow at the point (10-20 search data) where the cross-validated mean squared errors displayed an elbow (Figure 2). This elbow indicates that any additional search data used receive only very small weights.

Figure 4—Effect of the number of search data on the kriging efficiency and theoretical slope of regression for the copper case study
Checks and measures of performance for kriging estimates

Scatter plots of the estimated values versus the true values are shown for simple and ordinary kriging with 10 and 90 data in Figure 6. As expected, the observed conditional bias for simple kriging with any number of data is very low, as is the conditional bias with ordinary kriging and a large number of data. Note that the metrics presented here, including the slope of regression, are not the average of the theoretical values as they are in Figure 5, but are calculated using the known true values from cross-validation. As the number of search data used for simple kriging increases, the average weight to the simple kriging mean decreases, resulting in an increased standard deviation of the estimates.

Finally, the effect of the number of search data on kriging metrics was quantified for the zinc case study (Figure 7). As with the mean squared error, it can be observed that the simple kriging and ordinary kriging efficiencies quickly converge. In this test case, the slope of regression exceeds 1 for ordinary kriging; it is not common for the theoretical slope of regression to exceed 1, but this has been documented in the literature (Boyle, 2010).

Discussion

There are a number of useful observations that can be gathered from the test cases. Increasing the number of search data will generally decrease the mean squared error for simple and ordinary kriging. This is contrary to the notion that the search should be restricted to relax the assumption of global stationarity and make better estimates. Increasing the number of search data for ordinary kriging will implicitly increase the accuracy of the estimate of the local mean and improve the estimates made. In the case studies examined here, using a large number of data resulted in estimates with a lower mean squared error when examined with cross-validation. For the majority of the deposit where the truth is not known, kriging metrics, including kriging efficiency and the slope of regression, provide a reasonable proxy for assessing the departure from theoretical optimality. If increasing the number of search data results in a large increase in kriging efficiency, or the slope of regression, then increasing the number of search data will also be likely to reduce the mean squared error of the estimate. This is observed when the effects of the number of search data on the mean squared error and kriging metrics are compared for each of the case studies.

In both the oil sands and copper case studies, ordinary kriging with a large number of search data resulted in a lower observed mean squared error compared with simple kriging. In the zinc case study, the results of ordinary and simple kriging with a large number of data were comparable. Theoretically, simple kriging will always result in a lower mean squared error compared with ordinary kriging. The better performance of ordinary kriging relative to simple kriging is attributed to better estimation of a local mean compared to the global mean used by simple kriging. If the mean is not globally stationary in the domain, then using a local stationary mean with ordinary kriging will result in better estimates. This is supported by the plots of the mean weight given to the simple kriging mean in the case studies (Figures 4, 5, and 7). In the copper (Figure 4) and oil sands
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(Figure 5) case studies, the simple kriging mean receives weights of up to 0.1 when a very large search neighbourhood is used. In these cases simple kriging was outperformed by ordinary kriging. Conversely in the zinc case study (Figure 7), the simple kriging mean receives a negligible weight when a large search neighbourhood is used. In this case the results of simple and ordinary kriging were almost identical. Using a very large number of search data resulted in a very accurate estimate of the local mean. Increasing the number of search data did not decrease the accuracy of this estimate, as very distant data points received negligible weight.

Increasing the number of search data had the expected effect on the kriging metrics; it decreased the conditional bias, which was accessed through the slope of regression, and increased the kriging efficiency as the kriging variance approached the global simple kriging variance. When the number of search data is restricted to reduce smoothing and produce estimates that more closely match with the histogram predicted by volume-variance relationships, a conditional bias will be introduced into the estimates. For the purposes of estimating resource and reserves, this conditional bias is generally accepted since the cost of using a very smooth kriged map is often drastically under- or over-estimating of reserves unless the estimates are post-processed with a local conditioning approach such as uniform conditioning.

Ultimately, the kriging search strategy depends on the estimate type. Estimate types can be broadly classified as (1) visualization and geological understanding, (2) interim estimates, or (3) final estimates. When the goal is visualization and gaining a level of geological understanding, a very smooth map is desired since this is easier to understand. For this purpose global simple kriging (to prevent any search artifacts) could be considered. An alternative approach would be to use local kriging with a smoothing method (Gribov and Krivoruchko, 2004; Rivoirard and Romary, 2011) applied.

Interim estimates are used for long-term planning. The goal is to anticipate the information effect for the future and consider volume variance relations. For interim estimates a restricted search to increase the variance of the estimates could be considered. This increases understanding of what variability can be expected in the future. This approach is straightforward and commonly implemented, but not the only available approach. Emery (2006) compared the use of Monte Carlo integration with the point-support multigaussian model and the commonly used discrete Gaussian model for use with ordinary multigaussian kriging. Either method could be used as an alternative to evaluate ore deposit reserves without introducing a bias.

Final estimates come down to the decision of ore or waste. These decisions must be made with minimum conditional bias and should have a minimum mean squared error. From the results of the case studies presented, the practitioner should consider the use of ordinary kriging with a large number of search data. A series of cross-validation case studies could be done with the data-set to confirm that this was the best approach. These results are similar to the results obtained by Boyle (2010), who demonstrated that a large number of samples increased the accuracy of estimates, although this improvement was marginal. Boyle emphasized that dividing the area into suitable domains is likely to be far more important than the selection of the precise number of data to use when estimating, a suggestion that we echo here. The incorporation of additional data to decrease estimation error and conditional bias must be done within a stationary domain.

Conclusions

There are a number of useful kriging measures, including Krige’s efficiency, the slope of regression, and the statistically-based kriging efficiency proposed in this paper. The proposed kriging efficiency considers the estimation variance compared to the global simple kriging variance, which is the theoretically minimum possible estimation variance. In the case studies considered, the lowest mean squared error in cross-validation was normally for ordinary kriging with a very large number of search data. In this case, the kriging efficiency was close to 1. The kriging metrics used in this paper can be used to ascertain the departure from theoretical optimality of a kriging implementation. If the kriging efficiency or slope of regression is significantly lower than 1, then the mean squared error and conditional bias will likely be high. This may be acceptable if the goal of the estimate is to assess the proportion of the deposit above a certain cut-off grade, in which case the smoothing introduced by using a very large number of search data may be unacceptable. Ultimately, the decision of how many search data to use and search strategy depends on the type of estimate being made. The kriging metrics and guidelines introduced in this paper should assist the practitioner in making these decisions.

References

GARDON, L.S. 1965. Objective analysis of meteorological fields. (translated from Russian), Israel Program for Scientific Translations for the US Department of Commerce and the National Science Foundation.
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Validity range of the discrete Gaussian change-of-support model and its variant
by J.-P. Chilès*

Introduction
In many situations decision-makers are faced with selectivity: extract ore above some cutoff grade, remediate soils whose pollution exceeds some threshold, restrict traffic speed when the ozone concentration exceeds some limiting value. In each case, data relates to a small support (e.g., a core, an air pollution sensor) that we regard as a point, whereas the decision is taken for a much larger support – selective mining units, decontamination units, average ozone concentration across a city in a given time interval (typically with a low threshold applied to the daily average concentration and a higher threshold for an hourly average concentration). To predict the effect of selectivity it is necessary to take into account that the grade or concentration distribution becomes less dispersed as the support becomes larger. In the framework of random function theory, the result depends on the whole spatial distribution. It can be obtained by means of Monte Carlo simulations (non-conditional simulations in the global case, conditional simulations in the local case). Even if Monte Carlo simulations are more accessible now than in the past, there are always situations where they require excessively intensive computation. It is therefore useful to have access to the approximate solutions provided by change-of-support models. Many such models have been developed by Georges Matheron (a synthesis is given in chapter 6 of Chilès and Delfiner, 2012). The most popular model is the discrete Gaussian model. It should not be used in just any situation because it has been developed for random functions such as transforms of stationary Gaussian random functions. The initial model was proposed in 1976 by Matheron (1976a). A simpler variant was proposed more recently by Emery (2007). We investigate here the accuracy of this change-of-support model for the modelling of the marginal distribution of block values. This investigation is carried out in the special case of lognormal random functions, which constitute a large and important class of random functions, studied notably by Krige (1978).

We first recall the assumptions of the discrete Gaussian model (DGM) and its variant, and explain why they lead to approximations to the true solution. We then describe the principle of the validation method. Finally, we explore the validity range of both models depending on space dimension, variance and covariance function of the logarithmic variable, and support size. In the conclusion we give some indications on the modelling of the local block distribution.

The discrete Gaussian model
Let us consider a stationary random function (SRF) $Z(x)$ that can be expressed as the transform of an SRF $Y(x)$ with standard normal marginal distribution. It is therefore of the form $Z(x) = \phi(Y(x))$ with the transformation function $\phi = F^{-1}G$, where $F$ is the marginal cumulative distribution function (c.d.f.) of $Z(x)$ and $G$ the standard normal c.d.f. Similarly, we can consider that the mean grade $Z(v)$ of the block $v$ is of the form

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\( \varphi_v(Y_v) \) where \( Y_v \) is a standard normal random variable and \( \varphi_v \) the block transformation function that we want to determine (because we are here interested only in determining \( \varphi_v \), we consider a single block \( v \) but assume strict stationarity of \( Y \) to ensure that \( \varphi_v \) does not depend on the block location).

**Model DGM1**

Consider a uniform random point \( x \) within the block \( v \). The random variable \( Z(x) \) has for c.d.f. the marginal distribution \( F \) of the SRF \( Z(\cdot) \) and can be expressed as the transform \( \varphi(\text{Y}(x)) \) of the random variable \( Y(x) \). The basic assumption of the discrete Gaussian model proposed by Matheron (1976a), hereafter referenced as model DGM1, is that the bivariate distribution of the \( \{Y(x), Y_v\} \) pair is Gaussian, with a positive correlation coefficient \( r \). Matheron deduces from Cartier’s relationship that the block transformation function \( \varphi_v \) is given by

\[
\varphi_v(y_v) = \int \varphi(r_y + \sqrt{1-r^2}y) f(y) dy \quad [1]
\]

This defines the distribution of \( Z(v) \).

In practice \( \varphi \) is expressed through its Hermite polynomial expansion

\[
\varphi(y) = \sum_{n=0}^{\infty} \varphi_n \chi_n(y) \quad [2]
\]

where the \( \chi_n \) are the normalized Hermite polynomials (see, e.g. Chilès and Delfiner, 2012, Appendix A.5), and the coefficients \( \varphi_n \) are given by

\[
\varphi_n = \int \varphi(y) \chi_n(y) G(dy) \quad [3]
\]

Relationship [1] then implies that \( \varphi_v \) can be expressed in the form

\[
\varphi_v(y_v) = \sum_{n=0}^{\infty} \varphi_n r^n \chi_n(y_v) \quad [4]
\]

The variance of \( Z(v) \) can be derived from the coefficients \( \varphi_n^2 \) of the expansion of \( \varphi_v \) or from the covariance \( C(h) \) of the SRF \( Z \). For consistency, \( r \) is obtained by equating these two expressions, that is

\[
\sum_{n=0}^{\infty} \varphi_n^2 r^{2n} = \frac{1}{|v|^2} \int \int C(x-x') dx dx' \quad [5]
\]

Considered as an equation in \( r \), Equation [5] has a unique solution between 0 and 1. The correlation coefficient \( r \) is called the change-of-support coefficient.

Extensions of the model (not considered here) enable the local estimation of a block by conjunctive kriging or in a multivariate Gaussian framework. A step further makes it possible to estimate the ore tonnage and metal quantity that will be obtained when selecting blocks on the basis of future grade estimates (once blast-holes become available). This is the so-called information effect. On these points, see Matheron (1976a) and Chilès and Delfiner (2012, pp. 455–466).

**Model DGM2**

The variant DGM2 proposed by Emery (2007) is simpler but requires the additional assumption that the bivariate distribution of \( Y(y) \) and \( Y(x') \) for two independent random points within the same block \( v \) is Gaussian. In that case, Emery shows that \( r^2 \) is the variance of the average \( Y(v) \) of \( Y(v) \) in the block \( v \):

\[
r^2 = \frac{1}{|v|^2} \int \int \rho(x-x') dx dx' \quad [6]
\]

where \( \rho(h) \) is the covariance (here a correlogram) of the SRF \( Y(\cdot) \). Moreover, \( Y_v \) is simply the average \( Y(v) \) rescaled to a unit variance by the change-of-support coefficient \( r \):

\[
Y_v = Y(v) / r \quad [7]
\]

This induces large simplifications in the extensions of the model to local estimation (with and without information effect), notably in the framework of a multivariate Gaussian assumption (Emery, 2005, 2008; Chilès and Delfiner, 2012, pp. 455–466).

**Discussion of the assumptions**

Do there exist random functions satisfying the assumptions of models DGM1 and/or DGM2? It is easy to simulate Gaussian samples with pair-wise correlation \( r \), thus corresponding to samples of \( Y \) with independent random locations in the block \( v \) (see Emery, 2007). However, the author is not aware of any random function model for fixed locations leading to such correlations for random locations in \( v \). Should such a model exist, it would be specific to that support \( v \). The above assumptions should therefore be considered as approximate only.

Let us first consider the situation where \( Z \) is a Gaussian SRF, that is, \( \varphi \) amounts to an affine transformation (in that case we have no need of the discrete Gaussian model, but it is interesting to see what it would mean). Models DGM1 and DGM2 yield the same value for \( r \) because \( C(h) \) is proportional to \( \rho(h) \), and relationship [7] is exact. Since \( x \) is random in \( v \), the bivariate distribution of \( Y(x) \) and \( Y_v \) is a mixture of standard bivariate normal distributions with correlations

\[
\omega(x) = \frac{1}{r|v|} \int \int \rho(x-x') dx' \quad [8]
\]

for \( x \) having any possible location in \( v \). The average value of \( \omega(x) \) when \( x \) scans \( v \) is \( r \), so that the average correlation between \( Y(x) \) and \( Y_v \) is \( r \) but, depending on \( r \), it can be an average of very different values. A mixture of such bivariate Gaussian distributions is not a bivariate Gaussian distribution: it is a Hermitian bivariate distribution (Matheron, 1976b; see also Chilès and Delfiner, 2012, pp. 418 and 423). In the one-dimensional case, that is, when \( v \) is a segment of length \( L \), and for an exponential covariance with unit sill and with scale parameter \( a \), the change-of-support coefficient \( r \) and the function \( \omega(x) \) are respectively given by

\[
r^2 = \frac{2a^2}{L^2} \left[ \exp \left( -\frac{L}{a} \right) - 1 + \frac{L}{a} \right]
\]

\[
\omega(x) = \frac{1}{r} \int \frac{2 - \exp \left( -\frac{x}{a} \right) - \exp \left( -\frac{L-x}{a} \right)}{L^2} dx
\]
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\[ \omega(x) \] is minimal at the extremes (\( x = 0 \) or \( L \)) and maximum for \( x = L/2 \). The contrast between the largest correlation and the smallest one is

\[ \frac{\omega(L/2)}{\omega(0)} = \frac{2}{1 + \exp\left(-\frac{1}{2} \frac{L}{a}\right)} \]

This ratio is equal to 1 for \( L/a = 0 \), remains close to 1 when \( L/a \) remains moderate (with a value of about 1.245 for \( L/a = 1 \)), and increases to 2 when \( L/a \) increases to infinity. Figure 1 shows the graph of \( \omega(x) \) for some values of \( L/a \). The approximation of a constant correlation \( \omega(x) \) thus seems to be acceptable when \( L/a \) is not too large.

The situation is more critical for the bivariate distribution of \( Y(x) \) and \( Y(x') \) because it is a mixture of standard bivariate Gaussian distributions with correlations \( \rho(x-x') \), thus ranging from 1 when \( x = x' \) to \( \rho(L) \) when \( x \) and \( x' \) are at opposite corners of the block \( v \) and \( L \) denotes their separation vector (we assume here isotropy and monotonicity of the correlogram). In the above one-dimensional case, \( L \) is the length of the segment and this minimum correlation is equal to \( \exp(-L/a) \): it can be very small if \( L \) is large with respect to \( a \).

When \( Z \) is not Gaussian, the transformation function \( \phi \) is not linear. The variogram of \( Z \) is therefore not proportional to the variogram of \( Y \). Note that the change-of-support coefficient of model DGM2 depends only on the variogram of \( Y \) and does not depend on the transformation \( \phi \). This is not the case for model DGM1, because the covariance \( C \) of \( Z \) depends also on the transformation \( \phi \):

\[ C(h) = \sum_{n=1}^{\infty} \phi(n) \rho(h)^n \]

The correlograms \( \rho(h)^n \) are less and less structured when \( n \) increases. For example, if \( \rho(h) \) is an exponential covariance with scale parameter \( a \), \( \rho(h)^n \) remains exponential, but with scale parameter \( a/n \), and therefore tends to a pure nugget effect when \( n \) tends to infinity. As a consequence, the change-of-support coefficient \( r \) of model DGM1 is larger than that of model DGM2, with a difference that increases as the block \( v \) becomes larger and the terms \( \phi(n) \) with large \( n \) dominate in the development of the transformation function \( \phi \). The efficiency of model DGM2 will be similar to that of model DGM1 when the block size is small and the \( \phi(n) \) decrease rapidly with \( n \).

Note that the change-of-support coefficient \( r \) cannot simultaneously satisfy relationships [5] and [6] (except if \( Z \) is bivariate Gaussian). Unlike model DGM1, model DGM2 therefore does not preserve the variance of \( Z(x) \) expressed in the right-hand side of relationship [5], which is an important parameter of the block distribution. This may be critical if the ratio \( r_1/r_2 \) of the coefficients \( r \) provided by models DGM1 and DGM2 is significantly larger than 1.

Validation of the discrete Gaussian model

We will check the validity of the assumptions of the discrete Gaussian models for modelling the marginal block distribution. This corresponds to the so-called global change-of-support, in contrast with the local change-of-support which consists of predicting the distribution of block values conditional on data available in the block and its neighbourhood.

The result depends on the spatial distribution of the random function \( Z \) and we consider the ideal situation where this spatial distribution is known. In practice, we will consider the case where \( Z \) is a lognormal SRF (that is, its logarithm is a Gaussian SRF). We can assume without lack of generality that it has a unit mean. It is then characterized by its logarithmic standard deviation \( \sigma \) and by the correlogram \( \rho(h) \) of its logarithm. The random function \( Z(x) \) is thus of the form

\[ Z(x) = \exp\left(\sigma Y(x) - \frac{\sigma^2}{2}\right) \]

It can also be expressed as

\[ Z(x) = \sum_{n=0}^{\infty} (-1)^n \frac{\sigma^n}{\sqrt{n!}} \chi_n \left(Y(x)\right) \]

which corresponds to an expansion of the transformation function \( \phi \) with

\[ \varphi \equiv (-1)^n \frac{\sigma^n}{\sqrt{n!}} \]

The \( \varphi(n) \) decrease rapidly when \( n \) increases if \( \alpha \) is small, but less rapidly if \( \alpha > 1 \). For \( \alpha = 3 \), for example, \( \varphi(n) \) increases up to \( n = 8 \) and then decreases slowly. The logarithmic assumption thus includes very contrasted behaviours. This can also be seen on the variance of \( Z \): it is given by

\[ \sigma^2 = e^{\alpha^2} - 1 \]

Since \( Z(x) \) has unit mean, its coefficient of variation is close to \( \alpha \) when \( \alpha \) is small (0.55 for \( \alpha = 0.5 \)) but takes large values when \( \alpha \) is large: 1.31 for \( \alpha = 1 \), 7.32 for \( \alpha = 2 \), 90.01 for \( \alpha = 5 \). A logarithmic standard deviation as large as 5 is far beyond what is seen in mining applications (such a value has been reported for permeability in hydrogeology (e.g. Zimmerman et al., 1998) but the main variable in that case is log-permeability rather than permeability). When dealing with ore grade or pollution concentration, we are of course interested in \( Z(x) \) itself rather than its logarithm, and such a logarithmic standard deviation would be extreme. Would it correspond to an actual situation, we would be in a very uncomfortable position because a simple parameter such as the arithmetic mean of \( Z \) would require a large number of observations to be estimated reliably. Of course, it is possible...
Validity range of the discrete Gaussian change-of-support model and its variant

to deduce the mean of \( Z \) from the mean and the variance of 
\( \log Z \), but then we rely heavily on the assumption of a
lognormal distribution, which cannot be taken for granted.

A specificity of lognormal SRFs is that DGM models lead
to a lognormal distribution for \( Z(v) \), with logarithmic variance 
\( r^2 \sigma^2 \): This is the well-known permanence of lognormality. It
can be shown easily by application of Equations [1] or [4].

Another specificity is that the correspondence between the
coefficient \( C(h) \) of \( Z \) and the correlogram \( \rho(h) \) of \( Y \) takes the
simple form

\[
C(h) = \exp \left( \sigma^2 \rho(h) \right) - 1,
\]

which facilitates the computation of the right-hand side of
Equation [5].

Validation method

Several authors have already checked the validity of model
DGM1 or DGM2 in the lognormal case (Matheron, 1981;
Cressie, 2006; Emery, 2007; Chilès and Delfiner, 2012). The
principle is to compare the block transformation function
obtained with the DGM model with the ‘exact’ block transformation
function – and the correlogram \( \rho(h) \) of \( Y \). The
results of this check were limited to the one-dimensional case with an
exponential covariance (the random function is then a
Markov random process), and used a limited number of
simulations for \( \sigma \) ranging from 0.01 to 1.0, with
a slight bias for \( \sigma > 2 \) when \( \sigma = 1 \), and a significant bias when
\( \sigma = 0.1, 1, \) and 10.

Note that this approach, fully similar to that of Matheron
(1981), is slightly different from that used by Emery (2007)
to check model DGM2. Indeed, Emery simulated standard
Gaussian values with pairwise correlations all equal to 1, thus
corresponding to the values of \( \rho \) at independent random
points in \( v \).

Results

We focus on a spherical correlogram for \( Y \), and on three
contrasted block sizes \( L \) with respect to the range of \( a \) of
the correlogram: \( L / a = 0.1, 1, \) and 10.

When \( L / a = 0.1 \), the change-of-support coefficient \( r_2 \) of
model DGM2 is close to 1 (0.980 in 1D, 0.951 in 2D, 0.933 in
3D) and the coefficient \( r_1 \) of model DGM1 remains very close
to \( r_2 \) even for a large \( \sigma \) value (for \( \sigma = 3 \) we obtain 0.981 in
1D, 0.955 in 2D, 0.938 in 3D). Both models lead to similar
block distributions, very close to the true one (see Chilès and
Delfiner, 2012, p. 454 for the 2D case with \( \sigma = 1.5 \)).

When \( L / a = 1 \), the coefficient \( r_2 \) is equal to 0.791 in 1D,
0.59 in 2D, and only 0.46 in 3D. Moreover, \( r_1 \) increases
significantly with \( \sigma \) when \( \sigma \) exceeds 1, as can be seen in
Figure 2.

Figures 3 and 4 show the results obtained with 100 000
simulations for \( \sigma = 1 \) and 2 respectively. Model DGM1 quite
perfectly reproduces the true transformation function – and
thus the block distribution. Model DGM2 gives good results
as far as large values are not considered but comes with a
slight bias for \( \sigma > 2 \) when \( \sigma = 1 \), and a significant bias when
\( \sigma = 2 \).

When \( L / a = 10 \), the coefficient \( r_2 \) is equal to 0.271 in
1D, 0.077 in 2D, and only 0.022 in 3D, and like in the
preceding case \( r_1 \) can have much larger values when \( \sigma \)

Note that we do not consider the average value at \( M \) grid
nodes as an approximation to the average in a block \( v \) of the
\( d \)-dimensional space: we substitute the problem in the
discrete space to the problem in the continuous case, so that
there is no approximation in the approach. The simulations
are generated with the discrete spectral method, which
produces perfectly Gaussian simulations (up to the quality of
the pseudo-random number generator). If it were not possible
to exactly reproduce the desired covariance for \( Y \), we would
replace it by a covariance as close as possible to it and check
this model (this is likely to occur with Gaussian covariances,
for example). Note that an alternative would be to build
simulations based on covariance matrix decomposition (e.g.
Chilès and Delfiner, pp. 493–494); this method also produces
perfectly Gaussian simulations, meets any covariance model,
but has stronger limitations than the discrete spectral method
in terms of size of the grid. For consistency, the integrals in
Equations [5] and [6] are replaced by discrete sums. We use
a large number of simulations (up to 100 000).

Figure 2—Graphs of the change-of-support coefficient of model DGM1
for \( L / a = 1 \) as a function of the logarithmic standard deviation \( \sigma \) in the
1D, 2D, and 3D cases. The value at the origin coincides with the
change-of-support coefficient of model DGM2, which does not vary
with \( \sigma \).
Validation range of the discrete Gaussian change-of-support model and its variant

Figure 3—Validity of the DGM approach for a 2D block with size $I$, equal to range $a$ and a logarithmic standard deviation $\sigma_L = 1$: ‘True’ block transformation function $\phi_v$ (determined from 100 000 simulations) and approximations provided by models DGM1 and DGM2.

Figure 4—Validity of the DGM approach for a 2D block with size $I$, equal to range $a$ and a logarithmic standard deviation $\sigma_L = 2$: ‘True’ block transformation function $\phi_v$ (determined from 100 000 simulations) and approximations provided by models DGM1 and DGM2.

exceeds 1. Model DGM1 presents a slight bias at the extremes of the distribution, whereas DGM2 is biased everywhere but close to the median (see Chiles and Delfiner, 2012, p. 454 in the 2D case with $\sigma = 1.5$).

Conclusion

This validation exercise shows that the original DGM1 model of Matheron (1976a) gives a very good approximation to the true block distribution, except for extreme values when the logarithmic standard deviation is very large. The variant DGM2 of Emery (2007) is also a very good approximation to the true distribution provided that we are in one of the following situations: (i) the block size is small with respect to the range, (ii) the logarithmic standard deviation is not too large, or (iii) we are not interested in the distribution of high grades. In such a case, this variant can be applied safely. Application of the models, and especially of DGM2, should be done carefully with highly skewed grade distributions and/or very large blocks or panels.

Further checks should be carried out. The approximation of the DGM models has to be quantified also in 1D (time series) and in 3D (the approximation is less valid as the space dimension increases). It is also interesting to examine other covariance models for $\rho(h)$. When $\sigma$ is fixed, the solution provided by DGM models depends only on $r$, but two covariance models that would give the same $r$ value do not necessarily give the same true block distribution. Finally, the presence of a nugget effect extends the validity of DGM models, but this has to be quantified.

We have addressed only the global change-of-support problem. The local change-of-support (prediction of the block distribution conditional on neighbouring data) has been examined by Cressie (2006), who considers unbiased lognormal estimators that are exponentials of the simple or ordinary kriging estimators of $Y(v)$, which amounts to the assumptions of model DGM2. The experiment compares this ordinary lognormal kriging with the optimal solution provided by conditional expectation, obtained by a Monte Carlo method. The results indicate that ordinary lognormal kriging performs well in situations where the block size is small with respect to the range, the lognormal standard deviation is not too large, and the neighbourhood is sparse. The first two conditions are required for the global model to be efficient. The third expresses the fact that conditional expectation makes better use of numerous data than an estimator whose form is limited to the exponential of a linear combination of the logarithms of the data.

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Bibliography


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Genetic algorithms and scenario reduction

by M. Armstrong*, A. Vincent*†, A. Galli*, and C. Méheut†

Synopsis
Scenario reduction is designed for selecting a representative subset of geostatistical simulations out of a much larger set. Three steps are involved: measuring the dissimilarity between two simulations; finding a metric to measure the distance between any subset of \( k \) simulations and the full set of \( N \) simulations; and finding an efficient algorithm for selecting the subset that minimizes the metric. This paper focuses on the third question. We show that genetic algorithms are an efficient way of approaching the minimum when the population of subsets to be sampled is large. Two case studies based on the Walker Lake data-set are presented: firstly choosing \( k=4 \) simulations out of a total of 100, and secondly choosing 20 out of the same 100 simulations.

In the first case it was possible to compute all possible combinations exhaustively and hence to demonstrate that the algorithm converges to the true minimum. This was not possible in the second case. Instead we demonstrate that it outperforms the random draw algorithm used in earlier work. A procedure for tracking individual selections during the iterative procedure was developed. This allows us to measure the evolution in the percentage of progeny resulting from crossing-over and from mutation that survived in the next generation. It gives valuable insight into how to choose the parameter values in the algorithm. Another key finding is that there is a trade-off between the number of individuals per generation and the number of generations required for the algorithm to converge.

Keywords
genetic algorithms, crossing-over, mutants, Walker Lake.

Introduction
Nowadays many more conditional simulations of orebodies, reservoirs, and aquifers can be generated than in the past. In some applications it is possible to post-process all of them but in others this is impossible. For example, in mining, pit optimization and scheduling are computer-intensive and time-consuming; similarly for fluid flow simulations and production optimization in the oil industry. When only a certain number of the conditional simulations (\( k \), say) can be post-processed, the question is how to choose a representative set of that size out of the full set of \( N \) simulations. This process can be split into three steps:
1. Measuring the dissimilarity between two simulations
2. Finding a metric to measure the distance between the full set of \( N \) simulations and any given subset of size \( k \)
3. Finding an efficient algorithm for selecting the best subset (i.e. the one that minimizes the metric)

Armstrong et al., (2010, 2013) proposed using a metric denoted by \( D(J,q) \), based on the scenario reduction metric\(^2\) developed by Heitsch and Romisch (2009), together with a random search algorithm. The procedure gave very encouraging results when it was used to select subsets containing \( k = 10, 12, \) or 15 simulations out of a total of \( N = 100 \) simulations. More recently we have started working on cases in the oil industry with larger values of \( k \) and \( N \), where the total population of possible subsets is much larger. Table 1 gives the total numbers of subsets for different values of \( k \) and \( N \). For very small values of \( k \), the best strategy is to test all the subsets exhaustively. Otherwise an efficient search procedure is required. The difficulty in finding the subset that minimizes the metric in such a large discrete population is that standard gradient-based methods cannot be used. Genetic algorithms seem better suited to this.

\(^1\)The procedure we developed for measuring the dissimilarity between two simulations and the metric \( D(J,q) \) is summarized in Appendix 1.
\(^2\)The German research group led by Heitsch and Romisch specializes in the stochastic optimization of large systems, using multi-stage programming with recourse (e.g. for electricity prices or hydroelectric systems). A branching tree structure is used to model the evolution of prices over time or of the water input into dams. The problem is that the number of branches in the tree increases exponentially with time, and sooner or later the tree has to be pruned. As each of the price paths is called a scenario, the procedure for pruning the tree is called ‘scenario reduction’. In their work, Heitsch and Romisch recognized the fact that the tree is not perfectly known and took this into account when developing their scenario reduction metric \( D(J,q) \).

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This paper proposes an improved algorithm that uses genetic algorithms for selecting the best subset. The procedure is tested on two examples: firstly for subsets with $k = 4$ simulations out of a set of 100 simulations and secondly for subsets of $k = 20$ from the same set of 100 simulations. In the first case, as the metric can be computed exhaustively for all possible subsets, we were able to check whether the genetic algorithm found the true minimum or was trapped in a local minimum. In the second case, we compared the genetic algorithm with the random search procedure developed earlier.

The paper is structured as follows. In the next section we describe the genetic algorithm that we have used. In the following section we give background information on the 100 simulations that are used in both case studies. Then the first case study is presented. We show that the genetic algorithm effectively reaches the global minimum. We also show that there is a trade-off between the number of individuals in each generation and the number of generations required to converge. The more individuals per generation; the fewer generations are needed. The second case study is presented in the following section. Even with a total population of $5.36 \times 10^{20}$ combinations, genetic algorithms give much better results than those obtained with the random search procedure. Other cases with even larger sets of simulations are currently being studied. Our experience to date has shown that the speed of convergence in large cases depends on the choice of the parameter values used, namely, the number of individuals in each generation and the number of progeny obtained by crossing-over and by mutation. In order to understand the impact of these parameters on the convergence we developed a method for tracking individual selections over time. The survival rates of parents, progeny from crossing-over, from mutations of a single gene and of all the genes, from one generation to another, were evaluated. We show how this evolves over time.

The conclusions are given in the last section. For the sake of brevity, the procedure developed for measuring the dissimilarity between two simulations and the metric $D(J,q)$ is given in Appendix 1. One of the reviewers of the paper asked why we focus only on minimizing the metric $D(J,q)$ instead of comparing the values of the objective function computed using the full set of simulations rather than on a subset of them. This is a very good question. Heitsch and Romisch (2009) showed that minimizing the metric $D(J,q)$ is equivalent to minimizing metric (2) (Appendix 2), for all objective functions $f$ in a fairly wide class. So we only need to find the subset $Q$ that minimizes $D(J,q)$. A more detailed explanation is given in Appendix 2.

**Genetic algorithm used**

Sastry, Goldberg, and Kendall (2005) provide a good description of genetic algorithms, which were first invented in the mid-1970s (Holland, 1975), together with a review of developments up to 2003. Their usefulness in solving many difficult real-world applications has been demonstrated over the past 40 years; for example, for electric power systems (Valenzuela and Smith, 2002; Burke and Smith, 2000); for scheduling university courses (Paechter et al., 1995, 1996) and university examinations (Burke and Newall, 1999); for rostering nurses (Burke et al., 2001); for warehousing (Watson et al., 1999), and for scheduling sports (Costa, 1995) and machines (Cheng and Gen, 1997).

The algorithm we use is similar to that of Cao and Wu (1999). Each individual in the population is represented by a vector of length $k$. These vectors are considered as ‘chromosomes’ with $k$ genes. The algorithm selects two individuals in the population to be parents, and gets them to mate by crossing the chromosomes to produce ‘children’. Occasionally a spontaneous mutation occurs. As in natural selection, the fittest of the children are more likely to be the parents of the succeeding generation. The probability of an individual being chosen to be a parent depends on its fitness.

In our application the $k$ genes in each chromosome represent the numbers of the $k$ simulations to be kept. At the outset 10,000 individuals (vectors) were generated by drawing $k$ numbers at random between 1 and $N$. The fittest 1000 of these were then selected to reproduce. Here the fitness is defined as $1/D(J,q)$. The algorithm selects two individuals to be parents. The gene cross-over is carried out by picking a cross-over point $P$ at random between 1 and $k$ (included). The first $P$ genes from one parent are then joined to the $(k-P)$ genes from the other parent. Figure 1 illustrates a crossing-over.

Mutation is another important feature of natural selection and of genetic algorithms. A position within each chromosome is selected at random. A gene (a simulation number between 1 and $N$) is selected at random to replace the gene at that location. Figure 2 illustrates a mutation in which simulation no. 66 is mutated into simulation no. 29. The others remain unchanged. Figure 3 summarizes the procedure that we have programmed.

This procedure of mating and mutation is repeated.

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**Table I**

| Number of combinations of $k$ simulations selected from $N$ simulations without replacement |
|---|---|---|---|---|---|---|---|---|
| $k = 1$ | $k = 3$ | $k = 4$ | $k = 5$ | $k = 10$ | $k = 12$ | $k = 15$ | $k = 20$ |
| $N = 100$ | 100 | 1.62e+5 | 3.92e+6 | 7.53e+7 | 1.73e+13 | 1.05e+15 | 2.53e+17 | 5.36e+20 |
| $N = 700$ | 700 | 5.69e+7 | 9.92e+9 | 1.38e+12 | 7.30e+21 | 2.63e+25 | 3.12e+30 | 2.49e+38 |
Genetic algorithms and scenario reduction

Mutation Point  \[ \rightarrow \]
Parent \[ [31 \ 18 \ 66 \ 99 \ 85] \]  \[ \Rightarrow \]  \[ [31 \ 18 \ 29 \ 99 \ 85] \]  Child

Figure 2—Mutation. A gene chosen at random is changed to another randomly selected gene. Here simulation no. 66 is replaced by simulation no. 29

Create \( NN \) new sets each containing \( k \) simulations (pure mutants)
Select the \( N1 \) of these having lowest values of \( D(J,q) \) as parents for next generation
Create \( N2 \) progeny by crossing-over
Create \( N3 \) new individuals by mutating 1 of parent’s genes.
Create \( N4 \) new individuals as pure mutants
New generation contains \( NN = N1+N2+N3+N4 \) individuals. Select best \( N1 \) of them

Repeat \( NGG \) times

Figure 3—The genetic algorithm that was used. Initially \( NN \) new sets of \( k \) simulations are created by drawing the simulation numbers at random (pure mutants). The best \( N1 \) of these are selected to be the parents for the next generation (i.e. those with the lowest values of \( D(J,q) \)). Next, \( N2 \) new individuals are created by crossing-over from the parents, then \( N3 \) individuals are created by mutating one gene in one of the parents (1-mutants), and lastly \( N4 \) pure mutants are created. The best \( N1 \) individuals are selected from the \( NN \) = \( N1+N2+N3+N4 \) individuals in that generation. This is repeated \( NGG \) times

through \( NGG \) generations. At each step the fitness of all the individuals in the population is computed by calculating the inverse of \( D(J,q) \) for each individual and the fittest are kept. It would be possible for individuals with repeated values of simulation numbers (e.g. 1, 1, 2, 3 . . .) to be created by mutation or crossing-over. As they turn out to have higher values of \( D(J,q) \) they are eliminated rapidly.

Background information on two case studies

The simulations used in both case studies were generated for a case study on the impact of hedging on a hypothetical gold mine in Australia. They were based on the Walker Lake data-set (Isaaks and Srivastava, 1989), except that the grades were modified to reproduce the statistical characteristics of the Kisladag gold deposit in Turkey and the size of the blocks and selective mining units was rescaled to match its annual production. See Armstrong et al. (2010 and 2013) for details. As the profitability of mines depends primarily on the recoverable reserves, especially the quantity of metal recovered, we chose to use the metal above cut-off in each panel to measure the dissimilarity between simulations. To be more precise, each simulation is represented by a vector

giving the metal above cut-off for a series of 16 possible cut-offs for each panel. In studies using the Walker Lake data, the area is usually divided into 30 panels each containing 100 selective mining units. So we computed the metal above cut-off for each panel for the 16 cut-offs corresponding to different possible gold prices. Each simulation was represented by a vector of length \( 480 = 30 \times 16 \). The underlying idea is that two simulations are ‘very similar’ if the metal above cut-off in one simulation is very close to that of the other simulation for every cut-off and for every panel\(^3\).

We say that these vectors are a proxy for the simulations because they encapsulate the essential characteristics of the simulation in a shortened form. In contrast to mining, the proxies for oil and gas reservoir simulations and aquifer simulations should reflect their fluid flow characteristics.

Having computed the proxy for each simulation, we compute an \( N \times N \) matrix \( D \) of the dissimilarities between simulations where \( N \) is the total number of simulations. Let \( S \) denote the selected subset of \( k \) simulations; let \( J \) be the subset of the remaining \((N-k)\) simulations. The metric \( D(J,q) \) between the subset \( S \) and the full set of simulations is computed using the procedure described in Appendix 1. In previous work (Armstrong et al., 2010, 2013), subsets containing 12 simulations were considered so there was a total of about \( 1.05 \times 10^{10} \) possible combinations. In the second example here we select subsets of 20 simulations so the population to be sampled contains \( 5.36 \times 10^{20} \) possible combinations, that is, there are 100 000 times as many candidate subsets. This is why we are looking for a more efficient sampling method.

Results for first case study: \( k = 4, N = 100 \)

The first step was to find the true global minimum for subsets containing 4 simulations out of a total of 100 simulations. This involved computing the metric \( D(J,q) \) exhaustively for about 3.9 million possible subsets. Figure 4 shows the histogram of all the values of the metric. The global minimum turned out to be 0.2564.

![Histogram of the values of the metric D(J,q) for exhaustive sampling](image)

Figure 4—Exhaustive histogram of the values of the metric \( D(J,q) \) obtained from 3 921 225 possible subsets of 4 simulations from 100 simulations. The minimum is 0.2564

---

\(^3\)Strictly speaking, this dissimilarity measure is not a distance because a zero value could be obtained for two simulations that were not identical simply by permuting the selective mining units within one or more panels.
Genetic algorithms and scenario reduction

The next step consisted of running the genetic algorithm 100 times with different initial sets of 1000 subsets. All of the runs of the genetic algorithm reached the global minimum after at most 8 generations. Figure 5 shows the evolution of the metric for all hundred runs of the genetic algorithm together with the global minimum (shown by the dotted red line). We had wondered if there was enough ‘genetic material’ in a population of this size to reach the global minimum. If not, the genetic algorithm might have been trapped in a local minimum and been unable to get out of it. Our worries were in vain.

We then repeated the test using a much larger population containing 10 000 subsets instead of 1 000. The 10 runs of the genetic algorithm all reached the global minimum but after 4 generations instead of 8 (Figure 6). This shows that there is a trade-off between the number of subsets considered in each generation and the number of generations required to reach the minimum.

Results for second case study: \( k = 20 \) \( N = 100 \)

Before running the genetic algorithm, we carried out 2 million random draws of 20 numbers between 1 and 100. The lowest value of \( D(J,q) \) found by the random search procedure was 0.1946 compared a mean of 0.2216 and a standard deviation of 0.0077. Figures 7 presents the histograms of all 2 million values of \( D(J,q) \) (left) and of the lowest 5000 values (right).

As the genetic algorithm was run for 50 generations, each with 10 000 subsets, about 500 000 subsets were evaluated. The minimum value of \( D(J,q) \) found was 0.1868, compared to 0.1946 after evaluating 2 million randomly selected subsets. This confirms that the genetic algorithm is more efficient at finding subsets with low values of the metric.

Figure 8 presents the evolution of the minimum value of the metric \( D(J,q) \) shown by the solid line and also the maximum (fine line) and the mean (dotted line). As expected the minimum decreases monotonically as a function of the number of generations. The mean also decreases steadily but the maximum is more or less constant. This is because new randomly drawn subsets are being included. The genetic algorithm has effectively succeeded in generating lots of promising subsets with values of the metric that are much lower than the random search procedure used earlier.
Gene tic algorithms and scenario reduction

Tracking individual selections
As both the genetic algorithm and the random search procedure are iterative procedures, there is no guarantee of their reaching the true minimum. In this example we found that 8009 of the 10,000 subsets in the 50th generation had a value of \( D(J,q) \) equal to the minimum 0.1868. We found that they were all identical but they had appeared at different times. Table II presents the 7 of these 8009 subsets with the value of the metric in the first column followed by the identity number of that subset, those of its parents, and then the generation number when it first appeared. The first of these subsets was first created in generation no. 32 as a result of a mutation. The ‘0’ for the second parent indicates a mutant.

The fact that the simulation numbers in all 8009 subsets are identical suggests that the algorithm has either reached the true minimum or is trapped in a local one.

Percentage of parents, progeny, and mutants surviving to the next generation
Having a method for tracking individual selections makes it possible to follow their evolution over time from one generation to another. This is important when choosing how many progeny, 1-mutants, and pure mutants to create each generation. To be more specific, we wanted to know how many of these four classes survived from one generation to another. Figure 9 shows the percentage of each class that survives over time from the first generation until the 50th. At the outset about 10% of each type of individual survived to the next generation. For the first few generations the pure mutants (mauve) are important, but this drops off rapidly. For the first 8–10 generations the 1-mutants (red) have a high survival rate. The survival rate for progeny (created by crossing-over) drops off more slowly. After 35 generations, most of the parents survive to the next generation. So the population is quite fit (with low values of the metric \( D(J,q) \)).

Looking at these results one might be tempted to reduce the number of pure mutants and 1-mutants after about 25 generations, but the last few drops in the value of the metric (i.e. after 35 generations) turned out to be due to mutants. We interpret this as meaning that new genetic material has to be introduced into the gene pool in order to find the minimum, or else it would become too narrow – ‘too inbred’, one might say.

Local minima
Two problems when using iterative algorithms are the criterion for stopping and avoiding being trapped in local minima. In our case, these problems increase in importance as the size of the subset to be sampled (and the computer

Table II
The ID number, those of their two parents and the generation in which they first appeared, for 7 of the 8009 subsets with the minimum value of \( D(J,q) \) found by the genetic algorithm. The subsets are all identical but were found by different paths, starting out from a subset created by a mutation in generation no. 32 (the ‘0’ for the second parent indicates a mutant).

<table>
<thead>
<tr>
<th>ID no.</th>
<th>Parent no. 1</th>
<th>Parent no. 2</th>
<th>No. of generations</th>
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<tbody>
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<td>265813</td>
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</tr>
</tbody>
</table>

Figure 8—The evolution of the minimum value of \( D(J,q) \) obtained by the genetic algorithm (solid line), and also the mean (dotted line) and the maximum (thin line). Each generation consisted of 10,000 subsets. As expected, the minimum decreases monotonically as a function of the number of generations. The mean also decreases steadily but the maximum is more or less constant. This is because new randomly drawn subsets are being included.
Genetic algorithms and scenario reduction

Time required) increases. In this section we show that while it would be tempting to assume that the algorithm has converged when the value of \( D(J,q) \) stops decreasing for some time, the algorithm may well be trapped in a local minimum. To illustrate this point, we ran the genetic algorithm ten times for the second case using the following parameters: 10,000 individuals per generation; 1000 were retained as parents \( (N1) \) for the next generation; 2000 progeny \( (N2) \) were created by crossing-over; 7500 individuals \( (N3) \) were obtained by mutating 1 gene; and a further 500 were pure mutants \( (N4) \). This was repeated over 500 generations. Figure 10 shows the minimum value of \( D(J,q) \) per generation for the first 50 of the 500 generations for the 10 repetitions. The value of \( D(J,q) \) did not drop after that point. Three distinct long-term minima were found for \( D(J,q) \): 0.1868349 (5 cases out of 10 repetitions), 0.1871285 (2 cases), and 0.1871484 (3 cases). At first we expected the algorithm to have identified one particular set of \( k \) simulations corresponding to each minimum. We were surprised to find that this was not the case. Figure 11 shows all the values of \( D(J,q) \) for the top 1000 individuals in each generation. Note how the range of values narrows as the number of generations increases, until all the individuals have the same \( D(J,q) \) value. In fact, the individuals are identical. This means that new genetic material brought by the 1-mutants and the pure mutants was not ‘fit’ enough to be selected for the following generation; nor were the new individuals created by crossing-over.

Looking back at Figures 10 and 11, it is now clear that we had wasted time and effort continuing to run the algorithm for 500 generations; it had reached a local minimum after 50–60 generations. The algorithm was unable to get out of the local minimum because all the individuals were the same. It could not create ‘fit enough’ individuals to survive the Darwinian selection process. We are currently modifying certain aspects of the algorithm to overcome this problem.

Conclusions and perspectives for future work

In the two tests presented and in others that we have carried out, the genetic algorithm outperformed the random search method used earlier. We believe that this is true in general. However, we still need to know more about its performance characteristics. Firstly, what criterion should be used to stop the algorithm? An arbitrary number of generations, and in that case how many? Or some criterion based on the algorithm’s performance?

Secondly, we need a better understanding of how the genetic algorithm functions, particularly when the number of combinations in the space to be sampled is large. Does the crossing-over mechanism contribute more than mutations? Is one more efficient early on and the other more useful later on? How many individuals should there be in each generation? In the first example, we showed that there is a trade-off between the number of generations required and the number of individuals per generation. The more individuals per generation, the faster the value of \( D(J,q) \) drops – but at the cost of more computations. Further work is required to clarify these points and also to find ways to create ‘fit enough’ individuals in order to avoid getting trapped in local minima. Having said that, while it is important from a theoretical point of view to understand the convergence properties, this is not primordial in practice because the sets of simulations that make up the local minima are all very similar.

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Appendix 1: The dissimilarity matrix D and the metric D(J,q)

Dissimilarity matrix D

The first step is to choose a proxy to represent the simulations. This is a vector that encapsulates the main features of simulations. For example, in open-pit mining applications, it could be the quantity of metal in each panel above a set of cut-offs; in petroleum engineering it could be streamline fluid flow simulations. The next step is to construct the square \(N \times N\) matrix of distances between pairs of proxies. This matrix is of course symmetric with zeros down the diagonal.

Metric D(J,q)

Let \(\mathbb{P}\) be the probability measure associated with the initial set of \(N\) geostatistical simulations which are denoted by \(\mathcal{J}\), for \(i = 1, \ldots, N\). Let \(p_i\) be the probability of the \(i\)th simulation.

In general geostatistical simulations are considered to be equally probable so \(p_i = 1/N\). We want to find a probability measure \(Q\) with only \(k\) simulations, that is as close as possible to \(\mathbb{P}\). Let \(S\) be the set of the numbers of the \(k\) simulations to be retained; let \(J\) be the set of the numbers of the \((N-k)\) others. Let \(q_j\) be the probability of the \(j\)th simulation to be retained. In contrast to \(p_i\), the probabilities \(q_j\) are no longer equally likely. We use the metric developed by Heitsch and Römisch:

\[
D(J,q) = \mu \left( \sum_{i=1}^{N} p_i \delta_{q_j} \right) \sum_{j \in J} q_j \delta_{q_j}
\]

where \(\mu\) is the Kantorovich functional.

When this metric \(D(J,q)\) is transposed to geostatistical simulations, it is computed from the dissimilarity matrix in the following way. The rows and columns of the dissimilarity matrix are rearranged so that the \(k\) simulations to be retained (e.g. in \(S\)) are placed before the other \((N-k)\) simulations (e.g. in \(J\)). The dissimilarity matrix can now be partitioned as:

\[
\begin{bmatrix}
D_{SS} & D_{S}\bar{J} \\
D_{\bar{J}J} & D_{JJ}
\end{bmatrix}
\]

Computing the new probabilities \(q\)

In order to evaluate \(D(J,q)\) we first determine the probabilities \(q_j\) in the new measure. This is done by taking the simulations in \(J\) one by one and finding the member of \(S\) that is closest to each one. The probability of the simulation being eliminated is then assigned to the closest member of \(S\). Speaking figuratively, each member of \(S\) ends up as the ‘head’ of a group consisting of itself plus those members of \(J\) that are closer to it than any other member of \(S\). So its new probability \(q_j\) is the sum of its own initial probability \(p_i\) plus those of the others in its group. Some members of \(S\) find themselves at the centre of a large group; other groups have only a few members, while some are loners (singletons). Having determined the new probabilities, it is easy to compute the metric \(D(J,q)\).

A short example

The easiest way to illustrate these two steps is via a short example. Suppose that we want to select 5 simulations out of a total of 20 equally probable geostatistical simulations. Suppose that \(S =\{2, 7, 12, 13, 15\}\) and \(J =\{1, 3, 4, 5, 6, 8, 9, 10, 11, 14, 16, 17, 18, 19, 20\}\). The rows and columns in the dissimilarity matrix are rearranged. Table III (a) gives the sub-matrix \(D_{SS}\).

Taking the members of \(J\) one by one, find the lowest value in each row in \(D_{SS}\). For example, the closest member of \(S\) to simulation no. 1 (in \(J\)) is simulation no. 12. In fact, simulation no.12 is the closest one to 9 of the simulations in \(J\) (no. 1, 3, 5, 6, 8, 10, 16, 17, 19, 20). These are highlighted in yellow in Table IIIa. So its new probability \(q_j\) is the sum of its own initial probability \(p_i\) plus those of the others in its group. Some members of \(S\) find themselves at the centre of a large group; other groups have only a few members, while some are loners (singletons). Having determined the new probabilities, it is easy to compute the metric \(D(J,q)\).
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Table III

(a) Five simulations no. 2, 7, 12, 13, and 15 are to be retained. The matrix \( D_{JS} \) below is used for assigned the other 15 simulations to the closest one out of the 5 retained, as shown by the coloured highlighting.

(b) Minimum value (left column) and closest simulation (right column). \( D(J,q) \) is computed by summing the row minima each multiplied by their probability. As the 20 geostatistical simulations are equally probable, it suffices to sum the minima and multiple by 0.05

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<tr>
<td>9</td>
<td>.800</td>
<td>.291</td>
<td>.353</td>
<td>.397</td>
<td>.464</td>
</tr>
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<td>10</td>
<td>.554</td>
<td>.331</td>
<td>.267</td>
<td>.843</td>
<td>.633</td>
</tr>
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<td>11</td>
<td>.601</td>
<td>.238</td>
<td>.259</td>
<td>.654</td>
<td>.741</td>
</tr>
<tr>
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<td>.795</td>
<td>.284</td>
<td>.309</td>
<td>.616</td>
<td>.511</td>
</tr>
<tr>
<td>14</td>
<td>.416</td>
<td>.340</td>
<td>.264</td>
<td>.616</td>
<td>.505</td>
</tr>
<tr>
<td>16</td>
<td>.485</td>
<td>.281</td>
<td>.299</td>
<td>.778</td>
<td>.575</td>
</tr>
<tr>
<td>18</td>
<td>.800</td>
<td>.184</td>
<td>.299</td>
<td>.641</td>
<td>.497</td>
</tr>
<tr>
<td>19</td>
<td>.779</td>
<td>.536</td>
<td>.447</td>
<td>.862</td>
<td>.699</td>
</tr>
<tr>
<td>20</td>
<td>.443</td>
<td>.299</td>
<td>.290</td>
<td>.563</td>
<td>.477</td>
</tr>
</tbody>
</table>

\( D(J,q) = 4.422 \times 0.05 = 0.2211 \)

Table IV

The new probabilities \( q \) for the 5 simulations selected

<table>
<thead>
<tr>
<th>Simulation no.</th>
<th>2</th>
<th>7</th>
<th>12</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability ( q )</td>
<td>0.05</td>
<td>0.30</td>
<td>0.50</td>
<td>0.10</td>
<td>0.05</td>
</tr>
</tbody>
</table>

In order to compute the value of \( D(J,q) \) we go back to Table III. Each of the row minima is multiplied by the original probability of that simulation. Since there are 20 equally likely simulations, the probabilities are all 0.05. In this example the value of \( D(J,q) \) is 0.2211, that is the sum of the row minimums (0.422) multiplied by 0.05, as shown in Table IIIb.

Appendix 2: The stability property of the objective function

One of the reviewers of the paper asked why we focus only on minimizing the metric \( D(J,q) \) when the overall objective is the accuracy of the objective function computed over the selected subset of the simulations compared to the full set. The answer is because of the stability properties of the stochastic programming problem with respect to small perturbations. Here we summarize the explanation given in Heitsch and Romisch (2003) based on work by Dupacova et al. (2003) and Rachev and Romisch (2002).

Our overall objective when using a set of simulations is to optimize an objective function \( f \) over all the simulations weighted by their probabilities. This is reformulated in stochastic decision problems as

\[
\min \left\{ \int_\Omega f_0(\omega, x)P(d\omega) : x \in X \right\}
\]

where \( \Omega \subset \mathbb{R}^m \) is a nonempty closed convex set \( \Omega \) is a closed subset of \( \mathbb{R}^i \) the function \( f \) from \( \Omega \times \mathbb{R}^m \) to \( \mathbb{R} \) is continuous with respect to and convex with respect to \( x \) \( P \) is a fixed Borel probability measure on \( \Omega \).

Typically the integrand \( f \) is not differentiable but is locally Lipschitz continuous on \( \Omega \). In Dupacova et al. (2003) and Rachev and Romisch (2002) it is shown that model (1) is stable with respect to small perturbations in terms of the probability metric:

\[
\sup_{f \in F} \left\| f - f_0 \right\| = \sup_{f \in F} \left\| \int_\Omega f_0(\omega, x)P(d\omega) - \int_\Omega f_0(\omega, x)Q(d\omega) \right\|
\]

Heitsch and Romisch (2003) show that minimizing the metric \( D(J,q) \) is equivalent to minimizing (2) for all objective functions \( f \) in a fairly wide class. We only need to find the subset \( Q \) that minimizes \( D(J,q) \) in order to minimize the absolute difference between the approximate objective function obtained using the subset \( Q \), and that obtained using the full set of simulations. Readers can consult Heitsch and Romisch’s paper for the mathematical proofs, and the exact conditions on the objective function \( f \).

Interpreting these results in terms of geostatistical simulations

The probability measure \( P \) gives the probabilities of all the initial scenarios. In general the geostatistical scenarios are considered to be equally likely. The probability measure \( Q \) gives the probabilities of simulations in the reduced subset; these are not equally likely. Some simulations are ‘typical’ whereas as others are less likely. They could correspond to ‘bonanzas’ or ‘disasters’, which are both important for decision-makers.

Expression [2] gives an upper bound on the absolute difference between the objective function obtained using the full set \( P \) and the reduced set \( Q \) for all possible objective functions. ⇔
Kriging, indicators, and nonlinear geostatistics
by J. Rivoirard*, X. Freulon†, C. Demange†, and A. Lécureuil†

Synopsis
Historically, linear and lognormal krigings were first created to estimate the in situ mineral resources of blocks. Nonlinear geostatistics and indicator kriging were subsequently developed to evaluate also the portion recovered when applying a cut-off on selective mining units (SMUs) within blocks. In practice these methods are generally based either on the Gaussian model with a transformation generalizing the lognormal case or on the indicators above cut-offs. Very often the indicator approach is simplified by kriging separately each indicator, and when starting from a continuous variable, a practical advantage of the discretization into classes lies in the easy treatment of a zero effect and of the high values. However, a number of so-called isofactorial models have also been developed for a discrete or continuous variable, where the full cokriging of indicators (i.e. disjunctive kriging) simplifies to the separate kriging of factors. Moreover, these models are equipped with a change of support, allowing a consistent estimation of recoverable resources on SMUs.

Min-Max Autocorrelation Factors (MAF) analysis of the indicators offers a new approach for indicator modelling. In particular the first factor, the one with the highest spatial continuity, can help in choosing the type of model. For example a monotonic experimental first factor can be used directly as the basis of a discrete diffusion model, unless a continuous diffusion model such as the Gaussian model can be used on the original variable. This approach is illustrated on a uranium deposit mined selectively: estimates of recoverable resources by discrete disjunctive kriging and uniform conditioning in a Gaussian model are compared locally to short-term estimates based on two areas densely drilled.

Keywords
disjunctive kriging, indicator kriging, Min-Max Autocorrelation Factors, recoverable resources, discrete diffusion model.

Introduction
Linear kriging is a recognized and commonly used technique. It allows estimation of a regionalized variable (represented here by $Z(x)$ where $x$ denotes a point-support location) at target points, or estimation of sets of points such as blocks in the case of an additive variable.

Although it was a domain of intense research many years ago, nonlinear geostatistics (Rivoirard, 1994; Chilès and Delfiner, 1999) is still a complex and difficult part of geostatistics. In contrast with linear geostatistics, which considers only linear combinations of the variable, nonlinear geostatistics deals with transformations of the variable, for example a Gaussian transformation or indicators.

One purpose of nonlinear geostatistics is the estimation of the target variable itself at points or over blocks. Another one is the estimation of a transformation of the variable (Vann and Guibal, 2000). Typically it is used to predict the exceedence of the threshold at points (e.g., the indicator $\text{Ind}\{Z(x) \geq z\}$) or for blocks ($\text{Ind}\{Z(v) \geq z\}$ where $v$ denotes the block support). The latter case, which corresponds in particular to the recoverable resources of selective mining units (SMUs) above cutoff $z$, is more complex as it cannot be reduced to a simple block averaging and demands a change of support.

In practice today, the two most commonly applied nonlinear approaches are Gaussian-based methods such as uniform conditioning or multiple indicator kriging (IK) (Journel, 1982; Vann et al., 2000). The choice of one technique or the other is often made a priori, and the existence of other methods is ignored. In particular, the fact that local estimation is better supported by the transformation of the variable that presents the highest spatial correlation is largely unexploited.

In this paper we propose to use the Min/Max Autocorrelation Factors (MAF) (Switzer and Green, 1984; Desbarats and Dimitrakopoulos, 2000) of indicators as a tool to determine the transformation that yields the highest spatial correlation and to choose the model for nonlinear geostatistics accordingly. This model can then be used to compute local estimates using disjunctive kriging. This approach extends the normal practice of multiple IK as it takes into account cross-correlations between the indicators and includes an appropriate change-of-support model, rather than an arbitrarily chosen one.
**Kriging, indicators, and nonlinear geostatistics**

The paper is structured as follows. We first present an overview of the main nonlinear models and methods. Then we consider the use of MAF of indicators for the choice of the model. Finally, we present a case study for the local estimation of mineral resources on real data.

**Overview of methods and models**

The Gaussian transformed model is very commonly applied, both in nonlinear geostatistics and in conditional simulation. The transformation generalizes the lognormal distribution used at the origin of geostatistics (Krige, 1951, 1952, 1978).

Thanks to a multi-Gaussian hypothesis, the Gaussian model easily provides the conditional distribution (and so the conditional expectation) at target points (Verly, 1983). It is adapted to an original variable which has a continuous distribution and in particular has no zero effect (a high proportion of zeroes can cause a problem for the inversion into Gaussian values). The Gaussian model is also equipped with a change-of-support model. A variant technique is the uniform conditioning (UC) under the Gaussian model (Rivoirard, 1994; Deraisme et al., 2008), which predicts the distribution of SMU values in a panel, conditional on an estimate of this panel (relaxing the constraints on zero effect and stationarity).

In the indicator approach, the variable is either discrete or if continuous is discretized into classes. This can be convenient in case of a zero effect, as well as in case of high values that can be grouped into one class. IK consists of independently kriging the indicators of (cumulated) classes, or in estimating linearly each class with a common variogram. Its advantage relies on its simplicity. It does not, however, exploit the joint structure of indicators nor, when using a common variogram, the common deconstruction of high grades (Vann et al., 2000). In addition, it is often used with a posterior change of support that does not guarantee consistency (Emery, 2008). Furthermore, post-processing is required to obtain estimates of indicators at all possible cut-offs.

Disjunctive kriging (DK) is the original name for indicator cokriging (Matheron, 1976). The required knowledge of bivariate distributions between points is actually equivalent to the multivariate structure of indicators. DK is possible with both discrete and continuous distributions.

In the case of a mosaic model with independent valuations (Rivoirard, 1984), all structures are the same and DK reduces to independent kriging of each indicator class (this is the model where IK would be optimal at point support).

A number of so-called isofactorial models were developed years ago (refer in particular to the numerous papers by Matheron cited in Chiles and Delfiner, 1999). The factors have no spatial cross-correlation, so that DK is obtained by kriging these separately. Moreover, internally consistent change-of-support models exist for all of these models. An example of such an isofactorial model is the Gaussian model, where bivariate distributions are Gaussian, and factors are the Hermite polynomials of the Gaussian transformed variable. Another example is the gamma model, whose factors are the Laguerre polynomials (Hu, 1988).

Such models are diffusion-type models: intermediate values are met when going from low to high values and vice-versa. Moreover, they are based on a defined probability distribution: Gaussian, gamma, etc. The observation of elliptically-shaped scatter plots between points separated by a given distance, typical of bi-Gaussian distributions, for instance, is in practice a good vector for the choice of a Gaussian model, before modelling the variogram of the Gaussian transformed variable.

On the other hand, the indicator approach considers directly structural tools such as simple or cross-variograms. We have already seen the very particular case of the mosaic model. Two isofactorial models are especially interesting here, which correspond to finite discrete distributions and which are not based on a defined probability distribution. This makes them suitable to describe an indicator transformed data-set. One is the discrete diffusion model developed by Matheron (1976) and Lajaunie and Lantuéjoul (1989). Its modelling is based on the experimental determination of its first factor, the one which carries the highest spatial continuity. This should be monotonic with respect to the variable under study (this is a condition for diffusion, representing the transition from low to high values through medium ones). All other factors are derived from this first factor. The other model is the model with indicator residuals (IR) (Rivoirard, 1989). It is not a diffusion-type model, but a hierarchical one, where the first factor is one basic indicator and the other factors are the successive residuals of indicators.

Discrete disjunctive kriging (DDK), based on a discrete isofactorial model, can handle zero-effect or atoms (i.e. 0 or other values observed with a high proportion), extreme values, and includes a change of support. It can be viewed as an extended IK as it overcomes recognized limitations of the IK approach: specifically, it takes into account the cross-correlations between the indicators and includes a consistent change-of-support model. The parameters of these discrete models (discrete diffusion or IR) can be derived from the analysis of MAF computed on indicators, since the MAF of indicators correspond to an experimental version of the factors of an isofactorial model.

**Using MAF of indicators for the choice of model**

The fact that the local estimation should be driven by what corresponds to the major spatial continuity in the variable is largely unexploited in many geostatistical approaches. For instance, in a transformed Gaussian model, the Gaussian variable is known to be the transformed variable that has the highest continuity (and the estimation is based on the kriging of the Gaussian variable), but this fact is not used for choosing the model. This is where MAF of indicators (after discretization of the variable) can be useful (as linear combinations of indicators, MAF are the same whether cumulated indicators are used or not). Remember that MAF are multivariate statistics which are for spatial statistics or geostatistics what principal components (PCs) are for statistics: in both cases they are orthogonal linear combinations of the initial variables, but in the case of MAF they are based on spatial continuity instead of statistical variability in the case of PCs. MAF are ranked by decreasing spatial continuity at a chosen separation vector $h$, and their cross-correlation at this distance and at distance 0 is zero.
Modelling MAF assumes that their cross-correlation is zero for other distances, which can be checked on the experimental cross-variograms of the MAF. The absence of cross-correlation over all distances implies that an isofactorial model fits the data. In addition, computing the MAF of indicators provides valuable structural information. In particular the first MAF is the linear combination of indicators that presents the highest spatial continuity. It is a function of the discrete variable under study, and its composition in terms of values or classes on the original variable is meaningful.

A first factor which is a stepwise function of the variable (i.e. an indicator) could orientate towards an IR model. With a monotonic first factor, the structure is essentially due to the gradual transition from low to high values. This corresponds to a diffusion-type model, and could support the use of a discrete diffusion model or another diffusion-type model such as a Gaussian or gamma model. MAF that do not correspond to existing models (e.g. a first factor that is not monotonic) would require the development of new models, enabling kriging MAF and possibly including a change of support.

**Application to a uranium mine**

In the following section, discrete disjunctive kriging is applied to drill-holes from a uranium project in a horizontal stratabound sediment-hosted deposit: two areas were densely drilled to establish the grade variogram, especially for short distances, and to evaluate long-term resource models. The data-set used to illustrate the methodology contains composites derived from radiometric measurements in vertical drill-holes and has been split in two subsets: (a) holes drilled on a regular 50 m centred grid used to establish long-term models on 25×25×2 m³ panels (or long-term data-set); (b) holes drilled on a regular 12.5 m centred grid in two 50×50 m areas (or short-term data-set). Basic statistics on 2 m length composites are reported for both data-sets in Table I.

Two long-term resource models have been computed considering 5×5×2 m³ SMUs, the first one using the uniform conditioning in a Gaussian model (UC); the second using discrete disjunctive kriging (DDK); both are based on the same long-term data-set. Finally, a uranium grade model on 5×5×2 m³ blocks is computed using composites from the short-term data-set. This model, restricted to the 50×50 m areas, is similar to those used to guide the production and has been considered as the reference.

The application of uniform conditioning follows the classical approach (e.g. Rivoirard 1994); it is not described further and only the application of discrete disjunctive kriging and the comparisons are presented in more detail.

The discrete approach requires a disjunctive coding of the initial variable: a set of thresholds \((0 \leq z_1 < \ldots < z_i < \ldots < z_N)\) define indicators of ore classes and a discrete average grade can be calculated as follows:

\[
\hat{Z} = \sum_{i=0}^{N} \hat{z}_i \times (1_{Z=z_i} - 1_{Z=z_{i+1}})
\]

The histogram of the discrete grade is completely specified by the proportion and the average value of each class (Table II). In case of clustered data, a declustering procedure should be applied to derive the appropriate proportions attributable to each class interval.

The selectivity curves of the raw variable and its discrete counterpart are compared in Figure 1: the mean is preserved (i.e. the metal content) and the selectivity is slightly reduced (-3% of the selectivity index, i.e. the Gini coefficient). The reduction of the variance is higher (-50%) as the values of

<table>
<thead>
<tr>
<th>Class index</th>
<th>Limits (%)</th>
<th>Proportion (%)</th>
<th>Average grade (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[0, 0.2]</td>
<td>43.90%</td>
<td>0.08</td>
</tr>
<tr>
<td>1</td>
<td>[0.2, 0.3]</td>
<td>13.38%</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>[0.3, 0.4]</td>
<td>16.41%</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>[0.4, 0.5]</td>
<td>6.56%</td>
<td>0.45</td>
</tr>
<tr>
<td>4</td>
<td>[0.5, 0.6]</td>
<td>5.60%</td>
<td>0.55</td>
</tr>
<tr>
<td>5</td>
<td>[0.6, 0.7]</td>
<td>4.92%</td>
<td>0.65</td>
</tr>
<tr>
<td>6</td>
<td>[0.7, 0.9]</td>
<td>6.90%</td>
<td>0.79</td>
</tr>
<tr>
<td>7</td>
<td>[0.9, 1.1]</td>
<td>3.97%</td>
<td>0.99</td>
</tr>
<tr>
<td>8</td>
<td>[1.1, 1.5]</td>
<td>4.77%</td>
<td>1.28</td>
</tr>
<tr>
<td>9</td>
<td>[1.5, +]</td>
<td>5.48%</td>
<td>2.80</td>
</tr>
</tbody>
</table>

**Figure 1—Selectivity curves Q(T) of the continuous grade and its corresponding discrete distribution for 10 classes**

<table>
<thead>
<tr>
<th>Data-set</th>
<th>Number</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Variance</th>
<th>σ/μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-term</td>
<td>9 886</td>
<td>0.00</td>
<td>57.88</td>
<td>0.49</td>
<td>0.90</td>
<td>1.94</td>
</tr>
<tr>
<td>Short-term</td>
<td>4 090</td>
<td>0.00</td>
<td>36.17</td>
<td>0.70</td>
<td>1.55</td>
<td>1.77</td>
</tr>
</tbody>
</table>

**Table I**

**Statistics of uranium grade (‰) from drill-holes (2 m composites)**
Kriging, indicators, and nonlinear geostatistics

the upper tail of the histogram are summarized with a single value. The thresholds were selected between 0.2 and 1.5‰ for practical considerations. It is not necessary that the thresholds be evenly spaced. They could be defined as those minimizing the reduction of the selectivity index (or the variance) for a given number of classes, but such optimal thresholds were not considered here.

MAF (Switzer and Green, 1984) of the indicators have been computed for different lag values, along the drill-holes and horizontally. Figure 2 reports the experimental values of the first factor as a function of the class index. The first factor is fairly independent of the lag value used to calculate the MAF. This result shows that the isofactorial assumption is reasonable for this application.

The first factor is strictly monotonic, which is a good indication for the use of a discrete diffusion model. Details on this model and its implementation can be found in Lajaunie and Lantuéjoul (1989). The development of the model is sophisticated and out of the scope of the present paper. In contrast, the model is very easy to use for a practitioner having access to dedicated software. The reason is that the model is entirely defined by the marginal distribution of the discrete variable (i.e. the proportions of the N+1 classes) and by the first factor. The first factor is the function of the discrete variable which presents the highest spatial correlation: it is directly derived from data and coincides with the first MAF of the indicators. It has to be monotonic. All the other factors of the model are automatically deduced from this first factor. Figure 3 compares the experimental factors derived from the MAF analysis of the indicators and the factors of the fitted diffusion model. As we can see, the diffusion model describes accurately the experimental factors.

Experimental variograms of MAF are reported in Figure 4. They have been computed along the vertical drill-holes and horizontally (no anisotropy was observed horizontally). They show that the spatial correlation of factors decreases with their rank (this is consistent with their definition) and are significant only for the first three factors. It has been checked that the cross variograms show no significant spatial structure between factors.

In the case of the discrete diffusion model, the covariance functions of the factors ($\chi_i$) are derived from the covariance of the first factor, according to the following formula:

$$Cov(\chi_i(x), \chi_j(x+h)) = \rho(h) = \rho^h(h)$$

Figure 3—Comparison of Min/Max Autocorrelation Factors ($\Delta=2$ m) of the indicators and the factors of the discrete diffusion model. This is determined from the 1st MAF and the marginal distribution.

Figure 4—Experimental variograms of the diffusive factors and models. Factor models are derived from 1st factor model.

![Experimental variograms of MAF](image)

![First Min/Max Autocorrelation Factor derived from indicators](image)

![Comparison of Min/Max Autocorrelation Factors](image)
In this formula the power coefficients \(1 < \lambda_i < \lambda_{i+1}\) for \(i > 1\) are characteristics of the diffusion model, which are computed automatically from the marginal distribution and the first factor.

The covariance of the discrete grade can be derived also from the covariance of the first factor:

\[
\begin{align*}
C(h) &= \text{Cov}(\hat{Z}(x), \hat{Z}(x+h)) = \sum_{i=1}^{N} c_i^2 \times \rho^{\lambda_i}(h) \\
\end{align*}
\]

where the \(c_i\) are the linear coefficients to compute the discrete grade (with mean \(m\)) from the factors:

\[
\hat{Z}(x) = m + \sum_{i=1}^{N} c_i \times \chi_i(x)
\]

The variogram models fitted to the experimental variograms are reported in Figures 4 and 5: all models are derived from the covariance model of the first factor; this covariance is \(\exp(-\tau(h))\) where \(\tau\) is the variogram:

\[
\tau = \text{power} \ (\alpha=0.7, a_{xy}=100m, a_z=7m) + 1.5 \text{ power} \ (\alpha=1.5, a_{xy}=400m, a_z=7m)
\]

This model can be used to compute the cokriging of any linear combination of the factors, thus any function of the discrete grade. An application is the disjunctive kriging estimate of the grade on SMU or panels, i.e. the short-term model, or the \textit{in situ} resources of the long-term model.

To compute local estimates of the part of \textit{in situ} resources that can be recovered by selective mining, a change-of-support model is necessary. For the discrete diffusion model (Matheron 1984; Lajaunie and Lantuëjoul, 1989), the change of support is specified by a coefficient \(s\) \((s > 0)\) derived from the variance of the grade for the block support (calculated from the variogram of the grade):

\[
\begin{align*}
\text{Var}(\hat{Z}(v)) &= \sum_{i=1}^{N} c_i^2 \times \left( \frac{1}{1 + \lambda_m} \right)^s \\
&\geq 0
\end{align*}
\]

The variance of the discrete grade computed on the composites is 0.42. The variance of the grade for the block \(5\times5\times2\ m^3\) is derived from the sill of the regularized model; its value is 0.32. The coefficient of the change of support computed from Equation [5] is \(s = 0.33\).

Once this coefficient \(s\) has been computed, the covariance of the first factor of the blocks, \(\rho_v\), is deduced from the regularized model of the discrete grade of the composites:

\[
\begin{align*}
\text{Cov}(\hat{Z}(v), \hat{Z}(v')) &= \frac{1}{|v|} \int_{v} \text{C}(x-y)dx dy = \\
&\sum_{i=1}^{N} c_i^2 \times \left( \frac{1}{1 + \lambda_m} \right)^s \times \rho^{\lambda_v}(v,v')
\end{align*}
\]

Similarly to the model for composites, the model for the first factor of the blocks is \(\exp(-\tau_v(h))\) where \(\tau_v\) is the variogram:

\[
\tau_v = 0.5 \times \text{exp} \ (\alpha=20m, a_z=5m) + 1.5 \text{ power} \ (\alpha=1.1, a_{xy}=85m, a_z=5.5m)
\]

The simple and cross-covariance functions between point and block factors are then derived from the Cartier relation (Chiles and Delfiner, 1999). They are used to compute the kriging of any linear combinations of the block factors, in particular the disjunctive kriging of the metal and ore above a cut-off.

On the two areas densely drilled, the selectivity curves giving the metal vs. the tonnage at a given cut-off, \(Q'(T)\), for the Gaussian uniform conditioning and the discrete disjunctive kriging are reported in Figure 6. These estimates are based on the composites of the 50 m centered grid (i.e. the long-term data-set). They are compared with the selectivity curve of the short-term model computed using the composites of the 12.5 m centered grid (i.e. the short-term data-set).

Table III gives the comparison for cut-off 0.3‰ eU. The experimental selectivity curves of the short-term and long-term data-sets are reported also in Figure 6: there is a significant difference between the average grade of the long-term data-set, 0.5‰ eU, and the average grade of the short-term data-set, 0.7‰ eU. The local estimation of the recoverable resources fits; both methods, the local experimental mean and predicts the selectivity deduced from the short-term model.
Kriging, indicators, and nonlinear geostatistics

In the present application, two different nonlinear techniques, UC and DDK, give similar results. Both techniques are based on a diffusive model, a continuous one for UC and a discrete one for DDK.

Conclusion

Nonlinear geostatistics can be viewed as a research technique for the transformation of the variable leading to the highest spatial continuity and being consequently the most appropriate to drive local estimations. One approach to consider is the analysis of the MAF of indicators. This novel point of view makes a direct link between the elementary indicator approach and the more sophisticated models of nonlinear geostatistics. In the present application, the MAF of indicators can be seen to support the choice of a diffusion model.

MAF of indicators analysis also appears as a structural tool on which a methodology can be developed to estimate recoverable mineral resources using DDK. Practical applications may reveal the need to develop new models for MAF of indicators including their change of support. Other subjects of research concern the relaxation of stationarity and the extension to multivariate situations.

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References


Introduction and geological framework

The diamondiferous linear beach deposits of Namibia’s Sperrgebiet (Figure 1) form part of the world’s greatest marine diamond-bearing deposit.

The onshore beach components, which are now mostly mined out, extend northwards along the coastline from the Orange River mouth for about 100 km. Six raised beaches, from 30 m above present sea level, are developed at different elevations and extend to the current day sea-level. Strong geological continuity governed by sea-level stands in a coast-parallel direction is evident for all six beaches. Continuity in the direction normal to the coastline is not so strong. The geological delineation of the six raised beaches is based on 1 m trenches comprising 1×5 m sample paddocks and mega-trenches comprising 10×50 m sample paddocks (See Figure 2).

Through a process of beach accretion, the linear beach deposits are currently mined to 25 m below sea level. With the onshore in situ deposits largely depleted, new resources below sea level are targeted for exploration, and a sample optimization study is required.

Since there is very little sample data available for the submerged beaches, the onshore data is the only information available that can be used as analogue data with which to design optimized sampling programmes to be applied in the exploration of the submerged beaches. This is deemed acceptable based on the results obtained from the -25 m mining results and geological understanding of the depositional environment. Over time, as sample information from the submerged beaches becomes available, the sampling programmes will be reviewed.

Data

Traditional trench sampling campaigns used to delineate raised beach deposits are not feasible in a submerged beach environment. Current mining at depths of 25 m below sea level has, however, yielded enough evidence to show that data collected from onshore beach sampling campaigns can be used as proxy information (with some modifications) in the sample optimization studies.

Data from the 1 m wide trenches provides information regarding the nature and elevation of the footwall, diamond content, and gravel occurrence on a 1×5 m scale. Cliff lines (which are not always visible due to sand cover) and internal beach morphology are governed by sea-level stands. The cliff lines are not equally well developed for all sea-level stands and internal beach structures are sometimes destroyed during multiple cycles of sea-level transgression and regression.

Based on the simultaneous interpretation of a series of adjacent trenches, the six beaches were delineated and split into geological zones, each with its own statistical characteristics which are analysed separately.

Synopsis

At Namdeb, submerged beaches are earmarked for sampling and future mining and various sampling configurations are tested for optimality through the use of spatial simulations. For the creation of these virtual orebodies, basic statistics and variograms are needed, but in this specific instance no data exists from which the necessary parameters can be determined. The best that can be done is to use proxy data from onshore beaches, with adaptations where needed. The reworking of raised beaches during periods of rising and falling sea levels in some cases destroyed the internal beach structures, and it is very difficult to determine the variograms.

A method is proposed whereby simulated annealing is used to adjust the sample locations to align the data pertaining to beach crests or cliff lines, thus improving the variogram structure along the shoreline in the direction of the highest geological continuity.

Keywords

submerged beaches, simulated annealing, variography.
Improved variography using simulated annealing to adjust sample locations

The outlines of the geological zonation are straight lines which have, due to the 500 m spacing of the data, inherent inaccuracies. Previous work (Jacob, et al., 2006) showed that cliff lines are generally associated with zones of higher diamond concentration, and this feature of the depositional environment can be used to adjust data to align with the cliff lines. Adjustment of the data locations to align with the cliff lines (or beach crests) would result in the optimum variogram. This is equivalent to calculations along an unfolded structure, aligned to the underlying geological structures of the beach. The complexity of this method is underpinned by adjusting the data in such a way as to obtain the most robust structured variogram (clearly defined variogram type, range, and expected low nugget effect) through alignment of the higher grades in each sample line. To achieve this, a squared difference, weighted by the distance between the sample lines, is proposed while sample locations are adjusted through simulated annealing.

Problem statement

In order to create virtual orebodies for use in sample optimization studies, spatial simulations with associated variograms are used, calibrated to the statistics of each beach deposit. High ratios of anisotropy in the direction parallel to the current shoreline exist, but reworking of raised beaches during periods of rising and falling sea levels in some cases destroy the internal beach structures. The expected strong spatial correlation is thus not clearly visible in the data. This could also be a function of the data spacing, which is wide relative to the expected ranges for variograms of diamondiferous beach deposits, or else the jagged nature of the coastline. This makes the determination of variogram parameters challenging.

A method is proposed whereby simulated annealing is used to adjust the sample locations to align the data pertaining to beach crests or cliff lines, thus improving the variogram structure along the shoreline in the direction of the highest geological continuity. Figure 3 shows schematic trench positions on a Google Earth backdrop. The middle trench needs to be aligned to the cliff line.

The sample data is migrated onto a 1×5 m grid with the aim of aligning the highest data values per trench with the adjacent trenches subject to constraints and minimizing an objective function. The problem statement is illustrated in Figure 4.

By iterating the sample data locations (constrained to a -2 to +2 block movement in a north/south direction), the weighted grade difference between the lines can be minimized.
minimized, implying alignment of the data to the underlying geological structure. The solution to the problem illustrated in Figure 4 shows how the high-grade (red) blocks need to be adjusted in order for the grade data to honour the underlying geological structure.

Adjusting sample locations using simulated annealing

Simulated annealing is a perturbing search method, whereby a change in the system causes it to deviate slightly, which is used for finding a best outcome within a solution space. In this study an objective function is minimized using simulated annealing by changing one factor at a time. The deviations are evaluated and solutions accepted that reduce the objective function. This is done repeatedly in a structured manner until a best solution is found.

During the process, over and above accepting the solutions that improve the objective function, rejected solutions are randomly tested against the Metropolis criterion to occasionally accept less favourable solutions. This avoids the solution becoming entrapped in a local, sub-optimal solution. The probability of acceptance of a worse solution is determined as follows:

\[
P(\text{accept worse solution}) = e^{-\frac{(\text{obj}_{\text{new}} - \text{obj}_{\text{old}})}{T}} > \text{uniform}(0,1)
\]

where a temperature value \((T)\) and the new and old objective function values are used. As the number of iterations increases, the temperature is reduced according to an annealing schedule, and combined with a smaller difference between the two objective functions, it becomes less likely that worse solutions will be accepted. The annealing process eventually converges to a ‘best’ solution.

The simulated annealing algorithm as implemented by Goffe et al. (1994) was modified so that each random adjustment can contain only integer values.

To take cognisance of the distance \((d)\) between sample lines, the objective function includes a weighting based on the inverse distance between two adjacent sample lines. The objective function to be minimized is:

\[
\min \left[ \frac{1}{d_i} \times \sum_{j=1}^{\text{#samples per line}} (\text{stone grade}_{i,j} - \text{stone grade}_{i+1,j})^2 \right]
\]

The movement for each sample line is initially set to zero and after running the annealing process inspection of the results shows that the most logical solutions are obtained when the movements are not allowed to deviate too much from zero. These slight movements of the sample locations were thought to be realistic, as the geological boundaries are not expected to be radically misaligned with the underlying beach crests/cliff lines. The constraints of the annealing process thus allowed only relatively small adjustments to the sample locations and were restricted to not more than 20 m in the cross-beach direction.

Case study

The F beach is the oldest, highest grade, and most eastward of the six raised beaches. The samples covering the beach are spaced at 500 m in the north-south and 5 m in the east-west directions. With the samples closely spaced across the beaches, the east-west direction should be more representative in terms of the modelling of the nugget effect of the variogram.

Figure 5 shows the variograms before and after the data locations were adjusted.

The variograms are plotted using two different scales on the x-axes, as the anisotropy along and across the beach cannot otherwise be shown in a single graph. It can clearly be seen that after the sample locations were adjusted, the nugget effect of the along-beach variogram is in agreement with the nugget effect determined from the across-beach data.

Similar results (although slightly less impressive than from beach F) were obtained from some beaches, but not in all cases. The reason for this is thought to be the reworking of beaches during the multiple transgression and regression cycles of sea level stands, which destroyed so much of the beach structures that it is very difficult to reconstruct the linearity of the beach based on the sample grade only.

Examples of before and after data adjustments within sample lines are shown in Figure 6.

Results and conclusion

Through inspection of the example in Figure 4, an alternative solution of (-2, -1, 0, 0, -1) can be proposed. Similarly, the results after annealing the sample locations of the beaches must be contextualized within the framework of the more complex data configurations and the possibility that the annealing probably does not converge to a unique solution.
Improved variography using simulated annealing to adjust sample locations

The multiple outcomes of annealing runs are therefore analysed and a range of variogram parameters determined, leading to an array of simulations for use in the sample optimization studies. In the inspection of the output to determine if cliff lines or beach crests can be identified from the data, the minimized objective function and variogram parameters are used to establish the acceptability of the results. The outcomes thus far have been promising.

The research is ongoing and is focusing on eliminating outliers from the data, testing the effect of different constraints on sample movements, as well as testing the effect of using multiple random starting points in the annealing.

Acknowledgements

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References


Geostatistics has received a great deal of attention in the mining industry over the last few decades, as it offers varied tools and models to assess mineral resources and ore reserves (Journel and Huijbregts, 1978; David, 1988; Krige, 1999). Geostatistical analyses are based on exploration data and/or production data such as the grade assays in drill-hole or blast-hole samples. Data integrity is essential to obtain accurate estimates of resources and reserves. In this regard, data with extreme high values may be problematic, because of the effect on the sample variograms and resource and reserve estimates (Krige and Magri, 1982; Armstrong, 1984). The modeller is often tempted to consider such extreme data as outliers or erroneous measurements and to omit them from geostatistical analyses. Such a procedure is questionable if the extreme data corresponds to true values, because it may lead to underestimate the resources or reserves. The bias can be severe when estimating the grades of elements with heavy-tailed distributions, such as gold or silver, for which removing the highest values can affect the economic appraisal of the ore deposit.

To mitigate this impediment, a number of sophisticated robust estimation procedures have been proposed to reduce the influence of extreme high values (Journel and Arik, 1988; Parker, 1991; Arik, 1992; Costa, 2003; Machado et al., 2011, 2012). A simple alternative is to truncate the extreme high values to some threshold or top-cut value, a procedure known as ‘capping’ or ‘cutting’ (Sinclair and Blackwell, 2002; Rossi and Deutsch, 2014). As an example, Costa (2003) proposes to truncate the values that deviate by more than their cross-validation standard deviation error. With this method, the same high value can result in different truncated values, depending on the kriging neighbourhood, and no fixed top-cut is used. In practice, a fixed top-cut value is often considered, chosen (with some arbitrariness) in the last percentiles of the data distribution. This practice is widely used in the evaluation of precious metal deposits (David, 1988; Krige, 1999; Dagbert, 2005) and is accepted in current international codes for reporting mineral resources and ore reserves (SAMREC, 2007; JORC, 2012).

However, although less detrimental than removing extreme high values, considering truncated values omits some part of the data information and is likely to provoke a bias in the estimates, which has to be assessed and, if possible, fixed. On this subject, Rivoirard et al. (2013) recently presented a model in which a...
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fixed top-cut value is used and the estimates are corrected for bias. The model is attractive from several points of view, as it allows choosing the ‘optimal’ top-cut value and working with truncated indicator data, thus avoiding the use of extreme high values for calculating sample variograms and for performing spatial interpolation. This work presents an application of this top-cut model to a case study in mineral resources evaluation, together with proposed guidelines for practical implementation. Before dealing with the case study, a short review of the model is presented. Mathematical details and proofs can be found in the paper by Rivoirard et al. (2013).

Methodology

Spatial interpolation with top-cut model

Let \( x \) denote the vector of coordinates associated with a spatial location, \( Z \) the variable targeted for estimation (typically, the grade of an element of interest) and \( z_e \) a top-cut value. The variable (original grade) can be split into two components, a truncated grade and an excess, as follows:

\[
Z(x) = \min(Z(x), z_e) + (Z(x) - z_e)1_{Z(x)>z_e} \tag{1}
\]

where \( 1_{Z(x)>z_e} \) denotes an indicator function:

\[
1_{Z(x)>z_e} = \begin{cases} 
1 & \text{if } Z(x) > z_e \\
0 & \text{otherwise}
\end{cases} \tag{2}
\]

In turn, the excess can be written as the sum of its regression onto the indicator and a residual:

\[
Z(x) = \min(Z(x), z_e) + \{m^*(z_e) - z_e\}1_{Z(x)>z_e} + R_{z_e}(x) \tag{3}
\]

where \( m^*(z_e) \) is the mean value of \( Z \) above \( z_e \), so that \( m^*(z_e) - z_e \) is the mean excess above \( z_e \), and \( R_{z_e}(x) \) is the regression residual. The proposed model assumes that this residual has no spatial correlation with the indicator function or with the truncated grade:

\[
\forall x, x', \text{corr}(1_{Z(x)>z_e}, R_{z_e}(x')) = 0 \tag{4}
\]

and

\[
\text{corr}(\min(Z(x), z_e), R_{z_e}(x')) = 0 \tag{5}
\]

Under such an assumption, the estimation of the original grade can be made by jointly estimating the truncated grade and the indicator via cokriging (CK) and separately estimating the residual via kriging (K):

\[
Z(x)' = \min\{Z(x), z_e\}^{\text{CK}} + \{m^*(z_e) - z_e\}1_{Z(x)>z_e}^{\text{CK}} + R_{z_e}(x) \tag{6}
\]

If, furthermore, the residual has no spatial correlation structure, its estimate is equal to its mean value, i.e. zero. This results in the following estimate for the original grade:

\[
Z(x)' = \min\{Z(x), z_e\}^{\text{CK}} + \{m^*(z_e) - z_e\}1_{Z(x)>z_e}^{\text{CK}} \tag{7}
\]

The model therefore amounts to cokriging the truncated grade and the indicator function. Both of them no longer have extreme high values, in contrast to the original grade, which lends more robustness to the calculation of sample variograms and to the final estimates.

Validation of model assumptions

As assumed in Equation [4], the residual should be spatially uncorrelated with the truncated grade and the indicator. In other words, the high grade values (above \( z_e \)), which account for the non-zero residual values, should be independent of the low grade values (below \( z_e \)) and of the geometry of the set \( A(z_e) \) of locations with grade values greater than \( z_e \). Rivoirard et al. (2013) state that this occurs when there is no edge effects within \( A(z_e) \), i.e. when the value observed at a location of \( A(z_e) \) does not depend on whether or not this location is close to the boundary of \( A(z_e) \).

The absence of edge effects can be verified by examining the indicator variograms. Let us denote by \( \gamma_{z_e}(h) \) the variogram of the indicator function \( 1_{Z(x)>z_e} \) and by \( \gamma_{z_e,c}(h) \) the cross-variogram between the indicators \( 1_{Z(x)>z_e} \) and \( 1_{Z(x)<z_e} \) with \( z \) greater than \( z_e \) (under an assumption of second-order stationarity, these variograms are functions of the lag separation vector \( h \)). Then, in the absence of edge effects, one obtains (Rivoirard, 1994)

\[
\frac{\gamma_{z_e,c}(h)}{\gamma_{z_e}(h)} = \text{constant independent of } h \tag{8}
\]

Choice of optimal top-cut value

The previous ratio of indicator variograms is the main tool for finding the most appropriate top-cut value \( z_e \). Indeed, one can select several thresholds (candidate top-cut values) \( \{z_1, ..., z_k\} \) and calculate the ratios between the indicator cross- and direct variograms associated with successive thresholds. The first threshold for which the ratio is approximately constant (independent of the lag separation vector) corresponds to the minimal acceptable value (\( z_{\text{min}} \)) for choosing the top-cut value \( z_e \).

Also, it is convenient (although not compulsory) to choose the top-cut value so that the residual variability above \( z_e \) is pure nugget effect. In such a case, there is no advantage in choosing a higher top-cut value, since this would amount to incorporating poorly structured grade values in the truncated variable. In other words, when the residual variability above \( z_e \) is pure nugget effect, there is little or no loss of information entailed by working with the truncated grade \( \min\{Z(x), z_e\} \) and the indicator \( 1_{Z(x)>z_e} \) instead of the original grade \( Z(x) \). Following Rivoirard et al. (2013), this condition can be checked by calculating a residual indicator variogram defined as a difference between normalized indicator variograms:

\[
\frac{\gamma_{z_e}(h)}{T(z)^2} = \frac{\gamma_{z_e,c}(h)}{T(z)^2} \tag{9}
\]

where \( z \) and \( z' \) (with \( z < z' \)) are thresholds greater than \( z_{\text{min}} \), while \( T(z) \) and \( T(z') \) are the proportions of grade values above \( z \) and \( z' \), respectively. The threshold \( z \) for which the above difference is pure nugget provides the maximum acceptable top-cut value (\( z_{\text{max}} \)).

In summary, the examination of indicator variograms allows definition of an interval [\( z_{\text{min}}, z_{\text{max}} \)] in which to choose the ‘optimal’ top-cut value \( z_e \). In addition to these considerations (Rivoirard et al., 2013), the following additional checks are proposed once the top-cut value has been chosen:

1) Calculate the truncated grade, indicator, and residual at each data location
2) Calculate the autocorrelation function (correlogram) of the residual, in order to determine whether or not it is a pure nugget effect. If so, the contribution of the
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residual can be omitted in the estimation of grade (Equation [6]). Otherwise, one may prefer the estimator that accounts for the residual (Equation [5])

3) Calculate the cross-correlogram between the residual and the truncated grade, as well as the cross-correlogram between the residual and the indicator, and check that these are close to zero for every lag separation vector. This corroborates that the residual has no spatial correlation with the truncated grade and with the indicator, as it is assumed in the model (Equation [4]).

The use of sample correlograms and cross-correlograms is suggested because of their robustness to outliers or to data with extreme high values (Isaaks and Srivastava, 1988), although these tools may be biased estimates of the true underlying spatial correlations because of the centring and normalization of the data for each lag separation vector (the magnitude of the bias depends on the number of data and their spatial distribution). Unbiased estimates of the spatial correlation could be obtained with the variogram or with the non-centred covariance, at the price of a loss of robustness (Rivoirard et al., 2000, Chilès and Delfiner, 2012).

Type of cokriging

Let us return to the estimator in Equation [5]. This estimator relies on kriging the truncated grade and the indicator and, if not nugget, kriging the residual. Rivoirard et al. (2013) consider the use of simple and ordinary kriging/cokriging, depending on whether the mean values of the variables are deemed known or not.

The latter case (unknown mean values) is interesting in practice, insofar as the means can vary in space, provided that they remain approximately constant at the scale of the kriging or cokriging neighbourhood (assumption of local stationarity). When omitting the residual term and using the estimator in Equation [6] with ordinary cokriging, one further assumes that the mean grade above top-cut value, \( m'(z_0) \), is constant in space and known (but the probability that \( Z(x) \) exceeds \( z_0 \) may vary locally and is unknown), so that the local mean value of the residual is zero everywhere (Rivoirard et al., 2013). Now, if one denotes by \( m(z_0) \) the mean grade below top-cut value, it is possible to establish a relationship between the mean values of the truncated grade and indicator, as shown in Table 1.

Accordingly, the relationship between the mean values of the indicator \( (m_1) \) and truncated grade \( (m_2) \) can be modelled in the following fashion:

\[
(m'(z_0) - z_0)m_1 + m_2 = m'(z_0)
\]  

[9]

Even when considering that \( m_1 \) and \( m_2 \) are unknown, it is reasonable to assume that the previous relationship remains valid and to incorporate this relationship in the ordinary cokriging system (Emery, 2012). This is more restrictive than traditional ordinary cokriging, which assumes that \( m_1 \) and \( m_2 \) are unknown and unrelated (a somehow naive assumption, because of the strong dependence between the indicator and the truncated grade, as highlighted in Table 1), but more versatile than simple cokriging, which assumes that both mean values are known without any uncertainty. For the coefficients of the relationship between \( m_1 \) and \( m_2 \) to be constant (Equation [9]), it is assumed that the mean grade below top-cut value, \( m'(z_0) \), is constant in space and known, while the probability that \( Z(x) \) exceeds or falls short of \( z_0 \) may be locally variable and unknown.

In the next section, these tools and models are applied to a case study in mineral resources estimation.

Case study: porphyry copper-gold deposit

Presentation of the data set and deposit

The available data consists of samples taken from 167 exploration drill-holes over an area of about 0.24 km² in a porphyry copper-gold deposit (Figure 1A). The mineralization is disseminated and subvertical in orientation, with a main direction dipping about 60° with respect to the horizontal plane. Mineralization comprises chalcopyrite, bornite, digenite, chalocite, covellite, molybdenite, and pyrite. Gold mineralization occurs as native gold associated with gangue minerals, with particle sizes up to 160 µm, and as blebs within bornite and chalocite.

In the following, it is of interest to estimate the gold grade within the deposit. To this end, the available samples have been composited to a length of 5 m. The distribution and statistics of the grade data are summarized in Figure 1B and Table II, indicating a long-tailed distribution with a median of 3 g/t.

Choice of top-cut value

A set of thresholds (from 3 g/t to 6 g/t) are considered as candidate top-cut values and the associated indicator variograms are calculated. From these, it is determined that, in every case, the ratio of cross-to-direct variograms (Equation [7]) does not vary significantly with the lag separation distance (Figure 2A, 2C, and 2E), denoting the absence of edge effects within the set of locations with grade greater than the thresholds under consideration. Following Rivoirard et al. (2013), any top-cut value greater than or equal to 3 g/t is therefore eligible. On the other hand, the variograms of indicator residuals (Equation [8]) display some spatial structure at short scales (up to approximately 20 m) (Figure 2B, 2D, and 2F). Accordingly, the maximum

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean of indicator ( (m_1) )</th>
<th>Mean of truncated grade ( (m_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>True grade less than or equal to ( z_0 )</td>
<td>0</td>
<td>( m'(z_0) )</td>
</tr>
<tr>
<td>True grade greater than ( z_0 )</td>
<td>1</td>
<td>( z_0 )</td>
</tr>
</tbody>
</table>
eligible top-cut value is not well-defined, since the residual variability above the candidate top-cut values is not pure nugget effect.

Based on these premises, a top-cut value of 3 g/t has been finally chosen, corresponding to the 92% percentile of the gold grade distribution. This choice instead of a higher top-cut value (4, 5, or 6 g/t) is motivated by the fact that the truncated gold grade is likely to exhibit a better spatial continuity, as it is no longer ‘contaminated’ by outlying data. The gold grade can therefore be split into three components (a truncated grade, an indicator associated with the chosen top-cut, and a residual), as per Equation [3]. To validate the model, it is convenient to check that there is no spatial correlation between the truncated grade and the residual, as well as between the indicator and the residual. This is done by calculating the cross-correlograms between the residual and the truncated grade and between the residual and the indicator, and verifying that these correlograms are close to zero for every separation distance (Figure 3).

Variogram modelling

The direct and cross-variograms of the truncated grade and the indicator are calculated along the identified main anisotropy directions and are fitted with a spherical structure (Figure 4A, 4B, and 4C):

\[
\begin{pmatrix}
\gamma_{AA} & \gamma_{AI}
\end{pmatrix}
= 
\begin{pmatrix}
0.9 & 0.25 \\
0.25 & 0.115
\end{pmatrix}
\times \text{sph}(7m, 7m, 40m)
\]

\[ \gamma_{residual} = 1.0 \times \text{sph}(7m, 7m, 28m) \]  

The above equation gives a valid coregionalization model, as the eigenvalues of the sill matrix are non-negative (Wackernagel, 2003). The direction of main continuity (with a correlation range of 40 m) is dipping 60° with respect to the horizontal plane, in agreement with the known direction of mineralization, whereas the variograms are found to be isotropic with a correlation range of 7 m in the plane orthogonal to this direction. Note that the fitting relies mainly on the sample variograms at lag distances greater than 5 m (composite length), since few data pairs are involved in the calculation of the very first experimental point along each direction (only 1 pair for the direction of main continuity and 36 pairs for the orthogonal plane, while all the other experimental points involve several hundreds to thousands of data pairs).

It is also interesting to determine whether or not the residual is spatially correlated, in order to determine which estimator (Equation [5] or Equation [6]) is best suited to the data. To this end, instead of the traditional sample variogram of the residual, we calculated its sample correlogram, which is a more robust spatial continuity measure (Isaaks and Srivastava, 1988). This sample correlogram is then converted into a standardized variogram, showing the existence of a spatial correlation structure, although with a shorter range (28 m) than the truncated grade or the indicator. The residual variogram can be modelled by an anisotropic spherical structure, with the same anisotropy directions as the indicator and truncated grade (Figure 4D):

\[
\gamma_{residual} = 1.0 \times \text{sph}(7m, 7m, 28m)
\]
Capping and kriging grades with long-tailed distributions

Figure 2—(A, C, E) ratios of indicator variograms (Equation [7]); (B, D, F) variograms of indicator residuals (Equation [8]) (omnidirectional calculations)

Figure 3—Cross-correlogram between (A) truncated grade and residual, and, B) indicator and residual (omnidirectional calculations). Horizontal black line indicates zero correlation
Capping and kriging grades with long-tailed distributions

Resources estimation
According to the previous models, three approaches for estimating gold grade are compared:

1) Ordinary kriging of the truncated gold grade, corresponding to the traditional capping approach
2) Cokriging of the truncated gold grade and indicator, as proposed in Equation [6]. As stated previously, the mean values of both variables are assumed unknown but linearly related (Equation [9])
3) Cokriging of the truncated gold grade and indicator, as above, together with ordinary kriging of the residual, as proposed in Equation [5]. This third estimator is tested because the residual exhibits a spatial structure, as reflected in Figure 4D.

In each case, the same search neighbourhood is used, consisting of an ellipsoid with semi-axes 400 m along the main anisotropy direction and 100 m along the orthogonal directions. This ellipsoid is divided into octants and up to eight data are searched for in each octant. The dimensions of the search ellipsoid as well as the large number of searched data (up to 64 for each target block) have been chosen in order to obtain as accurate and conditionally unbiased estimates as possible, even if they inevitably yield smoothed grade models (Rivoirard, 1987; Krige, 1996, 1997b; Vann et al., 2003).

The traditional approach (kriging of truncated grade) leads to an estimate that is mostly less than the chosen top-cut grade (3 g/t) (Figure 5A), although the true grade is likely to exceed this top-cut, as suggested by the data histogram in Figure 1B. In contrast, with the cokriging approaches (Figure 5B and 5C), the grade estimates are no longer limited by the top-cut grade, which looks more realistic. The locations of high grade estimates are controlled by the indicator and by the residual, which correspond to the two corrective terms introduced in the traditional estimate (Equations [5] and [6]).

The differences between the three approaches can be assessed globally, by calculating the mean grade above different cut-off grades (Table III). It is seen that, for all the cut-offs, the traditional approach (kriging of truncated grade) yields biased estimates in comparison with the other two approaches. The bias can be explained because the traditional approach works only with the truncated grade and neglects the excess above the top-cut grade, whereas the other two approaches account for such an excess via the covariates (indicator and residual). Although the differences between the two top-cut models (with and without residual) are distinguishable on the maps drawn in Figure 5, their effect on the global statistics on the estimates is not significant, at least for low cut-offs; this is explained because, by construction, the residual has a zero mean value.

Cross-validation
In order to compare the model performances, we realized leave-one-out cross-validation: each data is successively removed and re-estimated from the surrounding data (Journel and Huijbregts 1978). The statistics on the cross-validation errors (mean error, mean absolute error, and mean squared error) are reported in Table IV, while the scatter diagrams between true and estimated gold grades are shown in Figure 6. These results confirm the bias of the traditional approach (mean error of -0.28 g/t), for which the estimates are practically limited by the top-cut grade. These estimates are also conditionally biased, insofar as the regression of the actual gold grade upon the estimated grade is significantly

Figure 4—Sample (dots and dashed lines) and modelled (solid lines) direct and cross-variograms for truncated gold grade and indicator (A, B, and C) and direct variogram of residual (D). Black: direction of main continuity (dip 60°); red: orthogonal plane
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Table III

<table>
<thead>
<tr>
<th>Cut-off (g/t)</th>
<th>Traditional capping</th>
<th>Top-cut model without residual</th>
<th>Top-cut model with residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.465</td>
<td>0.874</td>
<td>0.869</td>
</tr>
<tr>
<td>0.5</td>
<td>0.831</td>
<td>0.879</td>
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Figure 5—Gold grade estimates at a given elevation obtained with (A) traditional approach, (B) and (C) top-cut model without and with residual

Figure 6—Comparison of true vs. estimated gold grades with (A) traditional approach, (B) top-cut model without residual, and (C) top-cut model with residual. Solid blue lines: linear regression of actual grades upon estimated grades. Solid black lines: main diagonal
Capping and kriging grades with long-tailed distributions

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References


Capping and kriging grades with long-tailed distributions


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Mineral resource classification: a comparison of new and existing techniques

by D.S.F. Silva* and J. B. Boisvert*

Synopsis
A survey of 120 recent NI 43-101 technical reports was conducted to evaluate the current state of practice regarding resource classification techniques. The most common classification techniques are based on search neighbourhoods (50% of recent reports), drill-hole spacing (30% of recent reports), and/or kriging variance (6% of recent reports). Two new techniques are proposed. The first is based on kriging variance and involves removing one or more drill-holes with the highest weights while performing kriging and using the resultant kriging variance for classification. This technique has the advantages of variance-based techniques and reduces artifacts. The second technique is based on conditional simulation and uses a moving window approach for classification at the desired selective mining unit resolution based on larger production volume criteria. This technique has the advantage of accounting for heteroscedasticity, which is a common characteristic in mineral deposits, and also reduces artifacts since a production volume scale is considered for the actual classification. The drill-hole spacing, search neighborhood, kriging variance, and simulation-based techniques are described and compared for 2D and 3D examples with regular and irregular drilling patterns to highlight the advantages and disadvantages of each method.

Keywords
mineral resource, resource classification, NI 43-101, national instrument, technical reports, kriging variance, simulation, moving window, cross validation, variance.

Introduction
The economic assessment of mining projects includes many factors and resource classification is critical at any stage of mining. The quality of resource classification is a key requirement for accurate economic and environmental risk evaluation. The results of economic assessment are usually reported by companies in order to attract investors. Mineral resource classification standards were created in order to define rules for public disclosure of mineral projects, providing investors with reliable information to assist in making investment decisions. The key idea behind classification standards is to provide a general definition of different categories based on a quantified level of geological confidence so that a qualified/competent person can judge the uncertainty based on their past experience with similar deposits.

The estimation of quality/geological confidence depends not only on the quantity of available data, but also on its quality. A number of different quality parameters are discussed by Yeates and Hodson (2006), Postle et al. (2000), and Dominy et al. (2002). According to the CIM standards on mineral resources and reserves, the classification of mineral resources is dependent on “… nature, quality, quantity and distribution of data…” (Postle et al., 2000). Often companies adopt high standards of quality control in the early stages of projects in order to be able to support Measured resources; therefore, data quality is not considered in this work, all data is assumed to be error-free.

A number of techniques exist for the evaluation of mineable resources based on the quantity and distribution of data. Based on a survey of 120 recent NI 43-101 technical reports, geometric techniques are the most common and typically include drill-hole spacing and search neighbourhood. Techniques based on geostatistics are not as popular, but there are a number of proposals for resource classification, mostly based on ordinary kriging variance.

Typically, the kriging variance is used as a classification criterion by applying thresholds based on the variogram. The application of these thresholds to the kriging variance in order to define the categories was recommended by Royle (1977), Sabourin (1984), and Froidevaux et al. (1986) (as cited in Sinclair and Blackwell, 2002). More sophisticated techniques based on kriging variance were proposed by a number of authors. The relative kriging standard deviation, defined as the ratio between kriging standard deviation and the estimated value of a block, can be used (David, 1988). Arik (1999) proposed a classification based on a combination of the ordinary kriging variance and the weighted average of the squared difference between the estimated value of a block and the data values.

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used in its estimation. This combined variance is also used in the calculation of a resource classification index proposed later by the same author. The resource classification index includes the estimated value of the block and a calibration factor (Arik, 2002). Yamamoto (2000) proposed a classification technique based on interpolation variance, which is the weighted average of the squared difference between the estimated value of a block and the data values used in its estimation; the weights used are the ordinary kriging weights. Mwasinga (2001) gives a brief description of some other geostatistical classification approaches such as variogram range, kriging variance pdf, confidence limits based on normal and lognormal models, block efficiency, Isobel Clark’s classification index, and linear regression slope.

There is also a movement towards the use of conditional simulation techniques in order to support resource classification. Dohm (2005) proposed a methodology that uses conditional simulation to estimate the coefficient of variation (CV) of different production volumes: local (SMU), monthly, and annual. The estimated CVs are later used to define change-of-support factors, which accounts for the correlation between the blocks. These factors are used to define the threshold between classification categories. A block (SMU) with a CV (given by its kriging standard deviation and kriging estimate) small enough to support a monthly production volume with a precision of ±15% with 90% confidence (assuming Gaussian distributions) is classified as Measured. The annual production volume is used to define the Indicated category, and the remaining blocks are assigned to the Inferred category. The main drawback of this methodology is the assumption of normality and generalization of the coefficient of variation since the distribution, whether normal or not, can be assessed after the generation of an enough number of realizations. The use of conditional simulation for classification is also covered by Deutsch et al. (2016), Dominy et al. (2002), Snowden (2001), and Wawruch and Betzhold (2005).

The output of the survey of recent NI 43-101 technical reports motivated a comparison between the most common techniques, and this comparison motivated the development of new techniques that can take advantage of the recent advances in geostatistics. Although simulation is not used for resource classification in current practice, its possible benefits are investigated. The objective of this paper is to compare the most common techniques and the proposed methods in order to highlight the advantages and disadvantages of each; hopefully, motivating the use of more advanced geostatistical techniques in resource classification.

The proposed kriging variance and cross-validation classification was conceived for the purpose of (1) reducing artifacts that are observed while applying standard kriging variance or regression slope classification, (2) to improve ease of application (fewer subjective parameters) compared to combined or simulation-based techniques, and (3) to retain the advantages of variance-based approaches.

The methodology that uses probabilistic criteria applied to the conditional simulation realizations (1) makes use of the advantages of simulation, (2) applies meaningful probabilistic criteria, and (3) increases the resolution of classification based on criteria applied to large scales.
Mineral resource classification: a comparison of new and existing techniques

conditional simulation as the processing of multiple realizations for mine design is difficult (Dominy et al., 2002); however, it is becoming more common (Snowden, 2001). Each realization generated by simulation is an equally probable representation of the mineral grades and the full set of realizations must be treated as an ensemble, but has the benefit of being able to quantify the uncertainty in the variable under consideration.

The realizations can be scaled to any volume of interest, which is often a selective mining unit (SMU) or a production volume over some time period of interest. The scaled models can be used to evaluate the distribution of grades at a specific support, allowing a meaningful utilization of probabilistic criteria for resource classification. It is up to the qualified person to determine the criteria that would define each category. There are at least three critical parameters to be defined: volume under consideration, precision, and confidence interval (e.g. the values of a quarterly production volume must fall within ±15% of the mean 95% of the time in order to be classified as Measured).

A further advantage of using simulation-based techniques is the possibility of including many other important factors that should be considered for resource classification such as the incorporation of all identified sources of error (Dominy et al., 2002). Moreover, a significant proportion of current geostatistical research is focused on generating better conditional simulations; using simulation for classification allows practitioners to take advantage of the numerous advances being made in this field of study.

The use of conditional simulation for resource classification is suggested by many authors such as Wawruch and Betzhold (2005), Dohm (2005), Dominy, et al. (2002), and Snowden (2001), which presents it as a better approach to access uncertainty when compared to the kriging variance and other techniques, while Deutsch, et al. (2006) recommends its use only as a supporting tool while the final classification criteria should remain geometric. The reason for this is because the results of classification are highly dependent on the modeller assumptions and the parameters chosen, making resource disclosure less transparent to investors.

Methodology

Even with geometric-based classification there are a number of subjective parameters, i.e. drill-hole spacing classification can be automatically calculated by a computer algorithm or handle-defined bench-by-bench. A description of two popular techniques is provided – drill-hole spacing and search neighbourhood; a description of two proposed techniques follows based on cross-validation variance and conditional simulation.

Drill-hole spacing

As mentioned, the calculation of drill-hole spacing is not straightforward for irregular drilling patterns. Here, the calculation of drill-hole spacing is based on Equation [1], which is calculated with a circular search and corrected to represent a squared spacing i.e. 50 m × 50 m (DHS = 50 m).

\[ \text{DHS}(u) = R(u) \left( \frac{2}{n} \right)^{\frac{1}{2}} \]  

where \( u \) is the location of the block to be classified, \( \text{DHS}(u) \) is the calculated drill-hole spacing at location \( u \), \( n \) is the user-defined parameter of the \( n \)-closest drill-holes that intersect the horizontal plane, and \( R(u) \) is the average distance between the centre of the block and the \( n \)th and \((n+1)\)th drill-holes. In order to reduce artifacts and generate smoother classification maps, a new technique is proposed where multiple values of \( n \) are considered and the resulting \( \text{DHS}(u) \) is averaged (ADHS) to provide more spatially consistent results in the case of irregular drill-hole spacings.

Search neighbourhood

Two parameters are required to define a search neighbourhood; the search radius and the minimum number of drill-holes. The classification performed in this way is similar to classification based on DHS using a single threshold value equal to the equivalent drill-hole spacing (EDHS), which can be calculated using an equation similar to the equation used for DHS considering the search radius (\( R \)) and the minimum number of drill-holes (\( n_{min} \)) used as parameters for search neighborhood classification (Equation [2]).

\[ \text{EDHS}(u) = \frac{2}{n_{min}} \left( \frac{1}{R} \right)^{\frac{1}{2}} \]  

Kriging variance

In this work, the thresholds that define different categories are selected according to a desired drilling spacing. The kriging variance of a block located at the centre of a regular grid that would support Measured resources becomes the threshold between Measured and Indicated, and the value of the kriging variance of a block in the centre of a regular grid that would support Indicated resources becomes the threshold between Indicated and Inferred resources.

Cross-validation variance

A new classification technique is proposed in order to retain the advantages of the kriging variance over the geometric techniques and to reduce artifacts. The cross-validation variance (CVV) is calculated by removing one or more drill-holes with the highest weights while performing block kriging and using the resultant kriging variance to classify the blocks. This technique is suitable for regular and irregular drilling patterns; accounts for spatial structure and redundancy between data; and reduces artifacts caused by using the kriging variance alone. Classification is done by (1) removing the drill-hole with highest kriging weight, (2) calculating kriging variance using the surrounding data, and (3) applying a threshold for classification.

The number of drill-holes to be removed and thresholds are defined by the user in order to minimize the undesirable ‘holes’ and ‘patches’ that are created with conventional kriging variance classification. An improved reduction of artifacts can be achieved by using the average CVV resulting from removing different numbers of drill-holes.

Moving window classification based on conditional simulation realizations

It is desirable to have a classification model at SMU scale
Mineral resource classification: a comparison of new and existing techniques (Wawruch and Betzhold, 2005), but one of the difficulties of using probabilistic criteria (i.e. the values must fall within ±15% of the mean 95% of the time) for resources classification is that in order to classify at the SMU scale, the probabilistic criteria have to be less restrictive in order to allow for measured resources. Moreover, artifacts are often generated close to drilling locations where blocks are classified as Measured even in sparsely sampled areas. These artifacts are undesirable (Deutsch et al., 2006). This is often remedied by classifying resources based on larger volumes which may represent monthly, quarterly, or yearly production. In this case the probabilistic criteria can be more restrictive, leading to more control at a meaningful scale with fewer artifacts.

The quarterly production volume is much larger than the SMU size and its shape, volume, and position are often unknown as these depend on a detailed mine plan that is certain to change as more data is collected. However, the shapes of these larger panels can be determined from prior experience with similar deposits in conjunction with relevant information such as a grade variability model (Wawruch and Betzhold, 2005). Different origins for this large-scale block model lead to different classification models, and because of its size the classification is made at low resolution (Figure 1).

In order to obtain the desired SMU-scale classification resolution while minimizing artifacts, a large production volume is required but the panel positioning is not deterministic at the stage of classification. A local classification is proposed that considers a window representing the production panel centred at each SMU block which is classified according to the classification of that panel (Figure 2).

The specific values for the probabilistic criteria to be used is out of the scope of this work; it is certainly case-specific and requires expert judgment, as with all classification approaches. The parameters usually range between ±10% to ±30% for precision and between 95% and 80% for confidence intervals (Dohm, 2005; Dominy et al., 2002; Wawruch and Betzhold, 2005; Yeates and Hodson, 2006). The criteria used in this work are within these ranges.

Survey of NI 43-101 reports

The public disclosure of mineral project results by companies listed on Canadian exchanges must follow the Canadian Institute of Mining (CIM) standards for mineral resources. The documents that contain this disclosure are known as NI 43-101 and are publicly available through the SEDAR website (SEDAR, 2013). A survey of NI 43-101 technical reports issued in 2012 was conducted to evaluate the current state of practice regarding techniques used in reserve classification. The collected information relevant to this work are the classification technique employed, the chosen criteria, and the drilling pattern. From a total sample of 281 reports, only 120 had sufficient information to determine the technique used for classification. The remaining 161 reports are: those without resource classification; reports with only Inferred resources; reports with classified resources but without clear explanation of the methodology applied; and, reports on the same deposit that were already included in database.

The most common classification techniques are (Table I): search neighbourhood (SN); drill-hole spacing (DHS); kriging variance (KV); a combination of drill-hole spacing and search

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Table I: Analysis of classification methods used in NI 43-101 technical reports published in Canada in 2012 (120 reports were considered)

Figure 1—Different classification results for different origins based on a larger production volume scale

Figure 2—Moving window classification. Left: the SMU block is not considered Measured as the uncertainty in the larger production volume (light grey) is large. Centre: the SMU block is considered Measured as there is low uncertainty in the larger production volume (light grey) due to the denser data. Right: SMU blocks considered Measured
neighbourhoods (DHS+SN); and a combination of search neighbourhood and kriging variance (SN+KV). In general the DHS technique was preferred when drilling was regular; as this is often the case in mining operations. Often the SN technique is used when drilling was irregular as defining a consistent DHS to apply to the deposit is difficult. Kriging variance is used to account for geological continuity when the variogram is considered well defined, but manual treatment of the results was required to remove artifacts in most cases.

Comparison between techniques

The drill-hole spacing, search neighbourhood, kriging variance, simulation, and cross-validation variance are described and compared for 2D and 3D examples with regular and irregular drilling patterns to highlight the advantages and disadvantages of each. The 2D model was generated by an unconditional sequential Gaussian simulation (SGS) and sampled with a regular and irregular grid (Figure 3). The 3D example uses data from drill-holes on a porphyry copper-gold deposit (Figure 4).

Classification for the 2D regular grid is trivial, but is included as a benchmark for the techniques. It was created to resemble a constant thickness (10 m) tabular deposit in which the modelling block size is 25 m by 25 m and the quarterly production is given by a block size of 150 m by 150 m. For the regular 2D example the model is sampled by three regular grids: 200×200 m; 100×100 m; and, 50×50 m. For the irregular 2D example a random component was added to the coordinates of the regular grid before sampling. The variogram of the data is a nested structure of two isotropic spherical models with ranges of 200 and 300 m with 25% and 75% of contribution to the sill respectively.

For the 3D example the variogram of the data is a nested structure with three spherical models and a nugget effect of 15% (Equation [3]).

\[
\lambda(h) = 0.15 + 0.18 \times \text{Sph}_{a=50m}^{a=25m}
+ 0.17 \times \text{Sph}_{a=100m}^{a=25m} + 0.50 \times \text{Sph}_{a=100m}^{a=50m}
\]

\[\text{[3]}\]

The 3D example has two nominal drill-hole spacings of 50×50 m and 25×25 m. The modelling block size for the 3D example is 15 m by 15 m by 10 m and the quarterly production is given by a block size of 150 m by 150 m by 60 m.

Results and discussion

2D regular

The synthetic 2D example with a regular drilling pattern is considered first to visualize the results of each technique (Figure 5). For DHS the Measured blocks are those within the area drilled at 50×50 m grid with extrapolation of half a spacing (25 m). Indicated blocks are those within the area drilled at 100×100 m with extrapolation of 50 m, and Inferred blocks are those within the area drilled at 200×200 m.

For the SN classification the parameters were chosen by a visual sensitivity analysis in order capture the areas considered Measured, Indicated, and Inferred. Blocks with at least 8 drill-holes within 100 m are considered Measured (EDHS = 50 m), Indicated blocks are those with at least 8 drill holes within 200 m (EDHS = 100 m).

For the KV classification the thresholds were defined based on same drill-hole spacing used for DHS classification. The threshold between Measured and Indicated is 13% of the sill and the threshold between Indicated and Inferred is 31% of the sill.

The number of drill-holes removed for the CVV method is one and the thresholds were chosen by a visual sensitivity analysis in order to reduce artifacts. The removal of drill-holes increases the kriging variance for each block, which leads to higher thresholds when compared to using the KV
Mineral resource classification: a comparison of new and existing techniques

The thresholds used were 21% and 45% of the sill. For the average CVV classification, the number of drill-holes removed was one and two and the thresholds were 24% of the sill and 50% of the sill, again selected based on visual inspection.

The classification based on conditional simulation was performed with the proposed technique. In order to define measured blocks the quarterly production panel must have a precision of at least ±15% with 95% of confidence, while indicated must have a precision of ±30% at 80% confidence interval.

It will be noticed that for this synthetic example, the DHS zones defined by hand (titled Drill Hole Spacing in Figure 5) were matched well by the majority of the techniques as this is a fairly easy set of drill-holes to classify. As expected, the KV performed well in classifying different zones but with the problem of artifacts (patches) close to drilling locations that are successfully removed using the proposed CVV methodology. Artifacts were also successfully avoided using the proposed methodology for conditional simulation. In this case there is no anisotropy and the proportional effect is not expressed.

2D irregular

The 2D example with irregular drilling is used to visualize the effect of parameters for each technique and to visualize the adequateness of each technique in situations in which classification is not straightforward.

Drill-hole spacing

A visual analysis of the parameters for DHS is shown in Figure 6. Increasing the number of data used in calculation reduces the artifacts but also increases misclassified blocks compared with the assumed correct manual classification (Figure 5 – upper left). There is no control on the search radius considered as it is a function of the block location and number of data searched (n). Data far from a block may inadvertently assign a higher category for a block; a small number of drill-holes is recommended to avoid this problem. The use of average DHS (Figure 6 – right) removes the reliance on selecting a single value of n. More accurate (closer to the known ‘by hand’ technique) and smoother (fewer holes and patches) maps can be achieved using the average.

Search neighbourhood

A visual analysis of the parameters for the SN technique is shown in Figure 7. Classification based on SN requires two parameters (search radius and minimum number of drill-holes) and performs similarly to DHS for irregular drilling patterns. The classification maps may require post-processing to reduce noise on the classification borders.

Kriging variance/cross-validation variance

A visual analysis of parameters for the CVV technique is shown in Figure 8. Blocks that are close to redundant drill-holes tend to stay in the same category as with the conven-

Figure 5—Classification results for the 2D regular grid. Axes dimensions: 2000 m by 2000 m. The upper left classification is the assumed correct classification for this simple synthetic example.

Figure 6—Sensitivity on DHS parameters. (Left: single parameter; right: average DHS searching 1 to 4, 1 to 6, and 1 to 20 drill-holes). Axes dimensions: 2000 m by 2000 m.
tional KV method; blocks that are located close to isolated drill-holes tend to be downgraded. This is a desirable characteristic but a balance must be achieved between removing ‘patches’ and creating new ‘holes’. In general, the technique reduces the artifacts compared to using the KV alone (Figures 5, 11, and 12). If the removal of one drill-hole is not sufficient for removing artifacts the average CVV may be considered.

Conditional simulation

A visual analysis of parameters for classification based on conditional simulation using the proposed methodology is shown in Figure 9. The conventional classification for small (SMU) and large scale (panel) is compared with the proposed methodology for classifying at a local scale resolution by using large-scale criteria (Figure 10). The chosen criteria for SMU scale classification were precision of ±30% with 90% confidence for Measured and ±30% with 50% confidence for Indicated. For the large scale the criteria were precision of ±15% with 95% confidence for Measured and ±30% with 80% confidence for Indicated. The proposed technique of centring a production volume on each SMU (Figure 10 right) reduces artifacts and does not have the undesirable reliance on a fixed large-scale grid, where panels clearly contain part Measured and part Inferred SMU blocks (Figure 10 centre).

Classification results

The result of classification for the 2D irregular case is shown in Figure 11 for all techniques and illustrates how different techniques considered perform in a non-straightforward way. The DHS was calculated using Equation [1]. Blocks with DHS less than or equal to 50 m are Measured, blocks with DHS less than or equal to 100 are Indicated, and the remaining blocks are Inferred.

For SN classification the parameters were chosen by a visual sensitivity analysis in order to take the best combination that captured the areas considered Measured, Indicated, and Inferred. Blocks with at least 8 drill-holes within 100 m are considered Measured (EDHS = 50 m), Indicated blocks are those with at least 8 drill-holes within 200 m (EDHS = 100 m).
Mineral resource classification: a comparison of new and existing techniques

For the KV classification the thresholds were defined based on a regular grid of 50 × 50 m for Measured and 100 × 100 m for Indicated. The threshold between Measured and Indicated is 13% of the sill and the threshold between Indicated and Inferred is 31% of the sill based on an equivalent DHS.

The number of drill-holes removed for CVV method is one and the thresholds were chosen by a visual sensitivity analysis in order to minimize artifacts. The thresholds used were 20% and 45% of sill. For the average CVV method the number of drill-holes removed was one and two and the thresholds were 23% and 52% of the sill.

The classification based on conditional simulation was performed with the proposed technique. In order to define Measured blocks the quarterly production panel must have a precision of at least ±15% with 95% confidence, while Indicated must have a precision of ±30% at 80% confidence interval.

3D example

The 2D examples are appropriate for vertically drilled holes, but mineral classification problems are often three-dimensional with a significant proportion being irregularly drilled as a high degree of geological confidence requires drill-holes intersecting the orebody in different directions (Yeates and Hodson, 2006). For the 3D example, a sensitive analysis similar to that made for the 2D irregular case was performed in order to select the parameters for various classifiers with exception of the classification based on conditional simulation. The probabilistic criteria used were a precision of ±15% with 95% confidence for Measured and ±30% with same confidence interval for Indicated. The classification models are shown in Figure 12.

For this example the grade values were estimated by ordinary kriging and the resources were calculated and classified with each technique. The results of resource calculation and classification are given in Figure 12 and Figure 13.

The quantitative results for geometric methods and proposed techniques were similar, with a slight increase in the Indicated category for the proposed techniques (CVV and SIM). Using the KV, there was a considerable increase in the Measured category due mainly to the 'patches' artifacts that are common with this classification technique. Ignoring the KV technique, it is interesting to note that the Measured and Indicated results are surprisingly consistent across all techniques. Of course, the benefit of incorporating simulation into classification is that local classification can be more accurate as data redundancy and anisotropy can be incorporated.

Conclusions

From a review of the most recent Canadian 43-101 reports, the most common techniques used for resource classification are geometric in nature. These techniques do not account for the spatial continuity of the variables nor redundancy between data, but typically result in classification maps that have less artifacts and are less sensitive to modelling parameters (i.e. kriging and simulation parameters).
The advantage of using variance-based techniques as opposed to geometric is the opportunity to account for grade continuity and data redundancy, which can significantly affect the local uncertainty that classification should be measuring. The kriging variance captures this information but often results in artifacts when used in classification. The combination of cross-validation with the kriging variance is able to reduce these undesirable features and incorporate known information on spatial continuity. Although the kriging variance incorporates these desired features, it does not account for the proportional effect, which is a significant limitation for the highly skewed distributions common in the mineral industry. Simulation-based classification has the potential to overcome this limitation. The proposed methodology is capable of performing classification at a typical block modelling scale (often SMU) but with reduced artifacts as a production volume scale is considered for the final classification. The main limitation of conditional simulation for classification is the sensitivity to key parameters such as the covariance function and trend model, which are very dependent on modelling assumptions, making resource disclosure less transparent to investors.

Many methodologies for classification have been proposed in recent years, but only a few of them are actually used in practice. The techniques proposed in this paper represent viable alternatives for resource classification. As with all resource classification techniques, it is the responsibility of the practitioner to assess the appropriateness of the final result based on knowledge of the deposit.

References


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Enhanced geological modelling of the Upper Elsburg reefs and VCR to optimize mechanized mine planning at South Deep Gold Mine

by K. Osburn*, H. Pretorius†, D. Kock†, N. King†, R. Pillaye†, and M. Hlangwane‡

Synopsis
South Deep Gold Mine, owned by Gold Fields Ltd., is situated near Westonaria in the Gauteng Province of South Africa and mines the conglomerate bands of the Upper Elsburg reefs (Mondeor Conglomerate Formation) of the Witwatersrand Supergroup and the Ventersdorp Contact Reef (VCR) of the Ventersdorp Supergroup. The stoping and underground developments are mechanized. The Upper Elsburg reefs are mined by a variety of mining methods, including mechanized drift and fill, modified drift and bench, longhole stoping, and low-profile mining. Optimal mine design and scheduling for deep-level mechanized mining are complex, and success is highly dependent on detailed, robust, and accurate geological and geostatistical models.

Geological structures significantly influence the sedimentological characteristics, distribution, and preservation of the Upper Elsburg reefs and VCR. Accordingly, particular emphasis is placed on the generation of a mine-scale structural model that accommodates the relationships between the older north-trending fault systems (West Rand and Panvlakte faults) and younger east-trending dextral wrench faults. Results from underground mapping, borehole intersections, and high-resolution three-dimensional seismic data have been integrated to produce coherent three-dimensional geological models.

The Upper Elsburg reefs suboutcrop against the VCR and comprise an easterly diverging clastic wedge, thickening from the suboutcrop position, to approximately 130 m at the mine’s eastern boundary. The Upper Elsburg reefs are characterized by conglomerate and quartzite bands forming multiple, stacked, upward-fining unconformity-bounded couplets. Palaeocurrent directions are dominantly from west-northwest to east-southeast, indicating that the more proximal deposits are preserved close to the suboutcrop, with distal facies to the east.

Sedimentological modelling is applied to individual stratigraphic units and caters for facies definition. This is achieved through channel width (CW) kriging and fitting of type sections to borehole and mapping data. Homogenous geological geozones for each stratigraphic unit are thus defined within individual structural blocks on the basis of those sedimentological parameters that have been found to have a positive spatial correlation to gold concentration. These geozones then serve as constraints to the evaluation of the orebody.

This contribution presents a summary of the modelling processes that are currently applied in the development of high-confidence, timeously produced geological models that are essential input for mineral resource estimation and mechanized mine planning and scheduling.

Keywords
Witwatersrand Basin, West Rand Goldfield, South Deep Gold Mine, Upper Elsburg reefs, VCR, geological modelling, mine planning and scheduling, gold distribution, channel width.
the auriferous reefs. Accordingly, the geological and resultant grade models must take cognisance of the position, displacement, and fault relationships by defining zonal boundaries that correlate with these structures as well as other geological variables.

On surface, the mine lease area comprises an undulating topography where the bedrock is largely covered by residual soils and alluvium. Outcrops are dominated by rocks belonging to the Pretoria Group of the Transvaal Supergroup that have a regional dip towards the south at less than 10 degrees. Poorly exposed outliers of younger rocks (Karoo Supergroup) have been observed in the northern portion of the lease area.

The youngest Transvaal strata are interbedded andesitic lava and tuff of the Hekpoort Formation that outcrop to the south of the mine infrastructure. They attain a maximum thickness of 260 m and are underlain by up to 550 m of quartzite and interbedded shale of the Timeball Hill Formation. These sediments are best exposed on the east-trending ridges that dominate the northern portion of the mine lease.

The Pretoria Group rocks are underlain by a 1,300 m thick sequence of sediments, predominantly dolomite and chert, belonging to the Malmani Subgroup of the Chuniespoort Group, which forms the basal unit of the Transvaal Supergroup. The lowermost 20 m of the
Enhanced geological modelling of the Upper Elsburg reefs

Chuniespoort Group is occupied by the Black Reef Quartzite Formation.

Below the sediments of the Transvaal Supergroup is a 1 500 m thick sequence of basaltic lava belonging to the Klipriviersberg Group of the Ventersdorp Supergroup. These rocks, in turn, form the hangingwall of the gold-bearing reefs (Figure 3).

The Ventersdorp Contact Reef (VCR) marks the contact between the Central Rand Group sediments and the Klipriviersberg lava and is deposited on a major regional unconformity.

Within the mine lease area, the VCR is best developed west of the Upper Elsburg suboutcrop. The Upper Elsburg Individuals and Massives, which are comprised of conglomerate with interbedded quartzite, form part of the Waterpan and Modderfontein Members that constitute the upper portion of the Mondeor Conglomerate Formation, Turffontein Subgroup (SACS, 1980; Figure 4). It is these auriferous conglomerate units that are the main mining target at South Deep. The Upper Elsburg reefs occur as a clastic wedge composed of 15 units with a rotational onlap and a disconformity between each unit. The clastic wedge diverges from the suboutcrop against the VCR in the central portion of the mining rights area to a maximum vertical thickness of approximately 130 m at the eastern boundary (Figure 4). The suboutcrop against the VCR trends at a bearing of approximately N23°E, forming a divergent clastic wedge at approximately right angles to the suboutcrop.

Mining methods

South Deep has evolved into a fully mechanized mining operation with the last conventional mining taking place in early 2008. Mining currently takes place up to 2 700 m below the surface. Currently, three mining methods are utilized to supply ore to the plant.

Drift and bench mining

Drifts are linked to drift accesses and advance at 3.7 m per blast during the primary drift mining phase. Bench accesses are mined from the opposite direction at a lower elevation (Figure 5). Benches are mined by drilling vertical blast-holes and cleaned through remote loading. Finally, when a bench is mined out, it is backfilled and the neighbouring drifts and benches are accessed and mined.

Longhole stoping

Top and bottom drives are mined to access the longhole stope (Figure 6). A slot raise is developed between the top and bottom accesses to create a free-breaking face. The slot is approximately 15 m in height, equal to the width of the planned longhole stope. Blast-holes emulating a fan are drilled from either the top or bottom access. These blast-holes are usually referred to as a ring, and typically three rings are blasted at one time, delivering approximately 4 060 t of rock per blast. After cleaning through remote loading, further rings are blasted until the entire stope is extracted.

Destress mining

Stope access drives are mined from the main access drive (Figure 7). Out of the stope access drives, stoping drives are mined sequentially and then backfilled. Upon completion of backfilling the stoping drive, the stopping access drive is advanced to access the next stoping drive.
The orebody has a shallow dip of between 15° and 20° towards the south. This dip is steeper than the maximum inclination at which mechanized mining can take place, therefore destress mining is done horizontally. Drift and bench mining cannot take place at an apparent dip of more than 8°, thus this mining method is used only in areas where selective mining is viable. Due to the scale of these mining methods, no attempt is made to separate the quartzites from the gold-bearing conglomerates. All extracted material (ore and waste) is sent to the plant for processing.

Domaining

It is recognized that there is a strong correlation between sedimentological parameters and gold distribution within the Upper Elsburg and VCR sediments. Sedimentological data is captured in both exploration and grade control drilling programmes and includes parameters that have been found to correlate closely with grade distribution. These are, in order of precedence: channel width (CW, in metres), percentage conglomerate (%), and average clast size (millimetres). Modelling considers both the proximal/distal relationship and channel morphology within a specific unit. Higher grades are associated with proximal rudaceous phases, while lower grades occur distally.

Spatial plots of grade and sedimentary data are employed to define homogenous areas for each parameter within a defined structural block and unit. The first step is to produce a plot of the raw data to create a reference point for future processing. All raw data is obtained from validated borehole logs and assays. Histograms and cumulative frequency plots of the raw data determine the optimal intervals for the generation of gridded, classed data plots and contour plans. These are ultimately utilized to define homogenous area boundaries (Figure 8).

These individual parameter boundaries are then overlain to define overall geological facies boundaries within a block. The geological facies in each block are then compared to those in other blocks and, where facies with similar characteristics
Enhanced geological modelling of the Upper Elsburg reefs

Figure 8—(a) Class plot of gridded Au values (MBB Reef), and (b) contour plot of Au values (MAC Reef). Current mining areas are in grey as shown in Figure 2

are identified, they are amalgamated to produce an overall geozone with a unique grade and sedimentary signature. Within each geozone, grades are assumed to be homogenous, and interpreted to be a realization of the stationarity random function (Duke and Hanna, 2001). When reconciled, a close resemblance between geozone estimates and actual reef characteristics, including grades, is encountered.

All estimation is confined within the boundaries of the defined geozones (‘hard boundary’). ‘Soft’ boundaries are drawn as a perimeter around the hard bundaries and have been introduced to facilitate estimation along the edge of a geozone where insufficient data is available within the defined search radius. The use of soft boundaries is applied only where a geological relationship exists with the adjacent geozone, i.e. where geological evidence suggests that the boundary in question coincides with a gradual transition between geological environments, such as proximal/distal relationships.

The soft boundary of a geozone will incorporate the samples of the transition zone, i.e. the area between the hard and soft boundaries (Figure 9). The incorporation of additional data improves the confidence of the estimate.

In the absence of geological continuity across a geozone boundary (i.e. where samples at close proximity to each other are deemed unrelated), such samples will be excluded by the hard boundary and have no influence on the estimation. Typically this occurs where geozones are separated by faults that are known to have a significant lateral displacement (Figure 9).

Prior to estimation, histograms and trend plots are generated to indicate any modality or drift in the data that new borehole intersections may have introduced into the geozone (Figure 10).

Further improvements to the stationarity of geozones are achieved by evaluating various corridors of consistent length and width along the area of interest. Comparison of the statistical characteristics of each corridor reveals, whether geozones are appropriately defined or if further refinement is required (Figure 11).

Geological modelling

Geological modelling history

A seismic survey of the mine lease area was conducted in the late 1980s. The information obtained from the seismic survey points formed the basis for the evolution of the early three-dimensional (3D) geological models. At the time, it was general knowledge that the orebody comprised the Upper Elsburg reefs as well as the VCR, which unconformably overlies the Upper Elsburg reefs. Since the early 2000s the Upper Elsburg reefs have been modelled as four major sedimentological and stratigraphic units. The VCR lava contact represents the seismic datum. Close to the suboutcrop these four units, predominantly composed of juxtaposed conglomerate, were modelled as one unit, and termed the ‘Shore Line Composite’ (SLC). These models considered bottom and top contact surfaces intersected by faults, to a resolution of 2 m.

In 2004, a second seismic survey was commissioned owing to vast improvements in data acquisition and processing. The survey results were utilized together with drilling and mapping information to develop an enhanced geological model. The four initial major sedimentary units were further defined and subdivided into first seven and later sixteen 3D-modelled units, with the seismic data locating the contact between the VCR and Ventersdorp lavas.

In 2008/09, the seismic data of the 2004 seismic survey was re-processed and integrated with the geological models. Notably, up to this stage, the 3D modelling consisted mainly of seismic survey interpretations, underground mapping, and straight-lining between successive borehole intersections. It is noted that the earlier modelling packages were not as flexible and user-friendly in constructing 3D geological models currently available software.

Figure 9—Schematic plan depicting ‘hard’ and ‘soft’ boundaries and their relevance for estimation of geozone ‘C’
During 2010, the limitation of straight-lining between borehole intersections became evident from irregularities between observed and modelled data. Available data revealed that the clastic wedge does not always diverge to the east at a constant rate due to reactivation of syn-depositional faults. In certain localities, the divergence ratio is greater than in other areas, while towards the east, the Upper Elsburg reefs reach a maximum thickness of approximately 130 m and the units become sub-parallel and conformable to each other.

These observations triggered the introduction of type sections, based on palinspastic reconstructions at regular intervals throughout the orebody. The sections were linked to the actual borehole intersections to obtain correct CW values for localized areas. These sections formed the basis of a representative depositional model, yielding robust CW values to be used during geological modelling and estimation.

The newly established methodology for CW estimation, detailed in the following section, ensures that different users will obtain identical results, with a full audit trail compliant with all reporting codes that enhances the transparency of the geological model construction process. Figure 12 provides a graphic illustration of the evolution of the South Deep geological models.

**Current geological modelling procedures**

**Channel width estimation**

Earlier methods used at South Deep were dependent, to some extent, on the users' experience and frame of reference. Therefore, the geological model was susceptible to subjectivity. This was mostly due to the limitations of digital terrain modelling software packages available in the geoscience environment at the time. As described previously, South Deep’s mine planning procedure is highly dependent on accurate and robust, fully 3D resource models, which in turn are explicitly a function of the quality and accuracy of the geological model.

![Figure 11—Representation of CW stationarity valuation showing two stationary geozones (blue and red) as well as a gradational unit (green)](image1)

![Figure 12—Graphic representation of the evolution of the South Deep geological model illustrating the increase in accuracy as well as confidence](image2)
Enhanced geological modelling of the Upper Elsburg reefs

To mitigate the subjectivity of geological modelling processes, emphasis has been placed on the estimation of the CW. Currently, uneconomic quartzitic units become thicker towards the east, while the mature conglomeratic units are more consistent in CW throughout the mine lease boundary (Figure 13).

The variograms for quartzite and conglomerate CWs perpendicular to the suboutcrop with the VCR are detailed in Figures 14a and b. In modelling the variograms and constructing the variogram contours, a specific unit of the respective lithology (quartzite or conglomerate) was selected that was representative of the deposit as a whole. The anisotropic variogram model for the conglomerate units indicates a range exceeding 800 m perpendicular to the suboutcrop, while the quartzite shows a range of 400 m.

From the variograms it can be determined that CW is mostly continuous in the suboutcrop direction (N23°E) with ranges in excess of 400 m in any direction. However, various wrench faults occur throughout the orebody, offsetting the clastic wedge in a dextral direction with lateral movement of 80 m to 300 m. Due to the character of the divergent clastic wedge, CWs across these wrench faults are not conformable.

To mitigate the influence of the wrench faults and the easterly trend in CW, the search ellipse has been set to two and four grade-control drillhole spacings (30 x 30 m) in an east-west and north-south direction (N23°E), respectively.

Due to the unavailability of drilling platforms, the 30 x 30 m drilling grid is not achieved in all of the current mine areas. Consequently, CW estimation is not achieved over the entire lease area. Therefore, a further limiting factor is applied during the kriging of CWs. If more than seven full-reef validated intersections are utilized in the estimation, the estimated value is considered valid. If not, then the CW attribute is obtained from the relevant depositional model, as described in the previous section. Due to all the constraints and limits put on the estimation of the CW, the best estimator to apply is ordinary kriging.

The CWs are then added as an attribute to a point file; from there the points are draped onto a wireframe surface of a reef unit. Once these points conform to the surface, they are translated by the CW value to the elevation where the following reef unit would be encountered. The digital terrain model obtained from the estimated locations is then further integrated with the actual drillhole intersections.

Stratigraphic sequence modelling of CW

In addition to kriging, stratigraphic sequence modelling, known at the mine as the type section method, is also applied.
Enhanced geological modelling of the Upper Elsburg reefs

to determine CW. Of the two methods, kriging yields a more accurate result and is preferred. However, owing to the scarcity of data in new mining areas, kriging estimation in these areas must be conducted with caution. Type sections of stratigraphic sequences are constructed for these areas and utilized in CW determination.

Stratigraphic sequence modelling entails the construction of palinspastic sections (Figure 15) conforming to borehole intersections. This results in a local CW model (Figure 16) for areas with insufficient data. These CWs are then utilized in the stratigraphic sequence modelling.

The importance of CW is highlighted when considering that eight conglomerate bands, intercalated by quartzite units, are mined. Thus an error of a few metres at the top of the stratigraphic sequence will propagate down and have an adverse effect on the spatial position of the lower conglomerate bands.

Once the thicknesses between stratigraphic surfaces have been modelled, mapping data, boreholes, seismic points, and isopach plots are integrated to create the final stratigraphic surface. Quality assurance and quality control techniques include visual checks on sections of the stratigraphy, as well as percentage borehole honouring analysis.

Future development of geological models

Previous geological modelling methods employed at South Deep were effective for building robust and accurate geological models, but were time-consuming. However, by applying the estimation techniques discussed above, utilizing modern and improved software, there will be significant staff and time reductions.

The new software – which is a proprietary software package and therefore not described – has the advantage that all data used is dynamic. Boreholes and mapping data are continually updated to refine the geological models of the sixteen stratigraphic surfaces (Figure 4). Updates are characterized by minimal turnaround times, resulting in the latest geological model being constantly available. The importance of regularly updated geological models to mining and mine planning are obvious.

Presently, only areas with current mining activity are dynamically built and are thus kept up to date as new information becomes available. Areas where mining occurred in the past will be converted from a digital terrain format into the dynamic format on a project basis.

Conclusions

Mechanized mining requires a high-confidence design throughout the planning and scheduling phase to limit off-reef mining. A reliable mine design is underpinned by up-to-date grade-tonnage models, which in turn are supported by accurate geological models. To achieve this, South Deep Mine implemented various innovations, including channel width (CW) estimation and dynamic and stratigraphic sequence modelling. These are accompanied by rigorous quality assurance and quality control processes. As a result, updated

Figure 15—Palinspastic type sections created for CW determination

Figure 16—Representative depositional model of South Deep
geological and grade control models are continuously produced, facilitating accurate and optimal mine planning, design and extraction of the orebody.

Given the multi-reef nature and structural complexities of the orebodies, geological modelling at South Deep Mine has progressively improved over the years. The implementation of the CW estimation process through kriging has also provided the mine with an auditable geological modelling process. Kriging of reef thicknesses is fundamental to the geological modelling, and as more drill-hole and mapping data becomes available, the construction of type sections and consequently stratigraphic sequence modelling will be converted into an estimated model.

A highly detailed geological model of the sixteen individual reef units enables resource estimation and consequently mining to be conducted at higher confidence levels.

References


BACKGROUND

Many a metallurgist fell in love with pyrometallurgy after witnessing a smelter being tapped. There is something magical in the combination of light, energy and danger that simultaneously stirs the primal instincts to ‘run for your life’ and ‘go closer and have a look’.

But tapping a smelter is not an easy task. Much engineering goes into the design of the taphole. Due to the aggressive nature of the process, material selection is as important as layout. The design process kicks off with a set of design criteria, which needs to be revised as the results of laboratory, computational fluid dynamics (CFD) and time-and-motion studies become available.

Designers have to design not only for installation and operability but also for maintenance—routine and breakdown. During the design stage, matters such as online monitoring of the taphole wear, handling of liquid products, extraction of fumes and periodic maintenance must be taken into account.

Though taphole life can be improved with good taphole design, a good design can be destroyed with incorrect tapping practices and equipment. Despite the harshness of the tapfloor environment, it requires precision equipment and operating practices. The design and maintenance of the drilling, tapping and plugging equipment and materials plays an equally important role in taphole life and tapfloor safety. As does protective equipment.

Operators want the taphole life to be as long as possible since taphole failures is often the cause for a reline—a very expensive exercise in an upcycle when the cost of a new lining is small compared to the loss in production. Managing the maintenance and reline schedule is a challenge with lessons often learned the hard way.

The first thing students are taught in pyrometallurgy courses is how to compile a mass and energy balance for a smelter. An accounting mass and energy balance is used not only to schedule furnace taps but also to make process decisions. Process variables measured during or after tapping are important inputs to a workable mass and energy balance.

Even though much has been done to make the tapping process as automatic as possible, tapping of smelters cannot be done without labour. Tap floor operators work in harsh environments where safety is of utmost importance. Selection of suitable personnel and intensive training is required.

No pyrometallurgical smelter can operate without some form of tapping system. It is the one thing all smelters have in common. A meeting point of science, technology and skill.

So let us talk about it.

OBJECTIVES
To provide an international forum to present and discuss the design, maintenance, safety and operating practices surrounding the tapping of pyrometallurgical smelters.

WHO SHOULD ATTEND
This conference is aimed at delegates from the pyrometallurgical industry, and includes:
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Production reconciliation of a multivariate uniform conditioning technique for mineral resource modelling of a porphyry copper gold deposit

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Synopsis
The paper provides a brief review of a multivariate uniform conditioning and a localized multivariate uniform conditioning (LMUC) technique, and presents a production reconciliation case study based on a porphyry copper-gold deposit in Peru. The reconciliation study compares the long-term LMUC mineral resources model (typical of new mining projects, which are invariably based on drilling data on a relatively large grid) to the corresponding production blast-hole grade control model, as well as the final plant production.

Keywords
discrete Gaussian model, Localized Multivariate Uniform Conditioning (LMUC), production reconciliation, smoothing effect, conditional biases, information effect, simple kriging panel conditioning.

Introduction
Indirect techniques to estimate recoverable resources during medium- to long-term planning derive the unknown selective mining unit (SMU) distribution estimates from the observed distribution of relative large kriged blocks (panels). The drawback of the indirect methods is that only the probability distribution of the SMUs within local panels can be derived, but not their individual spatial locations within the panel. A localized multivariate uniform conditioning (LMUC) post-processing technique has been proposed to enhance the indirect uniform conditioning by localizing the results at the SMU scale.

In this regard, the tonnages and metals represented by the grade-tonnage curves estimated by traditional indirect uniform conditioning (UC) are decomposed and distributed into the SMUs within respective panels according to a ranking of the main element grade estimate of the SMUs. UC consists of estimating the grade distribution on SMU support within a panel, conditioned to the estimated panel grade, usually based on ordinary kriging (OK) or simple kriging (SK) with local mean to accommodate a possible lack of stationarity (i.e. when the average grade varies within the deposit). The general framework that forms the basis of UC is the discrete Gaussian model of change of support, based in particular on the correlation between Gaussian-transformed variables. The mining industry’s acceptance of the UC method has been apparent for several years, and a good reconciliation is generally found between UC medium- to long-term estimates and production data.

The UC method has been extended to the multivariate case, where the correlations between main and secondary variables can be calculated on any support after transformation into Gaussian space (Deraisme et al., 2008). As the correlations between the different grades and the main element are taken into account in the multivariate uniform conditioning (MUC), the local scale estimates for the other multiple-commodity metals contained in the tonnage, which in this case are assumed to depend only on the main commodity grade, are immediately obtained from the metals attached to those tonnages. The resultant local SMU estimates are referred to as localized multivariate uniform condition estimates. The correlations between the different elements on the SMU support are reproduced by means of the correlations introduced in the multivariate change-of-support model. In addition, a rigorous formulation of the information effect on panel grade distribution has been developed that allows the heterogeneity of the expected production data configurations to be taken into account when estimating future SMU recoverable grades.

As highlighted above, the disadvantage of traditional indirect UC is that the outputs consist of panel-local grade-tonnage curves representing a number of non-localized SMUs within these panels. Therefore, it is difficult in practice to use these models for underground production reconciliation of a multivariate uniform conditioning technique for mineral resource modelling of a porphyry copper gold deposit

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Production reconciliation of a multivariate uniform conditioning technique

and open pit mine planning that requires a grade model on the SMU support. Abzalov (2006) proposed a solution using the grade-tonnage functions from the indirect univariate UC and then decomposing the panel-specific grade-tonnage data into a suite of individual SMU-sized units within the respective panels, according to a ranking of the main element grade estimate of the SMUs.

Deraisme and Assibey-Bonsu (2011) extended Abzalov’s proposal to the MUC case for medium- to long-term recoverable estimates for mine planning on a porphyry copper-gold deposit in Peru. A multivariate conditional simulation (MCS) comparative study with MUC by Deraisme and Assibey-Bonsu (2012) also showed that the fundamental assumption of the MUC technique, whereby the correlations of the secondary elements are dependent solely on the main element, is acceptable in practice. Although MCS takes into account the complete set of correlations, a significant amount of time is required for MCS; also, it does not readily provide a base model for practical mine planning as several equiprobable simulations are generated in the process.

This paper presents a brief review of the UC, MUC, and LMUC techniques and provides a production reconciliation case study for the same porphyry copper-gold deposit that was evaluated using the new LMUC technique (Deraisme and Assibey-Bonsu, 2011). In this case study the long-term LMUC estimates are compared to the corresponding production blast-hole grade control model, as well as the final production results. In order to avoid potential conditional biases of the medium- to long-term LMUC recoverable estimates, simple co-kriging with local means was used for the panel conditioning (Deraisme and Assibey-Bonsu, 2011).

Discrete Gaussian model applied to recoverable resource estimation

Review of the discrete Gaussian model

Let \( \nu \) be the generic SMU and \( Z(\nu) \) its grade, which will be used for the selection at the future production stage.

The recoverable resources above cut-off grade \( z \) for such blocks are:

- the ore: \( Q(\nu) = 1_{Z(\nu) \geq z} \)
- the metal: \( Y(\nu) = Z(\nu) 1_{Z(\nu) \geq z} \)

(\( 1_{Z(\nu) \geq z} \) stands for the grade indicator at cut-off \( z \), i.e.: \( 1_{Z(\nu) \geq z} = 1 \) if \( Z(\nu) \geq z \); \( 1_{Z(\nu) < z} = 0 \) if \( Z(\nu) < z \).

We use here the discrete Gaussian model for change of support (Rivoiraud, 1994). A standard Gaussian variable \( Y \) is associated with each raw variable \( Z \). Let \( Z(\nu) = \Phi_Y(Y(\nu)) \) be the sample point anamorphosis. The block model is defined by its block anamorphosis \( Z(\nu) = \Phi_Y(Y(\nu)) \), given by the integral relation:

\[
\Phi_Y(Y(\nu)) = \int \Phi_Y(y(\nu) + \sqrt{1-r^2} u) \Phi_Y(u) du
\]

where the change of support coefficient \( r \) is obtained from the variance of blocks.

Then, the global resources at cut-off \( z \) are:

\[
E[T(z)] = E[1_{Z(\nu) \geq z}] = E[1_{Y(\nu) \geq z}] = 1 - G(z)
\]

where \( g \) and \( G \) are the standard Gaussian probability density function (p.d.f.) and cumulative distribution function (c.d.f.), and \( y \) is the Gaussian cut-off related to \( z \) through \( z = \Phi_Y(y) \).

Review of the uniform conditioning in the univariate case

UC by panel grade (Rivoiraud, 1994) aims at estimating the recoverable resources on a generic selection block \( \nu \) randomly located within a large block or panel \( V \), conditioned on the sole panel grade, or for more generality, the panel grade estimate \( Z(V^*) \). Tonnage and metal at cut-off \( z \) are then:

\[
[T(\nu)]^* = E[1_{Z(\nu) \geq z} | Z(V^*)] = E[1_{Z(\nu) \geq z]} \Phi_Y(Y(\nu)) = \Phi_Y(Y(\nu)) = \Phi_Y(Y(\nu))
\]

\[
[Q(\nu)]^* = E[Z(\nu) 1_{Z(\nu) \geq z}] = E[Z(\nu)] \Phi_Y(Y(\nu))
\]

The estimation of the metal at zero cut-off must then satisfy the relation: \( E[Z(V) 1_{Z(V^*)}] = Z(V^*) \). This implies that the panel grade estimate \( Z(V^*) \) has to be conditionally unbiased i.e.

\[
E[Z(\nu) 1_{Z(V^*)}] = Z(V^*)
\]

The model also assumes that the Gaussian anamorphosis of \( Z(V^*) \) is linked to that of \( Z(\nu) \):

\[
Z(V^*) = E[\Phi_Y(Y(\nu)) | Y(\nu)] = \Phi_Y(Y(\nu)) = \Phi_Y(Y(\nu))
\]

Hence the relationship:

\[
S = r \rho \nu = r \rho(\nu, \nu)
\]

This is used to compute the correlation (‘corl’) between the block and the panel estimate:

\[
corl(Y(\nu), Y(\nu)) = \rho(\nu, \nu) = S / r
\]

The ore tonnage and metal at cut-off \( z = \Phi_Y(y) \) are then:

\[
[T(\nu)]^* = E[1_{Z(\nu) \geq z} | Z(V^*)] = E[1_{Z(\nu) \geq z}] \Phi_Y(Y(\nu)) = \Phi_Y(Y(\nu)) = \Phi_Y(Y(\nu))
\]

\[
[Q(\nu)]^* = E[Z(\nu) 1_{Z(\nu) \geq z}] = E[Z(\nu)] \Phi_Y(Y(\nu))
\]

Uniform conditioning in the multivariate case

MUC consists of estimating the recoverable resources of blocks \( \nu \) in panel \( V \) from the panel estimates \( Z_1(V^*), Z_2(V^*), \ldots \).

The problem is simplified by making the following assumptions (\( i \) denotes the index of a secondary variable 2, 3, \ldots):

- \( Z_i(V) \) is conditionally independent of \( Z_i(V^*) \) given \( Z_1(V^*) \), and so the UC estimates for the main variable correspond to the univariate case
- Similarly, \( Z_i(V) \) is conditionally independent of \( Z_i(V^*) \) given \( Z_i(V^*) \).
Production reconciliation of a multivariate uniform conditioning technique

$Z_1(v)$ and $Z_2(v)$ are conditionally independent of the other metal panel grades given $\{Z_1(v), Z_2(v)\}$. It follows that the multivariate case reduces to a bivariate case. In particular we have:

$$[Q_w(z)] = E[Z_1(v)1_{Z_1(v), Z_2(v)}]$$

The development of the equations makes practical computations achievable (Deraisme et al., 2008).

The important point is that the multivariate model requires correlations between all variables and one main variable. The choice of that variable is then of prime importance, particularly because the correlations between the secondary commodities are not directly modelled but are partly inferred through their respective relations with the main variable. It should be noted that the panel estimates must be calculated using co-kriging. As highlighted above, simple co-kriging has been used for the case study in this paper. The aim was to avoid conditional biases that were observed for the ordinary co-kriging estimates as a result of the limited resource drilling data on a relatively large grid.

Case study

Geology

The case study is based on a porphyry copper-gold deposit in Peru. The mineralization is found in intrusive rocks within sedimentary rocks. Oxidation, weathering, leaching, and subsequent secondary enrichment have led to the formation of four mineral domains with distinct different metallurgical behaviours.

The uppermost domain is the oxide domain. It is characterized by the complete removal of copper mineralization through oxidation and leaching. Gold mineralization within the oxide domain is characterized by some improvement in grade and is free-milling due to the complete breakdown of primary sulphide minerals.

All of the ore beneath the oxide domain makes up parts of the sulphide zone, which is separated into three domains on the basis of degree of oxidation and consequent change in sulphide mineralogical composition. The sulphide zone domains are, from top to bottom, the mixed domain, the supergene domain, and the hypogene domain. The supergene domain is an enriched copper blanket comprising chalcocite-covellite-chalcopyrite (Figure 1).

The production reconciliations presented in this paper covered mainly the supergene and hypogene domains, which have significant economic importance on the mine. The variables studied were gold (AUTOT), total copper (CUTOT), and net smelter return (NSR).

Database and analysis

The resource drilling data on average was on 25×25 m to 50×100 m drill spacing. The samples were composited on a 2 m basis and used to derive the LMUC estimates. The initial MUCs were based on simple co-kriging of 40 m × 40 m ×10 m panels assuming 10 m × 10 m × 10 m SMUs. The SMUs were based on equipment capacities and mining selectivity as applied at the mine. Figure 2 provides the drill-hole layout for the Annulus hypogene domain.

Figure 1—Plan view of the deposit showing geological domains
Production reconciliation of a multivariate uniform conditioning technique

In addition to the resource drilling data, a comprehensive 6 m × 5 m blast-hole data grid was available from mining. The blast-hole data was not used for the MUC/LMUC resource estimates. The three variables AUTOT, CUTOT, and NSR generally have a positively skewed distribution as shown in the hypogene annulus domain (Figure 3), with coefficients of variation from 0.65 to 0.85. Significant correlations of about 0.7 are observed between gold and copper. Declustering weights have been applied to calculate histograms and variograms. Figure 4 shows one of the typical variograms in the hypogene domain.

As proposed by Abzalov (2006) and based on the extended multivariate UC work done by Deraisme and Assibey-Bonsu (2011), the grade-tonnage functions from the indirect multivariate UC were decomposed into a suite of individual 10×10×10 m SMU-sized units within the respective panels. These decomposed 10×10×10 m SMU estimates are referred to as localizsed multivariate uniform conditioning (LMUC) estimates as highlighted above. The main advantage of the LMUC approach is to derive non-smoothed SMU grades with variability closer to the future production SMU block grades.

Change-of-support models for MUC and LMUC

The distribution of 2 m composites has been modelled using a Gaussian anamorphosis function decomposed into Hermite polynomials. The change of support on SMUs is then achieved; the coefficients are calculated according to the selected main variable. The interpretation of these coefficients (Table I) as coefficients of correlation between different
variables in the Gaussian space shows that the correlations between block values are slightly higher than the correlations on composites.

In providing the co-kriging panel conditioning estimates required for the MUC/LMUC, significant conditional biases were observed with ordinary co-kriging (OK) as demonstrated by the large negative kriging efficiencies (KEs) and poor slopes of regression associated with a substantial number of the OK-based estimates in Figure 5. The conditional biases observed for the OK estimates are a result of the limited available resource data. These significant conditional biases observed with the OK estimates have adverse consequences on ore and waste selection for mine planning as well as financial planning. As a result, simple co-kriging with local means was used for the panel conditioning in all cases.

Basis for the production reconciliations
The LMUC recoverable estimates were compared with the corresponding 'actual' grade control (GC) block values based on the available comprehensive 6 m × 5 m blast-hole data grid. The LMUC estimates were also compared with plant production data. Reconciliations have been analysed on a monthly, quarterly, and annual basis. The efficiency of the LMUC reconciliations is measured on the basis of the spreads of percentage errors defined as:

\[
\text{Percentage Error} = \frac{(\text{Actual} - \text{Estimate})}{\text{Estimate}} \times 100\%
\]

Actual represents either in situ GC block estimates based on 6 m × 5 m blast-hole data or plant production data (PD); and Estimate is the corresponding LMUC resource estimates before production.
Results
As highlighted above, the main advantage of the LMUC approach is to derive non-smoothed SMU grades with variability closer to the future production SMU block grades. Table II shows the LMUC-estimated SMU dispersion variances against the corresponding ‘actuals’ based on final production blast-hole data. The table shows that the LMUC dispersion variance estimates compare well with the ‘actuals’. Figure 6 further illustrates that the smoothing effect due to the information effect has appropriately been improved by the LMUC technique. However, as shown later, on an individual block basis the assigned LMUC grades sacrifice local accuracy, as noted also by Journel et al. (2000): ‘It appears that global accuracy (semivariogram reproduction) cannot be obtained without sacrificing local accuracy. [Proper] kriging, notwithstanding its smoothing effect, remains the best local estimator’. (See also Assibey-Bonsu et al., 2008).

Table III shows the production reconciliation of the monthly LMUC resource estimates with the corresponding plant results. The reconciliation results in Table III are provided on the basis of the spreads of the percentage errors. The analyses of the spreads of the monthly percentage errors show upper and lower 10% confidence limits of –12%/+10%, –6%/+14%, and –8%/+8% for tons, gold grade, and copper grade respectively (the lower and upper 10% confidence intervals have been read directly off the histogram of the percentage of errors as observed over the production period). Figure 7 further shows the analyses of the spreads of errors in a graphical form. The figure shows that during the monthly production periods the percentage errors were well within the above confident limits. (The top two benches of the LMUC resource model incorporate what is referred to on the mine as a ‘short-term model’, which is discussed later in this paper).

The results further show percentage errors of +6%, +2%, and -7% on a quarterly (ie 3 monthly) basis for tons, gold grade, and copper grade respectively (Table IV). The mine reports production results on a quarterly basis to shareholders. Over an annual production period, the observed percentage errors were –1%/+3%, demonstrating the narrowing of the observed percentage errors over the annual period.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated LMUC dispersion variance</th>
<th>‘Actual’ dispersion variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>Copper</td>
<td>0.08</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tons Grades</th>
<th>Limits</th>
<th>Limits</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower 10%</td>
<td>Upper 10%</td>
<td>Lower 10%</td>
</tr>
<tr>
<td>Gold</td>
<td>-12%</td>
<td>10%</td>
<td>-6%</td>
</tr>
<tr>
<td>Copper</td>
<td>-8%</td>
<td>8%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6—Typical example of CUTOT estimated grades on one bench. Anticlockwise from the top: kriged panels, kriged SMUs, SMUs indirectly estimated by LMUC.
The average global errors between the follow-up and the LMUC model are within acceptable limits (<6%, Tables VI and VII).

However, the individual LMUC selective mining block estimates, based on simple co-kriging conditioning (SK) using local means, show some conditional biases as reflected by the slope of regressions of 0.7 and 0.52 for Au and Cu respectively (Figures 8 and 9). The conditional biases are a result of the limited available resource data used for the LMUC resource estimates as well as certain geological model changes on waste and ore contacts, which were updated.

**Table V**

Reconciliation between resource models and the grade control model

<table>
<thead>
<tr>
<th>Period</th>
<th>Tons</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gold</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Copper</td>
</tr>
<tr>
<td>2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 months</td>
<td>-0.6</td>
<td>9.6</td>
</tr>
<tr>
<td>6 months</td>
<td>-0.6</td>
<td>6.5</td>
</tr>
<tr>
<td>Annual</td>
<td>-0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 months</td>
<td>-0.1</td>
<td>2.5</td>
</tr>
<tr>
<td>6 months</td>
<td>-0.4</td>
<td>6.5</td>
</tr>
</tbody>
</table>

**Reconciliation of LMUC and grade control models in the short-term model area**

The mine replaces the first two benches of the LMUC resource model with what is termed a short-term (ST) model. The ST model is developed by extrapolating the blast-hole data from the mined-out areas and is used as interim short-term estimates for the first two benches (i.e. before blast-hole data become available in the short-term model area). The observed reconciliations of the LMUC model with the GC and ST models show similar good results (Tables VI and VII, see also Table IV above on errors for respective periods). Furthermore, Figures 8 and 9 show the regression of the LMUC model SMU estimates on the corresponding ‘actual’ GC block values. As highlighted previously, the GC values are based on the available comprehensive 6 m x 5 m blast-hole data grid in the GC area. The figures demonstrate a reasonable general agreement of the LMUC individual block estimates with the corresponding follow-up production data, with correlations of 0.65 and 0.62 for Au and Cu respectively.
Production reconciliation of a multivariate uniform conditioning technique

Using the detailed blast-hole data. Additional significant conditional biases (i.e. significantly higher than that of SK co-kriging above) were observed when ordinary co-kriging (OK) conditioning was used as discussed in a previous section of the paper.

Conclusions

➤ Gaussian models (in this case multivariate uniform conditioning, MUC) used for calculating recoverable resources provide consistent results in modelling the change of support and the information effect in the multivariate case
➤ The production reconciliation results show the overall advantage gained by using localized multivariate uniform conditioning (LMUC) estimates based on SK co-kriging as demonstrated by the narrow spreads of the monthly percentage errors. The central 80% confidence limits of the monthly production errors were -12%/+10%, -6%+/14%, and -8%/+8% for tons, and gold, and copper grades respectively. The case study also showed percentage errors of +6%/+2%/-7% on a quarterly basis for tons, and copper and gold grades respectively. The narrowing of the observed confidence limits is also observed as shown by the reduced observed average percentage errors of -1%/+3% for the plant production reconciliations on a macro or long-term production basis
➤ The study further showed that on a local production scale (and especially for short- to medium-term planning), regression effects and conditional biases were still evident with the assigned LMUC individual SMU estimates, thus sacrificing local accuracy. Significant conditional biases were particularly evident with the ordinary co-kriging estimates, which were mainly due to the limited data that was available for the LMUC resource estimates (limited obtainable data is typical of all long-term and project resource estimates). In this regard, the simple co-kriging estimates based on local means showed more efficient panel conditioning estimates for the purpose of the MUC/LMUC resource assessment and the reconciliations.

Acknowledgements

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References


Table VII

<table>
<thead>
<tr>
<th>Model</th>
<th>Tons</th>
<th>Grade</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gold</td>
<td>Copper</td>
</tr>
<tr>
<td>GC vs ST12</td>
<td>-2.8%</td>
<td>3.1%</td>
<td>5.9%</td>
</tr>
<tr>
<td>GC vs LMUC12</td>
<td>5.3%</td>
<td>4.4%</td>
<td>-5.3%</td>
</tr>
</tbody>
</table>
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