PROMET MEI ENGINEERS SOUTH AFRICA

ESTABLISHING ROOTS IN THE NORTHERN CAPE.

Level 2 BEE company, ProMet MEI, is making inroads and enlarging its footprint in the Northern Cape. The 30% black owned projects house has established an office to support its customer base in the region.

ProMet MEI’s chairman and sampling specialist, Tshepo Molefe, is presently consulting on-site to South Africa’s largest iron ore producer on a full time basis. “We have the capacity and are capable of taking on complete projects from consulting on requirements, defining the scope through to execution and commissioning,” says Molefe.

According to Molefe, the company has also teamed up with a fabrication firm in the Northern Cape region. It has many years of experience and is renowned for its on-time project execution as well as keeping to budget. “We are confident that this winning formula combined with permanent committed staff and support on the ground, will produce the precise results we are known for.”

The company’s Africa Business Development manager, Noel Botha explains that the expansion into the area is also opening up opportunities in manganese, coal and other minerals, where PROMET MEI has a very strong presence and large knowledge base.

“We take great pride in our technical ability, which has resulted in the successful ratio of tenders that we have won. Our strong presence in manganese, backed up with our knowledge base, will also enhance our client offering in the area,” says Botha. “We have recently won an order for a project at South Africa’s only electricity producer for a complete sampling system revamp. This includes the engineering and implementation.”

A further proven competency is process engineering, including the full range of process options from primary beneficiation of magnetite, hematite and earthy ores, to the production of steel products, including agglomeration, indurating, direct reduction, blast furnace iron-making, direct smelting and steelmaking.

The company’s core expertise lies in providing sound process-oriented solutions. “Our founders and employees have many years of experience in operating and designing plants and processes covering the full spectrum for iron ore, diamonds, coal, gold, base metals and other industrial minerals. The support that our new office will have is immense,” concludes Botha.

ProMet MEI also has access to a wealth of knowledge and financial muscle from its parent company to make inroads into the coal industry. Its design staff has specific coal managerial, operational and design expertise gleaned from many years working in the global coal processing industry.

Local experience has been at collieries, dispatch facilities and power stations in the Witbank area including low ash export products, in Vryheid on coking coal and coking ovens and the Waterberg coalfield developments as well as across the borders in Botswana, Mozambique and Zimbabwe.

For further Information contact +27 (0) 21 556 9759
Email noel.botha@promet.co.za www.promet.co.za

ProMet MEI South Africa (Pty)Ltd
For eighty years, Mintek has been a global leader in minerals and metallurgical research and development.

TODAY THIS CENTRE OF TECHNOLOGICAL EXCELLENCE, with its teams of highly trained and experienced scientists, engineers, researchers and specialists, continues to develop and provide advanced technology for the more effective extraction, utilisation and beneficiation of our mineral wealth.

Mintek provides expertise in all commodities and metallurgical fields:

- One of the world’s longest-running and largest minerals technology facilities — based in Johannesburg, South Africa.
- Metallurgical support throughout all stages of project development.
- “One-stop shop” service — spanning all metallurgical disciplines.
- Unique, proprietary technologies and products.
- 250 engineers and scientists, and 500 support staff.
- Very large integrated test and pilot plant facilities.
- Comprehensive accredited analytical and mineralogical services.
- Extensive knowledge database of Southern African and worldwide mineral resources.
- 80-year track record in providing quality work.

Celebrating 80 years of excellence in mineral and metallurgical innovation.

200 Malibongwe Drive, Randburg, South Africa.
Private Bag X3015, Randburg 2125, South Africa.
Phone: +27 (011) 709 4111
Fax: +27 (011) 709 4326
E-mail: info@mintek.co.za
http://www.mintek.co.za
OFFICE BEARERS AND COUNCIL FOR THE 2013/2014 SESSION

Honorary President
Mark Guifani
President, Chamber of Mines of South Africa

Honorary Vice-Presidents
Susan Shahangu
Minister of Mineral Resources, South Africa
Rob Davies
Minister of Trade and Industry, South Africa
Derek Hanekom
Minister of Science and Technology, South Africa

President
M. Dworzanowski

President Elect
J.L. Porter

Vice-Presidents
R.T. Jones
C. Musingwini

Immediate Past President
G.L. Smith

Honorary Treasurer
J.L. Porter

Ordinary Members on Council
H. Barlott
N.G.C. Blackham
V.G. Duke
M.F. Handley
W. Joughin
A.S. Macfarlane
D.D. Munro

Past Presidents Serving on Council
N.A. Barcza
R.D. Beck
J.A. Cruise
J.R. Dixon
F.M.G. Egerton
A.M. Garbers-Craig
G.V.R. Landman

Branch Chairmen
DRC
S. Maleba
Johannesburg
I. Ashmole
Namibia
G. Ockhuizen
Pretoria
N. Naude
Western Cape
T. Ojumu
Zambia
H. Zimba
Zimbabwe
S.A. Gaihali
Zululand
C. Mienie

Corresponding Members of Council
Australia:
I.J. Corrans, R.J. Dippenaar, A. Croll, C. Workman-Davies

Austria:
H. Wagner

Botswana:
S.W. Diederichs

Brazil:
F.M.C. da Cruz Vieira

China:
R. Oppermann

United Kingdom:
J.J.L. Cilliers, N.A. Barcza, H. Potgieter

USA:
J-M.M. Rendu, P.C. Pistorius

Zambia:
J.A. van Huyssteen

PAST PRESIDENTS

*Deceased
* W. Betef (1894–1895)
* A.F. Crosse (1895–1896)
* W.R. Feldmann (1896–1897)
* C. Butters (1897–1898)
* J. Levey (1898–1899)
* J.R. Williams (1899–1903)
* S.H. Pearce (1903–1904)
* W.A. Caldecott (1904–1905)
* W. Callen (1905–1906)
* E.H. Johnson (1906–1907)
* J. Yates (1907–1908)
* R.G. Bevington (1908–1909)
* A. McA. Johnston (1909–1910)
* J. Moir (1910–1911)
* C.B. Saner (1911–1912)
* W.R. Dowling (1912–1913)
* A. Richardson (1913–1914)
* G.H. Stanley (1914–1915)
* J.E. Thomas (1915–1916)
* J.A. Wilkinson (1916–1917)
* G. Hildick-Smith (1917–1918)
* H.S. Meyer (1918–1919)
* J. Gray (1919–1920)
* J. Chilton (1920–1921)
* F. Wartenweiler (1921–1922)
* G.A. Wattermeyer (1922–1923)
* F.W. Watson (1923–1924)
* C.J. Gray (1924–1925)
* H.A. White (1925–1926)
* H.R. Adam (1926–1927)
* Sir Robert Kote (1927–1928)
* J.A. Woodburn (1928–1929)
* H. Pirow (1929–1930)
* J. Henderson (1930–1931)
* A. King (1931–1932)
* J.T. McIntyre (1932–1933)
* John V. Muller (1933–1934)
* Theo Meyer (1934–1935)
* P.N. Lategan (1935–1936)
* E.C. Ranson (1935–1936)
* C.J. Gray (1936–1937)
* T.K. Prentice (1937–1938)
* E.H.A. Joseph (1938–1939)
* P.E. Hall (1939–1940)
* R.S.G. Stokes (1940–1941)
* J.A. Wilkinson (1941–1942)
* J.H. Dobson (1942–1943)
* E.H. Johnson (1943–1944)
* J.T. McIntyre (1944–1945)
* F. Wartenweiler (1945–1946)
* V. Nimmo-Dewar (1946–1947)
* J.R. Williams (1947–1948)
* F.G. Hill (1948–1949)
* O.A.E. Jackson (1949–1950)
* W. Cullen (1950–1951)
* H.S. Meyer (1951–1952)
* W.E. Gooday (1952–1953)
* O.A.E. Jackson (1953–1954)
* F.G. Hill (1954–1955)
* H. Britten (1955–1956)
* Wm. Bloch (1956–1957)
* H. Simon (1957–1958)
* M. Barcza (1958–1959)
* W.S. Findlay (1960–1961)
* D.G. Maxwell (1961–1962)
* J. de V. Lambrechts (1962–1963)
* J.F. Reid (1963–1964)
* D.M. Jamieson (1964–1965)
* H.E. Cross (1965–1966)
* P. Lambold (1967–1968)
* V.C. Robinson (1970–1971)
* J.P. Hugo (1972–1973)
* F.W.J. van Rensburg (1973–1974)
* M.D.G. Salmon (1976–1977)
* D.J. van Niekerk (1977–1978)
* D.J. van Niekerk (1978–1979)
* H. Wagner (1986–1987)
* N.A. Barcza (1996–1997)
* M.H. Rogers (1999–2000)
* A.M. Garbers-Craig (2008–2009)
* J.C. Ngoma (2009–2010)
* G.V.R. Landman (2010–2011)
* J.N. van der Merwe (2011–2012)

Honorary Legal Advisers
Van Halsteys Attorneys

Auditors
Messrs R.H. Kitching

Secretaries
The Southern African Institute of Mining and Metallurgy
Fifth Floor, Chamber of Mines Building
5 Hollard Street, Johannesburg 2001
P.O. Box 61127, Marshalltown 2107
Telephone (011) 834-1273/7
Fax (011) 838-5923 or (011) 833-8156
E-mail: journal@saimm.co.za
Foreword—Danie Krige
by R.C.A. Minnitt ................................................................. iv–v
President’s Corner
by M. Dworzanski ............................................................... vii
Special Articles
Professor Danie Krige FRSSAf
R.C.A. Minnitt and W. Assibey-Bonsu ................................ viii–xi
Criteria for the Annual Danie Krige Medal Award
by G.L. Smith ................................................................. xi
Memories of Danie Krige, Geostatistician Extraordinaire
by E. Magri ................................................................. xii–xiii
SANCOT News: Advancement in Vertical Tunnelling in Mining
by H.J. Thieleczek ............................................................ xiv
Danie Krige Commemorative Edition—Volume II
Use of geostatistical Bayesian updating to integrate airborne radiometrics and soil geochemistry to improve mapping for mineral exploration
by J.M. McKinley, C.V. Deutsch, C. Neufeld, M. Patton, M. Cooper, and M.E. Young ........................................ 575
Witwatersrand gold reef evaluation: the ‘variancegram’ tool
by C. Lemmer and M. Mogilnicky ........................................ 587
Analysis of the dispersion variance using geostatistical simulation and blending piles
by D.M. Marques and J.F. Costa ............................................. 599
Geostatistics: a common link between medical geography, mathematical geology, and medical geology
by P. Gruaeran ...................................................................... 605
Investigating ‘optimal’ kriging variance estimation using an analytic and a bootstrap approach
by C. Thiert, M.Z. Ngwenya, and L.M. Haines ........................................ 613
Limitations in accepting localized conditioning recoverable resource estimates for medium-term, long-term, and feasibility-stage mining projects, particularly for sections of an ore deposit
by W. Assibey-Bonsu and C. Muller ........................................ 619
Geostatistical applications in petroleum reservoir modelling
by R. Can, Y. Zee Ma, and E. Gomez ........................................ 625
Designing an advanced RC drilling grid for short-term planning in open pit mines: three case studies
by J.M. Ortiz and E.J. Magri .................................................. 631
On localizing uniform conditioning estimates
by O. Rondon ....................................................................... 639
Iron oxide Cu-Au (IOCG) mineralizing systems: an example from northeastern Russia
by A. Kostin, A. Vedyaev, and G. Rafat .................................... 645
Factorial kriging for multiscale modelling
by Y.Z. Ma, J.-J. Royer, H. Wang, Y. Wang, and T. Zhang .................. 651
Application of a localized direct conditioning mineral resource modelling technique for medium- and long-term planning of underground mining operations
by W. Assibey-Bonsu, C. Muller, and H. Pretorius .......................... 659
On the reduction of algorithmic smoothing of kriged estimates
by L. Tomey ......................................................................... 667
Multivariate block simulations of a lateritic nickel deposit and post-processing of a representative subset
by J. Deraisme, O. Bertoli, and P. Epinoux .................................... 673
List of papers published by Professor Danie Krige .......................... xv–xvi

International Advisory Board

R. Dimitrakopoulos, McGill University, Canada
D. Dressinger, University of British Columbia, Canada
E. Esterhuizen, Norsk Research Organization, USA
H. Mint, McGill University, Canada
M.J. Nickel, Murdoch University, Australia
H. Potgieter, Manchester Metropolitan University, United Kingdom
E. Topal, Curtin University, Australia

THE INSTITUTE, AS A BODY, IS NOT RESPONSIBLE FOR THE STATEMENTS AND OPINIONS ADVANCED IN ANY OF ITS PUBLICATIONS.

Copyright © 1978 by The Southern African Institute of Mining and Metallurgy. All rights reserved. Multiple copying of the contents of this publication or parts thereof without permission is in breach of copyright, but permission is hereby given for the copying of titles and abstracts of papers and names of authors. Permission to copy illustrations and short extracts from the text of individual contributions is usually given upon written application to the Institute, provided that the source (and where appropriate, the copyright) is acknowledged. Apart from any fair dealing for the purposes of review or criticism under The Copyright Act no. 98, 1978, Section 12, of the Republic of South Africa, a single copy of an article may be supplied by a library for the purposes of research or private study. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means without the prior permission of the publishers. Multiple copying of the contents of the publication without permission is always illegal.

U.S. Copyright Law applicable to users in the U.S.A.

The appearance of the statement of copyright at the bottom of the first page of an article appearing in this journal indicates that the copyright holder consents to the making of copies of the article for personal or internal use. This consent is given on condition that the copy pays the stated fee for each copy of a paper beyond that permitted by Section 107 or 108 of the U.S. Copyright Law. The fee is to be paid through the Copyright Clearance Center, Inc., Operations Center, P.O. Box 763, Schenectady, New York 12301, U.S.A. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

Printed by
Camera Press, Johannesburg

Advertising Representative
Barbara Spence
Avenue Advertising
Telephone (011) 463-7940
E-mail: barbara@avenue.co.za

The Secretariat
The Southern African Institute of Mining and Metallurgy
ISSN 2225-6253

E-mail: journal@saimm.co.za
Fax (011) 838-5923
P.O. Box 61127
Marshalltown 2107
Telephone (011) 838-5923
P.O. Box 765, Schenectady, New York 12301, U.S.A. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

The Copyright Act no. 98, 1978, Section 12, of the Republic of South Africa, a single copy of an article may be supplied by a library for the purposes of research or private study. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means without the prior permission of the publishers. Multiple copying of the contents of the publication without permission is always illegal.
The initiative for this volume came from the Southern African Institute of Mining and Metallurgy with the aim of commemorating Professor Danie Krige’s work, his contribution to geostatistics, and the creation of new knowledge that he has stimulated. At the outset I would like to sincerely thank the authors for their contributions and for the swift and diligent responses to the call for papers. Since most of the work fell to the authors of the papers, compiling this volume has been a pleasure. The first invitation to contribute to the volume went out on 10 May 2013 and as the volume stands, it will contain 37 papers from 67 authors from around the globe, all completed in a period of nine months. The readiness and willingness of geostatisticians to contribute their research to this volume signals their appreciation for the work and person of Danie Krige.

Something worth noting is that most authors found the writing of articles in this volume an enjoyable experience. In fact one author commented that he hoped I would enjoy reading the paper as much as he enjoyed writing it.

I thought that a summary of the early development of geostatistics might be in order. Early publications by Danie Krige were translated into French and Russian and the effects of his new insights into mineral resource evaluation spread to mining centres around the world. From this interest the APCOM meetings arose, and as the need for a forum to discuss such issues in the South African and Australian mining industries became evident, the Geostatistical Association of South Africa and the Geostatistical Association of Australia were formed. Early development of the subject material (before the term geostatistics was coined) in the literature of the 1945 to 1965 period was couched in phrases such as ‘statistical approach’ or ‘statistical analysis’ and ‘mine valuation’. At that stage geostatistics was unknown as a subject, but terms such as lognormal, correlation, regression, trend surface, moving average, and interpolation procedures began to appear in this literature.

It is interesting to use the number of publications in the field of geostatistics as a proxy for the way the discipline of geostatistics has evolved over the years since its first developments in the early 1950s. In 1988 Patricia Sheahan produced a bibliography of geostatistics for the Economic Geology Research Unit at the University of the Witwatersrand, in which she lists 315 publications on geostatistics in the period 1951 to 1988. Her definition for geostatistics from the Glossary of Geology, published in 1987 by the American Geological Institute: ‘(a) statistics as applied to geology; the application of statistical methods or the application of statistical data for use in geology; (b) statistical techniques developed for mine valuation by the French school of Matheron’, guided her choice of what to include, noting that her bibliography refers to practical aspects of geostatistics with an emphasis on literature since the mid-1970s, and that it is by no means comprehensive. However it is the best record that we have for that period of geostatistical development, so the data for the number of publications related to the subject for each year between 1951 and 1988 (Figure 1) is source from Sheahan (1988).

The period between 1950 and 1987, when the number of publications on geostatistics reached 133, is of particular interest. Figure 1 shows a moderate growth in the number of publications after 1964, but an exponential rise in 1976. Publications output in the 1978 to 1981 period was relatively subdued, but from 1982 growth in interest and output was rapid. I have found no bibliography of works relating to geostatistics after 1988 that meaningfully captures publications output. Webster’s Timeline History 1962-2007 of Geostatistics (Parker, 2009) fails to capture many publications that were produced after 1987, and probably before that time as well. For interest, the numbers of publications by Danie Krige and Georges Matheron are also shown on the diagram.

It is probably dangerous to go down this road for risk of offending people by either getting the numbers wrong or by missing someone, but anyone with more than five publications in Sheahan’s bibliography for this period is listed below. Some who have made huge contributions to the advancement of geostatistics, for example Ed Isaacs and Mohan Srivastava, besides many others (M. David, Ch. J. Huijbregts, P. Delfiner, and J.P. Chiles), do not feature in the list. So it should be said that the number of publications does not necessarily reflect the importance of the contributions to
the development of geostatistics. Number of publications is not the only metric by which we should measure the development of a scientific discipline, but it really is interesting to see who was publishing geostatistical papers during the period (listed in the Table below).

The number and timeline of publications indicate two important features of Danie Krige as an outstanding researcher and academic – firstly the consistency, and secondly the number of publications produced during his lifetime.

Matheron and Kleingeld (1987), in their consideration of the evolution of geostatistics (APCOM 87, Geostatistics, vol. 3, pp. 9-12), note that the term ‘random function’ was not in use when Danie Krige and Herbert Sichel introduced statistics into ore evaluation. The term ‘random variable’ has been used fairly extensively in the past. For example, Paul Lévy (Ecole Polytechnique) wrote an article entitled ‘Wiener’s random function, and other Laplacian random functions’ (pp. 171-187 in Proceedings, Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, 1951), and in 1962 A.M. Yaglom wrote the book ‘An Introduction to the Theory of Stationary Random Functions’ (published by Prentice-Hall). This term was first introduced by B. Matérn in his original 1948 publication on forestry in Sweden and was also used in the 1960 translation (with updates) from the Swedish version. The term ‘géostatistique’ was introduced by Matheron in 1962, as was the term ‘kriging’, although the latter appeared as a French word in a footnote. Matérn and Matheron also note that the concern around the adoption and acceptance of geostatistics was not related primarily to the mathematical complexities, but rather to the poor communication about the ‘… conceptual and psychological interpretation of the given phenomena …’ between methodical mathematicians, practical mining engineers, and intuitive geologists.

They also identified three periods in the development of geostatistics, the earliest being 1945–1965 when linear geostatistics was developed. Application of the lognormal distribution to gold mining by Herbert Sichel was followed by Danie Krige’s application of regression analysis between sampling and mining blocks for resource estimation. These works allowed Georges Matheron to fully develop the linear geostatistics of his doctoral thesis in 1965.

The second period, between 1966 and 1974, saw the development of nonlinear geostatistics on location at Centre de Geostatistique in Fontainebleau. M. David and G. Matheron made important contributions, but A.G. Journel, Ch. J. Huijbregts, P. Delauné, P. Chauvet, and J.P. Chiles are said to have developed nonlinear geostatistics during this period. It is interesting to note how the number of publications escalated after the end of this period (Figure 1), but I leave the reader to interpret this observation. The third period between, 1974 and 1987, dealt with complex problems including ore/waste selection, change of support, transfer functions, parameterization of reserves, disjunctive kriging, and indicator kriging (Matheron and Kleingeld, 1987). Since that time, the interest and growth in geostatistics has shaped the extraction of minerals and metals from ore deposits around the world.

This volume is about geostatistics, its development, growth, and applications in the earth and natural sciences, but its importance lies in both the quality of the papers and the number and stature of the contributors in the geostatistical fraternity. This will indeed be a memorable volume. Of particular importance is the fact that the research presented here is at the forefront of the thrust in geostatistical research, and the collection of papers taken together indicates the forward direction for the frontiers of geostatistical research. I asked an eminent geostatistician where he thought the new frontiers of geostatistical research lay, and I believe that, collectively, the papers in this volume answer that question.

It is hoped that you will enjoy reading this volume as it commemorates the work of Professor Danie Krige and reflects the latest in global thinking about geostatistics.

R.C.A. Minnitt

### Table of Publications

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of publications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agterberg, F.P.</td>
<td>11</td>
</tr>
<tr>
<td>Armstrong, M.</td>
<td>6</td>
</tr>
<tr>
<td>Clark, I.</td>
<td>9</td>
</tr>
<tr>
<td>Dagbert, M.</td>
<td>8</td>
</tr>
<tr>
<td>David, M.</td>
<td>11</td>
</tr>
<tr>
<td>Dowd, P.A.</td>
<td>8</td>
</tr>
<tr>
<td>Froidevaux, R.</td>
<td>6</td>
</tr>
<tr>
<td>Journel, A.</td>
<td>19</td>
</tr>
<tr>
<td>Kia, Y.C.</td>
<td>10</td>
</tr>
<tr>
<td>Krige, D.G.</td>
<td>23</td>
</tr>
<tr>
<td>Marechal, A.</td>
<td>7</td>
</tr>
<tr>
<td>Matheron, G.</td>
<td>14</td>
</tr>
<tr>
<td>Philip, G.M.</td>
<td>6</td>
</tr>
<tr>
<td>Rendu, J.M.M.</td>
<td>16</td>
</tr>
<tr>
<td>Royle, A.G.</td>
<td>13</td>
</tr>
<tr>
<td>Sinclair, A.J.</td>
<td>8</td>
</tr>
<tr>
<td>Switzer, P.</td>
<td>6</td>
</tr>
</tbody>
</table>

R.C.A. Minnitt
calling all
MODERN
MINING
GEOLOGISTS
and
GRADE CONTROL
PROFESSIONALS
THREE-DAY
MINING GEOLOGY
and GRADE CONTROL COURSE

Attend global consultancy, Golder Associates’ mining geology and grade control course and keep up to date with the latest in grade control methods.

Now in its fifth year, this course has set the standard for mining geology and grade control worldwide. It will be presented by an international expert.

WHEN:
20 - 22 October 2014

WHERE:
Leriba Lodge, Centurion, South Africa

REGISTRATION & ENQUIRIES:
Linda Engelbrecht
lengelbrecht@golder.co.za
+27 11 313 1022

Golder Associates

EXCEPTIONAL QUALITY KNOWS NO DEPTHS.

For expert results, choose the highest quality hand tools with a lifetime guarantee. From spanners and wrenches to heavy duty toolsets, Gedore products are hot forged and precision crafted for absolute toughness – minimising injury and increasing productivity to ensure that you’ll never want to buy another tool again.

Gedore manufactures and distributes hand tools suited to a variety of industries throughout the African continent. We offer a lifetime guarantee and dedicated after sales service for all of our products.

CALL: 0861-GEDORE FAX: 011 608 2953
EMAIL: sales@gedore.co.za WEB: www.gedore.co.za

Visit our website for a FREE copy of our catalogue | www.gedore.co.za
Mentoring and coaching are two words that come up at many mining industry forums. Many mining companies have mentoring and coaching programmes. The Engineering Council of South Africa (ECSA) encourages candidate engineers to seek out a mentor, and the SAIMM has had mentoring and coaching programmes in the past. What is a mentor? An experienced and trusted advisor is the definition. What is a coach? A private tutor is the definition. While many people will regard the two as being one and the same, they clearly are not and, more importantly, both are vital.

When do we need an advisor? When we have limitations in experience or knowledge and thus need to seek advice. The advice will generally be broad in nature. A good example within a mining industry context would be a young graduate approaching a senior and experienced colleague and asking them for advice around possible career paths in mining. A good mentor will first point out the options, such as production, projects, research and development, technical consulting, and management. Secondly, they will explain what each option entails – the type of work involved, the qualifications needed, and prior experience required. Finally, they will highlight different career path choices together with the personal attributes required.

When it comes to career progression and opportunities arising for promotion, I am sure that most of us have come across the question, ‘have you ticked all the boxes?’ Without a good mentor you will not know what all the boxes are, nor will you know what combination of boxes you need for the different career paths. I realized early on in my career that I needed broad experience in terms of commodities, unit processes, and work types. And to this day I have not stopped broadening my experience. My advice will always be to gain broad experience, regardless of which career path you choose within the mining industry.

When do we need a coach? When we need to increase our knowledge in a specific area because our work responsibilities require more knowledge than we currently have. This generally entails finding an expert in the specific area. It is often perceived that expert knowledge can be obtained only from someone who is an academic. Although in many instances this is true, expertise is often linked to extensive experience and knowledge, without an academic background. I remember distinctly as a young graduate looking to the more senior individuals for their expertise. And I did not seek out the managers – they were my mentors, not my coaches. I sought senior foremen and consultants as my coaches. They taught me the ‘tricks of the trade’ that no textbook or lecturer would be able to reveal. Your tertiary qualification will provide your theoretical background, but you will have to seek out your practical knowledge and experience, and this is where the combination of mentors and coaches is a critical success factor.

I would like to end with a good example which has always stuck in my memory. Some years ago I visited a platinum concentrator and the plant manager was my host. One would assume that a plant manager would have sufficient knowledge and experience of flotation, given that it is one of the major unit processes in a platinum concentrator. But as we walked through the plant and went past the flotation cells he asked me, ‘as a matter of interest why are there different sizes of bubbles and why are some bubbles dark and others light?’ Those of you with a knowledge of flotation will be as astonished as I was as to how this individual became a plant manager! He clearly lacked coaching, and probably mentoring too. So my conclusion is do not underestimate the value of mentoring and coaching, and also appreciate that they are different and both are vital.

This is my last President’s Corner, since my term as SAIMM President has come to an end. The last 12 months have flown by, but they were busy. I am pleased to say that despite all the current turmoil in the mining industry, the Institute continues to flourish. Our membership grew by 10 per cent and around 2500 delegates attended our conferences over the last financial year. We are also about to establish a new branch in the Northern Cape. This all confirms that the mining industry regards the SAIMM as an important stakeholder and thus continues to support us. I would like to thank the SAIMM administration team for their fantastic support and hard work. I would also like to thank the office bearers, Council members, and all committee members for their invaluable contributions. It has been an honour to serve the Institute as President, and I have every confidence that the Institute will continue to grow and to serve its members optimally.

M. Dworzansowski
President, SAIMM
IN MEMORIAM

Professor D.G. Krige FRSSAf

R.C.A. Minnitt* & W. Assibey-Bonsu²

¹School of Mining Engineering, University of the Witwatersrand, South Africa
²Group Consultant, Geostatistics and Evaluation, Gold Fields Limited, Perth, Australia

*Author for correspondence email: Richard.Minnitt@wits.ac.za

Daniel Gerhardus ("Danie") Krige, whose name is world-renowned in the field of mineral resource estimation and evaluation, passed away peacefully on Sunday morning, 23 March 2013; he was 93 years old. The funeral at Constantia Kloof Dutch Reformed Church on 28 March was attended by more than 200 family members, friends and colleagues from industry and universities, with eulogies by Oscar Steffen, Richard Minnitt and Wинфред Assibey-Bonsu. Such was the renown of Daniel Krige that his death is recorded together with that of Margaret Thatcher in Wikipedia (Deaths in April, 2013).

Born in Bothaville, Free State, on the 26 August 1919, Daniel G. Krige grew up in Krugersdorp, matriculating from Monument High School in Krugersdorp in 1934 at the age of 15. He graduated with a BSc(Eng) degree in mining engineering from the University of the Witwatersrand at the end of 1938 (aged 19). In 1938 he joined Anglo Transvaal where he worked on a number of gold mines until 1943, gaining a wide range of valuable practical experience in surveying, sampling and ore valuation. He then joined the Government Mining Engineer's Department where he worked for eight years before returning to industry as Group Financial Engineer of the Anglovaal Group. He held this post until 1981, after which he spent 10 years as Professor of Mineral Economics at the Witwatersrand University. He remained a registered Professional Engineer although his activity as a consultant naturally diminished.

HONOURS AND AWARDS

His contributions were recognised by the Witwatersrand University through the award of the DSc(Eng) degree in 1963 and a DIng(HC) degree in 1981 by the University of Pretoria. He received further honorary doctorates from the University of South Africa in April 1996 and from the Moscow State Mining University in September 1997. He received many merit awards from the SA Institute of Mining and Metallurgy, including two gold medals in 1966 and 1980 and two silver medals in 1979 and 1993. In 1984 he received the Institute’s highest award, the Brigadier Stokes platinum medal. He was awarded the William Krumbine medal from the International Association of Mathematical Geology in 1984, the Gold Medal for Scientific and Technical Achievements from the Suid Afrikaanse Akademie vir Wetenskap en Kuns in 1982, the Distinguished Achievement Award from the AFGOM International Council in 1989 and in the same year the Percy Fox Foundation Award in South Africa. In 1987 he received from the American Society of Mining Engineers one of its highest awards, the Daniel Jackling Award, and in 1988 he was made a 'Distinguished Member'; in both cases he was the first, and to date, the only South African to receive these honours. In 1992 the University of Antofagasta in Chile also honoured him with a special award. The South African State President awarded him with the Order for Meritorious Service Class 1, Gold in 1989. In 1998 The Royal Society of South Africa awarded him the John F. Herschel Medal for outstanding contributions to science in South Africa.

In February 2010 his distinguished contributions to engineering were acknowledged by the United States National Academy of Engineering (NAE) and he was elected a Foreign Associate, Section 11, Earth Resources engineering. Danie was the first South African to ever receive this award from the NAE. In December 2010 he received an Honorary Doctorate from the University of the Witwatersrand, and in 2011 he was awarded the Order of the Baobab (Kretemart Silwer), by President Jacob Zuma.

CAREER ACCOMPLISHMENTS

During his period in government service Danie Krige handled several of the post-war lease applications in the Free State and Klerksdorp goldfields. The fact that decisions on new gold mines of critical importance to the State and the economy as a whole were being taken on a limited number of drillholes, without any scientific analysis of the risks of failure, stimulated him to start basic research into ore evaluation. His approach was based on the application of mathematical statistics to these problems, an approach of which very little was known worldwide at that stage but which had already
been initiated in South Africa by Herbert Sichel via the lognormal frequency distribution model. In Krige’s 1951 paper, published in the Journal of the Chemical, Metallurgical and Mining Society of South Africa, he covered the statistical explanation of the conditional biases in ore block valuations and stimulated the use by several gold mines of regression corrections for routine ore reserve valuations, a technique which, in effect, was the first use on an elementary basis of what is now known as kriging. This paper introduced, inter alia, the basic geostatistical concepts of ‘support,’ ‘spatial structure,’ ‘selective mining units’ and ‘grade-tonnage curves.’

Professor Georges Matheron.

As the Anglovaal Group’s Financial Engineer, he was responsible for the Group’s ore evaluation, mine surveying, financial analyses of mining projects and negotiations, share valuations and technical computing facilities. During the early 1960s he implemented geostatistical kriging procedures on the two large gold mines of the Group. This was the first routine application of the kriging of ore reserves in the world. Apart from ore evaluation, his career led to significant contributions in the fields of investment and financial analysis and mining taxation. This is evidenced by his contributions to the establishments of the original South African uranium contracts, and by a substantial number of local and overseas publications in his field. These include the publication in 1955, in Afrikaans, of what was probably one of the first papers on risk analysis for new mining investments.

As Professor of Mineral Economics in the Mining Engineering Department of the University of the Witwatersrand he was responsible mainly for postgraduate courses in geostatistics and mining economics and supervised many masters and doctoral theses. After retirement from the university he continued his research, lecturing and publication activities and remained active as a consultant in the valuation of resources and reserves of mineral deposits and financial analysis for several of the Mining Houses and various local international mining and consulting companies. He presented courses in geostatistics and/or lectured at local universities (Pretoria, UNISA, RAU and Rhodes) and overseas (Australia, Germany, Taiwan, Chile, Russia and China). He has participated in, and contributed to, many international mining congresses in South Africa, the USA, Canada, Germany, Spain, Chile, Colombia, Slovenia, Australia, the UK, Russia, France and China; in several cases as the keynote speaker.

OUTCOMES

Krige’s work led directly, or contributed largely, to the following:

1. His recognition worldwide in mining circles as the principal pioneer in modern statistical methods of ore valuation, or geostatistics as it is now called.
2. Since the early 1960s his surname has been used to describe the geostatistical techniques of ‘kriging.’ The term was coined by Georges Matheron and is now applied worldwide mainly in the fields of exploration and ore evaluation, but the environmental, petroleum, hydrology, agriculture and other disciplines.
3. The teaching of geostatistics in graduate and postgraduate mining engineering and other courses at universities worldwide.

PUBLICATIONS

Danie Krige published some 90 technical papers both locally and overseas, including Russia. His early research papers that had stimulated interest in several mining circles overseas were republished in French in 1955, resulting in a major research effort by French mining engineers in this field. A 1951 paper, based on his MSc(Eng.) thesis submitted to the Department of Mining Engineering to the University of the Witwatersrand, expounded his pioneering work in geostatistics in more detail. His 1978 publication was the first Monograph (Geostatistics) in the monograph series of the SA Institute of Mining and Metallurgy.

A complete record of all Krige’s publications is available on a CD disc from the SA Institute of Mining and Metallurgy. They are presented under the following headings:

1. Original basic concepts and developments
2. Routine block kriging on mines
3. Geostatistical techniques, Simple kriging versus Ordinary kriging, Conditional biases
4. Bayesian approach
5. Valuation of new mines from drillholes
6. Reference works
7. Reviews
8. Economic and Financial.
OTHER PROFESSIONAL CONTRIBUTIONS

As a Professional Engineer, Danie Krige served for many years on the mining committee of the Engineering Council of South Africa. For an extensive period he was honorary treasurer on the Council of the SA Institute of Mining and Metallurgy and became an honorary life member. He was also a mining engineering member of the Income Tax Special Court, a founder member of the International Association for Mathematical Geology and of the Geostatistical Association of Southern Africa, a founder-member and honorary life Fellow of the Statistical Association, an honorary life member of the Institute of Mine Surveyors of South Africa and a Fellow of the Royal Society of South Africa. He also served as a director of several mining companies, as well as for the South African Development Trust, the Lebowa Development Corporation, and the Lebowa Mineral Trust.

Danie served on the sub-committee of the Prime Minister’s Economic Advisory Council which investigated State Aid for marginal gold mines in 1967/8. He designed the State Aid formula which assisted a large number of gold mines to survive the period of low gold prices. This scheme significantly contributed to the stabilisation and growth of the gold mining industry and the economy as a whole during a difficult time. He also served for many years on various committees of the Chamber of Mines. In 1974 he was a Chamber-nominated member of the Government-Chamber mining mission to Iran which investigated aspects of a closer co-operation on mining matters. More recently, he served as a member of the Marais Committee on mining taxation and on the Melamet Commission of Enquiry into further State aid for the ERPM gold mine; he was also an observer for the State on this mine’s Management Committee and Board of Directors until early 1994. He was also a member of the SAMREC Working Committee which developed the South African Code for reporting of Mineral Resources and Reserves as published in 2000.

He was South Africa’s representative on the International APCOM Council from its inception and initiated the arrangements for the Symposia held in South Africa in 1972 and in 1987, and he assisted in the preparation of this Symposium in Cape Town in 2003. He served as Chairman of the International Council, the first non-USA member to be elected to this position, from 1990 to 1993.

The outstanding feature of Danie Krige’s contributions was his focus on, and dedication to, the basic tenets of geostatistics and the use, wherever practical, of large databases to undertake practical follow-up studies. This style of approach to statistical and geostatistical research became apparent in his initial 1950/2 work and consistently underpinned his research. His rigorous practice of verifying new geostatistical techniques using large data sets, allowed him to test and audit their applicability and interrogate alternative approaches. His high standards of research contributed significantly to the advancement of the science of geostatistics and provided many fruitful avenues for future research. His lasting contributions are a tribute to a lifetime of dedication and he was a worthy leader and an example to all who practice and research in the field of geostatistics.

DANIE KRIGE’S ACKNOWLEDGEMENT OF THE GRACE GIVEN TO HIM

Thoughts from an interview he gave to Richard Minnitt during 2012

With his weight of achievement and a life of distinguished contributions to science and engineering behind him, Professor Danie Krige was a devout Christian who also recognised and acknowledged that he had been the recipient of gifts of grace from the Creator. He drew attention to six specific areas in which he could identify the grace of the Almighty at work in his life and career. The first was a tribute to his parents for the practical application of a godly lifestyle, the establishment of a firm foundation, and a life philosophy that was modelled by them in every area of life. An example of this was that even with the limited resources of a pastor, his parents saw to it that seven of the nine siblings received a tertiary education.

The second of the gifts of grace that Danie acknowledged was the support he had received from his two spouses. He was happily married for 45 years to his first wife (until her death), and for 20 years to Ansie, his second wife.

The third gift of grace was the way in which his career developed, and the various turns in direction that it took as his research unfolded. Having graduated from Wits he was employed in the sampling and survey departments of the Anglovaal Group. In the interview Danie stated that the most important event of his career occurred when an extensive prospection programme of deep drilling was launched to determine the extent and value of the extensions of the Witwatersrand gold deposits westward to Klerksdorp and the Orange Free State. Mining companies involved in exploration applied to the Government Mining Engineer for mining leases which led to the opening up of the new goldfields. The GME enrolled additional Mining Engineers to handle this work and he was one of the lucky ones to be chosen. Danie was involved full time in collecting, analysing, and using statistical modelling of the data to determine the underlying patterns of the gold distribution in the widening fields. These patterns proved to be adaptable to modern statistical programmes and these were in turn developed to yield improved grade estimation procedures. This work was successfully submitted for a Master’s thesis at the University of the Witwatersrand and a technical paper that was published here and overseas, raised interest. A French team under Professor Georges Matheron in Paris translated the publication and republished it in French. Professor Matheron insisted for the worldwide acceptance of the term KRIGING for this new valuation method, a term for a practice that is now accepted internationally.
follow-up values demonstrated the advantages of the new methods. Following the clear demonstration of the validity of the new methods, Anglovaal fully supported the application of Danie’s methods on their mines, as well as the publication of these findings, both locally and internationally, particularly at international conferences. In addition, Wits University awarded Danie a DSc(Eng) degree in recognition of this work. This led to the award of two Honorary Doctorate degrees in South Africa, from the University of Pretoria and the University of South Africa (UNISA), and a third from the Moscow State University, in addition to numerous other awards both locally and overseas.

The fifth gift of grace Danie acknowledged was that on retirement from Anglovaal at the age of 60, he accepted the unexpected opportunity of taking up the chair of Professor of Mine Economics at Wits University, which he occupied for the next 10 years. This enabled him to teach and undertake extensive consulting work for mining companies both locally and internationally, and was, in his opinion, a great blessing. The final gift of grace that Danie recognised was that after his retirement from Wits University he was able to undertake extensive national and international consulting work which he believed kept him occupied and young for the following 20 years. Danie also acknowledged, with deep gratitude, that while the opportunities presented themselves to him, his was the responsibility to make good use of them, and that without these gifts of grace his life’s work would not have been possible.

Acknowledgement

This paper which was first published in August 2013 is republished with the permission of the Royal Society of South Africa, http://dx.doi.org/10.1080/0035919X.2013.826748.

---

Criteria for the Annual Danie Krige Medal Award

Daniel Gerhardus Krige (26 August 1919 – 3 March 2013), one of South Africa’s most influential mining engineers and geostatistician of international repute, passed away last year. Danie was a recipient of the Brigadier Stokes award in 1984 – this is the Institute’s highest recognition of contribution to the minerals industry.

Following discussions at Office Bearers and Council during 2013 it was agreed to honour his memory and contribution to the mineral industry through three activities:

• The publication of a Danie Krige Commemorative Volume of the Journal. This is planned for March 2014 with a number of papers (37) having been submitted to the publications committee to date
• An annual Danie Krige Memorial Lecture to be facilitated by the School of Mining Engineering at the University of the Witwatersrand
• The annual award of a Danie Krige medal for a qualifying geostatistics paper published by the SAIMM in the previous year.

Selection criteria

The Danie Krige Medal will be awarded annually to the author (or co-authors) of the best geostatistical paper published in the previous calendar year. Accordingly, SAIMM members would be invited to nominate and/or submit papers for consideration on an annual basis.

The following criteria will govern the award:

i. Papers on theoretical or applied geostatistics are eligible
ii. The papers must have been published in the Journal of the SAIMM in the preceding calendar year
iii. Nominations for the award may be made by a member of the SAIMM (who is not an author) or submissions may be made by the author(s)
iv. Nominations and submissions must be submitted by email in pdf format to the SAIMM for attention of the Chairperson of the Danie Krige Medal Committee;
v. An individual may only submit one paper (or be nominated, based on one paper) for the award in any year
vi. No award will be made if none of the papers in a given year meet the minimum standards of the Danie Krige Medal Committee. In evaluating papers, the committee will use the following criteria and apply their professional judgement:
   a. The impact and contribution to knowledge of the paper in its specific field
   b. How innovative are the ideas or techniques described in the paper
   c. The relevance of the problem being addressed
   d. How well the paper is written (language, structure, supporting figure etc.)
    vii. Only one paper, or one series of papers on a topic by the same author, per year will qualify for the award
viii. The decision of the Danie Krige Medal Committee on the award of the medal will be final
ix. Award of a Danie Krige Medal excludes the winning paper from consideration for any other SAIMM publications awards i.e. the SAIMM Gold and Silver medals for Journal papers.

The Danie Krige medal will comprise a 38 mm diameter medal in 9 carat gold in an engraved rosewood case and carry an impression of Danie Krige on one side and the SAIMM logo on the other.

G.L. Smith
Immediate Past President, SAIMM
Memories of Danie Krige

Geostatistician Extraordinaire

Let me start the story in 1969, when I was a final-year mining engineering student at the University of Chile. At that time, André Journel, a young man from the Paris School of Mines (Fontainebleau), came to Chile to teach a two or three months-long seminar on geostatistics. If I am not mistaken, that was the very start of geostatistics in Chile. This effort was continued in the early 70’s by André’s colleague Alain Merechal. In those days, geostatistics was highly mathematical and theoretical, as commercial software did not exist and enthusiastic users had to write their own code. It is not surprising that geostatistics in Chile had a distinct French flavor and Kriging was referred to as ‘Krigeage’. In my ignorance, I grew up thinking that Daniel Krige was a very clever Frenchman.

I had no further contact with geostatistics until 1975. At that time, I was working for Anglo American Corporation in Welkom, South Africa, in the Management Sciences Department, developing mixed integer programming models for mine planning. Of course, the gold grades assigned to the different mining areas had a large impact on the sequencing, as well as on the expected profits. My esteemed friends, the geologists, had the almost impossible task of predicting the gold and uranium grades of large blocks, based on perhaps half a dozen boreholes covering an area as large as 50 km². On the other hand, closely spaced underground sampling showed large variability and intricate grade patterns. Grade predictions based on a very limited number of drill hole results seemed almost impossible. This is what started my interest in the subject.

At that time, publications were few - many of them in French, with at least 50 integrals per page. It was not easy! I admit I was battling to get to grips with the subject. One fine day, my boss said ‘Why don’t you go and see Danie Krige, perhaps he can give you a hand’. Immediately I replied ‘Great, I’m off to France’. ‘I’m afraid not, Danie works for Anglovaal in Johannesburg, so just get in your car’, came the reply.

I visited Danie every three or four months. He introduced me to his co-worker, Jean-Michel Rendu. They were extremely helpful and shared their knowledge, papers and lots of advice. During one visit, Danie explained that he was rather sad since Jean-Michel had accepted and offer as Professor at a prestigious University in the USA. Before returning to Welkom that evening, I walked into Danie’s office and said to him ‘What about the job, then?’ I thought to myself that I was being rather cheeky. To my great surprise Danie said ‘If you want the job, it is all yours’. I was delighted and could not wait to get home and tell my wife Patricia the great news.

I joined Anglovaal in 1976 and worked with Danie for 10 years. Danie was a patient boss and explained the famous log-normal regression model – the first steps towards kriging - on more than one occasion.

In the early 80’s, we gave four one-week courses at the University of Clausthal-Zellerfeld in Germany. On one of these occasions, our wives Ansie and Patricia accompanied us. We stayed in the silver-mining medieval town of Goslar, in Lower Saxony, close to the beautiful Harz Mountains. The hotel was close to the central square, housed in a thousand year-old building. In the evenings, after classes, we would sit in a quaint little coffee shop on the square, enjoying coffee with wonderful pastries and listening to the Town Hall ‘glockenspiel’.

For a weekend, we visited Berlin and took a bus tour to East Berlin. We crossed the wall at Check Point Charlie. A uniformed lady, who looked and acted as if she were a member of the Gestapo, boarded the bus, checked all the passports and returned them to each passenger, except Danie’s and mine. She took our passports into an office and stayed there for about 20 minutes. The other passengers were all muttering in German and looking at us. We had a quiet chat with Danie: ‘We are going to be famous; you coming from the land of apartheid and me from General Pinochet’s dictatorship. We might be here for a long, long time’. In the end the police woman came back, gave us our passports with a dour look and the tour continued normally.
Memories of Danie Krige

Unfortunately, Ansie suffered from severe asthma and passed away in 1989. We remember her as a very kind and loving person. Some years later, Danie married a wonderful lady, co-incidentally also named Ansie.

South Africa has produced a number of great statisticians. On one occasion, we had lunch at the Johannesburg Club with Danie, D.M. Hawkins, H.S. Sichel and Peter Brooker from the UK. In my modest opinion, having had the honour and the pleasure of working with some of these great minds, they all seemed to have something in common: they are all unassuming, approachable, helpful and always willing to share their knowledge. It seems to be the mark of truly great people.

I remember those 10 years I worked at Anglovaal with Danie as very happy, friendly and productive. The company and Danie were very special and encouraged research, conferences, publications and teaching part time at the Mining Engineering Department of the University of the Witwatersrand. We developed a fine working relationship with my favorite clients and lifelong friends, the geologists; in particular with Jan Mostert.

In 1986, due mainly to family pressures, we moved back to Chile, where I have been working as a part time Professor at the University of Chile and as a mining consultant in geostatistics and sampling ever since. Over the years in Chile, our working relationship with Danie continued and we presented some short courses together, met at conferences and seminars and completed ore resource calculation reviews for several large Chilean copper mines. On one occasion, Danie was accompanied by Ansie and we went on holiday to the beautiful Chilean Lake District, some 900 km south of Santiago. On the way south, I got a speeding fine. A few kilometers further, another policeman stopped me for yet another traffic offence. Ansie could not bear it, got out of the car and gave the policeman a dressing-down in English. To our great surprise, he let us go, remarking that my driving was not at all brilliant that day.

Some years ago, I was teaching the undergraduate course on linear geostatistics to the mining engineering students. We had covered the estimation variance material and had still to continue with Kriging. By coincidence, Danie was in Chile and I asked him if he would be so kind as to give the Kriging lecture himself. Typical of Danie, he immediately accepted. As we walked into the class, there was silence. I told the students that the next topic was Kriging and the best person to present the subject was none other than Professor Krige himself. You should have seen the look on their faces ……. That evening, the students came to our home to socialize and enjoy a few drinks with Danie. They took many photos and had a wonderful time with Ansie and Danie.

In 2007, I chaired APCOM in Chile. Danie very kindly delivered the opening keynote address with Clayton Deutsch on ‘The Origins and Future of Geostatistics’. The conference went very well and many people almost queued to have their pictures taken with Danie. As a keepsake, I still have a bottle of good Chilean wine with Danie’s signature on the label.

In July 2007, we went on safari to Tanzania with our dear friends Jeanne and Jan Mostert. On our return to Johannesburg, Ansie and Danie invited us for dinner at their home. We had a very pleasant, friendly evening. That was the last time we saw Danie.

Through the years, I lost track of the number of well-deserved Honorary Doctorates and distinctions that Danie received. A concept that was started off by comparing estimated block values against follow up mining values (true block values) in the South African Gold Mines has developed into a global industry with perhaps thousands of users in mining and other applications, by software developers and researchers all around the world. Danie’s seed idea was picked up by research centers such as Fontainebleau, Stanford, Alberta, at various centers in South Africa and more recently in Chile, and numerous new applications are continually being proposed and developed. An aspect perhaps less known was Danie’s extensive knowledge and ability in subjects such as taxation and financial evaluation of mining projects. In fact, his company designation was that of Financial Engineer.

While travelling through Patagonia in early March, 2013, we received a message from Oskar Steffen saying that after an illness, Danie had passed away on Saturday March 2. It was a long, sad day.

Danie was my friend, my mentor. We remember him with great fondness and respect.

E. Magri
Advancement In Vertical Tunnelling In Mining

Design and construction of vertical tunnels, more commonly referred to as shaft sinking, continues to rely on particular methodologies that are selected in response to a combination of site requirements, cost models, and available technology. Industry best practice in horizontal tunnelling is well researched and documented; however, changing demands for efficient, quick, and correctly risk-managed shaft sinking projects have revealed the need for similar documented best practice in vertical tunnelling.

Shaft sinking methodologies typically vary between two extremes, on the one side being blind sinking, which is high in cost and labour requirements, yet carries the risk management benefit of being able to timeously respond to adverse geotechnical conditions. At the other extreme, raise boring provides an attractive, rapid, low-cost, and low-risk approach in suitable conditions and is usually preferred over more arduous methods whenever possible.

Strides in technology are being made continuously to enable deeper, larger diameter shafts to be bored as an alternative to conventional shaft sinking. A particular benefit of raise boring is being able to avoid the delays associated with installation of in-line support. However, this very benefit can become its Achilles’ heel in a large, deep shaft. Before boring is completed, it might become necessary to treat local breakout at depths exceeding current equipment capacities.

Outcomes of existing geotechnical investigation methods usually indicate that an alternative method to raise boring should be considered when rock mass conditions are expected to be unfavourable. However, it can also usually be assured that poor conditions will occur at some stage within a long shaft. Given the benefits in terms of time, cost, and safety, this has led to risk indicators being questioned and equipment developed to improve risk-management strategies. With tolerance for safety risk practically disappearing, technological innovation continues to be driven ever higher.

The question then presents itself: on what basis is the optimum shaft sinking method selected for a particular project?

Under the umbrella of the ITA (International Tunnelling Association), a workgroup was initiated through SANCOT to put forward a best practice handbook on vertical tunnelling. The handbook is intended to document the process of vertical shaft engineering in order to assist the selection of an appropriate methodology. The complete design process is addressed, including the initial geotechnical evaluation, assessment of shaft design and constructability, pre-sink requirements, and selection of the excavation methodology. Current methods are separated into either conventional or boring, including blind sinking, slipe and line, Alimak raising, blind boring, and boring with a pilot hole. A final chapter addresses the end-stage process for operational readiness.

Good progress was made during the course of 2013. Sub-groups comprising practitioners with substantial experience across the South African mining industry were assigned to gather data and brainstorm methodologies. Following the recent appointment of Ron Tluczek as Chairman, the SANCOT initiative is regaining momentum.

On completion, the handbook is expected to deliver a comprehensive presentation of current best practice, with references to new technological development for efficient, safe, and cost-effective approaches to vertical tunnelling in mining.

H.J. Tluczek
Use of geostatistical Bayesian updating to integrate airborne radiometrics and soil geochemistry to improve mapping for mineral exploration

by J.M. McKinley*, C.V. Deutsch†, C. Neufeld‡, M. Patton*, M. Cooper*, and M.E. Young‡

Synopsis

Mineral exploration programmes around the world use data from remote sensing, geophysics, and direct sampling. On a regional scale, the combination of airborne geophysics and ground-based geochemical sampling can aid geological mapping and mineral exploration. Since airborne geophysical and traditional soil-sampling data are generated at different spatial resolutions, they are not immediately comparable due to their different sampling density. Several geostatistical techniques, including indicator cokriging and collocated cokriging, can be used to integrate different types of data into a geostatistical model. However, with increasing numbers of variables the inference of the cross-covariance model required for cokriging can be demanding in terms of effort and computational time. In this paper a Gaussian-based Bayesian updating approach is applied to integrate airborne radiometric data and ground-sampled geochemical soil data to maximize information generated from the soil survey, enabling more accurate geological interpretation for the exploration and development of natural resources. The Bayesian updating technique decomposes the collocated estimate into two models: prior and likelihood models. The prior model is built from primary information and the likelihood model is built from secondary information. The prior model is then updated with the likelihood model to build the final model. The approach allows multiple secondary variables to be simultaneously integrated into the mapping of the primary variable. The Bayesian updating approach is demonstrated using a case study from Northern Ireland. The geostatistical technique was used to improve the resolution of soil geochemistry, at a density of one sample per 2 km²; by integrating more closely measured airborne geophysical data from the GSNI Tellus Survey, measured over a footprint of 65 x 200 m. The directly measured geochemistry data were considered as primary data and the airborne radiometric data were used as secondary data. The approach produced more detailed updated maps and in particular enhanced information on the mapped distributions of zinc, copper, and lead. The enhanced delineation of an elongated northwest/southeast trending zone in the updated maps strengthened the potential for discovering stratabound base metal deposits.

Keywords

geostatistics, Bayesian updating, airborne geophysics, geochemistry, mineralization.

Introduction

Mineral and oil exploration programmes around the world utilize data from remote sensing, geophysics, and direct sampling. These techniques measure different attributes of the Earth, at a range of scales, the integration of which may provide more information than is immediately apparent, if correctly analysed. On a regional scale, the combination of geophysics (airborne magnetic and radiometrics) and geochemical sampling can aid geological mapping and mineral exploration (Smith et al., 1997, Morris et al., 2003). Integration of soil geochemistry, soil gas helium, and in situ radiometry (where the detector was placed directly over the soil) was used to delineate the subsurface extent of uranium zones in the Cuddapah Basin, India (Menon et al., 2009). Since airborne geophysical and traditional soil-sampling geochemical data are generated at widely different spatial resolutions, they are not immediately comparable due to different sampling densities. Previous work by Rawlins et al. (2007) on the integration of high-resolution radiometrics and detailed soil surveying from eastern England used multivariate geostatistical methods comprising coregionalization and residual likelihood (REML) modelling. In the present study, geostatistical techniques were applied to integrate regional multi-source geophysical and geochemical data. The application of a Bayesian updating approach provided the opportunity to investigate the relationship between the different data types. The threefold objectives were to:

- Investigate the use of a Bayesian updating approach to integrate different types of spatial geophysical and geochemical information
- Examine the controls (geological and parent material) on soil geochemistry
- Interpret the findings in terms of the development of the area’s natural resources.

© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
Use of geostatistical Bayesian updating to integrate airborne radiometrics

Study region and survey data acquisition

Geological setting

Northern Ireland, with an area of only about 14 000 km², is home to a remarkable diversity of geology (GSNI, 1997; Mitchell, 2004a) (Figure 1). The range of rocks present forms a stratigraphic record that commences in the Mesoproterozoic, comprising deformed and metamorphosed sedimentary and volcanic rocks formed between 900 and 600 Ma (Cooper and Johnston, 2004a). With the exception of the Cambrian, all geological systems are represented up to and including the Palaeogene, which comprises basalt lavas and associated rocks that formed between approximately 62 and 55 Ma (Cooper, 2004; Cooper and Johnston, 2004c). The basement rocks of Northern Ireland can be partitioned into three major terranes (Anderson et al., 2004). North of the Highland Boundary Fault occurs the Central Highland or Grampian terrane, which is host to the oldest rocks present in Northern Ireland and includes the Dalradian Supergroup (Cooper and Johnston, 2004a). An extension of the Midland Valley terrane of Scotland, south of the Highland Boundary Fault, hosts limited basement at outcrop, but includes the Tyrone Igneous Complex and small inliers of fossiliferous Ordovician and Silurian sedimentary rocks (Cooper and Mitchell, 2004; Cooper et al., 2011). Devonian and Carboniferous continental red-bed and marine Carboniferous sediments (Mitchell 2004b, 2004c) overlie the basement in the southwest of this terrane and are the focus of this study. The Southern Uplands-Down-Longford terrane lies to the south of the Southern Uplands Fault and is composed of Ordovician and Silurian rocks (Anderson, 2004) intruded by the Lower Devonian Newry Igneous Complex (Cooper and Johnston, 2004b) and Palaeogene Slieve Gullion and Mournes complexes (Cooper and Johnston, 2004c). In the northeast, straddling all three basement terranes, cover rocks of the Antrim Plateau are composed of Palaeogene basaltic lava with underlying Carboniferous, Permian, Triassic, Jurassic, and Cretaceous rocks exposed at the margins. Quaternary superficial deposits cover some 80% of the Northern Ireland and were mainly laid down over the last 120 000 years (Bazley, 2004). They include subglacial tills in the form of rib moraines and swarms of aligned drumlins, and proglacial deposits of clay, silt, sand, and gravel related to meltwaters. Post-glacial alluvium and peat continue to accumulate and obscure bedrock. Remote sensing using airborne geophysics and ground geochemistry are therefore particularly relevant to prospecting in this region.

Prospecting history

Northern Ireland has a varied mineral prospecting history dating from the 18th century (Arthurs and Earls, 2004). In the southwestern region, prospecting for base metals has largely been concentrated within the counties Fermanagh and Tyrone (Figure 1). County Fermanagh and the southern part of County Tyrone comprise shallow-water Carboniferous limestone sediments with interbedded sandstones (Mitchell, 2004c). The Carboniferous includes the non-fossiliferous Ballyness Formation and the Clogher Valley Formation. The Ballyness Formation comprises about 300 m of red and purplish-red sandstones and conglomerates with clasts of mainly white quartz pebbles. The Clogher Valley Formation (400 m) is divided into fenestral micrite, siltstones, sandstones, evaporate beds, and grey mudstone with miospores indicating an early Chadian age (Mitchell, 2004c).

Figure 1—Geological map of Northern Ireland (simplified after GSNI, 1997). The location of the study area is shown.
Use of geostatistical Bayesian updating to integrate airborne radiometrics

Previous geochemical surveys undertaken in the 1970s and 1980s revealed anomalies of lead, barium, and silver together with galena and barite in heavy mineral samples (Smith et al., 1996). The faulted sedimentary rocks of the Clogher Valley share similar features and a geological environment of Lower Carboniferous shallow-water sediments near a basin margin fault, with many of the major lead-zinc deposits of the Irish Midlands (e.g. Navan, County Meath - Republic of Ireland. Although the identification of anomalies in the 1970s stimulated prospecting in the area, the exploration methods available at that time lacked the sensitivity necessary to identify subtle geochemical anomalies in the areas covered by thick superficial deposits. This study aims to investigate whether the integration of high-resolution airborne geophysical and detailed geochemical data using a Bayesian updating approach can improve precision in mapping the geochemical variables of economic interest.

Due to the diverse nature of the geology on a regional scale in Northern Ireland, a local-scale study area was selected to investigate the Bayesian updating approach. The Clogher Valley area (shown on Figure 1) in the county of Fermanagh and the southern part of County Tyrone was chosen because:

- The area comprises lithostratigraphically distinctive Carboniferous and Devonian rocks
- The stratigraphy is controlled by basement tectonics and early Devonian terrane-bounding faults as evidenced by the fault-bounded Fintona Block
- This area is of particular economic interest due to an inferred relationship between base metals and basin faulting (Arthurs and Earls, 2004).

Survey data acquisition

The Tellus Project, managed by the Geological Survey of Northern Ireland (GSNI) and funded by the Department of Enterprise Trade and Investment and the EU’s Building Sustainable Prosperity Fund, was the most detailed geological mapping project ever undertaken in Northern Ireland. The data comprised multi-source airborne geophysics collected by a low-level geophysical survey aircraft and a ground geochemical survey of soil, stream sediments, and stream waters (GSNI, 2013). The airborne geophysical data included magnetics, electrical conductivity, and radiometrics. The results provide new insights into the geological underlay and delineation of faults and dykes, particularly where bedrock is obscured by glacial cover and peat. The geochemical data provide a baseline standard for 60 elements across rural Northern Ireland. The improved mapping of geology, soils, and natural resources has prompted a renewed interest in mineral prospecting and new licences have been awarded as a result. On release of the Tellus data, mineral licensing in Northern Ireland surged, with total coverage increasing from 17% to a peak of 73%. Prospecting junior Metalumm Exploration was awarded a number of licences across Northern Ireland, including the MR4 licence which is focused on the Clogher Valley and the MR2 and MR3 licences which have their northern limits in the area (GSNI 2012). The company sees the area as prospective for potential ‘Irish style’ base metal mineralization.

Geochemical survey

The GSNI Tellus Survey, completed between 2004 and 2006, provides a data-set combining comprehensive spatial soil sampling coverage with an extensive suite of soil geochemical analysis (Smyth 2007). Regional ‘rural’ soil samples were collected on a grid of one sample site every 2 km² across the rural areas of Northern Ireland. A parallel ‘urban’ soil sampling programme at a sample density of 4 sites per km² was completed across a number of selected urban areas, but these data are not used in this study. The soil samples collected at each rural sampling site included a surface sample collected from 5 cm to 20 cm below ground level (discarding surface organic litter and root zone where present) and a deep soil sample collected from 25 cm to 50 cm below ground level. The samples collected at each site represent a composite sample of five auger abstractions (completed with hand-held auger) at corner points and centre of a 20 m × 20 m sampling square. Rural soil samples were disaggregated prior to sieving to a 2 mm fraction and a representative sub-sample was obtained and milled for subsequent chemical analysis. Further details on the Tellus soil sampling programme, sampling methodology, and sample preparation including quality control procedures are summarized in Smyth (2007).

This paper uses data from the rural surface soil geochemical data set comprising a total of 6862 soil samples across Northern Ireland. As part of the Tellus Project, the rural soil samples were analysed for a range of up to 50 determinants. The soil samples used in this study were analysed using pressed pellet X-ray fluorescence spectrometry (XRF) for determination of major oxides and trace elements using wavelength dispersive XRF spectrometry (WD-XRF) and energy dispersive/polarized XRF spectrometry (ED-XRF) by the British Geological Survey (BGS), Keyworth, Nottingham. In a recent study, Dempster et al. (2012) demonstrated a very close geochemical match between soils, underlying glacial tills, and a range of bedrock types. Soil geochemistry will also reflect anthropogenic effects, anomalies associated with precious and base metal mineralization, and secondary environmental effect (i.e. absorption onto secondary Fe and Mn hydroxides, organic matter etc.). The Tellus data-set provides the basis for a comprehensive study with relevance for the whole of the UK.

Geophysical survey

The airborne surveys measured three geophysical parameters, obtaining high-resolution magnetic, radiometric, and electromagnetic data. The survey was carried out with a Twin Otter aircraft of the Joint Airborne-Geoscience Capability (JAC), a partnership of BGS and the Geological Survey of Finland (GTK). Flight lines had a line spacing of 200 m and an average flight altitude of 56 m (240 m over urban areas (Jones and Scheib, 2007). The high-resolution imagery generated by the geophysical survey provides complementary information on the geology and soil types of Northern Ireland and is particularly valuable where the bedrock is obscured by glacial cover and peat. In particular, the airborne survey improves the delineation of faults and dykes (Cooper et al., 2012). The high-resolution terrestrial gamma radiation was used in this study. Variations in
Use of geostatistical Bayesian updating to integrate airborne radiometrics

Gamma-ray emissions characterize near-surface rocks and soils. Terrestrial radiation was sampled every second using a gamma-ray spectrometer (Explorium GR-820/3), which resolves radiation emitted from the radioisotopes and daughters of potassium (K), uranium (U), and thorium (Th) (IAEA, 2003). These radioisotopes, due to their abundance and half-lives, are the main contributors of gamma radioactivity in rocks (Dypvik and Ericksen, 1983). Total count (TC) measurements, in units of counts per second (cps), were collected across the energy window from 0.41 to 2.81 MeV (Beamish et al., 2007). TC data comprise a spectral summation, including contributions from both natural and artificial radioactive sources, and so provide a higher signal/noise ratio than the individual radioisotope data (Beamish 2013).

Geostatistical methodology

The value of the Tellus data lies not only in the individual geophysical and geochemical data-sets produced, but equally in the potential for a comprehensive integrated mapping approach, as is undertaken in this paper. Airborne geophysics combined with soil geochemistry provides the opportunity to analyse and integrate data from multiple sources generated at varying spatial resolutions. The potential outcome of this is to maximize information generated by the Tellus project, generating even deeper and more accurate knowledge to support exploration and development of the natural resources of Northern Ireland.

Conventional geostatistical mapping comprises kriging data such as soil geochemistry to interpolate between sampled locations. Coefficients from variography are used to inform the kriged maps. Local uncertainty in the estimates is given by the kriging variance. In geostatistics, secondary data are often used to provide a quantitative measure at unsampled locations. For example, secondary information such as seismic data and geological interpretation are often used to improve the 2D modelling of sparse well data in the interwell regions (Ren et al., 2007). The geochemical survey forms only part of the Tellus geoscientific mapping project. Combining information from the Tellus soil survey with the results from the airborne radiometric survey affords the opportunity to significantly reduce the uncertainty in the estimation of key topsoil major and trace elements. Several geostatistical techniques can be used to integrate different types of data into a geostatistical model, including Gaussian-based Bayesian updating, indicator cokriging, and in particular collocated cokriging (Xu et al., 1992). Coregionalization was used by Rawlins et al. (2007) to model the direct and cross-variograms for soil measurements of K and Th and the corresponding radiometric measurements. However, as the number of variables increases, fitting the direct and cross-variograms simultaneously becomes increasingly difficult. With a large number of variables the inference of the cross-covariance model required for cokriging can be demanding in terms of effort and computational time (Ren et al., 2007). Use of a Bayesian updating approach is explored in this study due to the simplicity of data integration and the ability of the approach to take in account joint spatial and multivariate correlations between variables. This is particularly relevant for integrating the Tellus geophysical and geochemical data-sets, in that correlation between radiometric measurements and all of the geochemical elements can be explored and used if deemed appropriate. The strength of correlation between different geochemical elements along with the relationship between the radiometric and geochemical data can be taken into account and used to maximize the information generated from the geochemical survey data.

Theory of the Bayesian updating approach

The Bayesian updating technique for data integration decomposes the collocated estimate into a production of two models: the prior and likelihood models. Doyen et al. (1996) were the first to apply this form of Bayesian updating approach to integrate spatially correlated primary and secondary data. The prior model is built from primary information and the likelihood model is built from secondary information. The prior model is then updated with information from the likelihood model to build the final ‘updated’ model. The framework is Bayesian in the sense that different types of data are integrated together into a unified distribution of uncertainty. The terms ‘likelihood’ and ‘prior’ are somewhat arbitrary, but the mathematical framework of integration rests on a combination of these distributions in a (perhaps) non-convex manner; and therefore not a simple averaging. The approach by Doyen et al. enabled updating with only one secondary variable at a time. This approach for the Tellus Survey data would be time-consuming and ineffective in utilizing the correlation between the airborne radiometric survey elements and the multivariable geochemical data-set. Deutsch and Zanon (2004) proposed a similar approach that also allows the simultaneous integration of multiple secondary variables into the mapping of the primary variable. The advantage of this approach is easy implementation of mapping of multiple parameters using multiple secondary variables.

The development of the statistical theory of merging prior and likelihood distributions for the Bayesian updating approach is provided in detail by Doyen et al. (1996), Zanon and Deutsch (2004), Neufeld and Deutsch (2004), and Ren et al. (2007). In summary, Bayesian updating requires four distinct steps:

1. Calibration of the primary and secondary data to define the likelihood distribution
2. Calculation of the prior distribution using kriging
3. Updating the prior distribution with the likelihood distribution to obtain the updated distribution
4. Post-processing the posterior distribution, including summarizing local uncertainty and simulation for larger scale uncertainty.

Notation and Bayesian updating equations

A random function $Z(u_i)$ is the primary data available at $n$ locations in the area of interest where $u$ is the location vector and $i=1, ..., n$. There are then $m$ random functions $S_j(u)$ $j=1, ..., m$ as secondary data available at all locations in the entire study area. Bayesian updating is a Gaussian-based technique, so transformation of data is an integral stage of the approach. Following transformation, the $Y(u_i)$ value is the transformed primary data $Z(u_i)$ and the $X_j(u)$ notation represents the transformed secondary data $S_j(u)$. In the
context of Bayesian statistical analysis, the results of simple kriging using only the primary data provide the mean and variance parameters for a prior distribution of uncertainty parameterized by:

$$y^*_p(u) = \sum_{j=1}^{\infty} \lambda_j y(u_j) \quad [1]$$

The variogram is modelled by an authorized function (McBratney and Webster, 1986) and the weight \(\lambda\) is calculated from the simple kriging (or normal) equations:

$$\sum_{j=1}^{\infty} \lambda_j C(u_j - u_k) = C(u - u_k), \quad k = 1, \ldots, n \quad [2]$$

where \(C(u_k - u_l)\) is the covariance between primary data \(y(u_k)\) and \(y(u_l)\) at distances \(h\) away, and \(C(u - u_k)\) is the covariance between estimated location \(y(u)\) and primary data \(y(u_k)\) at distances \(h\) away. The kriging variance is given by

$$\hat{\sigma}^2_k(u) = \sigma^2 - \sum_{j=1}^{\infty} \lambda_j C(u - u_k) \quad [3]$$

The mean (Equation [1]) and variance (Equation [3]) define a Gaussian distribution at each location. The secondary data can be mathematically combined to provide an alternate estimate of uncertainty in the primary data at each location using the same normal equations (or simple cokriging equations) with the weights calculated from the correlations between different variables and between the secondary variable and primary variable:

$$y^*_s(u) = \sum_{j=1}^{m} \lambda_j y_j(u) \quad [4]$$

Here the weights \(\lambda_j(j = 1, \ldots, n)\) are given by:

$$\sum_{j=1}^{m} \lambda_j \rho_{j,k} = \rho_{j,0}, \quad k = 1, \ldots, m \quad [5]$$

where \(\rho_{j,k}\) is the correlation between different types of secondary data \(\rho_{j,0}\) and \(\rho_{j,0}\) is the correlation between the secondary and primary data. The likelihood estimation variance is then given by:

$$\hat{\sigma}^2_{l}(u) = 1 - \sum_{j=1}^{m} \lambda_j \rho_{j,0} \quad [6]$$

The mean (Equation [4]) and variance (Equation [6]) define a second Gaussian distribution of uncertainty at each location. This is referred to as a likelihood distribution, a term which is somewhat arbitrary; it is a second Gaussian distribution

The prior information (first distribution) and the likelihood information (second distribution) are then combined to yield a combined distribution of uncertainty that simultaneously considers sparse primary data and multiple secondary data

$$y^*_u = \frac{y^*_p \hat{\sigma}^2_p + y^*_s \hat{\sigma}^2_s}{(1 - \hat{\sigma}^2_p / \hat{\sigma}^2_s) + 1} \quad [7]$$

and the corresponding variance is calculated as:

$$\hat{\sigma}^2_u = \frac{\hat{\sigma}^2_p \hat{\sigma}^2_s}{(1 - \hat{\sigma}^2_p / \hat{\sigma}^2_s) + 1} \quad [8]$$

These results give the parameters of an updated distribution, also called the posterior distribution. These parameters define a Gaussian distribution that must be back-transformed to original units. The updating equation is interesting because it is not commonly applied in geostatistics for updating or merging two conditional distributions. These combination equations to obtain the updated mean and variance are the same as colocated cokriging (Chiles and Delfiner, 2012), that is, the underlying assumption behind their derivation is a Markov cokriging (Chiles and Delfiner, 2012) and the weight

The potential usefulness of this approach for the multi-source Tellus data is evident and would enable the relationships between the topsoil major and trace geochemical elements and the airborne radiometric survey elements (K, eTh, and eU) to be fully explored and used. In the Bayesian updating technique, data generated by both the geochemistry soil survey and airborne geophysical surveys are used to produce more detailed maps and in particular maximize information for mapped estimates of minerals of economic interest.

**Development of Bayesian updating approach for this study**

The Bayesian updating method was used in this study as a technique to improve the resolution of the more widely spaced soil geochemistry data (sampled on a grid of one sample per 2 km²) by integrating more closely-measured airborne geophysical data (resolution 65 x 200 m). The local case study for the Clogher Valley area, in the southwest of Northern Ireland, was used to investigate the effectiveness of the approach for integrating multiple variables of different types and different sources. The directly measured ground geochemistry survey data were considered as the primary data in the Bayesian approach and the airborne radiometric data was used as secondary data. For the Clogher Valley study area, the geochemistry survey comprised 589 data points (19 geochemical elements were selected) and the geophysical survey comprised 202 891 data measurements for four radiometric variables (eTh, eK, eU, and TC measurements). Within the study area, matching of paired geochemistry and geophysics was achieved for 589 points using a 1000 m radius. These points were used for the correlation matrix, which is discussed in due course.

**Results - Bayesian updating approach applied to the case study**

The Bayesian updating approach utilized soil geochemistry for 19 elements and airborne radiometric data (as illustrated later in the production of the correlation matrix for the likelihood model). The mapped outputs display the results for geochemical elements of particular economic interest: Cu (mg/kg), Zn (mg/kg), and Pb (mg/kg). In addition, the...
Use of geostatistical Bayesian updating to integrate airborne radiometrics

outputs for K$_2$O (%) are shown to exemplify the Bayesian updating approach. Summary statistics and histogram distributions for the selected elements for the Clogher Valley case study are displayed in Figure 2. The histogram for K$_2$O (Figure 2a) clearly indicates a bimodal pattern: K levels across the study area display a normal distribution of K in topsoils but with the presence of a secondary peak of low values. The summary statistics for Pb, Zn, and Cu indicate that the study area contains locations that are significantly anomalous with respect to these elements. Maximum levels for Pb, Zn and Cu are recorded as 487.5 mg/kg, 307.5 mg/kg, and 390 mg/kg respectively, set against a background pattern of lower values with minimum values of 7.5 mg/kg, 7 mg/kg, and 1 mg/kg for Pb, Zn, and Cu respectively). This is reflected in the strongly skewed histogram distributions (Figure 2b, c, and d). All data were transformed using normal score transformation prior to application of the Gaussian-based Bayesian technique (Deutsch and Journel, 1998).

Calculation of the prior distribution

Simple kriging (SK) was used to estimate the prior distribution of the geochemistry data. As the data had already been standardized (in the first pre-analysis step), the assumption of a random variable with known zero mean, as in SK, is justified. SK is equivalent to the normal equations and provides the exact solution to the parameters of a conditional distribution in a multivariate Gaussian context. If ordinary kriging was applied in this framework, this could lead to a bias in the estimates if the input distributions are highly skewed. Coefficients from variography were used for kriging the geochemistry data and enabled interpolation between the sampling locations. Omnidirectional variograms of the normal score transformed data for the selected geochemical elements (K$_2$O, Pb, Zn, and Cu) are displayed in Figure 2 (e–h). The variograms show very stable experimentally calculated points due to the large amount of available data. There is a relatively high nugget effect due to

![Image](https://via.placeholder.com/150)

Figure 2—Summary statistics and histograms for study area: (A) K$_2$O%, (B) Pb mg/kg, (C) Zn mg/kg, and (D) Cu mg/kg; omnidirectional variograms of the normal score transformed data estimated for (E) K$_2$O%, (F) Pb mg/kg, (G) Zn mg/kg, and (H) Cu mg/kg. γ is semivariance
Use of geostatistical Bayesian updating to integrate airborne radiometrics

short-scale variability and the large distance scale. Variogram analysis indicates a nugget:sill ratio (Isaaks and Srivastava, 1989) of 40% or more for all the selected geochemical elements. The model fitted to the variogram estimated from the Pb data shows the highest nugget:sill ratio (70%). This indicates higher spatial variability over short distances for the study area and short-range variation in the distribution of the elements in the topsoils, which was not detected by the 2 km² sampling spacing. Similar range values or correlation distances (7–10 km) are indicated for the Pb, Zn, and Cu data-sets. This suggests that the spatial distribution of these geochemical elements in the topsoils is comparable.

The kriged outputs for the K₂O, Pb, Zn, and Cu geochemical data-sets for the Clogher Valley area are shown in Figure 3. The kriged output maps indicate an elongated zone, trending NE/SW, of elevated K, Zn, and Cu with a secondary area or anomaly of elevated levels to the southeast of the area. The distribution of Pb is more sporadic but a similar pattern, although less well defined, can be identified.

Calculation of the likelihood distribution

The likelihood distribution defines the bivariate relationship between the primary and secondary data (Neufeld and Deutsch, 2004). Many of these relationships are too complex to be defined by a single parameter such as the correlation coefficient, so the production of the likelihood distribution comprised two parts: (1) the correlation matrix and (2) the prior distribution of the secondary data (in this case, the airborne radiometric data). The mean and variance for the likelihood data are derived from the combination of all the secondary data based on the correlation matrix.

Correlation matrix

The correlation matrix (Figure 4) was produced to provide the correlation coefficient for each pair of variables. This allowed joint spatial and multivariate correlations between the geochemical elements and the radiometric data to be taken into account in the Bayesian updating approach. For the meteorological and socio-economic data, the correlation coefficient values are given in the squares.
Clogher Valley case study the correlation matrix comprised the 19 geochemical elements along with four radiometric variables (eTh, eK, eU, and TC). Within the study area, matching of paired geochemistry and geophysics data was achieved for 589 points. These paired observations were used in the following analysis. Correlations between geochemistry data and the secondary radiometric variables were used for the likelihood calculation. The strength of correlation between different geochemical elements was taken into account together with strength of correlation between geochemical elements and radiometric variables. For example, the strength of correlation between eK and K₂O % \( r = 0.78 \) could be used in updating the output not only for topsoil K₂O but also for the geochemical elements of particular economic interest: Cu ppm, Zn ppm, and Pb mgkg⁻¹. Likewise, the strength of correlation between associated elements (e.g. \( r > 0.7 \) for Ni, Cr, and Co) can be used to reduce uncertainty in the final updated model. A moderate correlation is demonstrated between geochemical Th and eTh \( r = 0.67 \) and a weaker correlation for U \( r = 0.30 \). Strongly correlated variables were given more weight and weaker correlations less weight in the application of the Bayesian updating technique.

**Secondary radiometric data**

The secondary data required for the likelihood distribution were obtained from the radiometric data. Coefficients from variography were used for kriging the radiometric data. The variograms and kriged outputs estimated from the radiometric data are provided in Figures 5 and 6. The variograms estimated for the topsoil geochemical Th and U are provided for comparison. The overall shape of the variograms estimated from the geochemical and radiometric data, eTh, eU, and eK, are comparable (see Figure 2e for comparison with K₂O). The higher resolution of the radiometric data-sets than the geochemical data is immediately evident in the kriged estimates. The elongated NE/SW trending zone is evident in all outputs, but is most apparent in the kriged outputs for eTh and total radiometrics. The distinct borders of the anomalous gamma emission zone suggest a fault-bounded control. All secondary data can be mathematically combined based on its correlations (following the methodology provided previously) to provide a likelihood model.

**Bayesian updating of the likelihood and prior distributions**

Bayesian updating uses the mean and variance of the primary data given a specific value of the secondary data as the likelihood distribution (Neufeld and Deutsch 2004). The complex heteroscedastic and nonlinear relationships previously observed in the Tellus geochemistry data (McKinley and Leuanthong 2010) can be captured using this approach. The mean and variance for the prior distribution were derived from kriging the primary geochemistry data.

---

**Figure 5—Omnidirectional variograms of the normal score transformed data estimated for (A) Th mg/kg, (B) U mg/kg for geochemical data and radiometric variables (C) eK, (D) eTh, (E) eU, and (F) total radiometrics. γ is semivariance.**
The mean and variance of the primary geochemistry data, together with the secondary radiometric data, were used to define the likelihood distribution. Merging the primary information and the secondary information in this way provided an updated distribution containing information from both data-sets. This model gives a best estimate (in the context of minimum uncertainty) based on the primary and secondary data. Back-transformation was achieved by reversing the normal score transform with many quantiles sampled from the updated (or merged) conditional distributions.

Updated maps for the K, Pb, Zn, and Cu geochemistry data-sets are shown in Figure 7. The updated maps illustrate the benefit of integrating the geochemistry and radiometric data-sets. From a comparison of the original kriged maps (Figure 3, produced using only geochemical data) with the updated outputs (Figure 7) it is evident that greater detail is available on the updated maps. This is most obvious for K2O, but the mapped estimates of Zn, Cu, and to a lesser extent Pb also show greater definition of the elongated NE/SW trending zone. The strong correlation between K from the soil survey and radiometric K has contributed to maximizing the information in the updated map for geochemical K2O.

However, the Bayesian updating approach has also provided better delineation of a fault-bounded control for Zn and Cu. To validate the output maps, the kriging variances (in Gaussian units) or the calculated variance of the back-transformed quantiles can be mapped (Neufeld and Deutsch 2004). Validation of variances using the theta statistic or standardized squared prediction errors has been demonstrated by Lark (2000).

Discussion
The nature of the detailed mapping approach undertaken in the Tellus project and the systematic generation of ground survey and airborne data has provided a rich data-set of multi-source variables. In the Bayesian approach, the soil geochemistry survey data, measured on a 2 km² sampling spacing, provided primary information to build a prior model. Variography demonstrated evidence of spatial dependence in the selected geochemical data-sets of economic interest, Pb, Zn, and Cu. Consistency in correlation distances (range distances of 7–10 km) suggested that the spatial distribution of these geochemical elements in the topsoils is comparable. The kriged outputs provided smoothed estimate maps of the topsoil geochemistry. Similarity in the spatial distribution of
the elemental concentrations could still be surmised, with the indication of an elongated NE/SW trending zone (most obvious in the kriged estimates of Zn and Cu, Figure 3). In the Bayesian context, secondary data, in this case the radiometrics, were combined with the primary data to provide an estimate of the geochemical data at each location using weights calculated from the correlations between different variables and between the secondary variable and primary variable. The usefulness of this approach is that strength of correlation between variables, for example, between eK and K2O ($r = 0.78$) and between Ni, Cr, and Co ($r > 0.7$) can be taken into account to reduce uncertainty in the final updated output maps for the geochemical elements of particular economic interest. Strongly correlated variables are given more weight and weaker correlations less weight. Using multiple secondary variables can reduce the uncertainty, but a refinement of the approach would include reducing the number of variables in the correlation matrix to include only those with moderate to high correlations. A moderate correlation was recorded between geochemical Th and eTh ($r = 0.67$) and a weaker correlation for U ($r = 0.30$). Rawlins et al. (2007), in a comparative study of radiometric and soil data across eastern England, attributed limited correlation between geochemical and radiometric Th and U to greater analytical errors for these elements for both XRFs and estimations based on gamma emissions. The correlation between K and Th estimated by radiometrics ($r = 0.77$) is slightly lower than in the soil survey data ($r = 0.84$) for the Clogher Valley case study area. This is contrary to the findings of Rawlins et al. (2007) for the analogous study across eastern England. Rawlins et al. (2007) surmised that gamma emission measurements overestimated Th concentrations over Triassic (Mercia) mudstone parent material. The strength of correlation found in the current study between estimations based on gamma emissions and the soil survey is attributed to the high resolution of the radiometric data collected by the Tellus project and the association of high K and Th in Upper Devonian sandstones, siltstones, and mudstones. Besides the correlation between variables, the reliability of the secondary data can affect the estimation and uncertainty assessment of the updated model. In this case, terrestrial gamma radiation data provided high-resolution secondary data to include in the likelihood model with consistent quality over the entire study area. The value of using the high-resolution radiometric data is evident in the kriged mapped estimates, in that elevated levels of eTh and total radiometrics concur with the presence of an elongated NE/SW trending zone that was indicated in the kriged estimates of topsoil geochemistry.

Merging the primary and secondary information via Bayesian updating has provided more detailed updated mapped estimates for the selected variables of economic interest. This is most evident for K2O due to the strong correlation between eK and topsoil K. However, the geostatistical method also has maximized information on mapped estimates of Zn, Cu, and to a lesser extent Pb. Greater delineation of an elongated NE/SW trending zone in the updated maps strongly suggests a stratigraphic control for the geochemical data. Elevated K (strongly correlated with Th), Zn, and Cu are principally associated with sandstones, siltstones, and mudstones of the Upper Devonian Shanmullah Formation of the fault-bounded Fintona Block.

**Economic significance**

Greater definition of the orientation of terrane-bounding faults such as the Fintona Block is of particular significance due to the inferred relationship between base metals and basin faulting in the Clogher Valley area (Arthurs and Earls, 2004; McKinley and Leuanthong, 2010). In particular, previous exploration was focused on a target area adjacent to an interpreted fault line, called the Aghintain Fault, defined by elevated base metal values identified in earlier BGS geochemical surveys. Coincident stream sediment samples and panned concentrates indicated zinc anomalies associated with elevated lead, barium (baryte), copper (malachite), and zinc (sphalerite). The majority of the anomalous stream sediment and panned concentrate samples were found in the area northwest of the Clogher Valley close to this interpreted Aghintain-Lislane Fault System (Figure 1). However, drilling in the Aghintain area failed to prove the existence of the interpreted northwest-trending fault. The interpretation was that if the fault is present it must have thrown the uppermost Ballyness Formation on the hangingwall against the stratigraphically lower Ballyness Formation on the footwall. As a result, detection of the fault proved very difficult. The mapped estimates of elemental Tellus geochemistry data using the Bayesian updating approach has maximized the amount of information available for the Clogher Valley area, and in particular the inferred location of the Aghintain fault (Figure 8). This has provided useful information for subsequent geological, economic, and environmental investigations. The integration of primary information (soil geochemistry) and secondary information (radiometric data) merged with information from joint spatial and multivariate correlations has made best use of data from the Tellus project and in particular increased the definition and sensitivity necessary to identify subtle anomalies indicating prospectivity for base metals in areas of thick drift cover such as in the Clogher Valley.

**Conclusions**

The research has presented an innovative use of geostatistical analysis, and in particular a Bayesian updating approach, for the investigation, integration, and interpretation of geophysical and geochemical data from different sources. The advantage of the Bayesian updating technique is that multiple variables of different types and from different sources (in this case radiometric data and soil geochemistry) can be simultaneously integrated and applied to mapping of variables of interest. The primary information (in this case soil geochemistry) and the secondary information (radiometric data) can be shown separately, supported by a measure of uncertainty in the estimates, and the correlation between the data types and data sources can be tested. This research has utilized a global correlation to test the approach. A global correlation coefficient may be insufficient to describe local geological features and can be significantly affected by the presence of extreme values. Future work will include validating the uncertainty in the resulting model, using a jackknife or bootstrap approach and refinement of the approach to investigate the use of a locally varying coefficient to more accurately characterize complex local relationships between geochemical elements and radiometric data. A local
Use of geostatistical Bayesian updating to integrate airborne radiometrics

Figure 8—Faulted-bounded controls inferred from geostatistical analysis. The TM65 Irish grid coordinate system is used. The location of the map refers to the highlighted area in Figure 1

case study area has been selected using a limited data-set but the technique has the potential to be developed for the Tellus data for whole of Northern Ireland. This will enable a more meaningful interpretation of the nature of geochemical and radiometric variability and consequently any geological, environmental, and economic inferences.

Acknowledgements
The Geological Survey of Northern Ireland (GSNI) is thanked for access to the Tellus Survey data. The Tellus Project was carried out by GSNI and funded by the Department for Enterprise, Trade and Investment (DETI) and the Rural Development Programme through the Northern Ireland Enterprise, Trade and Investment (DETI) and the Rural Development Programme for Building Sustainable Prosperity. Thanks to GSNI for assistance with the preparation of figures. The helpful comments of Dr Barry Rawlins and Dr Don Appleton on an early version of the paper are gratefully acknowledged. The authors are grateful for the comments of two anonymous reviewers.

References


Use of geostatistical Bayesian updating to integrate airborne radiometrics


Introduction

The Witwatersrand gold reefs in South Africa are unique in terms of their continuity despite the relatively narrow reef widths. This aspect, together with the wealth of historical channel sample data captured, affords the rare opportunity of studying gold accumulation behaviour over square kilometres in extent. Such studies have in turn provided the backdrop against which large-scale forecasts can be attempted for a phenomenon as inherently variable as gold, and at such extreme mining depths. Due to the latter circumstance, forecasting of values ahead of a mining front is based almost exclusively on extrapolation. Although the channel sample data itself exhibit very high micro-scale variability (which all but masks any spatial continuity), it is well-known in the industry that, by averaging these data values into blocks of ever-increasing size, one can uncover spatial structures (variograms) with ever-increasing ranges to underpin the extrapolation. This practice serves the needs of the industry: resources within a short range of the just-mined blocks constitute the ‘annual forecast’ or Measured Resource for the coming year, and need to be estimated with relatively high resolution; resources beyond these constitute ‘medium term’ or Indicated Resources that may be mined within the following five years, and the resolution requirements for these resources are less stringent; finally, the remaining resources constitute the ‘long-term’ or ‘life-of-mine’ or Inferred Resources, and the resolution required for these last estimates is relatively crude.

In choosing the block sizes on which the averaging process will be based, one can be purely practical and choose the block dimension such that the resulting range will cover the area to be estimated. However, one is dealing with a geological phenomenon here, and there is an instinctive feeling that optimum block sizes – in terms of best elucidating the underlying spatial structures – will be peculiar to the reef or deposit studied. A tool called a ‘variancegram’ is proposed for determining the underlying optimum block sizes. There is a small latitude in the final determination of the sizes, allowing the very desirable feature of having the block sizes as integer multiples of each other and thus multiples of the smallest block size chosen. This feature forms a cornerstone for the development of an overall gold resource system, and the justification for considering the derived block sizes as ‘optimal’ will also be discussed.

Synopsis

The resolution with which the different categories of resources are forecast for Witwatersrand gold reefs should ideally tie in with block sizes that are optimal in terms of the variability structures of the reefs. A tool, called the ‘variancegram’, is proposed as a basis for block size choice. A variancegram is intrinsic to the particular reef and mine concerned. A further requirement is the ability to attach global confidence limits to weighted average estimates built up from combinations of local kriged estimates. Approximations to derive global kriging variances based on variables derived from local kriging deliver hugely inflated results if ordinary kriging is used, and markedly better, but not accurate, values if simple kriging is used. These approximations improve as the number of samples used in kriging each block is increased. It is shown that the behaviour of the different components of the global kriging variance with increasing number of samples, all differs, but they all link to the variancegram for the particular reef. The variancegram can thus be used to correct the different components to the values they would have had if all samples were used in kriging each block, and so deliver the ‘correct’ global kriging variance, even though only a limited number of samples were used in kriging each block. The desirability of having very stable solutions implemented in production systems is taken into account in the proposals. It is anticipated that the same variancegram findings will also hold for other densely sampled deposits, but this remains to be investigated.

Keywords

resources, classification, global kriging variance.

* Geological and Geostatistical Services, Johannesburg, South Africa.
† Asfalowa, Warsaw, Poland.
© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
Having forecast (kriged) the resources remaining to be mined with different degrees of resolution, the associated practical problem of the need for confidence limits to attach to average values built up from combinations of these varying supports is posed. In essence, this boils down to the intractable problem of the inability to derive a global kriging variance by a simple combination of local kriging variances. A first approximation to the problem, which has been mooted in the literature, is discussed.

The quality of the above-mentioned approximation is very dependent on the number of samples used to krig each of the local estimates. Theoretically, if all available samples were used to krig each local estimate, the resultant global kriging variance should be good. However, for most of the practical numbers of samples routinely used in such kriging, the approximation delivers a hugely inflated global kriging variance if ordinary kriging (OK) was done, and a markedly better, but not accurate, value if simple kriging (SK) was carried out.

The interesting finding is that, with increasing numbers of samples, the behaviour patterns of the different components of the global kriging variance, for OK or SK, differ; but they all link to the behaviour of the variancegram for the particular gold reef under consideration.

A method for correcting the above-mentioned inaccurate global kriging variances is proposed, based on the variancegram.

It is anticipated that the same variancegram findings will also hold for other densely sampled deposits, but this remains to be investigated.

**The variancegram**

Early work of Krige, published during the 1950s and summarized later (Krige, 1978), found that ‘the logarithmic variance of gold values shows a rising linear trend when plotted against the logarithms of the population areas concerned. This remarkable finding is, of course, true in theory and it would certainly be true for infinite deposits, but in practice one finds that the variance of the logarithms of the gold accumulation values, or logvariance for brevity, does not steadily increase without limit, but that it starts to deflect downwards towards an eventual ‘sill’.

For the purposes of determining optimum block sizes – to use in the routine evaluation of gold reefs – it was decided to investigate the above deflection points, with the view that they indicate points at which the logvariance decreases abruptly due to correlations between sample values that set in for areas beyond that particular size. The argument is that by averaging point values within an area indicated by, for instance, the first deflection point, one would absorb the first-structure variability within the block averages, and a variogram in terms of these averages would then elucidate longer-range structures.

Channel sample data (gold accumulation values) for a typical reef was available over several square kilometres in area (the ‘test data’). The logvariance of this ‘point’ test data was calculated within grid cells of side 7 m, 8 m, etc., up to cells of dimension 2000 m, and finally 5000 m. For each choice of cell size the logvariance for that cell size, averaged over the entire mine, was determined.

Figure 1 shows the graph resulting when the average logvariance per cell is plotted against the logarithm of the area of the cell.

As is clear from this graph, it was found that the series of deflections does not constitute sharp, unique bends. On the contrary, by drawing tangential lines to the graph, several sets of deflection points can be obtained.

A solution to this problem was found by plotting the average logvariance within square cells against the length of the side of the square. Figure 2 shows the resultant graph for the reef under consideration.

In contrast with Figure 1 this plot uses the effective block (cell) side length \( l = \sqrt{\text{Area}} \) to characterize the area under consideration. Such a construction will, for the purpose of this discussion, be called a ‘variancegram’ to distinguish it from the more familiar variogram of classical geostatistics. The distinction is all the more important since (i) the plot has the appearance of a variogram (which it is not), and (ii) in describing its main features one can draw on previous experience of fitting ordinary variograms. Point (ii) will be discussed in more detail below.

One notices from Figure 2 that the rate at which the logvariance varies with \( l = \sqrt{A} \) changes from a rapid to a much slower one as the side of the block involved is...
increased. If the initial rapid variation in the logvariance is interpreted as a reflection of the microstructure variability in the deposit, then this rapid increase in logvariance with block size would be expected to continue without limit as \( l \) increases, if this variability persists at all scales. Obviously this does not happen in the case of the test data. Instead, the logvariance increases less and less rapidly as \( l \) is increased, and dramatically so.

This empirical finding is to be interpreted as reflecting the presence of a set of superimposed micro-to-macro variability structures in the deposit, each exhibiting a characteristic range \( a \) over which it is operative, and beyond which it levels off to a constant value. For block sizes of dimension larger than \( a \), it would lead to a smaller logvariance than would otherwise have been found had the particular structure continued to increase at the same rate for all distances.

The preceding qualitative observation can be made quantitative by drawing on the understanding of the meaning of the spherical model in fitting variograms. As is well-known, the spherical model formula is equivalent to surrounding each point grade by a ‘sphere of influence’ with radius equal to one-half the variogram range \( a \), and arguing that the covariance of two grades at positions separated by \( h \) < \( a \) is proportional to the overlap volume of their respective spheres of influence. This assumption immediately gives

\[
C(h) = C(0)\left(1 - \frac{3}{2} \frac{h}{a} + \frac{1}{2} \frac{h^3}{a^3}\right), \quad h \leq a
\]

for the covariance, from which the standard spherical variogram formula follows,

\[
\gamma(h) = C(0)\left(\frac{3}{2} \frac{h}{a} - \frac{1}{2} \frac{h^3}{a^3}\right), \quad h \leq a
\]

\[
= C(0), \quad h > a
\]

*i.e.*, the variogram is proportional to the non-overlapping volume, and when the two spheres are separated by a distance of more than \( a \), the non-overlapping volume has reached a constant value.

These ideas can now be taken over to provide the basis for a quantitative method for describing the variancegram as well. For, by analogy with the variogram case, one can argue that the variability associated with a structure of range \( a \) in the deposit increases with square side \( l \) within a ‘sphere of influence’ of radius \( a \), beyond which the variability levels off once \( l \) exceeds \( a \). Then it immediately follows that the logvariance (due to all structures present), as a function of increasing square size, will gradually start to saturate as it approaches a final value, instead of increasing indefinitely. Calling the variancegram \( \Gamma(l) \) to carefully distinguish it from the usual variogram, one can write

\[
\Gamma_i(l) = C_i\left(\frac{3}{2} \frac{l}{a_i} - \frac{1}{2} \frac{a_i^3}{l^3}\right), \quad l \leq a_i
\]

\[
= C_i, \quad l > a_i
\]

for the influence of the \( i \)th structure in the variancegram. The variancegram itself is then constructed as follows:

\[
\Gamma(l) = \sum_{i=1}^{p} \Gamma_i(l)
\]

It is to be emphasized again that this is a parametrization of the variancegram that has been motivated by ideas suggested by the meaning of correlation in the case of the ordinary variogram. The big advantage of the approach is that one draws on existing expertise and programs to fit the curve.

In fitting the test data, it turned out that the spherical model gives an extremely good representation of the empirical variancegram. Figure 3 shows the experimental data points, with the model fitted.

The parameters of the fitted model are the following (using the standard spherical model notation):

\[
\begin{align*}
C_0 &= 0 \\
C_1 &= 0.710, \quad a_1 = 20 \text{ m} \\
C_2 &= 0.280, \quad a_2 = 60 \text{ m} \\
C_3 &= 0.190, \quad a_3 = 180 \text{ m} \\
C_4 &= 0.118, \quad a_4 = 540 \text{ m} \\
C_5 &= 0.205, \quad a_5 = 1620 \text{ m} \\
C_6 &= 0.097, \quad a_6 = 4860 \text{ m}
\end{align*}
\]

The advantage of fitting a nested spherical model is that it fits a continuous curve, even though there are six structures. There is thus no need to try to divine successive bends in the experimental curve.

In establishing a good fit, there is a small amount of latitude in choosing the ‘sills’ \( C \) and the ranges \( a \). However, the systematic increase in the ranges is no doubt an intrinsic property of the variability structures of the deposit, and arbitrarily chosen ranges will not deliver a fit.

As is clear from the above parameters, the small amount of latitude that does exist is used to stagger the ranges in such a way that they form integer multiples of each other.

The fact that the spatial variability of the phenomenon can be successfully decomposed in individual structures with finite ranges provides experimental justification for using these ranges as the optimum block sizes on which to base the different resolution levels of the resource.
calculations. It certainly provides a natural, coherent framework for an integrated system of increasing block sizes, as will become clear.

In the case of the test data, the block sizes implicated above turn out to be very convenient and appropriate. The 20 m block size has been used as the basis for annual resource forecasts; the 60 m block size provides variogram structures that are very appropriate for medium-term forecasts, and the 180 m block size could be useful for life-of-mine resources.

A note in passing: the variogram was fitted here up to a range of about 5000 m, which corresponds to the limit of experimental calculations. We will demonstrate that the variogram can be used as a tool to correct the global kriging variance approximations, as stated above.

What is very important, though, is that for the latter purpose the variogram has to be fitted over a very specific distance. Such a procedure does alter the parameters of the fit somewhat, i.e. they are not the same parameters taken over a shorter distance, as will be shown.

The problem of calculating global kriging variances

Having derived optimal block sizes on which to base forecasts for the different categories of resources, the kriging of these resources is straightforward. Additional considerations will determine whether OK or SK is used.

The main theme of the approach is thus not to use the original channel samples values as data to forecast different size blocks ahead, but to first average the point data into different size blocks to create data of different support. The new support now has a longer, more useful, range and, since the blocks to be forecast have the same support, point kriging is performed. The result is a resource forecast defined on blocks of varying support, where the blocks are integer multiples of each other in area. Naturally, each block has an associated local kriging variance as measure of the confidence that can be placed in its estimate.

The practical problem that formed the main thrust of this investigation stemmed from the requirement to be able to place confidence limits on the weighted average value calculated for a number of block estimates of varying support. These blocks could be specified by a single polygonal boundary that encloses them, or by several polygonal boundaries that are disjoint.

In the discussion that follows, the assumption is made in the first place that the forecast blocks are of different support, but that the data used in the forecast is all of the same support; say, the smallest data block used in the forecasts. The generalization to varying data supports will be made subsequently.

Assume that the total area(s) enclosed within the specified polygon(s) is denoted by \( V \), and the supports of the \( M \) enclosed local estimates are referred to as ‘blocks’ \( v_j \). A kriged estimate \( Z_j^* \) is available for each of these blocks. Then the ‘global’ kriged estimate that pertains to the entire area \( V \) is given by the weighted average (see for example, Journel and Huijbregts, 1978) of the \( Z_j^* \), i.e.

\[
Z^* = \frac{1}{V} \sum_{j=1}^{M} v_j Z_j^* \tag{5}
\]

The proviso on this formula is that the same set of all \( n \) data samples be used to krig the \( Z_j^* \). Similar relations then hold for the ‘global’ weight \( \lambda_i \) pertaining to the contribution of the \( i \)th data sample to the estimate \( Z^* \), and also to the Lagrange multiplier \( \mu \) in the event that ordinary kriging is employed:

\[
\lambda_i = \sum_{j=1}^{M} \frac{v_j}{V} \lambda_{i(j)}, \quad i = 1, 2, \ldots, n \tag{6}
\]

\[
\mu = \sum_{j=1}^{M} \frac{v_j}{V} \mu_j \tag{7}
\]

Recall again that \( M \) counts the number of blocks making up \( V \) and \( n \) the number of data points used to calculate the \( n \) weights \( \lambda_{i(j)}, i = 1, 2, \ldots, n \) that give the kriged estimate \( Z_j^* = \sum_{i=1}^{n} \lambda_{i(j)} Z_i \) of the \( j \)th block. From this, the above equation for the global weights follows,

\[
Z^* = \sum_{j=1}^{M} \left( \sum_{i=1}^{n} \frac{v_j}{V} \lambda_{i(j)} \right) Z_i \tag{8}
\]


A similar argument produces the ‘global’ Lagrange multiplier \( \mu \).

If it were simple to krig each block using all \( n \) available data values there would be no problem in producing the required global kriging variance associated with \( Z^* \), since this error variance is given by

\[
s^2 = \hat{C}(V, V) - \mu - \sum_{i=1}^{n} \lambda_i \hat{C}(V, v_i) \tag{9}
\]

where the Lagrange multiplier \( \mu < 0 \). In the case of SK, \( \mu = 0 \). The global weights \( \lambda_i \) and the global Lagrange multiplier \( \mu \) would be known from the above equations, and the covariances could be calculated straightforwardly from the variogram based on the smallest sample support. However, this is not the case.

As it stands, the implementation of the kriging equations to determine the \( \lambda_{i(j)} \) becomes impractical if the number of data points \( n \) is large (e.g. several hundreds or thousands, as would be the case here). Some of the weights become so small that rounding errors in the computer pose a serious problem.

The reduced data-set method (RDM)

An approximation to the above stringent requirements is clearly needed. One knows that only the ‘nearest data neighbours’ to the centre (or centre of gravity) of each block \( v_i \) are influential in determining \( Z_i^* \). Therefore an obvious approximation suggests itself: pretend that all the data values were used in kriging, but set artificially to zero the weights of those data samples that were not actually used to krig a specific block (and which are way out of variogram range anyway). Call this the ‘reduced data method’ or RDM for short. Although they used a different approach to calculating global kriging variances, Kim and Baafi (1984) alluded to this possibility, at least in theory. This dramatically reduces the number of data values required in kriging each block, and hence reduces the dimension of the corresponding matrix to be inverted.
This approximation yields results that are not too unreasonable for the global kriging variance in the case of SK, provided the number of data values n used per block is not too small.

For OK, the fact that the unbiasedness condition must be satisfied requires that the global weights sum to unity. This too is satisfied, provided the same requirement is met for the weights \( n' < n \), say, not set formally to zero:

\[
\sum_{i=1}^{n'} \lambda_i = 1, \quad j = 1, 2, \ldots, M
\]

\[
\sum_{i=1}^{n'} \lambda_i = \sum_{i=1}^{n'} \sum_{j=1}^{M} \frac{V_j}{V} \lambda_{i(j)} = \sum_{j=1}^{M} \left( \sum_{i=1}^{n'} \frac{V_j}{V} \lambda_{i(j)} \right) = \sum_{j=1}^{M} \frac{V_j}{V} (1) = 1
\]

As with most approximations, there is a price for introducing these simplifications: in artificially setting a selection of otherwise small but finite weights to zero, and implicitly leaving these data values out of the kriging equations, one finds that the global kriging variance is hugely overestimated.

Examples of the results obtained with this approximation for the test data are given next.

### Results obtained with the RDM

For ease of reference, the first term in Equation [9] (the global kriging variance formula) will be called the polygon covariance term, the second term the global Lagrange multiplier, or \( \mu \), and the last term the sample/polygon term, or \( sp \).

Channel sample data was averaged into 239 60 m × 60 m blocks, and variograms based on the raw average gold accumulation values showed good structure (the histograms of block averages are only slightly skewed, allowing direct calculation on raw values, instead of on logarithms). For the purposes of this illustration, both the data and the unknown blocks to be estimated taken to be of the same support, i.e. 60 m square blocks.

A rectangle of 18 × 5 (60 m) blocks, adjacent to the data blocks, was kriged (point kriging) using various schemes. Included in the latter was an exhaustive run for both OK and SK using all 239 blocks to krige every unknown block, with the idea that this would render the ‘true’ value for the global kriging variance in each case that one is striving for. This forms the last entry in the tables that follow. The other runs were based on using respectively \( n = 1, 2, 4, 8, 16, 32, 64 \) data samples to krige each block, for both OK and SK.

The polygon, for which the global kriging variance was estimated, included all 90 kriged blocks. The value obtained for the polygon covariance is, of course, a constant. The estimated, included all 90 kriged blocks. The value obtained

For values of \( n \) upwards of 16, a large number of negative weights are obtained and their impact is clear. Using a limited number of samples with which to krige each block does overestimate the global kriging variance, but not too dramatically.

The results obtained for OK, under otherwise exactly the same circumstances, are the following:

\[
\begin{array}{cccc}
n & \sum_{i=1}^{n} V_j \mu V_i & \sum_{i=1}^{n} \lambda_i \mu V_i & \sigma^2 V
\end{array}
\]

\[
\begin{array}{cccc}
1 & 166 827 & 153 594 & 1 710 247
2 & 979 379 & 154 284 & 1 020 663
4 & 576 688 & 152 143 & 620 113
8 & 380 837 & 144 328 & 452 077
16 & 257 711 & 137 838 & 315 441
32 & 192 633 & 132 324 & 255 877
64 & 138 827 & 123 468 & 210 927
239 & 44 929 & 97 512 & 142 985
\end{array}
\]

The amount by which the global kriging variance is overestimated climbs drastically as the number of samples \( n \) used in OK is reduced.

Faced with the above problem, a study was made of how \( sp \) (the second column in the above tables) in the case of OK, decrease/increase with \( n \), the number of samples used in kriging each block.

### The link to the variancegram in the case of OK

The link to the variancegram will be illustrated graphically in the first place. The link will be made even though the variancegram is based on logvariance calculations, and the kriging was done on raw values. In fact, raw variances can be used as basis for the variancegram too, but in the case of Witwatersrand gold reefs the logvariances probably result in a more stable graph, which is very important.

We require the logvariance as a function of the block side \( l \). As was demonstrated (Figure 3), this function \( f(l) \) can be faithfully reproduced by a set of nested spherical variogram structures, as given in Equations [3] and [4].

An important requirement for the link is that the variancegram has to be fitted specifically up to a square side that is indicated by the total area defined by all the data blocks. In the case of the test exercise the total number of data blocks is 239 and they cover 239 × 3600 m², i.e. 860 400 m². The side of an equivalent square is 928 m. Thus the variancegram has to be refitted only up to 928 m, which changes the parameters somewhat.

The required fit to the variancegram was accomplished using five spherical variogram structures with parameters:

\[
\begin{align*}
C_0 &= 0 \\
C_1 &= 0.710, \quad a_1 = 20 m \\
C_2 &= 0.280, \quad a_2 = 60 m \\
C_3 &= 0.190, \quad a_3 = 180 m \\
C_4 &= 0.090, \quad a_4 = 540 m \\
C_5 &= 0.180, \quad a_5 = 928 m
\end{align*}
\]

It is useful for the illustration to introduce the logvariance difference,

\[
f(l) = \text{Sill} - \Gamma(l)
\]

where the ‘sill’ in this equation can be any convenient value, since it will fall away in the final calculations.
Witwatersrand gold reef evaluation: the ‘variancegram’ tool

We chose a sill of \( f(0) = 2.2276 \), the overall logvariance of the data area.

Inserting the above values for the fit of the variancegram up to a square side \( l \) of 928 m, one obtains the graph in Figure 4 for \( f(l) \) versus \( l \).

The global Lagrange parameter \( -\mu \) in the case of OK

The remarkable finding is that the same behaviour as the above variancegram fit is displayed when \( -\mu(n) \) is plotted against \( (n - 1) \), where \( n \) is the number of samples used in kriging. Figure 5 gives the experimental points obtained for the test data.

It is immediately clear from Figures 4 and 5 that these two graphs can be scaled to coincide, and the scaling constants will give proportionality criteria. Furthermore, when \( -\mu \) is plotted versus \( f(l) \) (see Figure 11) the relationship is shown to be a linear one.

The variancegram is known beforehand, and therefore, three known points on the \( -\mu \) versus \( (n - 1) \) curve will determine the scaling constants for the entire curve. This in turn enables one to determine \( -\mu \) for a value of \( n \) of, say, 239 blocks, i.e. all the data blocks, which means that the correct value of the \( -\mu \) component of the global kriging variance can be computed.

The actual determination of the scaling constants will be discussed below.

The sample/polygon term \( sp \) in the case of OK

\( sp(n) \) also decreases with \( (n - 1) \), where \( n \) is the number of samples used in kriging. However, its behaviour is different from that of \( -\mu(n) \) versus \( (n - 1) \), shown in Figure 5.

Figure 6 gives the experimental points obtained for the test data when \( sp(n) \) is plotted versus \( (n - 1) \). There is not the immediate resemblance to the way in which \( f(l) \) decreases with \( l \).

The scaling behaviour of \( sp \) is clearly more complicated than was the case for \( -\mu \). When \( sp \) is plotted versus \( f(l) \) the result is a curve resembling \( f(l) \) versus \( l \). Figure 7 gives the former curve, and Figure 8 shows the latter for comparison.

The two curves can again be scaled to coincide, identifying the necessary scaling constants and allowing extrapolation to \( n = 239 \) for the correct value of \( sp(n) \).

The procedures are discussed below.

The link to the variancegram in the case of SK

In the case of SK there is of course only the sample/polygon term to correct in the equation for the global kriging variance,
since the Lagrange multiplier is absent. The procedure for correcting $\mu$ here is very similar to that used for correcting $-\mu$ in the case of OK, with only one difference: $sp$ increases with $n$ (see Figure 9) in a manner reminiscent of the way in which $\Gamma(l)$ increases with $l$.

This means that the only change necessary in the approach used for $-\mu$ is a change in what $f(l)$ in Equation [12] now stands for:

$$f(l) \rightarrow \Gamma(l)$$  \hspace{1cm} [13]

By empirically scaling the axes in the respective graphs of $sp$ versus $(n-1)$ and the new $f(l)$ versus $l$, the two curves can again be made to coincide. The relationship is linear.

Note that for $n = 64$ and $n = 239$ in Figure 9, the large number of negative weights obtained have an impact on the experimental value of the sample/polygon term. The star in the graph indicates the calculated ‘true’ $sp$ value for $n = 239$, which is unaffected by negative weights. Aside from that, it is not difficult to see that the above graph can be scaled to coincide with $f(l) = \Gamma(l)$.

**An approximate analytical model**

In this section a simple analytical model kriging system is examined to obtain some insight into the behaviour of $\mu$ and $sp$ for OK, for example, with increasing $n$. Analogous results can also be obtained for SK. The crux of the problem is to see if one can understand their behaviour from the structure and solutions of the kriging equations themselves. The OK kriging system to be solved is, as usual,

$$\sum_{k=1}^{n} C(v_i, v_k)\lambda_k + \mu = C(v_i, v)$$  \hspace{1cm} [14]$$

$$\sum_{k=1}^{n} \lambda_k = 1$$  \hspace{1cm} [15]

where $n$ data points have been used to krig block $v$. To obtain approximate analytic solutions for the values of $\lambda$ and $\mu$ for arbitrary $n$ replace the off-diagonal elements $C(v_i, v_k)$, for $i \neq k$, by their common average, and the right hand terms $C(v_i, v)$ by their average taken over all data points. Making the replacements

$$C(v_i, v_j) = C_i \quad \text{(no approximation)}$$

$$C(v_i, v_k) \rightarrow \bar{C}(v_i, v_k) = C_{d,v}, \quad i \neq k$$

$$C(v_i, v) \rightarrow \bar{C}(v_i, v) = C_{d,v}$$

in Equation [14], the resulting system of equations can be solved exactly. One finds that $\lambda_i = 1/n$ for all $i$,

$$\lambda_1 = \lambda_2 = \cdots = \lambda_n = \frac{1}{n}$$  \hspace{1cm} [17]$$

while $\mu$ is given by

$$-\mu(n) = \frac{C}{n} + C_{d,v}(1 - \frac{1}{n}) - C_{d,v}$$  \hspace{1cm} [18]$$

This exhibits the same qualitative behaviour as was seen empirically: for $n = 1$ the value is $\mu(1) = C - C_{d,v}$, dropping down to a lower limit of $\mu(\infty) = C_{dd} - C_{d,v}$. A graph of the solution for $-\mu(n)$ is shown in Figure 10 (compare to Figure 5).

In generating this illustrative figure, the typical estimates $C = 1.245 \times 10^6$, $C_{dd} = 0.187 \times 10^6$, and $C_{d,v} = 0.019 \times 10^6$ have been used.

In this connection it is therefore important to realize that the strong $n$-dependence of $\mu$ arises solely as a consequence of the restriction $\sum_{k=1}^{n} \lambda_k = 1$. This condition forces the common $\lambda$ values to equal $1/n$, and thus the $\mu$ in Equation [18] to become a function of $1/n$.
Correcting global kriging variances in the case of OK

The correction procedures referred to above will be illustrated using the same exercise that was used previously: the data-set consisted of average gold accumulation values for 239 60 m × 60 m blocks, and it was used to krige an adjacent rectangle of 18 × 5 (60 m) blocks. The relevant distance over which the variancegram had to be fitted was 928 m.

As before:

\[ f(l) = \text{Sill} - \Gamma(l) \]  \hspace{1cm} \text{(19)}

where an arbitrary 'sill' value of 2.2276 (in this case the overall logvariance of the data area) was chosen for convenience, and \( \Gamma(l) \) reproduced the variancegram by means of a model fitted with five nested spherical structures:

\[
\Gamma(l) = \sum_{i=1}^{\text{ens5}} \Gamma_i(l) \\
\Gamma_i(l) = C_i \left[ \frac{3l}{2a_i} - \frac{l^3}{2a_i^3} \right] \text{ for } l \leq a_i \\
= C_i \quad l > a_i
\]

\( l \) denotes the side length of the square for which each logvariance is calculated in the experimental variancegram.

Methodology for correcting −μ, the global Lagrange parameter

It was found that −μ decreases with \( n \), the number of samples used to krige each block, in a manner reminiscent of the way in which \( f(l) \) decreases with \( l \) (see Figures 4 and 5). By empirically scaling the axes in the respective graphs, these two curves can be made to essentially coincide.

Mathematically this means that the variable pairs \([-μ, f(l)]\) and \([n, l]\) are linearly related. Hence we may write

\[ l = α(n - 1) \]  \hspace{1cm} \text{(21)}

\[ -μ = βf[l] + γ = βf[α(n - 1)] + γ \]  \hspace{1cm} \text{(22)}

where the \( (n - 1) \) dependence on \( n \) is necessary since the minimum value of \( n \) is 1, and this has to be associated with the minimum value of \( l \) of the variancegram, which is zero. The function \( f \) was defined in Equation (19). The constants \( α, β, \) and \( γ \) perform the necessary scaling to cause the two curves in question to overlap. Since there are three of these unknowns, we need three sets of data input to determine them. From a practical viewpoint, one would expect to know values for small sample numbers \( n_1, n_2, n_3, \) say. Then, by manipulating Equation (22) one can solve in turn for \( α, β, \) and \( γ \) from

\[
\frac{-μ_1 + μ_2}{-μ_1 + μ_3} = \frac{f[α(n_1 - 1)] - f[α(n_2 - 1)]}{f[α(n_1 - 1)] - f[α(n_3 - 1)]} \\
\text{with } \begin{cases} 
-μ_1 = 0, \quad μ_2 = 0, \quad μ_3 = 0.5 \\
-μ_1 = 0, \quad μ_2 = 0.5, \quad μ_3 = 1 \\
-μ_1 = 0.5, \quad μ_2 = 0, \quad μ_3 = 1 \\
-μ_1 = 0.5, \quad μ_2 = 0.5, \quad μ_3 = 1.5 \\
-μ_1 = 1, \quad μ_2 = 0, \quad μ_3 = 1 \\
-μ_1 = 1, \quad μ_2 = 0.5, \quad μ_3 = 2 \\
-μ_1 = 1, \quad μ_2 = 1, \quad μ_3 = 3 \\
-μ_1 = 1, \quad μ_2 = 1.5, \quad μ_3 = 3.5 \\
-μ_1 = 1.5, \quad μ_2 = 0, \quad μ_3 = 2 \\
-μ_1 = 1.5, \quad μ_2 = 0.5, \quad μ_3 = 2.5 \\
-μ_1 = 1.5, \quad μ_2 = 1, \quad μ_3 = 3 \\
-μ_1 = 1.5, \quad μ_2 = 1.5, \quad μ_3 = 3.5 \\
-μ_1 = 2, \quad μ_2 = 0, \quad μ_3 = 3 \\
-μ_1 = 2, \quad μ_2 = 0.5, \quad μ_3 = 3.5 \\
-μ_1 = 2, \quad μ_2 = 1, \quad μ_3 = 4 \\
-μ_1 = 2, \quad μ_2 = 1.5, \quad μ_3 = 4.5 \\
-μ_1 = 2.5, \quad μ_2 = 0, \quad μ_3 = 4 \\
-μ_1 = 2.5, \quad μ_2 = 0.5, \quad μ_3 = 4.5 \\
-μ_1 = 2.5, \quad μ_2 = 1, \quad μ_3 = 5 \\
-μ_1 = 2.5, \quad μ_2 = 1.5, \quad μ_3 = 5.5 \\
-μ_1 = 3, \quad μ_2 = 0, \quad μ_3 = 5 \\
-μ_1 = 3, \quad μ_2 = 0.5, \quad μ_3 = 5.5 \\
-μ_1 = 3, \quad μ_2 = 1, \quad μ_3 = 6 \\
-μ_1 = 3, \quad μ_2 = 1.5, \quad μ_3 = 6.5 \\
-μ_1 = 3.5, \quad μ_2 = 0, \quad μ_3 = 5.5 \\
-μ_1 = 3.5, \quad μ_2 = 0.5, \quad μ_3 = 6 \\
-μ_1 = 3.5, \quad μ_2 = 1, \quad μ_3 = 6.5 \\
-μ_1 = 3.5, \quad μ_2 = 1.5, \quad μ_3 = 7 \\
\end{cases}
\]

where \( α, β, \) and \( γ \) denote the side length of the square for which each logvariance is calculated in the experimental logvariancegram.

Methodology for correcting sp, the sample/polygon term

As was shown above, the sample/polygon term, \( λαC(V, v_α) \), also exhibits a scaling behaviour, but one which is different from that of −μ. We first exploit the known \( l = α(n - 1) \) relation of Equation (21) to convert

\[ sp(n) = \sum_{α=1}^{n} λαC(V, v_α) \]  \hspace{1cm} \text{(27)}
to a function of \( f = f(\alpha(n-1)) \) instead of \( n \). (This is what gave a straight line for \( -\mu \)). In the case of \( sp \), however, one finds a curve resembling \( \Gamma(l) \) versus \( l \) (see Figures. 7 and 8). In the case of \( sp \), however, one finds a curve resembling \( \Gamma(l) \) versus \( l \).

The two curves, \( sp \) versus \( f \) and \( \Gamma \) versus \( l \) can again be scaled empirically to coincide. This means that

\[
sp = b\Gamma[f] + c = b\Gamma[a\{\alpha(n-1)\}] + c
\]

where \( a, b, c \) are three new scaling constants. The equations determining these have the same structure as Equations [23]–[25] with \( sp = sp(n) \) substituted for \( -\mu_i \) and \( \Gamma[f(\alpha(n_i - 1))] \) for \( f(\alpha(n_i - 1)) \). Thus

\[
\frac{sp_1 - sp_2}{sp_1 - sp_3} = \frac{\Gamma[a\{f(\alpha(n_i - 1))] - \Gamma[a\{f(\alpha(n_i - 1))] \} \Gamma[a\{f(\alpha(n_i - 1))] - \Gamma[a\{f(\alpha(n_i - 1))] \}
\]

and

\[
c = \left( \frac{\Gamma[a\{f(\alpha(n_1 - 1))] - \Gamma[a\{f(\alpha(n_1 - 1))] \} \Gamma[a\{f(\alpha(n_1 - 1))] - \Gamma[a\{f(\alpha(n_1 - 1))] \} \right)
\]

The least squares fit modification

Instead of relying on the assumption that the straight line of \( -\mu \) versus \( f \) in Equation [22] can be forced to pass through three of the data (experimental) points, which may lead to no real root for \( \alpha \) in Equation [26], one can try a ‘milder’ approach of only requiring a fit in the least squares sense. This more robust procedure should always produce a solution, which is important for a production system. To do so one requires that the sum over the differences squared of \( S \) sample points that are going to be used to determine the scaling constants,

\[
F(\alpha, \beta, \gamma) = \sum_i [b\alpha(n_i - 1)] + \gamma - (-\mu_i)^2
\]

be minimized as a function of the parameters \( \gamma, \beta \), and \( \alpha \). This means that

\[
\frac{\partial F}{\partial \gamma} = 2b \sum_i f_i + 2\beta \gamma + 2 \sum_i \mu_i = 0
\]

\[
\frac{\partial F}{\partial \beta} = 2b \sum_i f_i^2 + 2 \beta \gamma f_i + 2 \sum_i \mu_i f_i = 0
\]

\[
\frac{\partial F}{\partial \alpha} = 2 \sum_i (n_i - 1)f_i^2 + \gamma - \sum_i \mu_i = 0
\]

where \( f_i = f(\alpha(n_i - 1)) \) and \( f_i \) is the derivative of \( f(x) \) with respect to its argument \( x = \alpha(n_i - 1) \). The first two equations are the usual least square conditions, arrived at by differentiating w.r.t. \( \beta \) and \( \gamma \). The last equation is new. It arises upon differentiating w.r.t. \( \alpha \) and requiring that this too should vanish. Then, upon defining

\[
D = S \left( \sum_i f_i^2 \right) - \left( \sum_i f_i \right)^2
\]

one finds, as usual, that

\[
\beta = \left[ -S \left( \sum_i \mu_i f_i \right) + \left( \sum_i \mu_i \right) \left( \sum_i f_i \right) \right] \frac{1}{D}
\]

\[
\gamma = \left[ \left( -\sum_i \mu_i \right) \left( \sum_i f_i^2 \right) + \left( \sum_i \mu_i f_i \right) \left( \sum f_i^2 \mu_i \right) \right] \frac{1}{D}
\]

Substituting these two values into Equation [33], one finds an equation that determines the as-yet unknown \( \alpha \) as a root of an algebraic equation, very much as in Equation [26].

This equation can be recast as

\[
Q(y) = \beta \left( \sum_i (n_i - 1)f_i f_i^2 \right) + \gamma \left( \sum_i (n_i - 1)f_i \right) + \left( \sum_i (n_i - 1)\mu_i f_i \right) = 0
\]

i.e. the value of \( y \) at which \( Q(y) \) vanishes is the preferred value of \( \alpha \). Once this has been determined, the preferred values of \( \beta \) and \( \gamma \) follow from Equation [35]. Next, one has to check that the solution so obtained indeed leads to a minimum value of \( F(\alpha, \beta, \gamma) \), evaluated at these values of \( \alpha, \beta, \gamma \). The condition for this to be the case is that all three second partial derivatives are positive there. Doing the calculations, one finds the three conditions

\[
\frac{\partial^2 F}{\partial \gamma^2} > 0
\]

\[
\frac{\partial^2 F}{\partial \beta^2} > 0
\]

\[
\frac{\partial^2 F}{\partial \alpha^2} > 0
\]

which, in combination with the vanishing first derivatives, guarantee a minimum or a maximum.

The derivatives of \( f \) that are required come from Equation [19] with Equation [20] inserted for \( \Gamma(l) \). Note that this consists of a sum of individual spherical variogram forms that have zero derivatives once the argument exceeds the range. This should be borne in mind in programming the derivative. One has

\[
\frac{df}{dl} = \frac{3C_j}{2a_j} \left( 1 - \frac{l^2}{a_j^2} \right) l \leq a_j
\]

for the \( j \)th structure. Hence

\[
f' = \frac{df}{dl} = -\sum_j \frac{3C_j}{2a_j} \left( 1 - \frac{l^2}{a_j^2} \right) \theta(a_j - l)
\]

where the step function \( \theta \) is merely a reminder that each term in the sum will contribute only provided \( l \) does not exceed its particular range \( a_j \). The expressions for the required derivatives are

\[
f'_i = -\sum_j \frac{3C_j}{2a_j} \left( 1 - \frac{(n_i - 1)^2 a_j^2}{a_j^2} \right) \theta(a_j - l_i)
\]
Witwatersrand gold reef evaluation: the ‘variancegram’ tool

\[ f_i = \sum_{j=1}^{p} \frac{3C_j}{\alpha_j^2} \left( (n_i - 1) \frac{\alpha_j}{\alpha_j} \right) \theta(a_j - l_i) \]  \[ 46 \]

Notice that both derivatives are a sum over the \( p \) structures making up \( f \) and \( f' \), evaluated at the particular value \( l = k = (n_i - 1) \alpha \).

It is interesting to observe that in all cases where low values of \( n_1, n_2, n_3 \) were considered to obtain the scaling constants, the least squares method gave exactly the same values for the scaling constants as were found from the (largest) root of \( R(x) \) previously.

**Calculation of a corrected \(-\mu\)**

To calculate the scaling constants for the test data the values for \(-\mu(n_i)\) of \( [n_1, n_2, n_3] = [1, 2, 16] \) are used. Figure 11 shows the plot of \( R(x) \) versus \( x \), the positive root of which will determine \( \alpha \).

The value of the positive root in this case is \( x = 10.7968 \). This means that

\[ \alpha = 10.7968 \]  \[ 47 \]
\[ \beta = 867276 \]  \[ 48 \]
\[ \gamma = -256987 \]  \[ 49 \]

Knowing these constants, we can write the way \(-\mu\) is expected to vary with the sample number \((n-1)\):

\[ -\mu(n) = 867276 \ f[10.7968(n-1) - 256987] \]  \[ 50 \]

Extrapolating with this formula to \( n = 239 \), one finds

\[ -\mu(239) = 489988 \text{ versus } 44929 \]  \[ 51 \]

by direct calculation.

**Verification of the scaling hypothesis for \(-\mu\)**

In order to verify that these scaling factors are indeed correct, we show the values of \( f[10.7968(n-1)] \), and the corresponding values of \(-\mu(n)\) found in the kriging run:

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Samples used</th>
<th>Calculated ( f[10.7968(n-1)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\mu \times 10^{-6})</td>
<td>( n )</td>
<td>( f[10.7968(n-1)] )</td>
</tr>
<tr>
<td>1.6683</td>
<td>1</td>
<td>2.2276</td>
</tr>
<tr>
<td>0.9794</td>
<td>2</td>
<td>1.4241</td>
</tr>
<tr>
<td>0.5767</td>
<td>4</td>
<td>0.9544</td>
</tr>
<tr>
<td>0.3808</td>
<td>8</td>
<td>0.7484</td>
</tr>
<tr>
<td>0.2517</td>
<td>16</td>
<td>0.5395</td>
</tr>
<tr>
<td>0.1926</td>
<td>32</td>
<td>0.4865</td>
</tr>
<tr>
<td>0.1388</td>
<td>64</td>
<td>0.3755</td>
</tr>
<tr>
<td>0.0489</td>
<td>239</td>
<td>0.3528</td>
</tr>
</tbody>
</table>

The number pairs \([f', -\mu]\) are plotted as closed circles in Figure 12.

The straight line is given by Equation [22], plotted as a function of the variable \( f \). The agreement is truly remarkable!

The predicted point at \( n = 239 \) has also been included to illustrate that increasing values of \( n \) ‘pile-up’ on each other along the straight line of \(-\mu\) versus \( f \).

It is useful to point out again that the inclusion of a sill to define the logvariance difference \( f[l] = \text{Sill} - \Gamma(l) \) is merely one of convenience. From the previous development it is clear that one could equally well have considered \(-\mu\) as a function of the variancegram \( f[l] = \alpha(n - 1) \). Then

\[ -\mu = \beta(\text{Sill} - \Gamma(l)) + \gamma = -\beta \Gamma[\alpha(n - 1)] + \gamma \]  \[ 52 \]

Here \(-\mu\) is a linear function of the variancegram with gradient \(-\beta\) and a revised constant \( \gamma' = \text{Sill} \times \beta + \gamma \). Thus the entire exercise of determining the scaling constants could equally well have been implemented by using the variancegram: the choice of a sill is irrelevant.

**Calculation of a corrected \( sp \)**

By cross-multiplying in Equation [29] one finds a function that is the exact analogue of \( R(x) \) in Equation [26]. The root of the former equation now gives the value of \( a \) in Equation [28] for fixed \( \alpha \). Using the input values of \( sp \) for \( [n_1, n_2, n_3] = [2, 4, 16] \) one determines the scaling constants as

\[ a = 71.03 \]  \[ 53 \]
\[ b = 233614 \]  \[ 54 \]
\[ c = -283409 \]  \[ 55 \]

Knowing these constants we can similarly write down an equation for \( sp \) versus \( n \):

\[ sp(n) = 233614 \Gamma(71.03/f[11.914(n-1)]) - 283409 \]  \[ 56 \]

The extrapolation to \( n = 239 \) gives

\[ sp(239) = 116883 \text{ versus } 97512 \]  \[ 57 \]

Notice a small but important point: the value of \( a = 11.914 \) appears here instead of \( a = 10.7968 \). This happens because the samples for \( [n_1, n_2, n_3] = [2, 4, 16] \) were used for the \( sp \), since \( sp_1 \) and \( sp_2 \) lie too close to give a useful determination of \( a, b, c \). One must thus use the same \( n \)s for the coefficient \( \alpha \) to remain consistent.

**Verification of the scaling hypothesis for \( sp \) in the case of OK**

We verify the linear behaviour again by evaluating \( \Gamma(n) = \Gamma(\alpha(n - 1)) \) for the values at which the \( sp \) are known from the kriging runs.

\[ \text{Figure 12—} \ -\mu \text{ versus } f[l] \]
Witwatersrand gold reef evaluation: the ‘variancegram’ tool

Knowing these constants, one can again write down the way \( sp \) is expected to vary with the sample number \( (n-1) \):

\[
sp(n) = 26564 \cdot f[8.058(n-1)] + 34214
\]  

Extrapolating with this formula to \( n = 239 \), one finds

\[
sp(239) = 83983 \quad \text{versus} \quad 81195
\]

by direct calculation.

This estimated value for \( sp \) (239) is indicated by a star in Figure 9. It would indeed seem that this would have been the value of \( sp \) if the influence of negative weights were absent.

The scaling hypothesis for \( sp \) in the case of SK is verified next by evaluating \( f(n) = f[8.058(n-1)] \) for the values at which the \( sp \) are known from the kriging runs.

\[
sp \times 10^{-5} \quad n \quad f[71.03f[11.914(n - 1)]]
\]

\[
1.536 \quad 1 \quad 1.874
\]

\[
1.543 \quad 2 \quad 1.874
\]

\[
1.521 \quad 4 \quad 1.864
\]

\[
1.443 \quad 8 \quad 1.837
\]

\[
1.378 \quad 16 \quad 1.803
\]

\[
1.323 \quad 32 \quad 1.765
\]

\[
1.235 \quad 64 \quad 1.711
\]

\[
1.169 \quad 239 \quad 1.713
\]

The straight line character of the plot \( sp \) versus \( \Gamma \) is verified in Figure 13.

Resulting estimated global kriging variance in the case of OK

The directly calculated (‘true’) OK global kriging variance for the test data is

\[
\sigma_{OK}^2 = 195568 + 44929 - 97512 = 142985 \quad [58]
\]

The estimated OK global kriging variance based on the proposed correcting method is

\[
\sigma_{OK}^2 = 195568 + 48988 - 116883 = 127673 \quad [59]
\]

As is clear from comparison of the above two equations, the estimated \( -\mu \) and \( sp \) values are both higher than the true values, and since they are subtracted from each other, the resultant error in the global kriging variance should be reduced. However, the main source of discrepancy is the difference in the \( sp \) values. This could be ascribed to the fact that the true value of 97 512 is anomalously low due to the impact of negative weights, whereas the impact is absent in the estimated value.

Nevertheless, the estimated value exhibits an error of less than 11% of the true value, whereas the uncorrected RDM value based on kriging with 16 samples showed an error of 121%.

Correcting global kriging variances in the Case of SK

The correction procedure here focuses only on \( sp \), since the Lagrange multiplier is absent. It was illustrated in Figure 9, that for SK, \( sp \) increases with \( n \) in a manner reminiscent of the way in which \( \Gamma(l) \) increases with \( l \). This again means that the variable pairs \([sp,f(l)]\) and \([n,l]\) are linearly related.

Hence the rest of the formulism is exactly the same as was the case for \( -\mu \), except that \( -\mu \) is everywhere replaced by \( sp \), and \( f(l) \) is reinterpreted as being \( \Gamma(l) \).

The least squares fit modification of the method to ensure a solution in a production environment follows immediately with the same replacements in the formulism of the relevant subsection above.

The experimental values obtained for \( sp \) in the case of SK based on kriging with \( n = 1, 2, 4, 8 \), etc. are given above. Using \( n_1 = 1 \), \( n_2 = 2 \) and \( n_3 = 16 \), the following scaling constants were obtained when solving the equivalent of Equations [23]–[25], replacing \( -\mu \) by \( sp \):

\[
\alpha = 8.058 \quad [60]
\]

\[
\beta = 26564 \quad [61]
\]

\[
\gamma = 34214 \quad [62]
\]

The least squares fit modification of the method to ensure a solution in a production environment follows immediately with the same replacements in the formulism of the relevant subsection above.

The experimental values obtained for \( sp \) in the case of SK based on kriging with \( n = 1, 2, 4, 8 \), etc. are given above. Using \( n_1 = 1 \), \( n_2 = 2 \) and \( n_3 = 16 \), the following scaling constants were obtained when solving the equivalent of Equations [23]–[25], replacing \( -\mu \) by \( sp \):

\[
\alpha = 8.058 \quad [60]
\]

\[
\beta = 26564 \quad [61]
\]

\[
\gamma = 34214 \quad [62]
\]

The least squares fit modification of the method to ensure a solution in a production environment follows immediately with the same replacements in the formulism of the relevant subsection above.
Witwatersrand gold reef evaluation: the ‘variancegram’ tool

The deviations from the straight line at high \( n \) values are once more a manifestation of the impact of negative weights on the experimental \( s_p \) values when kriging is actually performed with such large numbers of samples.

The directly calculated (‘true’) SK global kriging variance for the test data is

\[
\sigma_{SK}^2 = 195568 - 81195 = 114373 \tag{65}
\]

The estimated SK global kriging variance based on the proposed correcting method is

\[
\hat{\sigma}_{SK}^2 = 195568 - 83983 = 111585 \tag{66}
\]

The estimated SK global kriging variance is within 2\% of the true value, whereas the uncorrected RDM SK global kriging variance based on kriging with 16 samples is within 4.5\% of the true value.

Conclusion

With research into new mining methods opening up the possibility of going to unprecedented depths in the Witwatersrand gold mines, the legacy of Krige’s work remains as relevant as ever.

References


Celebrating Kimberley’s Firsts

1871: First diamond mining begins in what becomes the City of Kimberley

1873: Not always first! Kimberley was once the second largest town in South Africa after Cape Town

1881: Kimberley Royal Stock Exchange, in the country’s first industrial centre

1882: First city in the Southern Hemisphere to install electric street lighting

1887: South Africa’s first school of mines opens giving a foundation to three other major South African Universities

1904: First electric tram in South Africa

1912: First woman trained as a pilot

1913: South Africa’s first aviation school opens

1914: The Big Hole closes after 34 years reopening as the first South African mining heritage museum

1931: First airport to install lighting equipment and first night landing

1969: First female judge: Miss Justice Leonora van den Heever

1992: De Beers Consolidated Mines empowered with Ponahlalo Holdings

2006: First woman head of Kimberley based De Beers Sightholder Sales South Africa: Mpumi Zikala

2013:

De Beers becomes the first major South African mining company to empower its trading business. Empowerment of De Beers Sightholder Sales South Africa with Ponahlalo Holdings.

Our diamond trading business is an empowered entity in Kimberley, creating skilled employment and economic activity; sorting and valuing most of South Africa’s diamond production. Trading diamonds in Kimberley to all our South African clients’ factories.
Introduction
All mineral deposits have variable grades in their composition. Depending on the type of mineralization, there are different scales of variability. Various mining methods can be employed to try to reduce this variability, attempt to reduce the dilution that occurs in mining, and remove the waste from the ore, to maintain the mean grade that feeds the processing plant as constant as possible.

Obviously, an optimal mining method exists for each deposit that yields a product of required average grade for downstream processing at a reasonable cost and a suitable rate of production (Parker, 1979). However, there are some cases where a proper mining method is not sufficient to ensure that the ROM (run of mine) has the characteristics desired for the feeding of the processing plant or the final product.

Common to all mining methods is the notion of the selective mining unit (SMU) – the smallest practical volume that can be classified as ore or waste (Parker, 1979). Considering a SMU of small volume, it is natural that there are some with high grades and others with low grades. These differences in grades among SMUs can be measured by the variance of the SMUs. As the size of the SMU increases, it will tend to include a mixture of high and low grades, reducing the variance. Note that the mean remains constant in all cases.

Krige’s relationship or additivity of variances (Krige, 1951, 1981) is a volume-variance relationship, found experimentally by D.G. Krige using data from the gold deposits of the Witwatersrand. In this relationship, the dispersion of a small unit, \( v \), within the deposit is equal to the sum of the dispersion of \( v \) within a bigger unit, \( V \), and the dispersion of these units, \( V \), within the deposit, \( D \).

The relationship between these three variances is:

\[
\sigma^2\left(\frac{v}{D}\right) = \sigma^2\left(\frac{v}{V}\right) + \sigma^2\left(\frac{V}{D}\right) \quad [1]
\]

Knowing the principles of Krige’s relationship and the volume-variance relationship, it would be possible to analyse the reduction of variability using blending piles of various sizes. Blending (homogenization) piles can be used as part of a system of quality control in order to reduce the variability of the material fed to the processing plant.

This is possible only because of the heterogeneity of constitution of the material. Heterogeneity is a primary structural property of all matter, i.e. all particulate solids, dry, wet, suspended in water or in air, are heterogeneous. According to Gy (1998), when the portions forming a material are not strictly identical, the material is considered heterogeneous. The heterogeneity can be analysed...
Analysis of the dispersion variance using geostatistical simulation and blending piles

from two different aspects, namely relating to the constitution and to the distribution of the material. The heterogeneity of the constitution refers to the intrinsic characteristics of the material, and consists of the differences that exist between particles or constituent fragments of a lot, \( L \). The mixing or blending of the constituent particles of the lot does not have any influence on the heterogeneity of the constitution. This heterogeneity is responsible for the occurrence of the Fundamental Sampling Error.

In order to reproduce the fluctuations in grades (heterogeneity of constitution and distribution) within an orebody (for a daily, monthly, or other time period), conditional simulations can be used (Journel, 1974). By applying this technique, the grades with their spatial continuity and variability are reproduced in models that mimic the real deposit. Some previous investigations involving blending, homogenization piles and geostatistical simulations can be found in Abichequer et al. (2010), Beretta et al. (2010), Binndorf (2013), Costa et al. (2007), Marques et al. (2009), Marques and Costa (2013), and Ribeiro et al. (2008). The seminal work by Schofield (1980) addresses homogenization piles using linear geostatistics (ordinary kriging).

This work presents a study developed on two iron ore deposits located in the Iron Quadrangle region of Brazil. The aim is to investigate the volume-variance relationship for different SMUs (blending piles) with Krig's relationship (analytical solution) and compare it with the results obtained using a numerical simulation. The variable silica (\( \text{SiO}_2 \) in \% ) was chosen for this analysis, as it constitutes the most critical contaminant for this iron ore.

Methodology

The grades of silica in the feed to the processing plant often fluctuate too much. The first decision is whether to use blending piles (homogenization), and if used, what is the ideal size. To help answer these questions, this study proposes the use of multiple geostatistical simulated models (Journel, 1974) allied to mine planning. Using the sequence of blocks extracted, obtained by mining planning, the fluctuations of the grades in each simulation can be evaluated and a band of uncertainty defined for the grades, considering all possible simulated scenarios.

Multiple outputs provide the means to evaluate the variability of the \( \text{SiO}_2 \) head grades feeding the processing plant. From this temporal sequence emulating the grades feeding the plant it is possible to analyse the volume-variance relationship, as well the dispersion variance of the blocks in the blending piles.

The original data (drill-holes) was used to simulate the grades, simulate the time series, and reconcile the models against the reference values obtained from the historical data from the blocks mined in one year. The simulations in a mining context mimic the characteristics of the mineral deposit, creating an array of values with the same statistical and spatial characteristics as the true grades. A simulation, therefore, is not an estimate, but rather a set of values with the same general statistical characteristics as the original data.

The steps involved in this study are:

- Generation of multiple equally-probable 3D models (geostatistical simulations)
- Sequencing of the simulated 3D block model according to the production of one year
- Emulation of the feeding of the blending (homogenization) piles
- Calculation of the reduction of the variability in blending piles using Krig's relationship (dispersion variance \( v/V \))
- Emulation of the blending piles and calculation of the reduction of variability
- Comparison of the results obtained in the previous two steps.

Assessing block grade uncertainty

A conditional geostatistical simulation algorithm was applied to generate 50 equally probable scenarios of \( \text{SiO}_2 \) spatial distribution in \( 10 \times 10 \times 10 \) m blocks. Combining the block sequence from the two mines provides the grades which will feed the blending piles, i.e., one block of 1000 \( m^3 \) from each mine resulting in a \( 2 \times 1000 \) \( m^3 \) combined volume (new SMU or \( v \)). Grades are shown in a time series (Figure 1) sequentially through a year (as the historical data of the company). Each SMU in this time series has a volume of \( 2000 \) \( m^3 \) and a mass of approximately 7 173 t. The band of uncertainty associated with each SMU grade (maximum and minimum grades) is obtained through the simulated grades of silica (\( \text{SiO}_2 \)) for each block derived from the 50 equally probable scenarios ordered according to the mine scheduling.

Note the uncertainty in the grades (max–min.) and their fluctuation through the year. These time series values are required as the input to the algorithm used to simulate the homogenization piles allowing an assessment on the reduction of variability.

Inter-pile grade variability using a numerical solution

According to Marques and Costa (2013), each block grade differs from every other, and differs from the global mean of the blocks calculated for the entire year. By grouping various blocks and forming a pile, the average grade of each pile is moved closer to the global annual mean when compared to the grade of each individual block within a pile. This phenomenon explains why the grades in the reclaimed ore from the piles have less variance than the grades that would be obtained from a block-by-block scheme feeding the processing plant.

![Figure 1—Time series of the SMU grades feeding the homogenization piles. The red line is the maximum grade and the blue line is the minimum grade of \( \text{SiO}_2 \)](image-url)
Analysis of the dispersion variance using geostatistical simulation and blending piles

The first step is to calculate the reduction in the variability. To find the acceptable grade interval we need first to calculate the weighted mean grade using all blocks from the first geostatistical simulation, according to:

$$ T_{\text{sim}} = \frac{1}{M_i} \sum_{i=1}^{n} (T_i \times M_i) \quad [2] $$

where $T_{\text{sim}}$ is the mean grade of a geostatistical simulation, $M_i$ is the sum of the masses of all blocks in the simulated model, $T_i$ is the grade of the block $i$, $M_i$ is the mass of the $i$th block, and $n$ is the number of existing blocks in the geostatistical simulation.

Next, we need to calculate the weighted mean grade of each pile (for the same simulation) according to:

$$ T_p = \frac{1}{M_p} \sum_{i=1}^{n} (T_i \times M_i) \quad [3] $$

where $T_p$ is the mean grade of the pile, $M_p$ is the total mass of the pile, $T_i$ is the grade of the block $i$, $M_i$ is the mass of the block $i$, and $n$ is the number of blocks forming the pile.

For each pile formed, the squared difference between the pile mean grade and the mean grade of the simulation ($T_{\text{sim}}$) (all blocks throughout the year) is calculated. Next, these squared differences are averaged, leading to the variance of the pile grades throughout the year, for the mass of the homogenization pile selected (inter-pile variance). The entire process then starts again, but choosing a different simulation.

**Inter-pile grade variability using Krig’e’s relationship**

In many practical situations, it is necessary to know a regionalized variable which is the average over a certain volume or area, rather than at a point. The basic volume on which a regionalized variable is measured is called its support (Armstrong, 1998). When a change occurs in the support (increase), a new block size is created. This new block or SMU is related to the previous one via the global average, but their spatial structural characteristics are different. For example, simulations in blocks of 10 × 10 × 10 m have greater variability than the panel (combination of blocks) created by blending these blocks. As the support increases, the variability among the new panels (combination of various blocks) decreases. The question we need to answer is how much this variability changes at different supports.

Using Krig’e’s relationship, the dispersion variance of the SMUs ($\sigma^2$) within each panel ($V$) was measured. Knowing the total variance for the year ($\sigma^2/D$), it is possible to compute the variance among piles over the year ($\sigma^2/D$).

In this analysis, let us consider $v$, the smallest panel, as 2 000 m$^3$, or approximately 7 173 t, which is the size of the SMU; the unit $V$ as the different panels tested (from approximately 7 173 t to 1 000 kt – equivalent to the various pile sizes tested); and $D$ as the whole domain (comprising the sum of the 3 000 panels of 2 000 m$^3$ each). For calculation purposes, the panel is considered as a function of the volume of individual SMUs (same volume support, i.e. constant in each block), but all results are presented as a function of the mass (approximate). Block mass may vary depending on the density of the rock type predominant at each SMU.

The reduction in variability due to the increase in the mass can be calculated from the dispersion variance, according to:

$$ \sigma^2 (v/V) = \gamma (V/V) - \gamma (v/v) \quad [4] $$

where $\sigma^2 (v/V)$ is the dispersion variance of $v$ (SMU) within a bigger unit $V$ (blending pile), $\gamma (v/v)$ is the variance of $v$ (SMU) throughout the year, and $\gamma (V/V)$ is the variance of a bigger unit $V$ (blending piles) in the year.

Initially $\gamma (v/v)$ and $\gamma (V/V)$ are calculated. This can be done using the appropriated charts (Journel and Huijbregts, 1978) or obtained by averaging the variogram value between all possible pairs within a SMU (known as a ‘gammarbar’; Deutsch and Journel, 1998). For other examples on calculating dispersion variance, see Isaaks and Srivastava (1989), Pyrcz and Deutsch (2002), and Journel and Kyriakidis (2004). In this case, the value of $\sigma^2 (v/D)$ is known for each simulation analysed, and is equal to the variance of the time series (grades of the continuous flow feeding the processing plant).

Thus, for each simulation of a continuous flow of ore that feeds the processing plant, or a blending and/or homogenization stack, it is necessary to calculate the experimental variogram of the grades and model it. As each simulation of continuous flow is a possible representation of reality, and all are slightly different, there are 50 possible temporal variograms. In the following discussion the variograms for just two of them are presented (simulations #30 and #46), which can be seen in Figure 2a and Figure 2b. In these figures, the variogram model is represented by a continuous black line. The lag used in the calculation of the experimental variogram (or chrono-variogram) for the continuous flow grades is approximately 2 hours and 55 minutes.

The reduction in variability can be calculated from the dispersion variance, according to:

$$ \gamma (h) = \frac{C}{D} + 2 \frac{C^{*}}{D^{*}} \frac{h}{40} \quad [5] $$

$$ \gamma (h) = \frac{C}{D} + 2 \frac{C^{*}}{D^{*}} \frac{h}{40} + \frac{C^{*2}}{D^{*2}} \frac{h}{50} \quad [6] $$

where each model is represented by a nugget effect, $C_0$, two spherical structures ($Sph$) with $C_1$ and $C_2$ as their sill contribution, and ranges, $a_1$ and $a_2, 1$ respectively [$\gamma (h) = C_0 + C_1 Sph (h/a_1) + C_2 Sph (h/a_2)$]. Once the variogram for each simulation has been fitted, it can be further used to calculate the dispersion variance. Table I presents the results for five panels (combination of multiple blocks sizes) using simulations no. 30 and no. 46.

![Figure 2—Temporal variogram for simulation (a) 30 and simulation (b) 46. Each h represents approximately a time (t) of 2 h and 55 min](image-url)
Analysis of the dispersion variance using geostatistical simulation and blending piles

Table I
Result of the dispersion variance of a 2 000 m$^3$ SMU within five increasing volumes – panels within the year - (simulation no. 30 and no. 46)

<table>
<thead>
<tr>
<th>h</th>
<th>Panel volume (m$^3$)</th>
<th>Approximate mass (t)</th>
<th>Dispersion variance (%$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Simulation 30</td>
</tr>
<tr>
<td>1</td>
<td>2 000</td>
<td>7 173</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>10 000</td>
<td>35 867</td>
<td>6.31</td>
</tr>
<tr>
<td>8</td>
<td>16 000</td>
<td>50 214</td>
<td>7.25</td>
</tr>
<tr>
<td>15</td>
<td>30 000</td>
<td>107 601.2</td>
<td>9.09</td>
</tr>
<tr>
<td>29</td>
<td>58 000</td>
<td>208 029</td>
<td>8.93</td>
</tr>
</tbody>
</table>

Figure 3 and Figure 4 show the complete results for the dispersion variance of 2 000 m$^3$ SMUs within increasing pile sizes (or equivalent panel volumes) for simulations no. 30 and no. 46, respectively.

Results
The variance of the final grade which feeds the processing plant is the sum of variability of the sources $\sigma^2 (v/V)$ and $\sigma^2 (V/D)$. Figure 4 depicts the values for the total variability ($\sigma^2$) of the grades due to the two sources listed versus the different blending pile sizes. The variability reduction associated with the increase of mass using the dispersion variance formula (area above the line) is shown in red (continuous smooth line). The blue (jagged) line is the variability decay due to the increase of mass for different pile sizes using the numerical solution (numerical emulation of the pile grades). Note that although the blue line is less smooth than the red line, both share the same overall form. Also, Figure 5 and Figure 6 have different total variances (y-axis), but are equivalent in form and proportion.

The benefit of reducing the variability of the head grades obtained by using different blending pile sizes can be seen clearly in Figure 4. For instance, a 50 kt pile will lead to a
Analysis of the dispersion variance using geostatistical simulation and blending piles

reclaimed ore with a head grade variance approximately 50% less than would be obtained by feeding the plant directly with the grades of 7 173 t SMUs.

The calculation on variability reduction due to the increase in pile mass was repeated for 50 equally probable simulated block models using either Kriging’s relationship or numerical emulation (Marques and Costa, 2013). For the sake of comparison the total variance of each simulation was standardized (equal to 1). Figure 7 shows the results using Kriging’s relationship, and Figure 8 shows the results for the numerical simulation.

The plots suggest an exponential decay in grade variability with an increase in pile size. This reproduces the volume-variance relationship discussed in Parker (1979). Consequently, the grades measured in the reclaimed ore from the piles will have a reduced variance when compared to the grade variability that would be obtained from feeding the processing plant block-by-block (or SMU-by-SMU).

It should be kept in mind that this reduction in variability will be obtained only if properly designed and operated homogenization piles (of the proper size and with the proper number of layers) are in place. This analysis is based on the heterogeneity of distribution, i.e. the way the material is distributed within a pile. The parts forming the stack differ among themselves and are different from the average content of the stack. Organizing the block or SMU forming the stack in different numbers of layers leads to a reduction of variation in the grades that will feed the processing plant. For each increment of mass, it is possible obtained a new time series of the feed to the processing plant (from the formed homogenization piles). The larger the homogenization pile that is formed, the lower the uncertainty in this band of the time series (as long as the pile is adequately homogenized). A detailed proof of this topic can be seen in Marques and Costa (2013).

Conclusions

The data used for modelling the variogram needed to calculate the variance of dispersion derives from simulated continuous flow and does not have the spatial structure of the original mineral deposit. The grades along the continuous flow follow the mining sequence and no longer have the spatial characteristics of the deposit. However, the sequence of values has its own temporal autocorrelation structure.

The dispersion variance decreases as the support increases. This relationship (volume-variance) is well known and the results corroborate the principle embedded in Kriging’s relationship. The simulation of a continuous flow was used to model the temporal variogram and to obtain the dispersion variance for various pile sizes. Additionally, a numerical simulation of various pile sizes gave very similar results.

The algorithm designed to predict the variability in the blending piles reflects the uncertainty associated with in-situ grades. The use of estimated models (via any Kriging method, for instance) is not suitable for performing this type of analysis, as there is an excessive smoothing in the block grades.

The largest amount of variability reduction in the homogenization system is associated with the mass pile increase (inter-pile grade variability). However, the number of layers should also be also considered in the formation of a homogenization pile, as a pile with a large mass but improperly assembled will not lead to the expected reduction in variability.

References


Analysis of the dispersion variance using geostatistical simulation and blending piles


---

Proven Lubricants for Extreme Mining Conditions

Speciality Mining Lubricants
- Long wall roof support fluids
- Fire resistant hydraulic fluids
- Multifunctional gear lubricants
- Mineral oil based hydraulic fluids
- Speciality greases
- Wire rope lubricants
- Rock-drill and pneumatic lubricants
- Diesel scrubber conditioners
- Surface mining speciality lubricants

Fuchs Lubricants (South Africa) (Pty) Ltd

Tel: +27 (0)11 565 9600  Fax: +27 (0)11 392 5686  
Email: denzylo@fuchsoil.co.za

www.fuchsoil.co.za

The Journal of The Southern African Institute of Mining and Metallurgy

AUGUST 2014  VOLUME 114
Introduction

Etymologically, the term ‘geostatistics’ designates the statistical study of natural phenomena. The early developments of geostatistics in the 1950s and 1960s aimed to improve the evaluation of recoverable reserves in mineral deposits (Krige, 1951; Journel and Huijbregts, 1978). Its field of application expanded considerably to encompass nowadays most fields of geoscience (e.g. geology, geochemistry, geohydrology, soil science) and a vast array of disciplines that all deal with the analysis of space-time data, such as oceanography, hydrogeology, remote sensing, agriculture, and environmental sciences. The success of geostatistics resides in its ability to capitalize on the first law of geography, stating that ‘Everything is related to everything else, but near things are more related than distant things’ (Tobler, 1970).

Indeed, one of the main characteristics of the aforementioned data types is their structured distribution in space and time, which reflects the impact of various factors (e.g. geology, weather, human activities, land cover) operating at different spatial and temporal scales. Geostatistical spatio-temporal models (Kyriakidis and Journel, 1999) provide a probabilistic framework for data analysis and predictions that build on the joint spatial and temporal dependence between observations.

The main steps of a typical geostatistical study are summarized in Figure 1 using the well-known Swiss Jura data-set (Goovaerts, 1997, 1999). Analysis of spatial data typically starts with a ‘posting’ of data values. For example, Figure 1 (top graph) shows the location of 259 soil samples where the concentrations of topsoil cadmium concentration were recorded. Most applications of geostatistics are concerned with the prediction of measured attributes at unsampled locations. Such interpolation or extrapolation is made possible by the existence of autocorrelation in the data, which can be quantified and modelled using the semivariogram. Various kriging techniques are then available to derive estimated attribute values and the corresponding prediction error variance at unsampled locations using information related to one or several attributes. An important contribution of geostatistics is the assessment of the uncertainty about the attribute values at any particular unsampled location (local uncertainty) as well as jointly over several locations (multiple-point or spatial uncertainty). Models of local uncertainty usually take the form of a map of the probability of exceeding critical values, such as regulatory thresholds in soil pollution. Spatial uncertainty is tackled through stochastic simulation that allows one to generate alternative models of the spatial distribution of attribute values that reproduce features of the data (e.g. histogram, semivariogram). Last but

Synopsis

Since its development in the mining industry, geostatistics has emerged as the primary tool for spatial data analysis in various fields, ranging from earth and atmospheric sciences to agriculture, soil science, remote sensing, and more recently environmental exposure assessment. In the last few years, these tools have been tailored to the field of medical geography or spatial epidemiology, which is concerned with the study of spatial patterns of disease incidence and mortality and the identification of potential ‘causes’ of disease, such as environmental exposure, diet and unhealthy behaviours, economic or socio-demographic factors. On the other hand, medical geology is an emerging interdisciplinary scientific field studying the relationship between natural geological factors and their effects on human and animal health. This paper provides an introduction to the field of medical geology with an overview of geostatistical methods available for the analysis of geological and health data. Key concepts are illustrated using the mapping of groundwater arsenic concentration across eleven Michigan counties and the exploration of its relationship to the incidence of prostate cancer at the township level.

Keywords
geostatistics, medical geology, spatial epidemiology, groundwater, arsenic.

* BioMedware, Inc, Ann Arbor.
© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
Geostatistics: a common link between medical geography, mathematical geology

not least, this uncertainty assessment can be combined with expert knowledge for decision-making, such as delineation of contaminated areas where remedial measures should be taken or selection of locations for additional sampling.

Medical geography is defined as the branch of human geography concerned with the geographic aspects of health, disease, and health care (May, 1950). The idea that place and location can influence health is a very old and familiar concept in medical geography. One of the first demonstrations of the power of mapping and analysing health data was provided by Dr John Snow’s study of the cholera epidemic that ravaged London in 1854. Using maps showing the locations of water pumps and the homes of people who died of cholera, Snow was able to deduce that one public pump was the source of the cholera outbreak (McLeod, 2000). Since then, the field of medical geography has come a long way, replacing paper maps with digital maps in what are now called geographic information systems (GIS). Similarly, descriptive speculation about disease has given place to scientific analysis of spatial patterns of disease, including hypothesis testing, multi-level modelling, regression, and multivariate analysis.

Recently, geostatistical techniques (semivariograms, kriging, stochastic simulation) have been tailored to the study of spatial patterns of disease incidence and mortality and the identification of potential ‘causes’ of disease, such as environmental exposure or socio-demographic factor (Waller and Gotway, 2004; Goovaerts, 2007, 2009). Once again, health outcomes, such as cancer mortality or incidence of late-stage diagnosis, tend to follow the first law of geography, and maps are used by public health officials to identify areas of excess (e.g. cancer clusters) and to guide surveillance and control activities, including consideration of health service needs and resource allocation for screening and diagnostic testing. Data available for human health studies falls within two main categories: individual-level data (e.g. location of patients and controls) or aggregated data (e.g. cancer rates recorded at county or ZIP code level); see example in Figure 2. Although none of these data-sets falls within the category of ‘geostatistical data’ as classically defined in the spatial statistics literature (Cressie, 1993), geostatistics offers a promising alternative to common methods for analysing spatial point processes and lattice data. One of the most challenging tasks in environmental epidemiology is the analysis and synthesis of data collected at different scales and over different spatial supports. For example, one might want to explore relationships between health outcomes aggregated to the ZIP code level, census-tract demographic covariates, and exposure data measured at a few point locations. Geostatistics provides a theoretical framework for performing the various types of changes of support, while providing a measure of the reliability of the predictions (Goovaerts, 2010, 2012).

‘Hydrobiogeochemoepidemiopathoecology’, a term coined by scientists as an alternative to the most common medical geology, is defined as the science dealing with the relationship between natural geological factors and health in humans and animal, and understanding the influence of ordinary environmental factors on the geographical distribution of such health problems (Selinus et al., 2005).

Bowman et al. (2003) distinguished two branches of medical geology, depending on whether health problems are caused by the natural occurrence of elements in the geologic environment (e.g. ingestion of food grown in soils with element deficiency or toxicity) or the release of elements by natural hazards, such as earthquakes, volcanic eruptions, or...
Geostatistics: a common link between medical geography, mathematical geology

Elevated levels of naturally-occurring arsenic have been identified in regional patterns within the USA and are attributed to geochemistry, geology, climate, and glacial history (Welch et al., 2000). In the Michigan Thumb region, arsenopyrite (up to 7% As by weight) has been identified in the bedrock of the Marshall Sandstone aquifer, one of the region’s most productive aquifers (Westjohn et al., 1998). The present case study explores the association between the incidence of prostate cancer and groundwater arsenic level for eleven Michigan counties displayed in Figure 3. Epidemiologic studies have suggested a possible association between exposure to inorganic arsenic and prostate cancer mortality, including a study of populations residing in Utah (Lewis et al., 1999). Unlike the Utah study no individual-level data is available here, which prohibits any exposure reconstruction (i.e. length of exposure is unknown in the absence of information on residential history) and the incorporation of important covariates, such as age, smoking, diet, heredity, or socio-economic status. Note that the objective of the case study is to illustrate the application of geostatistics in medical geology, and a thorough epidemiologic study is beyond the scope of this paper.

The information available for this so-called ecological study (i.e. analysis of aggregated health outcomes) consist of: (1) 9 188 arsenic concentrations measured at 8 212 different private wells that were sampled between 1993 and 2002, (2) prostate cancer incidence recorded at the township level over the period 1985–2002, and (3) block-group population density that served as proxy for urbanization and use of regulated public water supply versus use of potentially contaminated private wells in rural areas. Figure 4A shows a close-up of these three data-sets in the northern part of the study area. This case study illustrates a common challenge in

Figure 3—Groundwater arsenic data-set: the 8 212 well data observations (μg/L) available for modelling, and the map of bedrock with the location of the Marshall Sandstone suboutcrop where the highest concentrations of arsenic were found. The outlines of townships are displayed in background and the study area is located within the state of Michigan.

Setting the problem

Arsenic (As) is one of the most toxic elements in our environment and is listed as the third most toxic substance, after lead and mercury in the US Toxic Substances and Disease Registry. Its adverse impact on human health can take many forms, including skin lesions, cardiovascular disease, hypertension, reproductive and neurological disorders, respiratory problems, and various types of cancer (e.g. skin, lung, liver, bladder, prostate, kidney). Sources of arsenic exposure vary from burning of arsenic-rich coal (China) and mining activities (Malaysia, Japan) to the ingestion of tainted food (e.g. rice) or water contaminated by natural sources such as bedrock containing arsenic (e.g. Bangladesh, India, Taiwan, Philippines, Mexico, Chile). Arsenic in drinking water is a major problem and has received much attention because of the large human population exposed and the extremely high concentrations (e.g. 600 to 700 μg/L) recorded in many instances. Few studies have, however, assessed the risks associated with exposure to low levels of arsenic (say < 50 μg/L) most commonly found in drinking water in the USA.
Geostatistics: a common link between medical geography, mathematical geology

environmental epidemiology – that is, the analysis and synthesis of spatial data collected at different spatial scales and over different spatial supports. Exploring the relationships between these incompatible data-sets will require the estimation of all three variables over the same set of geographical units (i.e. townships here); see Figure 4B. Following the terminology in Gotway and Young (2007), this change of support (COS) will involve upscaling (spatial aggregation) for arsenic data and side-scaling, a term used to refer to the prediction of values on one set of spatial units from data on another set of overlapping spatial units, for population density.

Mapping arsenic content

This study will use the geostatistical model of the spatial distribution of groundwater arsenic concentrations that was described in details in Goovaerts et al. (2005). Only the most salient features will be presented here. The modelling was based on all 9 188 well data. The small magnitude of temporal variation relative to the variability in space or arising from measurement error, as well as the absence of temporal trend or seasonality, led us to ignore the temporal dimension in this study. Since this database contains arsenic measurements requested by homeowners, sampling is denser in areas where higher pollutant concentrations were initially reported. This preferential sampling was corrected using the cell-declustering technique (Deutsch and Journel, 1998), which calls for dividing the study area into rectangular cells; then each observation within a cell is assigned a weight inversely proportional to the number of data within that cell. These declustering weights were used, instead of equal weights, in the computation of summary statistics, leading to a mean and standard deviation of 10.97 and 15.22 μg/L, respectively.

Arsenic concentration was estimated at the nodes of a 500 m spacing grid using soft indicator kriging (Goovaerts, 1997) and 22 threshold values. Soft information was derived by a calibration of geological data, such as type of bedrock and unconsolidated deposits, and proximity of wells to the Marshall Sandstone suboutcrop, where the highest concentrations of arsenic were found. The choice of a non-parametric approach over lognormal or multigaussian kriging was motivated by:

1. The presence of 737 measurements below the detection limit. Unlike other techniques, indicator kriging does not require assigning a subjective value (e.g. half the detection limit) to this data since the first threshold can simply be identified with the limit.
2. The change in the spatial connectivity of different classes of observations. Indicator semivariograms in Figure 5 measure the transition frequency between two classes of arsenic values as a function of the separation distance. The greater the indicator semivariogram value, the less connected in space are the small or large values. As the threshold increases, the short-range variability becomes more important, which indicates that high arsenic concentrations are less connected in space than low concentrations.
3. The results of a cross-validation study using 9 188 well data and a validation study using 73 new wells. In both cases, soft indicator kriging provided the smallest mean absolute error of prediction.

Figure 4—Example of change of support where kriging is used to estimate groundwater arsenic concentration and population density at the township level using private well arsenic concentrations (upsampling) and block-group census data (side scaling). This change of support allows the analysis of relationship between drinking of groundwater with high level of arsenic and the incidence of prostate cancer.

Figure 5—Experimental indicator semivariograms computed from arsenic well data using five threshold values. The solid line is the model fitted using least-squares regression. Note how the short-range variability increases for the upper quartile and ninth decile of the sample histogram, which reflects the smaller spatial connectivity of high arsenic concentrations.
Figure 6 (top graph) shows the mean of the local distributions of probability modelled using soft indicator kriging (E-type estimate). This map closely reproduces the spatial pattern of bedrock formation (Figure 3), yielding smaller estimates in the southern part of the study area and larger estimates at the location of Marshall Sandstone. Concentrations were averaged within each of the 342 townships to yield a map suited for linkage with health data (Figure 6, bottom graph).

Mapping population density
Township-level population density was derived from census tract data using areal weighting or proportional allocation (Gotway and Young, 2002). In other words, census tract male population was allocated to each township based on the relative area of the census tract included in that township. The implicit assumption was that population was uniformly distributed within the census tract. Population data were then divided by the township area to compute the population density. The final map (Figure 7) illustrates the low population density in the northern part of the study area and highlights urban centres, such as Detroit, Flint, Ann Arbor, or Lansing.

Mapping prostate cancer incidence
Figure 8A shows the map of prostate cancer incidence rates computed at the township level. The analysis was restricted to white males aged 65 years and over to minimize the impact of disparities in age distribution across the area and to attenuate the impact of variability in health coverage since all cases were covered by Medicare. In addition, 146 rates based on less than 10 cases were considered as unreliable and assigned a missing value. Even in townships with more than 10 cases rates can still be unstable, in particular in sparsely populated rural areas. This issue, known as the ‘small number problem’ in epidemiology, can be addressed geostatistically using a form of kriging with non-systematic measurement errors, called Poisson kriging (Goovaerts, 2009). The basic idea is to filter the noise attached to each rate using rates recorded in adjacent geographical units. Rates based on small population, hence less stable, receive less weight than rates recorded in densely populated areas in
the kriging estimator. A major benefit of Poisson kriging over traditional statistical smoothers is that it allows the estimation of missing rates in addition to the filtering of noisy rates.

As for arsenic concentrations, this mapping technique requires the computation and modelling of a semivariogram. Two major differences are: (1) observations are rates and so are composed of a numerator (number of cancer cases) and a denominator (male population), and (2) the spatial supports of the observations are not points (areal data) and are not uniform (townships have different sizes and shapes). The first problem is tackled by using a population-weighted semivariogram estimator to attenuate the impact of less stable rates in the modelling of the spatial variability. On the other hand, the spatial support of the data is accounted for using a form of block kriging, called area-to-area (ATA) kriging (Kyriakidis, 2004). The last issue is the fact that ATA kriging requires a point-support semivariogram model, whereas only a block-support semivariogram model is available since all observations are areal data. The derivation of a point-support semivariogram model from a block-support model is called deconvolution in geostatistics and is a well-known problem in the mining industry. Mining blocks tend, however, to be all squares of the same size, a situation very different from the administrative units that are manipulated in medical geography. A special iterative deconvolution procedure was recently developed for the case of irregular units (Goovaerts, 2008, 2011).

The map of noise-filtered rates (Figure 8B), called risks, displays much less variability than the original rate maps. In particular, some of the extreme incidence rates recorded in rural counties are no longer present. Higher incidences are observed in the northern area that is more rural (Figure 7) as well as in the cities of Detroit and Flint, which are less affluent than the college towns of Ann Arbor and Lansing. The map of the kriging variance (Figure 8C) looks similar to the map of population density (Figure 7) and reflects the greater reliability of rates estimated in densely populated areas compared to rates that were missing or estimated in rural areas. Following Goovaerts (2006), the probability distribution of the unknown risk can be modeled using a Gaussian distribution that has the Poisson kriging estimate and kriging variance as mean and variance. The probability of exceeding specific thresholds can thus be computed fairly easily and incorporates both the magnitude of the risk estimate and the associated uncertainty. For example, Figure 8D shows the probability that the area-wide incidence rate of 1709 cases per 100 000 habitants is exceeded.

**Correlation analysis**

The relationship between the health outcome and putative covariates (arsenic level and population density) was analysed using logistic regression. The dependent variable is an indicator variable that takes a value of 1 if the probability of exceeding the area-wide incidence rate is above 0.5, and zero otherwise. The main predictor is the township-level concentration of arsenic displayed at the bottom of Figure 6. Given that rural townships are less likely to have access to regulated public water supply, one should expect the potential relationship between groundwater arsenic level and incidence of prostate cancer to be stronger where population density is low. This hypothesis was tested by using the following interaction term in the regression model: arsenic concentration × density class, where eight equally probable classes of population density (i.e. including similar number of townships) were created from Figure 7.

Regression results are reported in Table I. An odds ratio is a relative measure of association between an exposure (e.g. arsenic in groundwater) and an outcome (e.g. area-wide incidence rate for prostate cancer is exceeded with probability above 0.5). More precisely, the odds ratio represents the odds that the outcome will occur given a particular exposure, compared to the odds of the outcome occurring in the absence of that exposure. In the present case where the predictor is a continuous variable, the odds ratio can be interpreted as the change in odds if the arsenic concentration increases by 1 ppm. Table I shows that the risk for a township to exceed the area-wide incidence rate for prostate cancer increases significantly (odds ratio with 95% confidence interval larger than 1) for the first two classes of population density, that is in rural townships where habitants are more likely to rely on private wells for drinking water. The odds ratio is lower for all other classes that include townships that are more densely populated. The significant odds ratio recorded for the most urbanized townships is likely linked to the largest prevalence of chronic disease in neighborhoods of lower socio-economic status. A more detailed analysis is, however, warranted to

<table>
<thead>
<tr>
<th>Population density (hab./ sq. mile)</th>
<th>Odds ratio</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.5-36.8</td>
<td>1.616</td>
<td>1.231-2.122</td>
</tr>
<tr>
<td>36.9-57.2</td>
<td>1.116</td>
<td>1.033-1.207</td>
</tr>
<tr>
<td>57.3-86.5</td>
<td>0.987</td>
<td>0.906-1.075</td>
</tr>
<tr>
<td>86.9-137.5</td>
<td>0.972</td>
<td>0.916-1.033</td>
</tr>
<tr>
<td>137.8-200</td>
<td>0.815</td>
<td>0.702-0.946</td>
</tr>
<tr>
<td>201-451</td>
<td>0.999</td>
<td>0.941-1.062</td>
</tr>
<tr>
<td>455-1,336</td>
<td>1.000</td>
<td>0.943-1.061</td>
</tr>
<tr>
<td>1,364-6,718</td>
<td>1.124</td>
<td>1.007-1.254</td>
</tr>
</tbody>
</table>
Geostatistics: a common link between medical geography, mathematical geology

Geostatistics, in its infancy, its growth cannot be sustained, or at least is meaningless, if it does not involve the end-users who are the subjects of considerable interest in our societies. This renewal of attention has led to the development of overlapping disciplines, such as geoevaluation; geoscience and public health; medical geography; epidemiology; medical geography; medical ecology; clinical ecology; environmental medical epidemiology; geomedicine; geoepidemiology; geology and health; geology, environment, and health; medical geography; and pathocology, to name a few. All these fields of study are complex and require a multidisciplinary approach that relies on a wide variety of specialists from geologists, geochemists, and medical doctors to biologists and veterinarians. A common thread is the recognition of the critical influence of place and location on the occurrence of health outcomes and environmental processes. To quote the Dutch philosopher Baruch Spinoza (1632–1677), ‘Nothing in Nature is random. ... A thing appears random only through the incompleteness of our knowledge.’ Interactive mapping of epidemiological data with geographic and environmental features is a critical tool that facilitates the formulation of hypotheses and the identification of relationships regarding the spatial patterns of disease. Geostatistical methodology is likely to play a major role in this endeavour because of its ability to take into account the double aspect of randomness and spatial structure in the characterization of regionalized variables.

The application of geostatistics to the promising field of environmental epidemiology presents several methodological challenges that arise from the facts that: (1) data is very diverse and typically recorded over overlapping geographies (e.g. ZIP codes, census tracts), and (2) health outcomes are often aggregated over irregular spatial supports and consist of a numerator and a denominator (i.e. population size). Everyday geostatistical tools, such as semivariograms or kriging, thus cannot be implemented blindly. The last decade has witnessed the emergence of tools and techniques tailored to this new type of data. Irregular spatial supports can now be tackled thanks to area-to-area kriging and iterative deconvolution procedures. Similarly, Poisson and binomial kriging combined with population-weighted semivariogram estimators allow the incorporation of both the numerator and denominator in the processing of rate data. It is noteworthy that the general formulation of kriging introduced half a century ago could already accommodate different spatial supports for both the data and the predicted unit. The development of geographical information systems and dramatic increase in computational power finally made possible the implementation of these theoretical concepts.

The field of environmental health geostatistics is still in its infancy. Its growth cannot be sustained, or at least is meaningless, if it does not involve the end-users who are the epidemiologists, geologists, and GIS specialists working in health departments, geological surveys, and cancer registries. Critical components to its success include the publication of applied studies illustrating the merits of geostatistics over current empirical mapping methods, training through short courses, and updating of existing curricula, as well as the development of user-friendly software. The success of mining and environmental geostatistics, as we experience it today, can be traced back to its development outside the realm of spatial statistics, through the close collaboration of mathematically minded individuals and practitioners. Environmental health geostatistics will prove to be no different.

Conclusions
The assessment of the health risk associated with environmental exposure has become the subject of considerable interest in our societies. This renewed attention has led to the development of overlapping disciplines, such as geoevaluation; geoscience and public health; medical geography; epidemiology; medical geography; medical ecology; clinical ecology; environmental medical epidemiology; geomedicine; geoepidemiology; geology and health; geology, environment, and health; medical geography; and pathocology, to name a few. All these fields of study are complex and require a multidisciplinary approach that relies on a wide variety of specialists from geologists, geochemists, and medical doctors to biologists and veterinarians. A common thread is the recognition of the critical influence of place and location on the occurrence of health outcomes and environmental processes. To quote the Dutch philosopher Baruch Spinoza (1632–1677), ‘Nothing in Nature is random. ... A thing appears random only through the incompleteness of our knowledge.’ Interactive mapping of epidemiological data with geographic and environmental features is a critical tool that facilitates the formulation of hypotheses and the identification of relationships regarding the spatial patterns of disease. Geostatistical methodology is likely to play a major role in this endeavour because of its ability to take into account the double aspect of randomness and spatial structure in the characterization of regionalized variables.

The application of geostatistics to the promising field of environmental epidemiology presents several methodological challenges that arise from the facts that: (1) data is very diverse and typically recorded over overlapping geographies (e.g. ZIP codes, census tracts), and (2) health outcomes are often aggregated over irregular spatial supports and consist of a numerator and a denominator (i.e. population size). Everyday geostatistical tools, such as semivariograms or kriging, thus cannot be implemented blindly. The last decade has witnessed the emergence of tools and techniques tailored to this new type of data. Irregular spatial supports can now be tackled thanks to area-to-area kriging and iterative deconvolution procedures. Similarly, Poisson and binomial kriging combined with population-weighted semivariogram estimators allow the incorporation of both the numerator and denominator in the processing of rate data. It is noteworthy that the general formulation of kriging introduced half a century ago could already accommodate different spatial supports for both the data and the predicted unit. The development of geographical information systems and dramatic increase in computational power finally made possible the implementation of these theoretical concepts.

The field of environmental health geostatistics is still in its infancy. Its growth cannot be sustained, or at least is meaningless, if it does not involve the end-users who are the epidemiologists, geologists, and GIS specialists working in

Conclusions
The assessment of the health risk associated with environmental exposure has become the subject of considerable interest in our societies. This renewed attention has led to the development of overlapping disciplines, such as geoevaluation; geoscience and public health; medical geography; epidemiology; medical geography; medical ecology; clinical ecology; environmental medical epidemiology; geomedicine; geoepidemiology; geology and health; geology, environment, and health; medical geography; and pathocology, to name a few. All these fields of study are complex and require a multidisciplinary approach that relies on a wide variety of specialists from geologists, geochemists, and medical doctors to biologists and veterinarians. A common thread is the recognition of the critical influence of place and location on the occurrence of health outcomes and environmental processes. To quote the Dutch philosopher Baruch Spinoza (1632–1677), ‘Nothing in Nature is random. ... A thing appears random only through the incompleteness of our knowledge.’ Interactive mapping of epidemiological data with geographic and environmental features is a critical tool that facilitates the formulation of hypotheses and the identification of relationships regarding the spatial patterns of disease. Geostatistical methodology is likely to play a major role in this endeavour because of its ability to take into account the double aspect of randomness and spatial structure in the characterization of regionalized variables.

The application of geostatistics to the promising field of environmental epidemiology presents several methodological challenges that arise from the facts that: (1) data is very diverse and typically recorded over overlapping geographies (e.g. ZIP codes, census tracts), and (2) health outcomes are often aggregated over irregular spatial supports and consist of a numerator and a denominator (i.e. population size). Everyday geostatistical tools, such as semivariograms or kriging, thus cannot be implemented blindly. The last decade has witnessed the emergence of tools and techniques tailored to this new type of data. Irregular spatial supports can now be tackled thanks to area-to-area kriging and iterative deconvolution procedures. Similarly, Poisson and binomial kriging combined with population-weighted semivariogram estimators allow the incorporation of both the numerator and denominator in the processing of rate data. It is noteworthy that the general formulation of kriging introduced half a century ago could already accommodate different spatial supports for both the data and the predicted unit. The development of geographical information systems and dramatic increase in computational power finally made possible the implementation of these theoretical concepts.

The field of environmental health geostatistics is still in its infancy. Its growth cannot be sustained, or at least is meaningless, if it does not involve the end-users who are the epidemiologists, geologists, and GIS specialists working in

Acknowledgements
This research was funded by grant 1R21 ES021570-01A1 from the National Cancer Institute. The views stated in this publication are those of the author and do not necessarily represent the official views of the NCI.

References


Geostatistics: a common link between medical geography, mathematical geology


Investigating ‘optimal’ kriging variance estimation using an analytic and a bootstrap approach

by C. Thiart*, M.Z. Ngwenya†, and L.M. Haines*

Introduction
The problem of accommodating correlation between observations made at different locations permeates design, analysis, and prediction for spatial data. In two seminal papers, Krige formulated a statistical model for such data, which comprises a deterministic component reflecting the underlying trends and an error component capturing correlations between observations at separate locations and on specific parameters. The parameters are usually treated as known, whereas in practice they invariably have to be estimated and the empirical BLUP (that is, the EBLUP) so obtained. The empirical or estimated mean square prediction error (EMSPE), or the so called ‘plug-in’ kriging variance estimator, underestimates the true kriging variance of the EBLUP, at least in general. In this paper five estimators for the kriging variance of the EBLUP are considered and compared by means of a simulation study in which a Gaussian distribution for the responses, an exponential structure for the covariance function, and three levels of spatial correlation – weak, moderate, and strong – are adopted. The Prasad-Rao estimator obtained using restricted or residual maximum likelihood (REML) is recommended for moderate and strong spatial correlation and the Kacker-Harville estimator for weak correlation in the random fields.

Keywords
kriging, covariance parameters, mean square prediction error.

Synopsis
Kriging is an interpolation technique for predicting unobserved responses at target locations from observed responses at specified locations. Kriging predictors are best linear unbiased predictors (BLUPs) and the precision of the BLUP is assessed by the mean square prediction error (MSPE), commonly known as the kriging variance. Both the BLUP and the MSPE depend on the covariance function describing the spatial correlation between locations and on specific parameters. The parameters are usually treated as known, whereas in practice they invariably have to be estimated and the empirical BLUP (that is, the EBLUP) so obtained. The empirical or estimated mean square prediction error (EMSPE), or the so called ‘plug-in’ kriging variance estimator, underestimates the true kriging variance of the EBLUP, at least in general. In this paper five estimators for the kriging variance of the EBLUP are considered and compared by means of a simulation study in which a Gaussian distribution for the responses, an exponential structure for the covariance function, and three levels of spatial correlation – weak, moderate, and strong – are adopted. The Prasad-Rao estimator obtained using restricted or residual maximum likelihood (REML) is recommended for moderate and strong spatial correlation and the Kacker-Harville estimator for weak correlation in the random fields.

Keywords
kriging, covariance parameters, mean square prediction error.

Introduction
The problem of accommodating correlation between observations made at different locations permeates design, analysis, and prediction for spatial data. In two seminal papers, Krige formulated a statistical model for such data, which comprises a deterministic component reflecting the underlying trends and an error component capturing correlations between observations taken at separate locations (Krige, 1951), and invoked his model to obtain predictions at unobserved locations (Krige, 1962). Krige’s work provided the basis for best linear unbiased prediction within the context of spatial data and was formalized more rigidly and mathematically by Matheron (1963). An excellent overview of the historical development of the methodology, broadly referred to as kriging, is provided by Cressie (1990).

The kriging approach has been adapted seamlessly to accommodate more modern statistical practice and is used extensively in a vast range of areas. A particular problem, that of using best linear unbiased prediction to predict a response at an unobserved location when the parameters describing the correlation structure between locations are unknown, however, is not fully resolved. To be specific, an obvious solution to this problem is to ‘plug’ an available estimate of the parameters into the expression for the predictor with parameters known. Unfortunately, the distribution of the resultant predictor, termed the empirical best linear unbiased predictor or EBLUP, is intractable, and more particularly the mean square prediction error associated with the EBLUP cannot be expressed analytically.

In the present study, five approaches to approximating the mean square prediction error of the EBLUP, the ‘plug-in’, those of Kacker and Harville (1984) and Prasad and Rao (1990), and two taken from den Hertog et al. (2006) and based on bootstrapping, are introduced and compared using a simulation study that accommodates weak, moderate, and strong spatial correlation. The paper is organized as follows. Basic notions and formulae underpinning the methodology are presented first. The simulation study is then described, and the attendant results summarized. Finally, some broad conclusions and pointers for further research are given.

Preliminaries

Kriging basics
Suppose that observations are made on an attribute $Z$ at the $n$ spatial locations $x_i, i = 1, \ldots, n$, in a designated geographic region $D$. Let $Z(x)$ denote the observed response at a
Investigating ‘optimal’ kriging variance estimation using an analytic and a bootstrap approach

generic location \( x_0 \) where \( x_0 \in D \subseteq R^2 \) and let \( Z(x) \) where \( x = (x_1, \ldots, x_n) \) denote the \( n \times 1 \) vector of responses at the \( n \) locations. Suppose further that interest centres on predicting the response \( Z(x_0) \) at a location \( x_0 \) in \( D \). Then it is usual to obtain such a predicted value by introducing a weighted sum of the observations of the form

\[
\hat{Z}(x_0) = \sum_{i=1}^{N(x_0)} \lambda_i \cdot Z(x_i) = \lambda^T Z(x), \tag{1}
\]

where \( \lambda_i \) is the weight associated with the \( i \)th observation \( Z(x_i) \) with \( \sum_{i=1}^{N(x_0)} \lambda_i = 1 \), \( \lambda \) is the \( n \times 1 \) vector \((\lambda_1, \ldots, \lambda_N(x_0))\) and \( N(x_0) \) is the number of points in a search neighbourhood around the prediction location \( x_0 \).

Now, assuming second-order stationarity, the kriging model can be formulated as

\[
Z(x) = F(x)\beta + e(x) \tag{2}
\]

where \( F(x) \) is an \( n \times (k+1) \) matrix with rows which usually depend on the spatial coordinates alone and/or other explanatory variables, \( \beta \) is a vector of \( k+1 \) unknown parameters, and \( e(x) \) is an \( n+1 \) vector of error terms with mean \( 0 \) and variance-covariance matrix \( \Sigma(\theta) \). The covariance between any two points is taken here to be a function of the Euclidean distance between the two points and the vector of parameters \( \theta = (\theta_0, \theta_1, \theta_2) \), corresponding, in order, to the nugget, the partial sill, and the range. Other functions for the covariance between points, such as the exponential and the Matérn, which depend on more or fewer parameters, can also be invoked (Diggle and Ribeiro, 2007). The covariance between any two points, \( x_i \) and \( x_j \) is denoted as \( Cov[Z(x_i), Z(x_j)] = C(X_i - X_j; \theta) \) and for \( i, j = 1, \ldots, n \) are the elements of the variance-covariance matrix.

The best linear unbiased predictor (BLUP) at location \( x_0 \) under the kriging model in Equation [2] is obtained by minimizing the mean square prediction error, \( E[\|Z(x_0; \theta) - Z(x_0)\|^2] \), with respect to the weights \( \lambda \) under the condition of unbiasedness, that is \( E[Z(x_0; \theta)] = Z(x_0) \). Thus, assuming the parameter \( \theta \) is known, the kriging weight vector \( \lambda \) is given by

\[
\lambda = \Sigma(\theta)^{-1} c(\theta) - F^T \Sigma(\theta)^{-1} c(\theta) - F \Delta^{-1}
\]

where \( c(\theta) \) is the \( n \times 1 \) vector \((C(x_0-x_1; \theta), \ldots, C(x_0-x_n; \theta)) \), \( \Delta = F^T \Sigma(\theta)^{-1} F \) and \( f_0 = F^T \lambda \). It then follows that the BLUP at \( x_0 \) is given by

\[
\hat{Z}(x_0; \theta) = \hat{\beta} + c(\theta)^T \Sigma(\theta)^{-1} (Z(x) - F \hat{\beta}),
\]

and its mean square prediction error, denoted MSPE and termed the kriging variance, by

\[
Var[\hat{Z}(x_0; \theta) - Z(x_0)] = C(0; \theta) - c(\theta)^T \Sigma(\theta)^{-1} c(\theta) + [f_0 - F^T \Sigma(\theta)^{-1} c(\theta)]^2. \tag{3}
\]

where \( \hat{\beta} \) is the generalized least squares (GLS) estimator of \( \beta \), that is

\[
\hat{\beta} = (F^T \Sigma(\theta)^{-1} F)^{-1} F^T \Sigma(\theta)^{-1} Z(x).
\]

In practice, however, the parameters \( \theta = (\theta_0, \theta_1, \theta_2) \) are rarely known and must be estimated. Early studies adopted least squares estimators of \( \theta \) based on fitting appropriate nonlinear models to the semivariogram data. However, a more modern approach involves maximizing the log-likelihood or the restricted log-likelihood with respect to \( \beta \) and \( \theta \) to give the maximum likelihood estimator (MLE), \( \hat{\theta}_{MLE} \), or the restricted maximum likelihood (REML) estimator, \( \hat{\theta}_{REML} \), of \( \theta \) respectively. The kriging predictor at the location \( x_0 \) is then obtained by ‘plugging’ an estimate of \( \theta \), denoted generically \( \hat{\theta} \), into the expression for the BLUP to give the ‘empirical’ BLUP, termed the EBLUP, as

\[
\hat{Z}_{EB}(x_0; \hat{\theta}) = \hat{\beta} + c(\theta)^T \Sigma(\theta)^{-1} (Z(x) - F \hat{\beta}).
\]

In the same spirit, the kriging variance or MSPE of the predictor at \( x_0 \) can be estimated by plugging \( \hat{\theta} \) into the expression for the MSPE with \( \theta \) known, that is into Equation [3], to give the ‘empirical’ MSPE, denoted EMSPE and more commonly referred to as the plug-in kriging variance estimator.

**Kriging variance estimators**

It is well known that the plug-in estimator of the kriging variance underpredicts the MSPE of the EBLUP, at least in general (Zimmerman and Cressie, 1992, den Hertog et al. 2006). More specifically, the MSPE with \( \theta \) and \( \Sigma(\theta) \) known is given in Equation [3] and can be expressed succinctly as

\[
m_1(\theta) = E[\|\hat{Z}(x_0; \theta) - Z(x_0)\|^2].
\]

Thus, for \( \theta \) unknown, the EMSPE, that is the plug-in kriging variance, is given by \( m_1(\hat{\theta}) \). Strictly, however, the mean square prediction error associated with the EBLUP is given by

\[
m_2(\theta) = E[\|\hat{Z}(x_0; \theta) - Z(x_0)\|^2] = m_1(\theta) + Var[\hat{Z}(x_0; \theta) - \hat{Z}(x_0; \theta)]
\]

where the trailing term in the second equality is algebraically intractable and therefore \( m_2(\hat{\theta}) \) cannot be evaluated. A number of approaches, both algebraic and computational, to the construction of estimators of the EBLUP kriging variance, which are approximate and which to some extent redress the negative bias in the EMSPE, have been reported. In this study the properties of four such estimators, namely the Kacker-Harville estimator, the Prasad-Rao estimator, and two bootstrap estimators, are investigated. These estimators are summarized as follows.

Kacker and Harville (1984) used a Taylor series expansion of the trailing term in \( m_2(\theta) \) to derive an estimator of the EBLUP kriging variance, albeit approximate, as

\[
m_{2(KH)}(\theta) \approx m_1(\theta) + tr[A(\theta)B(\theta)], \tag{4}
\]

where

\[
A(\theta) = Var[\frac{\partial \hat{\theta}(x_0; \theta)}{\partial \theta}] \quad \text{and} \quad B(\theta) = E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T].
\]

In practice \( \theta \) can be plugged into the terms \( m_1(\theta) \) and \( A(\theta) \) of Equation [4] and the term \( B(\theta) \), which is the mean square error of \( \hat{\theta} \) and is unknown, can be approximated by the inverse of the observed Fisher information matrix for \( \theta \), that is the inverse of the Hessian \( H(\hat{\theta}) \). Thus the Kacker-Harville estimator can be evaluated as

\[
m_{2(KH)}(\theta) = m_1(\theta) + tr[A(\theta)H(\hat{\theta})^{-1}].
\]
but, since this is essentially a plug-in estimator, it still remains negatively biased. To redress this bias, Prasad and Rao (1990) invoked some intricate theory to develop a further correction to $m_2(\theta)$ and, based on their results, introduced an estimator which approximates the MSE of the EBLUP as

$$m_{2(PR)}(\theta) = m_1(\theta) + 2tr[A(\theta)H(\theta)^{-1}].$$

Both the Kacker-Harville and the Prasad-Rao estimators reduce the negative bias of the plug-in kriging variance, but the improvement depends on sample size, spatial autocorrelation, Gaussian assumptions, an unbiased estimate of $\theta$, and on the assumption that the covariance function is linear in the elements of $\theta$ (Prasad and Rao, 1990; Harville and Jeske, 1992). However, the latter assumption is clearly not valid for a range of widely used covariance models, such as the Gaussian, the exponential, the spherical, and the Matérn. This observation prompted Wang and Wall (2003) to suggest bootstrapping as an alternative method for estimating the MSE of the EBLUP.

Two bootstrap estimators of $m_2(\theta)$ are investigated in the present study. Specifically, suppose that observations $Z(x)$ at the $n$ locations, $x_1, \ldots, x_n$, are taken from a Gaussian random field and that predictors at $m$ new or test locations, $x_{01}, \ldots, x_{0m}$, are required. The MLE or REML estimators of $\theta$ and $\beta$, denoted $\hat{\theta}$ and $\hat{\beta}$ respectively, can immediately be obtained from the data and, following den Hertog et al. (2006), bootstrap procedures based on the estimated distribution $Z(x) \sim G(F(\hat{\theta}, \Sigma(\hat{\theta}))$ can then be implemented.

The bootstrap estimator of the MSE of the EBLUP based on the assumption of a joint or unconditional Gaussian distribution for the responses at the observed and the test locations, termed the unconditional bootstrap estimator (UBE) and denoted $m_{2(UBE)}(\hat{\theta})$, is introduced here and can be obtained by implementing the following algorithm.

**Unconditional bootstrap estimation**

1. Sample from $G(F(\hat{\theta}, \Sigma(\hat{\theta}))$ at the observed locations $x_1, \ldots, x_n$ and at the test locations $x_{01}, \ldots, x_{0m}$ simultaneously.
2. Use the bootstrap sample at the $n$ observed locations to fit a kriging model and use this fitted model to compute predictions at the $m$ test locations $x_{01}, \ldots, x_{0m}$.
3. Calculate the squared prediction error at each test point.
4. Repeat steps 1 to 3 $B$ times. Then compute the unconditional bootstrap estimate of the MSE, that is, $m_{2(UBE)}(\hat{\theta})$, at each test point. Stop.

An alternative bootstrap estimator to the UBE, termed the conditional bootstrap estimator (CBE) and denoted $m_{2(CBE)}(\hat{\theta})$, is based on the fact that the conditional distribution of the responses $Z(x_0)$ at the test locations, $x_{01}, \ldots, x_{0m}$, given observations $Z(x)$ at the $n$ locations, $x_1, \ldots, x_n$, under a Gaussian assumption is well known. Specifically,

$$Z(x_0) \sim G\left(F(\hat{\theta}, \Sigma(\hat{\theta}))\right)$$

where the vectors $\mu = F(\hat{\theta})$, $\Sigma = \Sigma(\hat{\theta})$, $\Sigma_{01} = \Sigma(\hat{\theta})$, $\Sigma_{00} = \Sigma(\hat{\theta})$, $\Sigma_{11} = \Sigma(\hat{\theta})$, $\Sigma_{10} = \Sigma(\hat{\theta})$, with $\hat{\theta}$ evaluated at $\hat{\theta}$, are the vectors of estimated means at the $n$ observed and the $m$ test locations respectively and the variance matrix for $x = (x_1, \ldots, x_n)$ and $x_{01}, \ldots, x_{0m}$, partitioned conformably with these vectors, is given by

$$
\Sigma(\theta; x, x_0) = \begin{bmatrix}
\Sigma_{11}(\theta) & \Sigma_{10}(\theta) \\
\Sigma_{01}(\theta) & \Sigma_{00}(\theta)
\end{bmatrix}.
$$

The conditional bootstrap estimator is also introduced in the present study and can be obtained by implementing the following algorithm.

**Conditional bootstrap estimation**

1. Use the kriging model, $G(F(\hat{\theta}, \Sigma(\hat{\theta}))$ fitted to the observations $Z(x)$ to make predictions at the $m$ test locations $x_{01}, \ldots, x_{0m}$.
2. Take a bootstrap sample at the test locations $x_{01}, \ldots, x_{0m}$ conditional on the observations $Z(x)$, that is, sample from the conditional distribution $Z(x_0) | Z(x)$.
3. Calculate the squared differences of the predictions and the bootstrap realizations obtained from steps 1 and 2 respectively.
4. Repeat steps 2 and 3 $B$ times. Then compute the conditional bootstrap estimate of the MSE, that is, $m_{2(CBE)}(\hat{\theta})$, at each test point. Stop.

**Simulation study**

In order to investigate the performance of the five estimators for the MSE of the EBLUP introduced in this paper, namely the plug-in, the Kacker-Harville, the Prasad-Rao, the unconditional bootstrap, and the conditional bootstrap estimators, a simulation study incorporating conditions of spatial correlation specified by three Gaussian random fields was undertaken (Ngwenya, 2009).

Settings for the simulations are summarized as follows. The observed locations were taken as a regular grid of 112 points in a square domain $D = [0,15] \times [10,15]$ and are specified by the set

$$S = \{(u, v) : u = 2k, v = 2l + 1 \text{ and } u = 2l + 1, v = 2k \text{ for } k = 1, \ldots, 7, l = 0,1, \ldots, 7\}.$$

In addition, a set of five points, $(0,0), (3,1), (13,0), (8,8), (3,13), (15,15)$ within the domain $D$ was chosen to specify the prediction locations (Figure 1). The five prediction points were located so as to enable the effective study of location and data configuration on the various $m_2(\theta)$.
Investigating ‘optimal’ kriging variance estimation using an analytic and a bootstrap approach

estimators. We have chosen a point in the middle of the grid, one inside the grid but close to the edge of the domain \( D \), two on the edge of the grid, and one outside the grid but still in \( D \).

An ordinary kriging model with \( F(x) = 1 \) and a Gaussian distribution for the error terms, namely

\[
Z(x) = \mathbf{1} \beta + \mathbf{e}(x) \quad \text{with} \quad \mathbf{e}(x) \sim G(0, \Sigma(\theta)),
\]

was adopted. More specifically, the regression parameter \( \beta \) was taken to be zero, thus defining a simple kriging model, and an exponential covariance structure

\[
C(x_i - x_j; \theta) = \theta_1 + \theta_3 (1 - e^{-3h/\theta_3})
\]

where \( h \) denotes the Euclidean distance between the locations \( x_i \) and \( x_j \), \( \theta_1 \) the nugget, \( \theta_2 \) the partial sill, and \( \theta_3 \) the range, was adopted. Fixed values of \( \theta_1 = 0.25 \) and \( \theta_2 = 1 \) were taken and three values for the range, namely \( \theta_3 = 1.5, 2.5, \) and \( 4.2 \), corresponding to weak, moderate, and strong spatial correlations respectively, were introduced. Thus three datasets or random fields, denoted \( Z_1, Z_2 \), and \( Z_3 \), with increasing spatial correlation were created.

It is sensible to compare each of the five estimators of the MSPE of the EBLUP introduced in this study with the true value, \( m_2(\theta) \). As noted earlier, no analytical expression for \( m_2(\theta) \) is available. However, Monte Carlo simulation to approximate the true value \( m_2(\theta) \) to a chosen degree of accuracy can readily be performed. The procedure is summarized as follows.

Monte Carlo procedure to approximate \( m_2(\theta) \)

1. Set \( r = 1 \)
2. Generate a spatial process over the grid of observation and prediction locations using the true values of the parameters \( \beta \) and \( \theta \)
3. For the \( r \)th realization fit a kriging model by using the data at the observation locations to obtain estimates \( \hat{\beta}_r \) of \( \beta \) and \( \hat{\theta}_r \) of \( \theta \)
4. Calculate \( \left[ Z(x_0; \hat{\beta}_r) - Z(x_0) \right]^2 \), the squared difference between the predicted value \( Z(x_0; \hat{\beta}_r) \) and the observed value \( Z(x_0) \) at each of the prediction locations, \( x_0 \)
5. Repeat steps 2 to 4, updating \( r \) to \( r + 1 \), \( R \) times and then compute the average squared differences between the predicted and simulated values as

\[
\frac{1}{R} \sum_{r=1}^{R} \left[ (\hat{Z}(x_0; \hat{\beta}_r) - Z_r(x_0)) \right] \]

for each of the prediction locations, \( x_0 \). Stop.

A pilot study to determine the number of simulations \( R \) that gives \( m_2(\theta) \) to an accuracy of at least \( \pm 0.0001 \) for each of the three random fields, \( Z_1, Z_2, \) and \( Z_3 \), was performed. A maximum value of \( R = 250 \) 000 for the full Monte Carlo simulations was found and used in all cases.

The plug-in, the Kacker-Harville, and the Prasad-Rao estimators of the MSPE of the EBLUP at the \( m = 5 \) prediction locations for each of the random fields \( Z_1, Z_2, \) and \( Z_3 \), were based on 1000 realizations at the observation locations. Specifically, for each realization the MLE and REML estimates of \( \hat{\theta} \) denoted generically as \( \hat{\theta} \), were found and plugged into the appropriate expressions. The resultant estimates were then averaged over the 1000 realizations to give the estimators \( m_2(\hat{\theta}), m_{2\text{KH}}(\hat{\theta}) \) and \( m_{2\text{PR}}(\hat{\theta}) \) to a degree of accuracy determined by the number of simulations. The bootstrap estimators were based on \( B = 1000 \) bootstrap iterations for each of the 1000 realizations of the observed and, in the case of the unconditional bootstrap estimator, the observed and the prediction locations. The unconditional bootstrap method is computationally expensive since \( 1000 \times 1000 \times 2 = 200 \) 000 kriging models had to be fitted to the simulated data. This consideration limited the number of realizations.

Results

The values the MSPE of the BLUP, \( m_1(\theta) \), the MSPE of the EBLUP, \( m_2(\theta) \), and the EMSPE \( m_{1\text{BE}}(\hat{\theta}) \) together with the values of the four alternative kriging variance estimators, namely \( m_{2\text{KH}}(\hat{\theta}), m_{2\text{PR}}(\hat{\theta}), m_{2\text{UBE}}(\hat{\theta}) \), and \( m_{2\text{CBE}}(\hat{\theta}) \), under both MLE and REML estimation of the covariance parameters, were computed at each prediction location across the three random fields, \( Z_1, Z_2, \) and \( Z_3 \). The R programming language (R Development Core Team, 2012) was used to calculate these values and the results are tabulated in Ngwenya (2009). For conciseness, only the relative biases in the estimators of the MSPE of the EBLUP are presented here, with relative bias defined to be

\[
E \left( m_{2, r}(\hat{\theta}_r) \right) - m_2(\theta) \over m_2(\theta)
\]

where \( m_{2, r}(\hat{\theta}_r) \), is a generic estimator of \( m_2(\theta) \). Specifically, the relative biases under MLE and REML estimation of the covariance parameters \( \theta \) are summarized in Table I(a) and I(b) together with \( m_2(\theta) \) values obtained by the Monte Carlo procedure. In addition, the absolute values of the percentage relative biases are represented diagrammatically in Figures 2(a) and 2(b) respectively.

Certain features of the relative biases in the MSPEs of the EBLUP are worth noting. In particular, the performance of the conditional bootstrap estimator \( m_{2\text{CBE}}(\hat{\theta}) \) is almost indistinguishable from that of the plug-in estimator \( m_1(\hat{\theta}) \) under both MLE and REML estimation of \( \theta \) and at each point across the three random fields. These estimators seriously underestimate \( m_2(\theta) \), with the degree of underestimation increasing with an increase in the spatial correlation exhibited by the random fields.

The Kacker-Harville and unconditional bootstrap estimators, \( m_{2\text{KH}}(\hat{\theta}) \) and \( m_{2\text{UBE}}(\hat{\theta}) \) respectively, also exhibit very similar behaviour, but only when the spatial correlation is moderate to strong. It is interesting to note here that Zimmerman (2006), in simulation studies on optimal network designs, found that design criteria based on \( m_{2\text{KH}}(\hat{\theta}) \) and \( m_{2\text{UBE}}(\hat{\theta}) \) lead to similar designs. However, when the spatial correlation is weak, \( m_{2\text{KH}}(\hat{\theta}_{\text{MLE}}) \) has a propensity to lead to overestimates of \( m_2(\theta) \), while \( m_{2\text{UBE}}(\hat{\theta}_{\text{MLE}}) \) exhibits a high degree of accuracy, with an absolute relative bias as small as 0.3% at the prediction location \( (15, 15) \). In contrast, the estimators \( m_{2\text{KH}}(\hat{\theta}_{\text{REML}}) \) and \( m_{2\text{UBE}}(\hat{\theta}_{\text{REML}}) \) are more evenly matched when the spatial correlation is weak.

Values of \( m_2(\theta) \), at the points \( (5, 1), (15, 15), \) and \( (13, 0) \), which lie at an edge, a corner, and outside of the area circum-
scribed by the grid of observed locations, are greater than those for the prediction points (8,8) and (3,13), which are surrounded by observed locations. However it is also clear from the results summarized in Table I that this feature of the values of \( m_2(\theta) \) for the prediction points is not mirrored in the corresponding values of the absolute relative biases. Indeed, there seems to be no obvious relationship between these measures of bias and the location of the prediction points and the spatial configuration of the grid, in accord with the findings of Zimmerman and Cressie (1992).

The Prasad-Rao estimator \( m_{2PR}(\hat{\theta}) \) does not seem to exhibit behaviour that is very close to any of the other four estimators of the MSPE of the EBLUP investigated in this study. Specifically, when the spatial correlation is weak, the estimator exhibits a large positive bias of up to 11.7% with \( \hat{\theta}_{MLE} \) and 2.3% with \( \hat{\theta}_{REML} \). This observation can be explained by the fact that, for weak spatial correlation, the large sample covariance matrix overestimates the variability in the parameter estimates \( \hat{\theta} \), thereby inflating the trailing term in the expression for \( m_{2PR}(\hat{\theta}) \) (Abt, 1999). The same explanation can be given for the fact that \( m_{2KH}(\hat{\theta}) \) has a propensity to be positively biased when the spatial correlation is weak. In contrast, for moderate and strong spatial correlation, the Prasad-Rao estimator \( m_{2PR}(\hat{\theta}) \) exhibits the smallest absolute relative bias at all prediction locations when compared to the other four estimators of \( m_2(\theta) \) in accord with the findings of Zimmerman and Cressie (1992). In fact, the Prasad-Rao estimator performs well with both \( \hat{\theta}_{MLE} \) and \( \hat{\theta}_{REML} \) in these situations, with biases as small as 0.5% and 0.1% respectively, and is clearly to be preferred.

Conclusions

Some broad conclusions based on the simulation study are now drawn, but should be taken strictly within the context of that study. Thus the performance of all of the five estimators of the mean square prediction error examined here depend somewhat sensitively on the method of estimation, that is MLE or REML, and in most cases REML should be the desired choice. The plug-in estimator consistently underestimates the MSPE of the EBLUP, in accord with more general findings in the literature, and is clearly unsatisfactory. The Prasad-Rao estimator performs optimally for the random fields with moderate and strong spatial correlation but would seem to be erratic for the field with weak correlation. In the latter case the Kacker-Harville estimator is to be preferred overall. It is interesting to note that the absolute relative biases of the prediction points do not appear to be related in any meaningful way to the location of those points in the domain of the simulation experiment.

Empirical studies on the nature and performance of approximate estimators for the MSPE of the EBLUP based on simulation or real data can be construed as being endless. Clearly, therefore, there is scope for further theoretical research into constructing reliable and robust estimators of this MSPE. One such approach could involve third-, fourth- and higher-order approximations of appropriate Taylor series expansions. However, such derivations are intricate and nontrivial and indeed are fraught with subtle problems.

Acknowledgments

The authors would like to thank the reviewers for their...
helpful comments. They would also like to thank the University of Cape Town and the National Research Foundation (NRF) of South Africa, grants (UID) 85518 and (UID) 85456, for financial support. Any opinion, finding, and conclusion or recommendation expressed in this material is that of the authors and the NRF does not accept liability in this regard.

References


---

### Table I

Approximate relative bias of the five kriging variance estimators at each point under (a) MLE and (b) REML of the covariance parameters across the Gaussian random fields $Z_1$, $Z_2$, and $Z_3$ used in the simulation study. For each field the estimator with the smallest relative bias at each point is indicated in bold. Also shown are the values of $m_2(\theta)$ obtained by the Monte Carlo procedure.

<table>
<thead>
<tr>
<th>(a)</th>
<th>Random field</th>
<th>$x_0$</th>
<th>$m_2(\theta)$</th>
<th>$\hat{\sigma}^{MB}_{MLE}$</th>
<th>$\hat{\sigma}^{MB}_{REML}$</th>
<th>$\hat{\sigma}^{MB}_{ubs}$</th>
<th>$\hat{\sigma}^{MB}_{obs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$ weak</td>
<td>(3,1)</td>
<td>0.862</td>
<td>-0.030</td>
<td>0.020</td>
<td>0.070</td>
<td>-0.010</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td>(13,0)</td>
<td>1.067</td>
<td>-0.031</td>
<td>-0.017</td>
<td>0.004</td>
<td>-0.016</td>
<td>-0.030</td>
</tr>
<tr>
<td></td>
<td>(8,8)</td>
<td>0.806</td>
<td>-0.026</td>
<td>0.038</td>
<td>0.103</td>
<td>-0.006</td>
<td>-0.025</td>
</tr>
<tr>
<td></td>
<td>(3,13)</td>
<td>0.795</td>
<td>-0.011</td>
<td>0.053</td>
<td>0.117</td>
<td>0.008</td>
<td>-0.011</td>
</tr>
<tr>
<td></td>
<td>(15,15)</td>
<td>0.941</td>
<td>-0.023</td>
<td>0.014</td>
<td>0.050</td>
<td>0.003</td>
<td>-0.022</td>
</tr>
<tr>
<td>$Z_2$ moderate</td>
<td>(3,1)</td>
<td>0.676</td>
<td>-0.049</td>
<td>-0.031</td>
<td>-0.012</td>
<td>-0.036</td>
<td>-0.049</td>
</tr>
<tr>
<td></td>
<td>(13,0)</td>
<td>0.880</td>
<td>-0.040</td>
<td>-0.025</td>
<td>-0.011</td>
<td>-0.026</td>
<td>-0.041</td>
</tr>
<tr>
<td></td>
<td>(8,8)</td>
<td>0.625</td>
<td>-0.048</td>
<td>-0.029</td>
<td>-0.008</td>
<td>-0.032</td>
<td>-0.046</td>
</tr>
<tr>
<td></td>
<td>(3,13)</td>
<td>0.617</td>
<td>-0.036</td>
<td>-0.016</td>
<td>0.005</td>
<td>-0.021</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(15,15)</td>
<td>0.765</td>
<td>-0.044</td>
<td>-0.026</td>
<td>0.008</td>
<td>-0.029</td>
<td>-0.042</td>
</tr>
<tr>
<td>$Z_3$ strong</td>
<td>(3,1)</td>
<td>0.544</td>
<td>-0.064</td>
<td>-0.048</td>
<td>-0.029</td>
<td>-0.051</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td>(13,0)</td>
<td>0.708</td>
<td>-0.048</td>
<td>-0.032</td>
<td>-0.017</td>
<td>-0.032</td>
<td>-0.048</td>
</tr>
<tr>
<td></td>
<td>(8,8)</td>
<td>0.502</td>
<td>-0.064</td>
<td>-0.046</td>
<td>-0.028</td>
<td>-0.048</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td>(3,13)</td>
<td>0.495</td>
<td>-0.051</td>
<td>-0.032</td>
<td>-0.014</td>
<td>-0.036</td>
<td>-0.051</td>
</tr>
<tr>
<td></td>
<td>(15,15)</td>
<td>0.623</td>
<td>-0.059</td>
<td>-0.042</td>
<td>-0.024</td>
<td>-0.043</td>
<td>-0.058</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b)</th>
<th>Random field</th>
<th>$x_0$</th>
<th>$m_2(\theta)$</th>
<th>$\hat{\sigma}^{MB}_{REML}$</th>
<th>$\hat{\sigma}^{MB}_{ubs}$</th>
<th>$\hat{\sigma}^{MB}_{obs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$ weak</td>
<td>(3,1)</td>
<td>0.864</td>
<td>-0.039</td>
<td>-0.017</td>
<td>0.003</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(13,0)</td>
<td>1.070</td>
<td>-0.037</td>
<td>0.023</td>
<td>-0.009</td>
<td>-0.020</td>
</tr>
<tr>
<td></td>
<td>(8,8)</td>
<td>0.809</td>
<td>-0.040</td>
<td>0.015</td>
<td>0.010</td>
<td>-0.019</td>
</tr>
<tr>
<td></td>
<td>(3,13)</td>
<td>0.798</td>
<td>-0.026</td>
<td>0.001</td>
<td>0.023</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(15,15)</td>
<td>0.945</td>
<td>-0.029</td>
<td>-0.010</td>
<td>0.011</td>
<td>-0.008</td>
</tr>
<tr>
<td>$Z_2$ moderate</td>
<td>(3,1)</td>
<td>0.678</td>
<td>-0.044</td>
<td>-0.028</td>
<td>-0.012</td>
<td>-0.028</td>
</tr>
<tr>
<td></td>
<td>(13,0)</td>
<td>0.883</td>
<td>-0.035</td>
<td>-0.031</td>
<td>-0.006</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td>(8,8)</td>
<td>0.627</td>
<td>-0.045</td>
<td>-0.027</td>
<td>-0.010</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(3,13)</td>
<td>0.618</td>
<td>-0.031</td>
<td>-0.013</td>
<td>0.005</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(15,15)</td>
<td>0.767</td>
<td>-0.034</td>
<td>-0.016</td>
<td>0.001</td>
<td>-0.016</td>
</tr>
<tr>
<td>$Z_3$ strong</td>
<td>(3,1)</td>
<td>0.545</td>
<td>-0.051</td>
<td>-0.035</td>
<td>-0.020</td>
<td>-0.037</td>
</tr>
<tr>
<td></td>
<td>(13,0)</td>
<td>0.708</td>
<td>-0.038</td>
<td>-0.023</td>
<td>-0.008</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(8,8)</td>
<td>0.503</td>
<td>-0.052</td>
<td>-0.036</td>
<td>-0.020</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(3,13)</td>
<td>0.496</td>
<td>-0.036</td>
<td>-0.022</td>
<td>-0.006</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(15,15)</td>
<td>0.624</td>
<td>-0.042</td>
<td>-0.024</td>
<td>-0.006</td>
<td>-0.022</td>
</tr>
</tbody>
</table>
Limitations in accepting localized conditioning recoverable resource estimates for medium-term, long-term, and feasibility-stage mining projects, particularly for sections of an ore deposit

by W. Assibey-Bonsu* and C. Muller‡

Synopsis
A localized nonlinear recoverable resource estimate technique has been applied using typical feasibility or new mining drilling data configurations drawn from a massive database from a mined-out area on a hydrothermal gold deposit. The results were then compared with the corresponding 8 m x 5 m grid grade-control data in order to determine the efficiency of the approach and the validity of the recoverable resource estimates for mine planning and financial forecasts.

Keywords
localized conditioning, post-processing, nonlinear recoverable estimates, direct and indirect recoverable estimates, kriging efficiency, slope of regression, relative profit, inefficient kriged estimate, conditional bias.

Introduction
The problem associated with the use of limited search routines and inefficient kriged estimates when estimating recoverable resources for new mining projects have been highlighted in several papers (Krige 1994, 1996; Krige and Assibey-Bonsu, 1999a, 1999b). The critical importance of using efficient and non-biased direct and indirect recoverable estimates in mine plans, and the corresponding financial forecasts for mining projects, has also been demonstrated in these papers.

The direct and indirect techniques are supposed to provide local adjusted grade-tonnage estimates for mine planning, but cannot identify the final individual blocks to be selected for mining above a specified cut-off. Techniques for estimating recoverable resources at the exploration or early stage of a mining project, or for extensions of existing mines, all suffer from the main problem of not being able to identify those specific blocks that are to be selected finally for mining above any cut-off. Thus any mine planning exercise based on these long-term recoverable estimates will, unavoidably, involve misclassification of blocks.

However, direct and indirect recoverable estimates based on inadequate data, as well as conditionally biased and inefficient estimates, can be badly misleading for the prediction of recoverable resource estimates for global life-of-mine estimates, as well as for sub-sections of the orebody. These will have obvious negative economic implications on the orebody to be mined in short-, medium-, and long-term periods, and hence also for the pattern of production and financial forecasts over the life of the mine, as well as for any grade control planning.

This paper analyses the effect of using inefficient localized conditioning recoverable resource estimates for project mine planning and financial forecasts.

Geology and the database used for the case study
The net effect of using inefficient direct recoverable estimates was analysed using historical production data from sections of a 3D hydrothermal gold deposit, which has been mined for several years from an open pit with grade control data on an 8 m x 5 m grid. The database for the case study also included resource drilling data. From this historic data, a set of resource drilling data, equivalent to exploration drill-hole values on an approximate grid of 40 m x 20 m, was selected to represent the data at the early stage of a new mining project.

Only resource drill-holes (RD) were selected for the initial resource modelling in this case study, and their orientations were predominantly steep as the drill-holes were oriented to intersect the main mineralization at better angles as far as practically possible (this has been continuously improved over time). The follow-up 8 m x 5 m grade control (GC) drill-holes, which were used as ‘actuas’ for the comparison with the direct recoverable estimates, were vertical.

---

* Group Geostatistician and Evaluator, Gold Fields Ltd, Perth, Australia.
† Minesoft Group, Johannesburg, South Africa.
© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
Limitations in accepting localized conditioning recoverable resource estimates

The set of 40 m x 20 m data was used to provide a geological wireframe model as well as ordinary kriged (OK) conditioning panel estimates. The panel size was 40 m x 20 m x 6 m.

In order to provide the recoverable resource estimates for the orebody, the localized conditioning (LUC) technique, as proposed by Abzalov (2006), was utilized. Abzalov proposed using the grade-tonnage functions from the large panel indirect estimates (derived from uniform conditioning (UC)) and then decomposing the panel-specific grade-tonnage data into a suite of individual SMU-sized units within respective panels. Abzalov suggested that the individual parcel grades of the SMUs derived from the decomposed UC approach be assigned, in this case, to the SMU-size blocks within the respective 40 m x 20 m x 6 m panels. In this case study the SMU used is 5 m x 5 m x 3 m.

The estimated localized recoverable conditioning SMU estimates (LUC) as derived from the 40 m x 20 m data were compared with the corresponding ‘actual’ SMU block values based on the available comprehensive 8 m x 5 m data grid from production grade control, in order to determine the efficiency of the localized recoverable estimates approach and the validity of the estimates for mine planning and financial forecasts.

The mine exploits oxidized and fresh ore in Tarkwaian sediments, which constitute a significant portion of the stratigraphy of the Ashanti Belt in southwest Ghana, West Africa. The hydrothermal mineralization occurs in two fault blocks of the Banket Sandstone. In each block, gold mineralization is concentrated in three to four structural zones with different orientations. The gold mineralization is associated with pyrite and pyrrhotite alteration and is usually located adjacent to en-echelon quartz veins. These alteration zones are often linked, and may result in significant volumes characterized by intense veining and gold mineralization. The model of the gold deposit is illustrated in Figure 1. A robust geological model/interpretation of the deposit was therefore available for the case study. In addition, there was further geological domaining aimed at providing practical homogeneous domains for the geostatistical assessment.

Basis for comparison of LUC estimates against actuals

The LUC recoverable estimates were compared with the corresponding ‘actual’ (GC) block values based on the available, comprehensive 8 m x 5 m grade control data grid in order to determine the efficiency of the approach (no resource drilling data was used for deriving the GC block values). Comparisons are provided for ore mined during 6 months, one year, and three years respectively in this paper. The efficiencies of the LUC estimates and the corresponding financial profiles are measured on the basis of the spreads of percentage errors defined as:

\[
\text{Percentage error} = \left(\frac{\text{Estimate}}{\text{Actual}} - 1\right) \times 100\%
\]

Actual represents in situ grade control block estimates based on 8 m x 5 m data, and Estimate is the corresponding LUC individual block estimates against Actuals were analysed. The financial profiles were estimated using the relative profit approach, which is discussed later.

Results

Table I shows the global in situ (zero cut-off) grades of the 40 m x 20 m x 6 m panels, and GC and LUC comparison for the main section of the study. The table does not reflect any in situ global grade distribution biases for the resource drilling panels, LUC, and the GC follow-up kriged outputs. The in situ panel resource and GC kriged block estimates in Table I further provide a declustered perspective of the input resource drilling and GC data grades. Table II shows typical variogram parameters observed in the study area for the Banket Sandstone and phyllite mineralization domains (similar variograms were used for the resource and the GC kriging).

Figure 2 shows a typical swath plot observed during the study. Although the figure shows slight deviation from data in one area, there are no obvious general biases.

However, in providing the ordinary kriged panel estimates, which were used for conditioning of the localized recoverable estimate, it was observed that a significant

![](image_url)

**Figure 1—Schematic cross-section showing the main geological formations and structural controls**

**Table I**

<table>
<thead>
<tr>
<th>Cut-off (g/t)</th>
<th>5x5x3m grade control OK ‘actual’ values (g/t)</th>
<th>40x20x6m OK panel estimate used for LUC (g/t)</th>
<th>5x5x3m LUC estimates (g/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.0</td>
<td>2.1</td>
<td>2.1</td>
</tr>
</tbody>
</table>
number of the panel kriged estimates had poor kriging efficiencies (KEs) and low regression slopes (RSs), due mainly to limited available data during the resource modelling (See Krige 1996 for further details on KEs). This is typical of most exploration and new mining projects. An adequate search routine was used within the context of the resource drilling data available for the panel estimates. Figure 3 shows typical KE and RS correlation in this regard. It is important to stress that although the kriged panel estimates are globally unbiased, as demonstrated above (Table I), most of the individual panel estimates show poor/negative KEs and poor RSs (Figure 3) due to limited available data for local panel estimates. As highlighted previously, this (i.e. invariably limited resource data) is typical of most exploration and new mining projects. These negative KEs and poor RSs reflect significant conditional biases. The situation observed for the KEs and RSs worsened when smaller panel block sizes were used (which is to be expected).

As stressed by Krige and Assibey-Bonsu (1999), the ‘efficient’ application of any non-linear post-processing technique requires the need for all the block estimates (large or direct SMU) to be conditionally unbiased, which means the approach used by some practitioners in limiting search routines to increase the dispersion variance artificially cannot be justified. This implies the need for the use of an adequate search routine, and where this becomes impractical, the use of simple kriging (SK) with the global or a more local mean. Deraisme et al. (2014) have demonstrated the presence of these biases in resource modelling even in a more continuous porphyry gold-copper orebody (these biases were also due to limited available data); the authors have demonstrated a practical solution to this problem by implementing SK with local means. From our perspective, adequate work is not readily found in the literature where practitioners effectively test the presence of these critical biases, including the use of KE and RS, when applying nonlinear post-processing techniques.

### Analyses of ‘actual’ GC values used for comparison with LUC resource estimates

OK estimation technique was used for the follow-up GC block

### Table II

<table>
<thead>
<tr>
<th>Domain</th>
<th>Sill</th>
<th>Angle1</th>
<th>Angle2</th>
<th>Angle3</th>
<th>Nugget %</th>
<th>Range1 X</th>
<th>Range1 Y</th>
<th>Range2 X</th>
<th>Range2 Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.6</td>
<td>-15</td>
<td>-20</td>
<td>0</td>
<td>34</td>
<td>17</td>
<td>17</td>
<td>50</td>
<td>88</td>
</tr>
<tr>
<td>2</td>
<td>7.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>44</td>
<td>19</td>
<td>19</td>
<td>36</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>17.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>40</td>
<td>14</td>
<td>14</td>
<td>43</td>
<td>83</td>
</tr>
</tbody>
</table>
Limitations in accepting localized conditioning recoverable resource estimates

Simple kriging (SK) of the GC blocks gave similar results to those obtained for OK GCs, as both techniques were based on a comprehensive 8 m x 5 m grade control data grid. The efficiency of GC estimates is shown in Figure 4, as demonstrated by the high RS and very good KE values.

**Correlation of ‘actual’ grade control values with the corresponding LUC estimates**

Figure 5 shows the correlation of the localized conditioning SMU estimates (LUC) on the x-axis and the corresponding ‘actual’ block estimates (GC) based on the comprehensive 8 m x 5 m grade control production data in a typical section of the orebody. Figure 5 demonstrates the inherent systematic biases in the LUC recoverable estimate model, which constitutes a fundamental problem. The correlation coefficient and the slope of regression should be close to unity if the model is conditionally unbiased and robust, which would also mean a high level of correlation between the LUC resource estimates and the actual grade based on the grade control. The trend line between the LUC resource estimates and the grade control values shows a slope of 0.1, reflecting significant conditional biases (Figure 5). In effect, there is almost no correlation between the individual localized LUC feasibility estimates and the ‘actual’ block values.

In addition, all the LUC estimates to the right of the intersection of the trend line and the 45º line (in red) are all over-estimated in terms of grade, while all the LUC estimates to the left of the intersection lines have been under-estimated.

Table III shows that all the LUC resource estimates with grades of 12 g/t ended up on average just above 3 g/t based on the grade control block values. Similarly, LUC block estimates of 6 g/t are on average about 2.3 g/t based on grade control data. Conversely, LUC estimates of 0.3 g/t (i.e., material classified as waste by LUC, e.g., based on a 0.6 g/t cut-off), ended up on average just below 2 g/t, indicating that they have been significantly underestimated and misclassified, since they are actually ore when compared to the grade control block values. These reflect serious conditional biases.

As the size of the unit volume is increased to that of ore mined over periods of 6 months, one year, and three years, the position shows certain improvements as shown in Table IV and Figure 6. However, the conditional biases for these macro sections’ grade and tonnage estimates are still serious and show that these estimates provide a misleading pattern of mine production and financial forecasts for short- and long-term planning.

**Table III**

<table>
<thead>
<tr>
<th>Recoverable LUC SMU estimates of blocks for grade categories</th>
<th>Corresponding average grade for actual blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recoverable LUC SMU estimates</td>
<td>Corresponding average grade for actual blocks</td>
</tr>
<tr>
<td>12.0</td>
<td>3.0</td>
</tr>
<tr>
<td>6.0</td>
<td>2.3</td>
</tr>
<tr>
<td>0.3</td>
<td>1.7</td>
</tr>
</tbody>
</table>

**Table IV**

Percentage errors of ore mined over respective time periods

<table>
<thead>
<tr>
<th>Cut-off (g/t)</th>
<th>6 Months</th>
<th>1 Year</th>
<th>3 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tons (%)</td>
<td>grade (%)</td>
<td>tons (%)</td>
</tr>
<tr>
<td>0.6</td>
<td>-15%</td>
<td>31%</td>
<td>-15%</td>
</tr>
<tr>
<td>0.7</td>
<td>-14%</td>
<td>31%</td>
<td>-16%</td>
</tr>
<tr>
<td>1.0</td>
<td>-3%</td>
<td>27%</td>
<td>-16%</td>
</tr>
</tbody>
</table>
to medium-term mining plans. The general and significant underestimation of tonnage and the corresponding overestimation of grades for the respective sections of the orebody are clear, as demonstrated in Figure 6. These show how misleading any planning based on such tonnage and grades estimates could be.

Discussion

The analysis shows that the main reason for the systematic underestimation of tons and the corresponding overestimation of grades for the individual LUC block estimates, as well for the sections of the orebody, is the significant conditional biases, which, in turn, are a result of the invariably limited resource drilling data available for the LUC estimates. Krige et al. (2008) observed similar biases for localized block estimates. Also, as noted by Journel et al. (2000): It appears that global accuracy (semivariogram reproduction [and non-smoothing]) cannot be obtained without sacrificing local accuracy. [Proper] kriging, notwithstanding its smoothing effect, remains the best local estimator.

The limited resource data issue is typical of most feasibility studies, as well as new mining projects or extensions of existing mines. Where the data-set is larger, the grade-tonnage bias problem could be less serious. However, the evidence sounds a valid warning not to rely on LUC conditioned by inefficient ordinary kriged panel estimates with significant conditional biases for feasibility mine planning and financial forecasts, unless such estimates can be shown to be effectively conditionally unbiased. As demonstrated elsewhere (Assibey-Bonsu and Searra, 2014; Deriasme et al., 2014), the corresponding simple kriging with local means provides better conditioning panel estimates for recoverable estimates.

Relative profits

Although the estimated grades to be mined above cut-off are of critical importance, the corresponding ore tons and metal content are also important. However, the final overall measure for a project's feasibility study, which incorporates all these variables, is the profile of the estimated profits to be earned relative to the capital investment. The analyses on the LUC grade estimates, as discussed above have, therefore, been extended to cover the corresponding estimates of relative profits, defined as:

\[
\text{Relative profit} = \frac{\% \text{ above cut-off} \times (\text{grade above cut-off} - \text{cut-off grade})}{2}
\]

Figure 7 shows, for the different production periods, the percentage errors of the profiles of relative profits for different cut-offs based on the LUC recoverable estimates compared with the corresponding 'actual' block values based on the available comprehensive 8 m x 5 m grade control data grid.

Figure 7 also demonstrates that the percentage errors associated with the profit profiles for different production periods are very significant. In the short-term production period (i.e. 6 months), the percentage errors range from 36%-74% for cut-offs from 0.6 g/t to 1.0 g/t. Although the error percentages decrease as the production volumes increase, the percentage errors associated with these production periods are still substantial and any financial forecast based on these estimates can be misleading.

Conclusions

From the results of the study, the dangers of accepting localized direct conditioning estimates (LUC) of tonnage,
grade, and profit figures for short- to medium-term production periods are obvious, particularly where the data-set is small. This is because the much smaller data-set effectively available for individual block and LUC estimates by ordinary kriging, as well as for the sections to be mined in these periods, could prevent efficient and reasonable estimates for planning purposes. In addition, these estimates could be subject to significant conditional biases and invalidate any estimates and patterns of financial forecasts for new projects or for extensions of existing mines.

References


Geostatistical applications in petroleum reservoir modelling

by R. Cao*, Y. Zee Ma† and E. Gomez†

Introduction

Although geostatistics was initially developed for mining resource evaluation (Krige, 1951; Matheron, 1963; Journel and Huijbregts, 1978), it has been extended to other geoscience applications, including forestry, environmental science, soil science, and petroleum science and engineering. This paper presents methods, workflows, and pitfalls in using geostatistics for hydrocarbon resource modelling and evaluation. Examples are presented of indicator variogram analysis of categorical variables, lithofacies modelling by sequential indicator simulation and hierarchical workflow, porosity modelling by kriging and stochastic simulation, collocated cokriging for integrating seismic data, and collocated cosimulation for modelling porosity and permeability relationships. These methods together form a systematic approach that can be effectively used for modelling natural resources.

Keywords

facies modelling, propensity, multilevel or hierarchical modeling, object-based modeling, collocated cosimulation, porosity, permeability.

Synopsis

Geostatistics was initially developed in the mining sector, but has been extended to other geoscience applications, including forestry, environmental science, soil science, and petroleum science and engineering. This paper presents methods, workflows, and pitfalls in using geostatistics for hydrocarbon resource modelling and evaluation. Examples are presented of indicator variogram analysis of categorical variables, lithofacies modelling by sequential indicator simulation and hierarchical workflow, porosity modelling by kriging and stochastic simulation, collocated cokriging for integrating seismic data, and collocated cosimulation for modelling porosity and permeability relationships. These methods together form a systematic approach that can be effectively used for modelling natural resources.

Keywords

facies modelling, propensity, multilevel or hierarchical modeling, object-based modeling, collocated cosimulation, porosity, permeability.

Facies and lithofacies modelling

Facies and lithofacies are discrete variables in which the rock property describes categories of the rock quality (Ma, 2011). An indicator variable represents a binary state with two possible outcomes: presence or absence. For three or more lithofacies, the indicator variable may be defined in terms of one lithofacies and all the others combined to indicate the absence of that selected lithofacies.

Indicator variogram

Most indicator variograms show the second-order stationarity with a definable plateau (Jones and Ma, 2001). The lithofacies variogram observed across stratigraphic formations is commonly strongly cyclical with lag distance. This has been termed a hole-effect variogram in geostatistics (Ma and Jones, 2001). Cyclicity and amplitudes in hole-effect variograms, and presents methods for modelling lithofacies, including sequential indicator simulation that integrates propensity analysis of descriptive geology, hierarchical facies, or lithofacies modelling workflows.

Reservoir models of continuous properties in hydrocarbon resource evaluation generally include porosity, fluid saturations, and permeability. This paper briefly discusses porosity modelling by using kriging and sequential Gaussian simulation, and permeability modelling by using collocated cosimulation. Based on the scale of subsurface heterogeneities, hierarchical modelling frameworks are presented not only for a two-step facies-lithofacies modelling workflow, but also for dealing with the more general multilevel modelling methodology, from the categorical variable of facies to continuous variables of porosity and permeability.

* China University of Petroleum, Beijing, China.
† Schlumberger, Denver, Colorado, USA.
© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
Geostatistical applications in petroleum reservoir modelling

effect variograms are strongly affected by the relative abundance of each lithofacies and by the means and standard deviations of the sizes of lithofacies objects (e.g. sandstone and shale bodies).

Sample density is very important for accurately describing an experimental variogram (Ma et al., 2009). In a reservoir with a low sandstone fraction, if individual sandstone or other lithofacies bodies are sampled densely (e.g. several data samples for each sand body), the experimental variogram will likely show spatial correlations and possibly a certain degree of cyclicity. However, if individual lithofacies bodies are sampled with only one observation each, the indicator variogram will appear with a strong nugget effect. This is consistent with the notion that geology is by no means random, but that sparse sampling can make it appear random.

Consider an experimental horizontal variogram generated while only a few wells have been drilled in the area. The closest spacing of the wells may be so great that a variogram cannot be determined for short lag distances. For instance, if the object size varies substantially for one lithofacies, the variability may override any signal that the periodic stratigraphy may carry. If the experimental variogram is calculated isotropically or with a wide tolerance around a specified direction, non-isotropic bodies (e.g. channels) will produce variograms with a mixture of spatial continuities.

For applications, an experimental variogram is typically fitted to a theoretical model. The spherical and exponential models, possibly with a partial nugget effect, are most commonly used in practice. These models imply monotonically increasing variance with increasing lag distance.

Sequential indicator simulation

Sequential indicator simulation (SIS) is one of the most commonly used methods to model lithofacies because of its capabilities in integrating various data. In particular, lithofacies probability maps or volumes can be used to constrain the positioning of the geological objects in a SIS model. As a matter of fact, because of the complexity of depositional history, a geological process is simultaneously indeterministic and causal. Such a phenomenon can be described by the propensity, which enables quantitative integration of descriptive geology with lithofacies frequency data at wells (Ma, 2009). Such integration can produce lithofacies probabilities that convey the descriptive geology, and at the same time honour the available data. Subsequently, these lithofacies probabilities can be used to constrain stochastic modelling for building realistic geological models. The boundaries in the propensity zoning are often interpretive, in which case the geologist can incorporate the conceptual depositional model.

When no probability is used for conditioning, the lithofacies model is driven only by the indicator variogram and data from the wells. As a result of the scarcity of the data, the facies objects are often distributed unrealistically randomly, especially in the areas of no control points. In the example shown in Figure 1A, reef facies should occur only in the eastern rim of the shelf, but the model generates the reef facies everywhere (Figure 1B).

By integrating the propensity analysis (Figure 2A) based on the conceptual depositional model, the facies probability maps were generated (Figures 2B to 2E). The facies model constrained by these probability maps shows most reef facies in the east of the area (Figure 2F). The propensity influence is obvious in the model.

Hierarchical facies modelling

Apart from SIS, other techniques for modelling facies include object-based modelling (OBM, see e.g., Ma et al. 2011), Boolean, and truncated Gaussian methods (Deutsch and Journel, 1992; Falivene et al., 2006). All these categorical modelling techniques can be hierarchized in two or more levels for multilevel facies modelling. For example, facies may
be first modelled using fluvial OBM or the truncated Gaussian method, and then modelled again using SIS based on the model already constructed. This is because the OBM and truncated Gaussian methods generally produce larger facies objects in the model, and SIS can be further used to model small-scale heterogeneities. OBM is suitable for clear shape definitions of geological features such as channels and bars. The truncated Gaussian method is suitable when the order of the facies is clearly definable.

The workflow of modelling lithofacies by SIS following a truncated Gaussian modelling or OBM has been applied to a number of hydrocarbon field development case studies, including carbonate ramps and shallow marine depositional environments. Here, an example of constructing a facies model using OBM based on the model constructed by SIS is presented. A facies model of sand-shale (Figure 3A) was first built using SIS with facies probability maps. An OBM method was then used to generate channels with splay (Figure 3B). Splay occurs on the edges of the channels, but are not invariably present. Notice that channels erode both previously deposited shale and sand.

**Kriging and stochastic simulation of porosity**

Porosity is one of the most important petrophysical variables in hydrocarbon resource characterization, as it describes the subsurface pore space available for fluid storage. Geostatistical methods for modelling porosity include kriging and sequential Gaussian simulation (SGS). Kriging generally produces smoother results, as the variance of the kriging model is commonly smaller than the variance of the data used in the kriging. SGS can be considered to be a two-step modelling workflow that performs a stochastic simulation based on kriging results. Sometimes, cokriging or cosimulation can be used when more densely sampled seismic data is available and can be calibrated with porosity.

In addition, the lithofacies model is often used to constrain the spatial distribution of porosity using SGS, because in the hierarchy of subsurface heterogeneities, depositional facies govern spatial and frequency characteristics of porosity to a large extent. Even though porosity can still be quite variable within each facies, the porosity statistics by facies generally exhibit less variation (Ma et al., 2008). Figure 4 compares two porosity models constructed with the two different lithofacies models presented earlier (Figure 2).

**Collocated cokriging and collocated cosimulation**

Collocated cokriging is a simplified version of cokriging (Xu et al., 1992). Instead of using a number of data points from the secondary variables, collocated cokriging uses only one data point that is collocated (i.e. at the same position) with the estimation point of the primary variable. The estimator by collocated cokriging is such that:

\[ Z(x_0) = m + \sum_{i=1}^{k} \lambda_i [Z(x_i) - m] + \lambda_0 [Y(x_0) - m_Y] \]  

where 
- \( m \) is the mean of the primary variable
- \( \lambda_i \) are the weights of the data points of the primary variable
- \( \lambda_0 \) is the weight of the collocated data point of the secondary variable
- \( m_Y \) is the mean of the secondary variable
- \( Y(x_0) \) is the collocated secondary variable.

The simple kriging solution to Equation [1] is a linear system of equations obtained by the least-square method that minimizes the estimation error. That is, in a block matrix form:

\[
\begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix}
\begin{bmatrix}
\lambda_{xx} \\
\lambda_{xy}
\end{bmatrix}
= \begin{bmatrix}
c_x \\
C_{xy}
\end{bmatrix}
\]

where
- \( C_{xx} \) represents the sample covariance matrix
- \( C_{yx} \) or its transpose, \( e_{xx} \) is the vector of covariances between each of the data points in the primary variable and the collocated data point of the secondary variable
- \( C_{00} \) is the variance of the secondary variable (equal to 1 after normalization)

**Figure 3**—(A) Facies model built using SIS with facies probability maps. (B) Facies model constructed hierarchically using OBM utilizing the facies model in (A) as the first-step model

**Figure 4**—Comparison of two porosity models constructed using SGS. (A) Porosity model constrained to the unconstrained lithofacies model (Figure 1B). (B) Porosity model constrained to the constrained lithofacies model (Figure 2F)
Geostatistical applications in petroleum reservoir modelling

\( \lambda_k \) is the vector of the simple kriging weights for the primary variable

\( \lambda_0 \) is the weight for the collocated data point of the secondary variable

\( c_z \) is the vector of covariances between each of the data points and the estimation point of the primary variable

\( C_{zy} \) is the covariance between the primary and secondary variables.

Special cases:
(1) When the collocated secondary variable is not used, the method is reduced to simple kriging
(2) When there is no sample data in the kriging neighborhood, collocated cokriging uses the sole data point of the secondary variable at the location, and becomes the linear regression. Indeed, \( \lambda_0 \) is obtained as the correlation coefficient between the two variables (scaled by their ratio of standard deviations), which is the solution of the linear regression
(3) An advantage of collocated cokriging compared to linear regression is that it has the exactitude property, inherited from all the kriging methods, while linear regression does not. That is, it honours all the sample data. The proof is quite straightforward. Making one of the known points in Equations [1] and [2] the estimation point will lead to that point having a weight equal to 1 while all the other weights become zero.

Collocated cosimulation is somewhat similar to sequential Gaussian simulation, except that their kriging estimation equations are slightly different (see Equations [1] and [2]). It can also be formulated as the summation of an estimation and a simulated error.

It is noteworthy that an (auto-) covariance function is symmetrical about the origin (zero lag distance), but a cross-covariance function is not necessarily symmetrical. In such a case, cokriging cannot be simplified to collocated cokriging because the delay effect (see e.g. Papoulis, 1965) in correlation between the primary and secondary variables cannot be easily conveyed into the auto-covariance function. Moreover, even if the cross-covariance is symmetrical, there are some important approximations in reducing cokriging to a collocated cokriging (Deutsch and Journel, 1992). Theoretically, only some special cases verify the assumptions (Rivoirard, 2001). In practice, the simplification in collocated cokriging and co-simulation make these methods very useful in honouring the relationships between two variables.

Collocated cokriging can be quite effective in integrating seismic data into petrophysical property models (Xu et al., 1992). Collocated co-simulation is generally more useful for modeling permeability as it can honour the porosity-permeability relationship, often termed the phi-K relationship. An example of modelling the phi-K relationship is discussed below in comparison to linear regression.

The phi-K relationship in the data is often a clouded nonlinear correlation (Figure 5A). Imposing a linear transform between the porosity and logarithmic permeability can significantly distort the frequency statistics of the permeability. In fact, the difference in frequency distribution between the permeability model generated using a standard linear regression and the well log data is commonly striking, as shown by the example in Figure 5B. The phi-K transform, in which the logarithm of permeability is estimated from the porosity using a linear transform, reduces the permeability because the exponential of the mean is smaller than the mean of the exponential (Delfiner, 2007). This effect of reduction can be easily seen by comparing the frequency distribution of the permeability from the linear regression to the original frequency distribution of the data. Indeed, a linear transform creates more low permeability values and skews the histogram toward the lower permeability values (Figure 5B).

Figure 5 (A) Cross-plot between the horizontal permeability (millidarcy) and porosity (fractional). Green data points are based on samples, and red on the model using cocosim. The solid line is the linear regression between porosity and the logarithm of permeability. (B) Histograms comparison of the well log permeability (green) and permeability model (blue) from the phi-K linear transform (logarithmic scale). Note the large discrepancies between the two histograms. (C) Comparison of the well log permeability and permeability model generated by cocosim (logarithmic scale)
Geostatistical applications in petroleum reservoir modelling

Instead of using a phi-K transform or cloud transform, collocated cosimulation or 'cocosim' (Ma et al., 2008) can be used to honour the relationship between the permeability and porosity. The histogram of the permeability model using cocosim closely matches the histogram of the well log permeability data (Figure 5C). Besides avoiding the bias introduced in a phi-K transform (Ma, 2010), there are other advantages in using cocosim, including the 3D porosity model constraining the 3D permeability model, and honouring the data at the well locations.

Conclusion

In 3D modelling of subsurface processes, there is commonly a significant lack of hard data. To generalize the data to a 3D model, critical inferences must be made. The quality and accuracy of the model depend on not only the quality and quantity of data, but also how the inference is drawn and made. It is often said that ‘garbage in, garbage out’. However, it should be noted that the ‘data in’ does not necessarily mean a good model will result; it can still result in ‘garbage out’ due to erroneous inference from the data to the model. Geostatistical methods provide tools for better inference from limited data in constructing a 3D reservoir model. These include incorporation of depositional interpretation using propensity analysis, variogram analysis, and the hierarchical modelling framework.

Propensity analysis can help the transition from qualitative description to quantitative analysis, bridge the gap between the descriptive geology and quantitative modelling, and provides useful constraints to condition the facies model to be geologically realistic. Variogram analysis can help characterize the continuity of rock properties, including geological object size and anisotropy. A broad hierarchical modelling workflow is an efficient way of modelling multi-scale subsurface heterogeneities, from large-scale structural and stratigraphic heterogeneities, to intermediate-scale facies heterogeneities, to smaller-scale petrophysical properties. Furthermore, some discrete variables in one category of a broad hierarchical workflow, such as facies, can be hierarchically modelled with two or more levels of modelling by different methods. An example of combining object-based modelling and SIS was given in this paper.

Geostatistical models of discrete lithofacies variables are important because of their use in constraining porosity and permeability models. Collocated cosimulation is an effective way to model porosity-permeability relationship while honouring the known permeability data and constraining the permeability model to the previously generated porosity model.

References


Subscription
to the
SAIMM Journal for the year
January to December 2014

The SAIMM Journal
all you need to know!

- R1 717.20
local
- US$477.00
overseas per annum per subscription.

- ¥ Less 1.5% discount to agents only
- ¥ PRE-PAYMENT is required
- ¥ The Journal is printed monthly
- ¥ Surface mail postage included
- ¥ ISSN 2225-6253

In the new world of work, we all have
-F to achieve more
-F at a faster pace
-F with less resources
-F against greater competition
-F in a global economy tougher than ever
before

The SAIMM Journal gives you the edge!
-F with cutting-edge research
-F new knowledge on old subjects
-F in-depth analysis

For more information please contact:
The Southern African Institute of Mining
and Metallurgy

Kelly Matthee
The Journal Subscription Department
27-11-834-1273/7

P O Box 61127, MARSHALLTOWN,
2107, South Africa

kelly@saimm.co.za or
journal@saimm.co.za
Website: http://www.saimm.co.za

SAIMM
THE SOUTHERN AFRICAN INSTITUTE
OF MINING AND METALLURGY

120 YEARS OF TECHNICAL EXCELLENCE 1895 – 2015

THE MINING INDUSTRY TRUSTS BOOYCO ELECTRONICS TO PUT IT ALL TOGETHER!

0861 BOOYCO
(0861 266926)

+2711 823 6842 • www.booycoelectronics.co.za
Introduction

Short-term decisions in mining operations are related to defining the destination of material blocks, either to the processing plants, low-grade stockpiles, or the waste dump. These decisions are usually based on the grade control estimates built from information gathered from samples taken specifically for this, from blast-holes (Deutsch et al., 2000; Ortiz et al., 2012). However, it is well known that the quality of these samples is poor due to many reasons. Firstly, blast-hole samples are taken within the production cycle, with significant time and space constraints, since this must be done before charging the blast-holes with explosives. Chemical analysis results, in some cases, are not returned in time to build the short-term plan, and the destination of the blocks may be defined without having the analytical results at hand, with a significant risk of misclassification. Secondly, sampling is done by a variety of methods, such as using a radial bucket placed under the drilling rig platform; taking a number of increments from the cone by means of a tube in order to complete the weight of the required sample; or taking a channel sample from the cone, among others. A correct sample must comply with the requirements of sampling theory. Experience has shown that this seldom occurs; in fact, most of these methods suffer from severe delimitation and extraction errors, not to mention segregation-related biases.

It is well known that sampling is an essential part of the resource modelling process, since if samples are of poor quality, the entire model is weak (Rolley, 2000). Furthermore, sampling operators are in many cases poorly trained and do not really understand the importance of applying a rigorous procedure during sampling. Night shifts make things worse, as little control is possible during these shifts, and there is frequently insufficient light for the operators.

The significance of poor blast-hole sampling has already been demonstrated (Magri and Ortiz, 2000). A random error around the true block values affects the decision regarding the destination of such blocks: to either the processing plants, to a...
stockpile, or to the waste dump. Furthermore, a systematic bias may have large economic impacts on a project and due to the nature of blast-hole drilling and sampling, bias can originate from multiple sources. Therefore, sampling theory must be followed with as much care as possible to reduce the losses due to poor sampling (Francois-Bongarcon, 1983; Pitard, 1993; Assibey-Bonsu, 1996; Magri, 2007; Pitard, 2009).

Poor samples lead to poor short-term plans. Furthermore, short-term plans should incorporate as many variables as possible that are relevant for ore processing, and estimation techniques that satisfy some requirements such as global and conditional unbiasedness and precision. The effect of all economic and detrimental elements must be taken into account to arrive at the best decision as to whether a block is profitable or not. Thus, a precise estimation of all the relevant variables is required in order to obtain a proper assessment of the best destination of each block. In addition to this already challenging setting, when different variables are correlated, and they all have an impact on the process, their spatial distributions should be co-simulated so as to consider the cross-correlations in space for the best assessment of the blocks’ value. In summary, in order to maximize profits, the final models built for deciding the destination of the blocks must be globally and conditionally unbiased and as precise as possible. Unfortunately, misclassification is not directly reflected in the company’s books since it leads to hidden losses: some ore blocks end up in low-grade stocks or in the waste dump, while some waste blocks are processed and do not pay for their treatment.

The following section contains a brief review of geostatistical techniques and presents the methodology for performing conditional simulations to assess the economic benefit of improving sampling quality and determine the best advanced drilling spacing to improve the financial returns of the mine, in addition to determining the best grade control approach to achieve this goal.

The geostatistical framework

Geostatistics was originated to solve prediction problems in gold mines in South Africa (Krige, 1952). The apparent random, yet structured, behaviour of gold grades triggered the formulation of a probabilistic approach, where the grade value at an unsampled location, $z(u)$, is related to a random variable, $Z(u)$, characterized by a probability distribution. The structured behaviour is accounted for by relating random variables at different locations by means of a random function, $Z(u), u \in D)$. This random function is in turn characterized by its statistical moments, which must be inferred from the available data, that is, the values gathered at sample locations $\{z(u_\alpha), \alpha = 1, ..., n\}$. The spatial distribution of actual values of the variable within the domain is interpreted as a realization of this random function.

The geostatistical paradigm consists of estimating the expected value of the random variables at every location to obtain a map suitable for local optimum prediction or constructing other realizations of the random function to characterize the uncertainty associated with unsampled locations, preserving the spatial relationships between locations. This is achieved through estimation and simulation techniques respectively. Results of these estimation and simulation processes are controlled by the definition of the domains where the random functions are defined. These domains are established by understanding the geology, and integrating this knowledge to the statistical analysis done during the exploratory data analysis. Poorly defined geological domains for estimation and simulation will inevitably lead to poor estimates and inaccurate uncertainty quantification.

Estimation is done by considering a linear estimator that depends on the surrounding information available. In geostatistics, this estimator is called kriging in honour of Danie Krige, who proposed this approach in the early 1950s. Kriging is the best linear unbiased estimator (Journel and Huijbregts, 1978; Isaaks and Srivastava, 1989). The kriging estimator is constructed by successively imposing these features (linearity, unbiasedness, optimality). Variations of the estimate are achieved by imposing a known or unknown mean, and allowing local variations of it (Goovaerts, 1997). Kriging is the best estimator in the least-squares sense that it, imposes the minimization of the error variance. The simple kriging estimate assumes the mean known and constant, and is at the heart of simulation procedures:

$$Z^K(u_0) = \sum_{\alpha=1}^{n} \lambda^K_\alpha Z(u_\alpha) + (1 - \sum_{\alpha=1}^{n} \lambda^K_\alpha) \mu$$

The estimation variance results in:

$$\sigma^K(u_0) = \sigma^2 - \sum_{\alpha=1}^{n} \lambda^K_\alpha C(u_\alpha, u_0)$$

where $\sigma^2$ is the variance of the population, which is estimated from the sample data, $C(u_\alpha, u_\beta)$ is the covariance between the data located at $u_\alpha$ and the location of interest $u_0$, and $\lambda^K_\alpha$ are the optimum weights to minimize this error variance. This variance gives a basic measure of uncertainty of the estimated value. However, as seen in Equation [2], this does not depend on the sample values themselves, but only on their locations. Therefore, the kriging variance measures the uncertainty at the estimation location due to the spatial configuration of the available data for its estimation, rather than based on the dispersion of the values.

The kriging weights are obtained from solving the following linear system of equations, which arises from imposing the minimization of the error variance:

$$\sum_{\alpha=1}^{n} \lambda^K_\alpha C(u_\alpha, u_\beta) = C(u_\alpha, u_0)$$

In practice, simple kriging is not used, but rather, the mean of the random variable is assumed unknown, but constant within the neighbourhood where samples are searched to be used in the estimation. This is a convenient choice, since it provides a robust estimator to changes in the local mean. This estimator is known as ordinary kriging. Its derivation follows the same steps as in simple kriging (linearity, unbiasedness, optimality), but owing to the assumption that the mean is unknown, it requires the kriging weights to add up to unity, to ensure unbiasedness. The estimate and estimation variance for ordinary kriging are:

$$Z^K(u_0) = \sum_{\alpha=1}^{n} \lambda^K_\alpha Z(u_\alpha)$$

$$\sigma^K(u_0) = \sigma^2 - \sum_{\alpha=1}^{n} \lambda^K_\alpha C(u_\alpha, u_0) - \mu$$

The system of equations requires an additional constraint over the weights, to prevent a weighting bias:
Designing and advanced RC drilling grid for short-term planning in open pit mines

\[ \sum_{i=1}^{n} \lambda_{i}^{SK} C(u_{i}, u_{0}) + \mu = C(u_{0}, u_{0}) \]  \\
\[ \forall \alpha = 1, \ldots, n \]  \\
\[ \sum_{i=1}^{n} \lambda_{i}^{SK} = 1 \]

In these equations, \( \mu \) is the Lagrange multiplier required to impose the constraint over the weights.

Ordinary kriging is commonly used to estimate block grades for long-term planning, using the samples from the exploration drilling campaign, and also in the short term to build the grade control model (although other geometric estimation techniques may also be used, such as inverse distance weighting). When blocks are estimated, average covariances are used to obtain the point to block relationship in Equations [5] and [6], instead of the point–to-point covariances presented. These average covariances are computed numerically by discretizing the block into points and averaging the covariances between these points and the sample location. Additionally, the block variance must be used in Equation [5], which can be calculated by volume-variance relations.

Estimation allows obtaining the best prediction in terms of precision. The idea of simulation is to provide alternate realizations of the random function, in order to quantify the uncertainty over a transfer function that acts over different locations on the domain. The actual values are interpreted as one possible realization of this random function, therefore each resulting realization performs like the actual deposit and can be used for risk assessment and uncertainty quantification. The main difference between estimation and simulation is that the former looks for the best local estimate, while the latter is concerned with reproducing the spatial characteristics that relate multiple locations.

There are several approaches to simulating a random function; most of them are based on a multigaussian assumption that relieves the inference of the probability distribution characterizing every location. Under this assumption, the random variable can be linked to a Gaussian-shaped probability distribution, whose expected mean and variance are identified with the simple kriging mean and kriging variance (Equations [1] and [2]). Simulated values are drawn directly from the conditional distributions at the Monte Carlo step. Changing the random path, and by changing the values of the previously simulated nodes composed by the normal scores of the sample data and normal scores is 0. The conditioning information is also included from the normal score sample data. The variance of the point support is

\[ E(Y(u_{i}))(n)) = Y_{SK}^{u_{i}} = \sum_{i=1}^{n} \lambda_{i}^{SK,Y} Y(u_{i}) \]  \\
\[ Var(Y(u_{i}))(n)) = \sigma_{SK,Y}^{2}(u_{i}) = \sigma^{2} - \sum_{i=1}^{n} \lambda_{i}^{SK,Y} C_{V}(u_{i}, u_{0}) \]

where the index \( Y \) has been added to emphasize that these are the estimate and variance of the transformed variable; the covariance function is also inferred from the normal score sample data. The variance of the point support distribution is

4. Draw a simulated value from the conditional distribution characterized by a mean equal to the simple kriging estimate, a variance equal to the simple kriging variance, and Gaussian shape. This is done by Monte Carlo simulation, that is, a uniform random value between 0 and 1 is generated and the corresponding quantile of the conditional distribution is taken as the simulated value of the \( Y \) variable. The simulated value is used as conditioning for all subsequent nodes in the random path.

5. Back-transform the Gaussian simulated values to the grade distribution by reversing the transformation on step 1.

\[ Z(u_{i}) = \phi^{-1}(Y(u_{i})) \]

where \( \phi \) is the quantile transformation from the reference (representative) \( Z \) distribution to a standard Gaussian distribution \( Y \), and \( N \) is the total number of samples

2. Define a random path over the \( M \) locations \{\( u_{i}, i = 1, \ldots, M \)\} where the variable is simulated.

3. At every location, compute the simple kriging estimate and kriging variance of the transformed grade \( Y \), using Equations [1] and [2]. These allow the exact estimation of the conditional expectation and conditional variance of the random variable under the multigaussian assumption. Note that the mean of the normal scores is 0. The conditioning information is composed by the normal scores of the sample data and the Gaussian values of the previously simulated nodes within the search neighbourhood.

Note that the result of this process is a conditional realization of the random variable at the simulation locations \{\( u_{i}, i = 1, \ldots, M \)\}. New realizations can be obtained by changing the random path, and by changing the values drawn from the conditional distributions at the Monte Carlo simulation step.

Other methods to simulate multigaussian random functions exist and could also be used (see for example Chiles and Delfiner, 1999).

Simulation trades off the local precision obtained in kriging to reproduce the spatial continuity of the variable. The distribution of simulated values at every unsampled location provides a measure of uncertainty and, contrary to the kriging variance, this uncertainty is data-value dependent. This is highly convenient in most applications where some relationship is seen between variability and local mean. More importantly, each realization can be treated jointly to compute the output to any transfer function, where the joint distribution at different locations has an impact on the final output. This occurs on flow, mine planning, and pollution problems.
Designing and advanced RC drilling grid for short-term planning in open pit mines

Once each realization has been processed to infer a given response to a process, the set of responses over the suite of realizations provides a quantification of the distribution of this response, which is useful for risk assessment (Glacken, 1997; Journel and Kyriakidis, 2004).

Methodology
Evaluating the financial output of a given drilling grid requires understanding the spatial variability of the grades within the deposit. Conditional simulation helps create possible realizations of the true distribution. Each one of these realizations can be used to compare the performance of a given combination of drilling density and sample quality.

The process is demanding, since the economic performance assessment must be repeated for each of the realizations built with conditional simulation. In order to obtain realistic financial outputs, several input parameters must be known: economic parameters such as mining and processing costs, drilling, sampling and assaying costs, sale price of the elements of interest, as well as metallurgical parameters in order to assess the short-term performance, etc.

The general methodology is as follows.

**Determine the current and expected sample quality considering the drilling equipment available**
Sampling errors emanating from blast-hole data can be assessed by QA-QC analysis of field duplicates and fundamental error calculations. This error becomes the current value and is added to the simulated grades that represent exact values, since they are obtained from exploration data which has very small errors. However, the blast-hole drilling recovery, which is very poor at the top of the bench already broken up, cannot be assessed by QA-QC, therefore the value computed from field duplicates can be considered lower than the actual error.

Additionally, sampling errors from more sophisticated RC equipment can be assessed. The use of sampling theory and the design of the sampling and sample preparation protocols will provide the calculation of the fundamental error of the new procedure. This value can be seen as the achievable improved situation. These errors will also be added to the simulated grade values that represent exact values.

**Build multiple dense conditional realizations of the distribution of grades within the domain, considering all relevant short term planning variables**
A set of dense conditionally simulated realizations of the grades within the geological units of the deposit must be built using simulation techniques and appropriate software. If significant cross-relationships exist between variables, then co-simulation should be tried, in order to preserve those relationships. Conditioning is provided by the exploration drill-hole samples, which may be diamond drill-holes and reverse circulation holes.

The sampling, sample preparation, and assaying error of these samples must also be assessed, although this error usually is significantly lower than the one associated with blast-hole drilling recovery and blast-hole samples, and could be considered negligible, except when coarse gold is present.

The goal of having dense realizations is to be able to sample them at several spacings, each representing a drilling spacing for advanced RC drilling, and evaluating the economic benefit of using that information and a given grade control methodology to select blocks to be processed, assessing their metallurgical performance and final revenue.

Furthermore, these dense realizations are block-averaged to obtain a representation of true block grade distributions, which in turn will be used to evaluate the short term plan performance.

**Sample the dense realizations at different spacings emulating the blast-hole grid as well as the advanced reverse circulation drilling grids and add the corresponding sampling error to each of them**
By sampling from each of the realizations, simulated values are obtained for locations representing regular sampling grids. The spacing of these grids represents the different drilling grids. The values obtained from this ‘sampling procedure’ over the dense realizations can be considered as the exact true grade at the corresponding locations. However, in practice, samples carry error. Therefore, a random Gaussian error is added to these exact values, to reflect the information quality, which can vary from minimum error samples obtained through high-technology RC equipment to large errors for traditional blast-hole sampling. It should be mentioned that blast-hole samples also carry non-random errors which may have significant consequences. These errors are hard to quantify, thus they have not been accounted for in this analysis. However, final results should be discussed considering this issue.

The simulated value with sampling error is obtained by Monte Carlo simulation from a Gaussian distribution with mean equal to the original simulated value at the location of the blast-hole sample: \( Z(u_l) \), where \( l \) represents one of the realizations; and variance equal to the fundamental error relative variance computed in the previous step multiplied by the sample value squared: \( \sigma_{FE}^2 \cdot (Z(u_l))^2 \).

For each realization \( l = 1, ..., L \), multiple sample data-sets are obtained, one for each drilling grid spacing and sampling error evaluated.

**Emulate the short-term plan strategy over each of the realizations**
Each of the sample data-sets obtained in the previous step, which represent a combination of information quantity (sample spacing) and quality (sampling error), can be used to predict the block grades, using the short-term planning strategy that is to be assessed. Different estimation procedures (inverse distance weighting, ordinary kriging) and parameters (minimum and maximum number of samples to be used in estimation, minimum number of samples per octants, number of octants informed, search radii, etc.) can be considered at this stage.

The result of this step is a model of estimated block grades similar to the one used for grade control, which will be more precise when more information is available and when this information is of higher quality, but this will come at a higher cost due to the additional drilling required for dense grids. Accuracy, however, is not necessarily better than in the
Designing and advanced RC drilling grid for short-term planning in open pit mines

original model if the multiple potential biases that occur during blast-hole drilling and sampling are not accounted for. This suggests much more research is required to understand, quantify, and model all the sampling errors present in blast-hole sampling.

**Compute the revenue for each combination of sample quantity, sample quality, and estimation procedure, and compare to the maximum unachievable profit that could be obtained without misclassification i.e. using the true block grades stored in step 2.**

For each drilling grid spacing and sampling error considered, the block grades can be estimated and the profit of processing those that are considered ore and dumping those that are considered waste (based on the estimated block grades, which are obtained with the samples with error) can be calculated. Revenue considers the following components:

\[
Revenue = Metal_{\text{revenue}} - Cost - Royalties
\]

where

\[
Metal_{\text{revenue}} = Metal_{\text{grade}} \times Tonnage \times Recovery \times Metal_{\text{price}}
\]

\[
Cost = Metal_{\text{grade}} \times Tonnage \times Recovery \times Cost_{\text{unitary}} + Cost_{\text{drilling}}
\]

In these equations, Royalties depends on the mining tax laws of each country, Recovery is usually a function of the Metal_grade and other detrimental elements, as well as the type of process considered. Cost_unitary is the cost per weight unit of the commodity under consideration. This cost will usually be composed of several items:

\[
Cost_{\text{unitary}} = Cost_{\text{mine}} + Cost_{\text{mill}} + Cost_{\text{process}}
\]

The cost of drilling and sampling (includes sample preparation and analysis) is added to the equation to account for the additional cost required to acquire the information given by a specific drilling grid.

The ideal (unachievable) case where all true block grades are known can also be assessed for each realization and can serve as an upper bound of the profit. This case can be evaluated because the ‘true’ block grades can be obtained from averaging the densely simulated points without adding any sampling error (step 2).

The best drilling grid with respect to the maximum revenue can be defined by comparing revenues obtained from the grade control model based on the blast-hole samples (with large sampling error), and those based on the RC sampling grids (with minimum error), for each of the grid spacings considered.

**Three case studies**

In this section, the methodology is applied to three case studies.

**Case 1: Large porphyry copper deposit**

The first application is performed on a porphyry copper deposit operated by open pit mining and producing both oxides and sulphides. The deposit shows a fairly typical configuration of lithology, including andesite, porphyry, and breccia, covered by ignimbrites and gravel. Mineralization units are characterized by a sequence of barren gravels at surface, followed by a leached cap, oxides, secondary sulphides, and a hypogene zone. Alteration zones are mainly due to the presence of clay, which defines ore types that have different behaviour during leaching.

The long-term geological model is used as a reference to identify the estimation units. Total and soluble copper are the relevant variables. Since their relationship is complex due to the mineralogical constraint (total Cu ≥ soluble Cu), solubility ratio and total copper are simulated at point support, on a 2 × 2 × 10 m grid, over a domain representing the next 5 years of production. For each of these realizations, soluble copper is deducted and block averaging is performed over the grades (not over the solubility ratio) to 10 × 10 × 10 m blocks. Global relative errors for total and soluble copper are obtained from duplicate blast-hole sampling data. These are 14% and 15% respectively. These numbers are optimistic, since they do not account for recovery problems.

Sampling grids ranging from 6 × 6 m to 20 × 20 m are assessed and relative errors are added to emulate the blast-hole sample quality. Also, the case of advanced RC drilling is considered by not adding error. It is assumed that the RC error is negligible and already included in the realizations since these were obtained from exploration data that had a high proportion of RC drill-holes.

Short-term planning is currently done with inverse distance squared estimation with few samples (minimum 1 and maximum 6). An estimation plan based on ordinary kriging is assessed considering a larger number of samples (minimum 4, maximum 16).

Revenue and profits were calculated using actual costs and the two short-term planning methodologies mentioned earlier. Several sensitivity analyses were performed to evaluate the robustness of the proposed advanced RC grid spacing. In particular, the decision was tested by changing the commodity price, the drilling cost, by adding a systematic underestimation bias to all blocks, and by considering a reduction in the geological classification of each block, from 5% to 2% over the entire domain.

The current situation, that of blast-holes on an 8 × 8 m grid, can be improved by considering the increased cost of RC drilling and the increased profit due to the better geological classification of the blocks and higher precision in the estimation of the grades. Results are shown in Table 1, where the economic losses in eight cases of drilling grids are presented, as well as the four main grade control estimation cases: inverse distance squared weighting and ordinary kriging, with no added error (emulating an advanced RC drilling system dedicated to obtaining samples for short-term planning) and with added error (emulating conventional blast-hole sampling). It should be noticed that for the 8 × 8 m grid, losses of inverse distance and kriging when considering the added error situation are low because they do not require additional drilling other than that already considered in the mine cost for blasting. If the error is too reduced by advanced RC drilling, then these holes have to be drilled specifically for this purpose, incurring additional cost.

The final recommendation was to move from the current blast-hole sampling on an 8 × 8 m grid, to drilling with the RC system on an 18 × 18 m grid, which carries a reduction in losses due to misclassification of ore blocks that reaches US$150 million over a 5-year period.
Designing and advanced RC drilling grid for short-term planning in open pit mines

Table I

<table>
<thead>
<tr>
<th>Case</th>
<th>Grid spacing (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6x6</td>
</tr>
<tr>
<td>ID2 No error</td>
<td>210.47</td>
</tr>
<tr>
<td>OK No error</td>
<td>184.51</td>
</tr>
<tr>
<td>ID2 Added error</td>
<td>346.68</td>
</tr>
<tr>
<td>OK Added error</td>
<td>299.58</td>
</tr>
</tbody>
</table>

Table II

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Advanced drilling grid (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6 x 6</td>
</tr>
<tr>
<td>Optimum (unattainable)</td>
<td>709.06</td>
</tr>
<tr>
<td>Samples without added error</td>
<td>671.89</td>
</tr>
<tr>
<td>Samples with added error</td>
<td>667.58</td>
</tr>
<tr>
<td>Difference</td>
<td>4.31</td>
</tr>
</tbody>
</table>

Case 2: Exotic oxide copper deposit

In this case study, the procedure was applied to an exotic copper deposit, which is mined by an open pit operation using 7.5 m high benches. The ore is processed through heap leaching, solvent extraction, and electrowinning to produce copper cathodes. Drilling is done by conventional DTH drilling since the presence of moisture and clay impeded the use of the advanced RC drilling system that was in place. In this case study, a production period of 5 years was evaluated.

Ten realizations were built respecting the geological domains, which are based on grade shells for the case of total copper grades, and on clay-controlled units, for carbonates. These realizations were built in a 2 × 2 × 7.5 m grid. The short-term model consists of blocks 6.25 × 6.25 × 7.5 m in size. Estimation of the selective mining units is done by ordinary kriging with a minimum of 3 and a maximum of 12 samples.

Sampling errors are assessed from duplicate data, resulting in 16% relative error for total copper and 20% relative error for carbonates.

Advanced sampling grids at 6 × 6 to 14 × 14 m centres in increments of 2 m were analysed.

The economic performance of each combination of advanced sampling grid and sampling error was analysed using the grade control estimation parameters as well as the mining and economic parameters provided by the operations and metallurgy teams. These include acid consumption estimation based on the total copper and carbonate grades, and copper recovery, which also depends on these grades.

Results in Table II show that the current sampling error obtained with the conventional DTH drilling rig costs about US$5 million for the 5-year period and the 8 × 8 m drilling grid, which is the closest to the unattainable optimum (highlighted in bold). This money could be used for improving the sampling system, to move towards a more sophisticated RC drilling rig that can deal with the moisture encountered.

Case 3: Complex gold deposit

The third case study involves a structurally controlled gold and silver massive deposit. Estimation units were defined based on the orientation of the controlling structural trends, which generated vertical mineralized volumes with widths ranging from about 10 m to 30 m. In addition to these structures, a central breccia contains disseminated mineralization. Bench heights of 8 m and 16 m were considered for the structurally controlled and disseminated mineralization areas respectively.

Automated sampling procedures were recommended for this operation due to the climatic conditions under which drilling is to be done. High winds and low temperatures hinder the possibility of obtaining good blast-hole samples, especially during the winter months. Therefore, a dedicated RC drilling rig with an automated sample capture system was considered in order to improve the quality of the information used to build short-term models and to ensure representative samples free of segregation error due to loss of fines and particle size segregation. Inclined RC drilling (60°) produces relative errors lower than 8% using sampling ratios of 4% and 2% in 8 m and 16 m benches respectively.

In this particular case, since blast-hole sampling is not feasible, the advanced drilling grid spacing optimization is reduced to balance out the advanced RC drilling, sampling and assaying costs versus block misclassification. Thus, a dense RC grid reduces losses due to misclassification but is more expensive; on the other hand, a sparse drilling grid is inexpensive but increases the losses due to misclassification.

The procedure was applied over the two distinct areas of the mine, producing slightly different results, due to the different bench heights and grade continuity in each zone. Sensitivity analysis was performed to assess the effect of changing the short-term estimation parameters to obtain the block grades from the samples obtained from the advanced drilling grids. Ordinary kriging with a minimum of 4 and a maximum of 16 samples outperformed all other estimation methods (inverse distance, nearest neighbor estimation and kriging with fewer samples).

Results in Table III show an improvement of US$50 million over a 5-year period when using ordinary kriging over inverse distance estimate on a 10 × 10 m advanced RC drilling grid.
Designing and advanced RC drilling grid for short-term planning in open pit mines

Table III
Economic losses (in million US dollars) for different drilling grids and estimation methods, complex gold deposit

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Advanced drilling grid (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6 x 6</td>
</tr>
<tr>
<td>Ordinary kriging with 4/16 samples</td>
<td>276.1</td>
</tr>
<tr>
<td>Polygonal estimation</td>
<td>365.0</td>
</tr>
<tr>
<td>Inverse distance squared with 4/16 samples</td>
<td>328.8</td>
</tr>
</tbody>
</table>

Conclusions
Simulated models of the distribution of grades and geological units can be built with conventional geostatistics software. Dense simulated models can be used to represent the true distribution of grades within a deposit. Several procedures can be emulated over these dense realizations, to evaluate the cost and benefit of many decisions. In this paper, we show that these models can be used to assess the cost of changing the drilling technology for short-term planning, and improving the sampling and sample preparation errors.

Three case studies are presented, showing that moving away from blast-hole sampling, which is well known for producing poor-quality drilling recovery and poor-quality samples, provides significant economic benefits that reach millions of dollars per annum.

The change in drilling and sampling technology for short-term planning results in better drilling recovery and better samples, without loss of fines, with increments that correctly represent the lot, and with improved health and safety conditions for drilling rig operators. The benefits of this change can be evaluated by processing the geostatistical realizations built (which represent the spatial variability of the orebody) in order to emulate the sampling procedure, sample quality, and estimation procedure. Grid optimization is carried out by quantifying the costs and revenues related to the different sampling grids.

The three case studies reveal that when operating conditions allow for a dedicated drilling rig, it is worth considering investing in a sophisticated sampling system mounted on an RC drilling rig to operate well in advance, thus providing timely data for building short-term models that can include several additional relevant variables.

Once validated dense simulations are available, sensitivity analyses can be easily performed in order to ensure that the recommended drilling spacings are robust with respect to changes in the economic and metallurgical conditions.

The results presented in this paper are optimistic in the sense that they do not account for additional errors due to poor recovery, and thus can be considered as a minimum in terms of the losses that the mines actually experience due to sampling problems.

Acknowledgements
The authors would like to acknowledge the Department of Mining Engineering at University of Chile, particularly the ALGES laboratory, as well as the Advanced Center for Mining Technology at Universidad de Chile. Comments and corrections made by Dr Francis Pitard and an anonymous reviewer are greatly appreciated by the authors.

References
A trusted industry leader

For 90 years, SCAW, a South African industry leader, has partnered with leading mining houses to ensure safety critical standards and productivity targets are consistently met. Whether it’s hoisting, grinding or excavating, Scaw produces an extensive range of products that drive mining safety and productivity.

Backed up by teams of qualified engineers with extensive experience in all aspects of grinding media, steel wire rope, chain and cast products, Scaw’s product specialists are available to advise on the design, selection, installation and maintenance of safety critical and mining products manufactured by Scaw.

www.scaw.co.za
On localizing uniform conditioning estimates
by O. Rondon*

Synopsis
Localized uniform conditioning is a technique developed to spatially locate selective mining unit grades that have been derived using uniform conditioning for the assessment of recoverable resources. The technique has the advantage of producing selective mining unit estimates conforming to the uniform conditioning panel-specific grade-tonnage curve while introducing spatial information at the scale of the selective mining units.

This paper describes an alternative technique to localized uniform conditioning which does not explicitly require the uniform conditioning panel-specific grade-tonnage curve to localize the selective mining unit estimates. The technique can therefore be implemented in mining software where uniform conditioning is not available.

Keywords
uniform conditioning, recoverable resources, change of support, localized UC, Gaussian distribution.

Introduction
During the early stage of a mining project the drill-hole data is frequently located on a relatively large grid, which is generally sufficient for estimating tons and grades using large panels whose dimensions reasonably conform to the drill spacing. However, such drill spacing is insufficient for the estimation of the recoverable resources required to properly assess mining projects at the stage of selective mining.

Uniform conditioning (UC) allows assessing the recovery functions of selective mining units (SMUs) inside a panel by using the estimated panel grade. However, UC does not provide any information about the spatial location of those SMUs, which is important in the mining industry to assess the spatial distribution of recoverable resources.

Localized uniform conditioning (LUC) is a relatively recent technique that has been used to spatially locate the grades of SMUs inside a panel. LUC uses the panel recovery functions given by UC and, through a ranking procedure of direct SMU estimates, derives and assigns SMU grades that reproduce the UC recovery functions (Abzalov, 2006).

This paper presents an alternative approach to LUC that does not require a ranking of direct estimates and does not explicitly use the recovery functions provided by UC to localize the SMU estimates. Comparative results between both approaches are discussed using copper grades from a porphyry deposit.

First, a brief summary of UC and LUC is presented where the concepts and notation used are introduced. Secondly, the alternative technique to LUC is shown in detail, and finally, comparative results for both approaches are presented.

Uniform conditioning
UC is a nonlinear technique for estimating recoverable resources inside a mining panel using the estimated panel grade (Rivoirard, 1994). The key to UC is a conditionally unbiased estimate of the panel grade and a sound change-of-support modelling relating the grades $Z(x)$ at point support, $Z(v)$ at SMU support, and the estimated grade $Z^*(V)$ at panel support.

The discrete Gaussian change–of-support model introduced by Matheron (1976) is used in UC to carry out the change of support. The model expresses $Z(x)$ and $Z(v)$ as functions of two standard Gaussian variables $Y(x)$ and $Y_v$,

$$Z(x) = \Phi(Y(x))$$  \[1\]

where $\Phi$ is the point anamorphosis function derived from the point support data (Rivoirard, 1994), and

$$Z(v) = \Phi_v(Y_v)$$  \[2\]

where $\Phi_v$ is the SMU anamorphosis function, which is derived via Cartier’s relation by further assuming that $Y(x)$ and $Y_v$ have joint Gaussian distribution with correlation coefficient $r > 0$. This implies that $\Phi_v$ is given by

* Optiro Pty Ltd, West Perth, Perth, Australia.
© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
On localizing uniform conditioning estimates

\[ \Phi_r(y) = \int_{-\infty}^{+\infty} \Phi\left(rv + u\sqrt{1 - r^2}\right) g(u) \, du \quad [3] \]

where \( g \) denotes the standard Gaussian density (Lantuéjoul, 1994; Rivoirard, 1994). The Hermite polynomial expansion of the point support anamorphosis allows the SMU anamorphosis \( \Phi_r \) to be explicitly computed as

\[ \Phi_r(y) = \sum \phi_n r^n H_n(y) \quad [4] \]

with \( H_n \) the Hermite polynomials and \( \phi_n \) the corresponding Hermite coefficients. The coefficient \( r \) corresponding to the variance correction factor from point to SMU support is chosen so as to respect the theoretical variance at the SMU support by inverting

\[ \text{Var}(Z(v)) = \sum \phi_n^2 r^{2n} \quad [5] \]

An alternative method has been proposed by Emery (2007), which allows the computation of \( r \) without inverting Equation [5].

Assuming that a similar approach holds for changing from SMU to panel support (Rivoirard, 1994), the estimated grade \( Z'(V) \) at panel support is expressed as

\[ Z'(V) = \Phi_r(Y_{V'}) \quad [6] \]

where \( Y_{V'} \) is a standard Gaussian variable and the panel anamorphosis function \( \Phi_r \) is given by

\[ \Phi_r(y) = \sum \phi_n r^n H_n(y) \quad [7] \]

The correction factor \( s \) is chosen so as to respect the variance of \( Z'(V) \) by inverting

\[ \text{Var}(Z'(V)) = \sum \phi_n^2 s^{2n} \quad [8] \]

If it is further assumed that \( Y_v \) and \( Y_{V'} \) have joint Gaussian distribution with correlation \( R \), then \( R = \rho \) and the conditional distribution of the Gaussian equivalent SMU grades \( Y_v \), given the Gaussian equivalent panel grade \( Y_{V'} \) is known. For a panel with \( Y_{V'} = y_{V'} \), this distribution is Gaussian with mean \( R y_{V'} \), variance \( 1 - R^2 \). This key result allows computation of the panel-specific recovery functions given by UC (Rivoirard, 1994). This is the core of the alternative technique to LUC proposed here.

The information effect in UC (Deraisme and Roth, 2001) allows modelling of the fact that the SMUs will ultimately be selected on an estimated SMU grade \( Z'(v) \) instead of the real \( Z(v) \) grade.

The modelling is carried out as before by expressing \( Z'(v) \) as a function of a standard Gaussian variable \( Y_{V'} \), i.e.

\[ Z'(v) = \Phi_r(Y_{V'}) \]

where \( Y_{V'} \) is obtained by inversion as before, so as to respect the theoretical variance of the SMU estimates.

Similarly, assuming that \( Y_{V'} \) and \( Y_{V'}' \) have joint Gaussian distribution with correlation coefficient \( R' > 0 \), the conditional distribution of \( Y_{V'} \), given that \( Y_{V'} = y_{V'} \), is Gaussian with mean \( R' y_{V'} \) and variance equal to \( 1 - (R')^2 \) (Deraisme, 2001). It is possible to show, by further assuming that \( Y_v \) and \( Y_{V'} \) are conditionally independent given \( Y_{V'} \), that \( R' \) is equal to \( \sqrt{\rho} \) where \( \rho \) is obtained by inverting

\[ \text{Var}(Z'(v)) = \sum \phi_n^2 s_{V'}^{2n} \rho^n \quad [10] \]

Therefore, derivation of the UC panel-specific recovery functions accounting for the information effect requires only a minor modification to the correlation value used in the conditional Gaussian distribution.

Localized uniform conditioning

LUC was proposed by Abzalov (2006) as a way to spatially locate the SMU grades inside a panel using the panel-specific tonnage (\( T \)) and metal (\( Q \)) curves given by UC.

The idea is to first carry out a direct estimation of SMU grades from the drill-hole data. Ranking of these estimates within each panel in increasing order of grade allows the derivation of a set of proportion values, which are used in conjunction with the corresponding panel metal curve to derive the SMU grades that will be used instead of the direct SMU estimates. The process is depicted in Figure 1, assuming a continuous curve for panel-specific tonnage \( T \) and metal \( Q \) curves. Given an SMU with proportion \( p \), a cut-off \( z \) is identified from the tonnage curve. The cut-off \( z \) is then used in the metal curve to compute the corresponding metal value associated with the proportion \( p \), and the direct SMU estimate is replaced by the LUC grade given by the ratio between metal \( q \) and tonnage \( p \).

In practice UC does not provide a continuous curve for \( T \) and \( Q \) but a discrete set of tonnage and metal values for a set of predefined cut-offs. In this case, the discrete set of tonnage and metal values is used to define grade classes that are used to assign the SMU grades (Abzalov, 2006). Therefore, it is

\[ Figure 1—Derivation of SMU grades by LUC assuming a continuous panel-specific tonnage \( T \) and metal \( Q \) curves. After ranking the direct SMU estimates inside the panel, a proportion \( p \) is obtained and used in the tonnage curve to compute the corresponding cut-off \( z \). This cut-off value is used in the metal curve to compute the associated amount of metal \( q \). The grade to be assigned to the SMU with proportion \( p \) is \( q/p \)\]
On localizing uniform conditioning estimates

extremely important for the use of LUC to properly select a set of cut-offs for which the set of finite panel tonnage and metal values provide a suitable discretization of the panel-specific tonnage and metal curves.

It is worth noting that the mechanism of LUC is not restricted to UC and can be equally used with any recoverable resource technique for which panel-specific tonnage and metal curves can be obtained, for instance multiple indicator kriging (Hardtke et al., 2011). However, incorporation of the information effect is not straightforward when using multiple indicator kriging.

Abzalov (2006) has pointed out some limitations of the LUC process. Particularly important are the number of SMUs per panel and the ability of the direct SMU estimates to provide a reliable ranking criterion within each panel. The former impacts the resolution with which the panel-specific recovery functions can be reproduced. The bigger the number of SMUs per panel, the better the reproduction of the recovery functions will be. The latter is valid within continuous mineralization domains with relative low nugget effect.

However, as shown by Harley and Assibey-Bonsu (2007), in the presence of strong short-range variability, LUC does not appear to be capable of producing accurate estimates under the constraint of reproducing the panel-specific tonnage and metal curves given by UC.

Direct LUC

Assuming that UC is a plausible model for assessing recoverable resources, an alternative approach to LUC, referred to as direct LUC, can be derived without explicitly using the panel-specific tonnage and metal curves provide by UC or doing any ranking of the direct SMU grade estimates.

Contrary to LUC, direct LUC explicitly uses the conditional distribution of the Gaussian equivalent SMU grades inside a panel with Gaussian equivalent grade $Y'_V = y_V$. This distribution is Gaussian with mean $\lambda y_V$ and variance $1 - \lambda^2$, where $\lambda$ is equal to $R$ or $R^*$, depending on whether the information effect is accounted for or not.

Direct LUC works by using equivalent Gaussian grades at point support to carry out a direct estimate of Gaussian SMU grades, which are then corrected on a panel-by-panel basis to match the conditional distribution given by UC.

The process is summarized as follows:

- Transform the point support data $Z(x)$ into equivalent Gaussian grades $Y(x)$
- Estimate equivalent Gaussian SMU grades $y^*$ using $Y(x)$
- Transform the panel estimates $Z^*(V)$ into equivalent Gaussian panel estimates $Y_V$ using the panel anamorphosis function given by Equation [4]
- Correct the distribution of the equivalent Gaussian SMU direct estimates on a panel-by-panel basis to match the conditional distribution of equivalent Gaussian SMU grades knowing the equivalent Gaussian panel grade given by UC.

This correction step is simple and an affine-type correction (Equation [11]) can be used to avoid the ranking of the SMU grades

$$y^*_{corr} = \frac{\sqrt{1 - \lambda^2}}{\sigma} (y^* - \mu) + \lambda y_V$$

where $\mu$ is the mean of all SMUs inside the panel with Gaussian equivalent grade $Y'_V = y_V$, $\sigma$ is the corresponding standard deviation of SMU grades within the panel, and $y^*_{corr}$ is the corrected Gaussian SMU grade.

- Back-transform the corrected Gaussian SMU grades $y^*_{corr}$ using the corresponding SMU anamorphosis function given by Equation [4].

Key to direct LUC is a sound anamorphosis modelling for SMU and panel support as well as the correction of the equivalent Gaussian SMU estimates. In practice, the SMU anamorphosis function given by Equations [4] or [9], if taking into account the information effect, and the panel anamorphosis function given by Equation [6] are input into the process as lookup tables, and direct and back-transformation are carried out using linear interpolation between tabulated values.

It is worth noting that if direct LUC grades are used only to report the proportion of SMUs inside a panel that are above cut-off and their corresponding average grade, then one obtains the UC results.

Case study – a porphyry copper deposit

The deposit is a low-copper (Cu) porphyry deposit with minor molybdenum (Mo) and gold (Au) mineralization. The mineralized domain covers an area of approximately 750 m (east) by 1100 m (north) with depths up to 400 m below surface.

Based on a nominal drill spacing of 80 m (east) by 80 m (north), panel grades were estimated using ordinary kriging with a panel size of 60 m by 60 m by 20 m. The estimation strategy was designed to deliver panel grade estimates with minimal conditional bias. This results in regression slope values ranging from 0.7 to 0.97.

Following the estimation of panel grades, UC, LUC, and direct LUC were undertaken using a SMU size of 10 m (east) by 10 m (north) by 10 mRL.

Comparison between LUC and direct LUC is carried by mean of global and selected panel-specific grade tonnage curves.

Figure 2 shows the global grade-tonnage curve within the mineralized domain obtained using LUC and direct LUC. The latter closely reproduces the results from LUC.

A key property of LUC is that regardless of the direct SMU estimated grades, the panel-specific grade-tonnage curve given by UC is always reproduced if there is a large number of SMUs inside the panel. Unlike LUC, direct LUC requires not only a large number of SMUs inside a panel, but also that the corresponding Gaussian SMU direct estimates exhibit a reasonable variability so, after the correction step, its distribution resembles the panel-specific conditional distribution of equivalent Gaussian SMU grades given by UC.

Figure 3 shows the grade-tonnage curves obtained with LUC and Gaussian LUC in a panel containing 72 SMUs. Direct LUC provides a satisfactory match to the grade-tonnage curve given by LUC. Furthermore, as shown in Figure 4, direct LUC provides a more continuous range of grades and does not exhibit the artifact of constant grade values observed in LUC because of the use of grade classes.
On localizing uniform conditioning estimates

Figure 2—Global grade-tonnage curve for Cu (%) grades obtained with LUC (red) and direct LUC (blue)

Figure 3—Grade-tonnage curves for 72 SMU Cu grades inside a panel obtained with LUC (red) and direct LUC (blue)

Figure 4—Scatter plot between LUC and direct LUC Cu grades with grade-tonnage curves shown in Figure 3
Discussion
This paper presented the direct LUC approach as an alternative to LUC. The approach does not require a ranking of direct estimates and does not explicitly use the recovery functions provided by UC to localize the SMU estimates. Instead, direct LUC corrects the equivalent Gaussian direct SMU estimates on a panel-by-panel basis to match the panel-specific theoretical distribution given by UC. Final SMU estimates are obtained by back-transforming the corrected SMU grades using the SMU anamorphosis function.

The three main advantages of direct LUC over LUC are:
➤ Direct LUC does not require carrying out any UC estimate in advance. Therefore, it can be coded as a macro or script in commercial mining software where UC is not available.
➤ Direct LUC does not rely on a predefined set of cut-off values.
➤ Direct LUC produces on output both UC and LUC results.

However, as pointed out by Abzalov (2006), LUC can easily incorporate external information such as high-resolution geophysical data, which is not the case when using direct LUC.

References


Stronger than conventional steel linings. Stronger than traditional wear resistant tiles. Ceramite® stands up best to the heat and erosion challenges of heavy industry.

Free flow Ceramite®, containing bauxite and/or silicon carbide aggregate, cast with a minimum thickness of 12mm, has a extremely high wear resistance, and an operational life much longer than traditional components in this harsh environment. Ceramite® provides far greater reliability and lower maintenance costs.

Ceramite® has been used extensively in the power, cement, ferro-alloy and platinum industries, and has been remarkably successful with platinum smelter granulated slag slurries, pulverised coal pneumatic conveying, furnace ore feed chutes and many more applications. With a minimum of 48 hour workshop turn-around installation, there is no quicker and stronger method to protect your pipes and other industrial challenges.

Furthermore, Ceramite® can be precast into absolutely any shape to meet your needs. Ceramite® may be applied to existing component designs or Process Consultants will work with Refraline to assess your unique requirements, and design and install a cost-saving protection measure that delivers reduced downtime, less maintenance, lower overall capital costs, and higher production levels.

In tough operating conditions; where your assets, your people, your product quality and your output levels need protection; only Ceramite® shapes up. Put it to work for you.

Typical property ranges

- Wear resistance = 1.3 to 4 cm³
- Density = 2.6 to 2.8 kg/dm³
- Cold crushing strength = 70 to 220 MPa (20 to 1500°C respectively)
- Flexural strength = 7 to 28 MPa (20 to 1500°C respectively)
- Thermal conductivity range choice (0.6 to 14 W/mK)

FREE FLOW CERMITE PROVIDES EXTREME WEAR RESISTANCE IN PIPE LININGS - 48 HOUR MINIMUM WORKSHOP TURN-AROUND

Elkem
Manufacturers of Ceramite®

RREFRALINE
Group of Companies

Installation Contractor

Visit www.processconsultants.co.za for data sheets and case studies or contact us on 011 803 2013, 082 657 2610 or processconsultants@mweb.co.za

PROCESS CONSULTANTS
CONSULTANTS TO THE METALLURGICAL INDUSTRY
Manufacturing and Distribution Licensee
Iron oxide Cu-Au (IOCG) mineralizing systems: an example from northeastern Russia

by A. Kostin*, A. Vedyaev†, and G. Rafat‡

Synopsis
GIS and multivariate geometric distribution have been used to predict the association of high-level intrusives with hydrothermal alteration related to iron oxide Cu-Au (IOCG) mineralizing systems. Several examples in the Russian northeast are presented in this paper. IOCG ore deposits can have enormous geological resources containing significant reserves of base, precious, and strategic metals, and hence are economically attractive targets for mineral exploration. To date, examples of this type of mineralization have not been reported from the northeastern area of Russia. The Tarinskiy ore cluster, located in Eastern Yakutia, shows brecciated altered rocks with sulphide and iron oxide cement, which is typical for IOCG mineralization of the iron oxide Cu-Au-U type. This cluster was formed near surface and is linked with the Rep-Yuruinskiy pluton. It has potential to host new world-class precious metal deposits in northeastern Russia.

Keywords
GIS, multivariate, iron oxide, Au-U-Cu, IOCG, Tarinskiy, Yakutia.

Introduction
A fundamental problem of ore deposit geology is the prediction of and prospecting for new large-scale precious metals deposits. One type of these – the large group of iron oxide copper-gold (IOCG) hydrothermal ore deposits containing Cu, Au, Ag, ±U, ±REE, ±Bi, ±Co, ±Nb, ±P – has not yet been recognized in the Russian Far East (Figure 1), and the potential for discovery is currently unknown.

The recognition of IOCG deposit types began with the discovery of the giant Olympic Dam deposit in 1975 in Australia, which contains 2000 Mt of 1.1% Cu, 0.5 g/t Au, 4000 g/t U3O8, and 0.24 – 0.45% La + Ce (Roberts and Hudson, 1983). This was followed in 1987 by the La Candelaria in Chile (470 Mt at 0.95% Cu, 0.22 g/t Au, and 3.1 g/t Ag) (Marschik, Leveile, and Martin, 1985). This was followed in 1987 by La Candelaria in Chile (470 Mt at 0.95% Cu, 0.22 g/t Au, and 3.1 g/t Ag) (Marschik, Leveile, and Martin, 1985). IOCG deposits related to alkali and calc-alkali plutons can also be associated with porphyry-type Cu-Mo or Cu-Au deposits, Cu-Ag ‘mantle’ deposits, U, haematite, and Au-PGE and polymetallic Pb-Zn-Ag ±Au veins (Corriveau, 2007; Gandhi, 2004).

Since IOCG mineralization is characterized by heterogeneity of both ore and metals, the application of new methods such as multidimensional geometric distribution using GIS and multivariate distribution of deposits should be very promising as a guide to exploration. Classical sub-soil geometry as a method of mathematical mapping spatial distribution of single elements can lead, in the case of complex deposits, to a situation where each element has its own geometry, i.e. the outlines of anomalies of various indicators do not coincide. This reduces the reliability of any assessment because of errors in determining the type of mineralization and incorrect evaluations of metal inventory. Therefore, it was necessary to develop a multidimensional model of geometric distribution of mineralization, which delineates homogeneous areas of a prospect by all elements simultaneously, thus excluding the ambiguity in the construction of the boundaries (Wedjaev, Wedjaeva, and Rafat, 2009). In addition, this model provides the best estimate of the average values of the parameters of the mineralized bodies (element contents, thickness etc.), which is ensured by a new solution for the statistical problem of multivariate distribution (Rodionov, 1981). This approach allowed the types of precious metal anomalies of East Yakutia, associated with IOCG mineralization, to be assessed with greater confidence.

*Ivăne Coţ s and Precious Metal Geology Institute, Siberian Branch of the Russian Academy of Sciences (DPMGI SB RAS).
†LLC ‘Alkon-Europe’, Moscow Russia.
‡GeoConsult (Pty) Ltd, Duisburg, Germany.
© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
Iron oxide Cu-Au (IOCG) mineralizing systems: an example from northeastern Russia

Main features of known IOCG-type deposits

Hydrothermal features
All regions with IOCG deposits are characterized by large alteration halos, including Na (Ca) and K, ranging from 10 to 100 km² or more (Barton and Johnson, 1996, 2004; Hitzman, Oreskes, and Einaudi, 1992). The alteration areas are usually of greater extent than the areas of IOCG mineralization (Gandhi, 2004). A key feature of IOCG deposits is the association with high-K granites (Pollard, 2000).

Mineralization
Economic mineralization is represented by chalcopyrite ± bornite and native gold, hosted by iron oxides. Haematite characterizes upper levels of the mineralizing systems, and magnetite the deeper levels.

Geophysical features
Sulphide-poor deposits with large volumes of Fe oxides and hydrothermal alteration form reasonable targets for regional airborne magnetic and gravity surveys (Kostin et al., 2006; Kostin, 2008). In combination with field geological observations, gravity and magnetic data are very useful, but expensive, tools in IOCG prospecting.

External features on Landsat images
Landsat multispectral data is one of the most powerful tools for exploring and characterizing many aspects of the Earth’s surface, and is an inexpensive exploration method (Kostin, 2011, 2012; Kostin and Ospiov, 2012). Spectral analysis using red (0.65−0.69 μm), green (0.52−0.60 μm), and blue (0.45−0.52 μm) is a true colour band (3-2-1) combination that allows areas with elevated concentrations of iron oxides to be distinguished (Figure 2).

Geological age
Geological age does not appear to be critical (Nisbet et al., 2000). IOCG deposits are known to occur from the Archean (Salobo and Igarapé Bahia) to the Mesozoic (Chilean Iron Belt, Russian Far East) (Requia et al., 2003; Sillitoe, 2003; Tallarico et al., 2004).

Prerequisites for IOCG-style mineralization in East Yakutia
A review of samples resulted in the identification of ore types that could be assigned to IOCG mineralization, but which were previously considered as a product of oxidation of sulphidic ores.

The Pozolota’ zone (65.799669°N 129.596191°E) in the Nuektaminskiy cluster (Kostin et al., 2006) includes a 50 × 26 m stock-like body of milky drusy quartz with haematite cement and a gold content ranging from 1.2 to 19.8 g/t (Figure 3A). The field of the eruptive breccias of the

Figure 1—Distribution of IOCG districts and important deposits worldwide (after Corriveau 2007) (red dots) and Eastern Yakutia (Russia) – a new greenfield IOCG area (red rectangle). Australia: Gawler district (Olympic Dam, Acropolis, Moonta, Oak Dam, Prominent Hill, and Wirrda Well), Cloncurry district (Ernest Henry, Eloise, Mount Elliot, Osborne, and Starra), Curnamona district (North Portia and Cu Blow), Tennant Creek district (Gecko, Peko/Juno, and Warrego); Brazil: Carajas district (Cristalino, Alemão/Igarapé Bahia, Sa-lobo, and Sossego); Canada: Great Bear Magmatic Zone (Sue-Dianne and NICO), Werneck district (West Coast skarns), Central Mineral Belt, and Kwyjibo deposit; Chile: Chilean Iron Belt district (Candelaria, El Algarrobo, El Romeral, Manto Verde, and Punta del Cobre); China: Bayan Obo (Inner Mongolia), Lower Yangtze Valley district (Meishan and Daye); Iran: Bafq district (Chogust, Chado Malu, BS Seh Chahoon); Mauritania: Akjoujt deposit; Mexico: Durango district (Cerro de Mercado); Peru: Peruvian Coastal Belt (Raul, Condestable, Eliana, Monterrosas, and Marcona); Sweden: Kiruna district (Ki-irunasara, Loussavaara), Atitk deposit (also described as a porphyry Cu deposit); South Africa: Phalaborwa and Vergenoeg deposits; USA: Southeast Missouri district (Pea Ridge and Pilot Knob); Adirondack and Mid-Atlantic Iron Belt (Reading Prong); Zambia: Shimyoka, Kantonga, and Kitumba; Eastern Yakutia (Russia): IOCG greenfield areas with the Nuektaminskiy, Endybalskiy, Kis-Kuelskiy plutons, and Rep-Yuruinsky district

Figure 2—Example of elevated concentrations of iron oxide on surface – draped Landsat geo-image (colour band 3-2-1 combination) over to DTM
Endybalskiy cluster (65.673781°N 130.132772°E) includes large bodies of breccia with goethite as cement. These were not analysed for copper and gold, but have a strong visual resemblance to IOCG mineralization (Figure 3B). The Kis-Kuelskiy diorite-granodiorite intrusive (65.501242°N 130.280125°E) includes different types of IOCG-like mineralization (Kostin, 2010): brecciated granodiorite with iron oxide cement (Figure 3C) and brecciated hornfels with iron oxide and sulphide cement (Figure 3D). In the 2011 field season, one of the iron oxide anomalies identified in Landsat TM imagery was confirmed as IOCG-style mineralization. This anomaly is situated in the Rep-Yuruinskiy district (Tarynskiy cluster, 63.574957°N 143.275846°E) and is hosted by brecciated and metasomatized rocks. The Rep-Yuruinskiy sub-type of IOCG mineralization consists of granodiorite-associated, breccia-hosted bodies in which arsenopyrite mineralization is associated with iron oxide alteration of breccias (Figures 3E, 3F). The breccias include thick lenses of quartz-chlorite metasomatic lithologies with disseminated Cu sulphide mineralization. Breccia colour depends on the saturation of iron oxides, and changes in the supergene zone from dark brown to different shades of brown and yellow-brown. The breccias are commonly heterolithic and composed of sub-angular to rarely rounded lithic clasts or fine-grained massive material. The area of brecciation covers some 5 km², while density varies from 2.41 to 3.23 (average = 2.76 t/m³). Hence, the resource potential could be about 712 Mt applying a depth of 50 m. The grab samples assay results are presented in Table I.

![Image of mineralized samples](image)

**Figure 3**—Typical samples and outcrops of mineralized material. (A) Sample 5214 – gold-bearing mineralization from the Pozolota showing (Nuektaminskiy cluster) of milky drusy quartz with haematite cement; (B) sample 5307 – eruptive breccia from the Endybalskiy cluster with haematite cement; (C) sample 6279 – Kis-Kuelskiy pluton: brecciated granodiorite with iron oxide cement; (D) sample 6246-1 – Kis-Kuelskiy pluton exocontact zone: brecciated hornfels with iron oxide and sulphide cement; E) sample 7117 – brecciated hornfels from Rep-Yuruinskiy pluton exocontact zone with gold-bearing cement; (F) sample 7127 – brecciated hornfels from Rep-Yuruinskiy pluton exocontact zone with Au-and U-bearing iron oxide cement.

Qz – quartz; He – haematite; horn – hornfels; Ri – rhyolite; Grd – granodiorite; Fe-ox – iron oxide cement; Fe-ox+gal – iron-oxide + galena cement

### Table I

**Assay results of rock grab samples from the Rep-Yuruinskiy cluster**

<table>
<thead>
<tr>
<th>No</th>
<th>Sample ID</th>
<th>Density g/cm³</th>
<th>Au (g/t)</th>
<th>Fe (%)</th>
<th>Cu (%)</th>
<th>Bi (g/t)</th>
<th>Mo (g/t)</th>
<th>W (g/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Chlorite-quartz metasomatite</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3021</td>
<td>3.2</td>
<td>1.3</td>
<td>4.9</td>
<td>0.1</td>
<td>1000</td>
<td>NSR</td>
<td>490</td>
</tr>
<tr>
<td>2</td>
<td>7108</td>
<td>2.8</td>
<td>0.1</td>
<td>13.7</td>
<td>0.1</td>
<td>520</td>
<td>NSR</td>
<td>NSR</td>
</tr>
<tr>
<td>3</td>
<td>7143</td>
<td>2.7</td>
<td>0.7</td>
<td>48.5</td>
<td>0.6</td>
<td>890</td>
<td>NSR</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>7154</td>
<td>2.8</td>
<td>0.4</td>
<td>3.9</td>
<td>1.1</td>
<td>180</td>
<td>NSR</td>
<td>NSR</td>
</tr>
<tr>
<td>5</td>
<td>7157</td>
<td>2.7</td>
<td>3.7</td>
<td>7.7</td>
<td>0.2</td>
<td>890</td>
<td>NSR</td>
<td>NSR</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average:</td>
<td>2.8</td>
<td>1.2</td>
<td>15.74</td>
<td>0.4</td>
<td>696</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hornfels breccias with Fe oxide cement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3022</td>
<td>2.8</td>
<td>7.5</td>
<td>7.7</td>
<td>1.6</td>
<td>890</td>
<td>26</td>
<td>8250</td>
</tr>
<tr>
<td>2</td>
<td>3024-1</td>
<td>3.5</td>
<td>0.2</td>
<td>9.6</td>
<td>0.4</td>
<td>110</td>
<td>26</td>
<td>NSR</td>
</tr>
<tr>
<td>3</td>
<td>3058-3</td>
<td>2.8</td>
<td>6.4</td>
<td>4.4</td>
<td>0.0</td>
<td>110</td>
<td>26</td>
<td>NSR</td>
</tr>
<tr>
<td>4</td>
<td>7112</td>
<td>2.5</td>
<td>0.3</td>
<td>20.1</td>
<td>0.2</td>
<td>300</td>
<td>94</td>
<td>2800</td>
</tr>
<tr>
<td>5</td>
<td>7115</td>
<td>2.6</td>
<td>0.4</td>
<td>13.7</td>
<td>0.4</td>
<td>940</td>
<td>120</td>
<td>8250</td>
</tr>
<tr>
<td>6</td>
<td>7116</td>
<td>2.5</td>
<td>2.2</td>
<td>6.5</td>
<td>0.0</td>
<td>160</td>
<td>150</td>
<td>NSR</td>
</tr>
<tr>
<td>7</td>
<td>7117a</td>
<td>2.9</td>
<td>0.0</td>
<td>4.3</td>
<td>0.0</td>
<td>NSR</td>
<td>99</td>
<td>NSR</td>
</tr>
<tr>
<td>8</td>
<td>7117b</td>
<td>2.9</td>
<td>0.2</td>
<td>10.7</td>
<td>0.8</td>
<td>890</td>
<td>160</td>
<td>NSR</td>
</tr>
<tr>
<td>9</td>
<td>7118</td>
<td>2.6</td>
<td>1.7</td>
<td>7.9</td>
<td>0.2</td>
<td>99</td>
<td>37</td>
<td>160</td>
</tr>
<tr>
<td>10</td>
<td>7124c</td>
<td>2.5</td>
<td>0.4</td>
<td>30.2</td>
<td>0.2</td>
<td>890</td>
<td>26</td>
<td>490</td>
</tr>
<tr>
<td>11</td>
<td>7127</td>
<td>2.6</td>
<td>0.0</td>
<td>38.7</td>
<td>0.6</td>
<td>850</td>
<td>43</td>
<td>7000</td>
</tr>
<tr>
<td>12</td>
<td>7128</td>
<td>2.7</td>
<td>1.4</td>
<td>33.6</td>
<td>0.8</td>
<td>890</td>
<td>51</td>
<td>NSR</td>
</tr>
<tr>
<td>13</td>
<td>7129</td>
<td>2.4</td>
<td>0.1</td>
<td>6.2</td>
<td>4.2</td>
<td>890</td>
<td>NSR</td>
<td>NSR</td>
</tr>
<tr>
<td>14</td>
<td>7137</td>
<td>2.7</td>
<td>0.5</td>
<td>6.1</td>
<td>0.4</td>
<td>890</td>
<td>NSR</td>
<td>NSR</td>
</tr>
<tr>
<td>15</td>
<td>7139</td>
<td>2.7</td>
<td>0.0</td>
<td>13.7</td>
<td>0.2</td>
<td>NSR</td>
<td>NSR</td>
<td>NSR</td>
</tr>
<tr>
<td>16</td>
<td>7141</td>
<td>2.8</td>
<td>0.0</td>
<td>34.9</td>
<td>0.7</td>
<td>410</td>
<td>63</td>
<td>NSR</td>
</tr>
<tr>
<td>17</td>
<td>7142</td>
<td>2.9</td>
<td>0.6</td>
<td>4.8</td>
<td>0.9</td>
<td>760</td>
<td>NSR</td>
<td>NSR</td>
</tr>
<tr>
<td>18</td>
<td>7156</td>
<td>2.8</td>
<td>0.1</td>
<td>18.5</td>
<td>0.4</td>
<td>650</td>
<td>31</td>
<td>NSR</td>
</tr>
<tr>
<td>19</td>
<td>7159</td>
<td>2.6</td>
<td>2.9</td>
<td>56.4</td>
<td>0.1</td>
<td>890</td>
<td>36</td>
<td>NSR</td>
</tr>
<tr>
<td>20</td>
<td>7161</td>
<td>3.2</td>
<td>0.8</td>
<td>6.3</td>
<td>0.4</td>
<td>890</td>
<td>NSR</td>
<td>3800</td>
</tr>
<tr>
<td>21</td>
<td>7169</td>
<td>3.0</td>
<td>0.5</td>
<td>5.6</td>
<td>0.0</td>
<td>230</td>
<td>NSR</td>
<td>1900</td>
</tr>
<tr>
<td>22</td>
<td>7170</td>
<td>2.9</td>
<td>0.4</td>
<td>10.7</td>
<td>0.0</td>
<td>NSR</td>
<td>NSR</td>
<td>2200</td>
</tr>
<tr>
<td>23</td>
<td>7174</td>
<td>2.5</td>
<td>0.0</td>
<td>17.9</td>
<td>0.0</td>
<td>NSR</td>
<td>24</td>
<td>NSR</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average:</td>
<td>2.8</td>
<td>1.16</td>
<td>16.0</td>
<td>0.5</td>
<td>510</td>
</tr>
</tbody>
</table>

Note: NSR – no significant result
Iron oxide Cu-Au (IOCG) mineralizing systems: an example from northeastern Russia

Table II compares the Rep-Yuruinsky occurrence with major worldwide IOCG-type deposits in terms of the resource and grade of mineralization.

Application of multivariate geometric distribution to enhance the reliability of assessment using the Rep-Yuruinsky anomaly

The area of mineralization associated with the Verkhne-Chubukhulakhsky intrusive has been rock-chip sampled on a regular grid. The samples were crushed, pulverized, and analysed by atomic absorption spectrometry (AAS) for Au and by atomic emission spectrometry (AES) for Cu.

The results of geochemical sampling were tabulated in a Microsoft Access™ database, which is connected by the function join to the layer of geochemical sampling points. The key field for the connection is the geological number of the sample. The result is a system that uses the geostatistical analyst module for the interpolation of the surface by any element of the connected tables. Ordinary kriging is usually used for the interpolation method, by which means a surface for each element can be constructed.

It should be noted, however, that a single element may belong to various types of mineralization that may make up a deposit and determine its geological and industrial type. It is therefore difficult to characterize a deposit by the spatial distribution of a certain element. On the other hand, an attempt to divide a deposit into homogeneous areas in order to characterize its geometry also entails considerable difficulties, since the boundaries delineated by a single element may conflict with the boundaries built with the help of other elements (Rudenko and Vedyaev, 2009).

By contrast, the geometric distribution utilizes the full range of elements simultaneously, based on a probabilistic model of the mineralization and the new aggregated solution of the statistical problem by multivariate distribution (Rudenko and Vedyaev, 2009). This eliminates not only the conflict between boundaries, but also allows, on the basis of the statistical characteristics of the homogeneous groups of observations obtained, the use of mineralogical and technological mapping to establish types of mineralization. This makes it possible, even at early stages of exploration, to recognize the geological-mining type of the prospect.
Iron oxide Cu-Au (IOCG) mineralizing systems: an example from northeastern Russia

- Type 2 is located in the mass of the brecciated hornfels and is characterized by initially low permeability to hydrothermal solutions, leading to low concentrations of Au and U
- Type 3 is associated with the outermost zone of the exocontact of the intrusive massif, passing into unaltered sandstone
- Type 4 corresponds to the outer brecciated zone of the exocontact, where Fe hydroxide cement is weakly developed and veins are absent. This type is characterized by the highest content of U.

Exploration model for Arc GIS analysis

IOCG-type deposits usually occur in the vicinity of porphyry Cu-Au or Cu-Mo deposits, associated with alkali and calc-alkaline plutons from A (anorogenic) to I (igneous) types, enriched with U, F, Mo, and REE (Corryvieve, 2007). A-type granitoids are greatly enriched with Fe and comprise ferroan calc-alkaline and alkali types, most of them being metaluminous. I-type granitoids are mostly calc-alkaline or calcic and are of the magnesian type, while A-type granitoids are metaluminous, less often peraluminous (Frost et al., 2001).

The use of GIS to assess the potential of plutons plays an important role in the development of exploration models. Many deposits show a spatial relationship with plutons and this stimulates attempts to assess their potential for economic mineralization.

To date no exploration for IOCG deposits has been carried out in the territory of Eastern Yakutia, and the potential for this type of mineralization has not been discussed in the public domain. This paper represents the first attempt to assess the potential of the territory for IOCG-type mineralization.

For this purpose a GIS project was created, which includes:
- Basic geology with the location of plutons and their simplified geometry
- Geodatabase of major-element compositions of samples from the plutons (more than 4000 analyses)
- ArcGIS online service with 15 m eSAT images;
- Airborne magnetic data using a line spacing of 1 km.

The interpretation of Landsat images near plutons yielded a large number of areas with conspicuous colors indicating Fe oxides – from dark orange to reddish-brown (Kostin, 2011, 2012).

In the first phase of the investigation the prospective A- and I-type granitoids with Fe, Cu, Au, and U mineralization are recognized. These are then compared with the magnetic anomalies to guide further exploration.

In the second phase, the areas with elevated concentrations of Fe oxides are tested for their possible association with IOCG-type mineralization. Using the geodatabase of major-element compositions, samples from these areas are evaluated for their prospectivity for Fe, Cu, Au, and U mineralization.

Uranium

As exemplified by Australian deposits (Schofield, 2009), the major-element composition of rocks plays an important role in the behaviour of uranium. The highest uranium concentrations match to:

- Type 2 is located in the mass of the brecciated hornfels and is characterized by initially low permeability to hydrothermal solutions, leading to low concentrations of Au and U
- Type 3 is associated with the outermost zone of the exocontact of the intrusive massif, passing into unaltered sandstone
- Type 4 corresponds to the outer brecciated zone of the exocontact, where Fe hydroxide cement is weakly developed and veins are absent. This type is characterized by the highest content of U.

Copper and gold

For the assessment of the potential of plutons for Au-Cu and Cu-Mo mineralization, data from Mongolian copper-porphyry deposits (Gerel, 1995) was used. A K2O/Na2O ratio ranging from 0.3 to 0.7 was established for Cu-Mo magmatic systems, while for Au-Cu systems the range increases to 0.7–1.3.

From the geodatabase, samples with Au-Cu mineralization were selected based on ratios of K2O/Na2O from 0.7 to 1.3 using the inquiry function of the database. Therefore, classification of the GIS band was based on the feature pluton area, and the selected groups characterized by the extent of their erosional level.

In order to rank the number of prospects for Au-Cu mineralization, a filter has been used that selects plutons belonging to the high-potassium calc-alkaline and shoshonitic series.

The algorithm of the filter is based on formulae for the curves that describe the series of magmatic rocks in the diagram SiO2–K2O. The general form of the equation is K2O=k × SiO2 – b, and inquiries take the form of, e.g.:
- For rocks of the tholeiite series: K2O < (0.035346 × [SiO2] – 1.5)
- For rocks of the calc-alkaline series: K2O between (0.035346 × [SiO2] – 1.5) and (0.066507 × [SiO2] – 2.5)
- For rocks of the high-potassium calc-alkaline series: K2O between (0.066507 × [SiO2] – 2.5) and (0.169054 × [SiO2] – 7.12)
- For rocks of the shoshonite series: K2O > (0.169054 × [SiO2] – 7.12).

In addition to the inquiry using high-potassium calc-alkaline series plutons, further conditions have been applied:
- Size of area < 5 km2
- K2O/Na2O ratio 0.3–0.7 (for Cu-Mo systems) and 0.7–1.3 (for Au-Cu systems).

GIS query results


Using ArcGisOnline, the i-cubed 15 m eSAT images showed the presence of elevated concentrations of iron oxides in the vicinity of the Burgaliisky, Verkhne-Burgaliisky, Levo-Jolakanisky, Verkhne-Tirekhtyakhskiy, Druza, Pravo-Tuostakhskiy, and Takalkanskiy plutons.

Large magmatic systems with IOCG potential may have three types of Fe oxide mineralization – directly at the contacts with the intrusive rocks (bright yellow shades),
Iron oxide Cu-Au (IOCG) mineralizing systems: an example from northeastern Russia

along the edge of pluons (dark yellow and brown tints), and in hornfels (dark brown) – this is seen at Levo-Jolakagskiy, Burgalyisky, and Verkhne-Burgalyisky pluons as an example (Figure 5).

Conclusions

➤ The data obtained suggests that mineralization ascribed to IOCG-type deposits is widely manifested in Eastern Yakutia (northeast of Russia), but its economic potential and geological features are unknown

➤ An exploration model for IOCG mineralization has been developed using GIS methodology to define pluons with higher prospectivity and associated mineralization

➤ The potential for IOCG mineralization is estimated to be very high. For example, the Verkhne-Burgalyisky pluron contains several areas of iron oxide mineralization. The largest is about 155 km², which may yield an inventory of several billion tons of iron oxides. Even at low Au grades, this type of mineralization may be of economic interest

➤ A review of historic samples confirms the IOCG potential and a regional exploration model can be built. This may differ somewhat from the known analogues because of the geological history of Eastern Yakutia

Based on the classification of IOCG mineralization the first discovery in Eastern Yakutia is an iron-oxide-rich breccia with Au-U-Cu(±Bi±Mo±W) mineralization. It is located at the roof of the calc-alkaline/alkaline pluons, and is interpreted to correspond to a subtype of Olympic Dam (Australia)

The use of multidimensional geometric distribution features enhances the reliability of the assessment of mineralization type even at early stages of exploration with limited detailed data.

References


References


KOSTIN, A.V. 2008. Use of multidimensional geometric distribution features enhances the reliability of the assessment of mineralization type even at early stages of exploration with limited detailed data.

Figure 5—Potential IOCG- mineralizing system at Levo-Jolakagskiy, Burgalyisky, and Verkhne-Burgalyisky pluons as an example of GIS query for U and Au-Cu mineralization


Heterogeneity, stationarity, and kriging

Heterogeneity is an important concern when applying a statistical method to a spatial process, because it largely determines the choice of an appropriate modelling method and whether the chosen method can be effectively applied. Large heterogeneities generally cause stationary stochastic modelling methods to go astray (Delfiner, 1976; Ma et al., 2008). Universal kriging and intrinsic random function (IRF) techniques were proposed to deal with modelling nonstationary stochastic processes (Matheron, 1973). These techniques have been successfully used for topographic mapping and other applications (Delfiner et al., 1978; Chiles and Delfiner, 2012).

Stationary, locally stationary, and intrinsic random functions

IRF theory is a generalization of stochastic processes with independent increments. The latter implies uncorrelated first-order differences, such as a Brownian motion (Matheron, 1973; Papoulis 1965; Serra, 1984). A Brownian motion is not stationary, because the variance increases when the domain of study increases. This was coined an intrinsic random function of order 0 (IRF-0) by Matheron (1973). Besag and Mondal (2005) provided a bridge between spatial intrinsic processes (also called de Wijis process) and first-order intrinsic autoregressions. By a further extension, a stochastic process whose \((k+1)\)th differences constitute a stationary process is termed an intrinsic random function of order \(k\) or IRF-\(k\) (Matheron, 1973).

Let \(A\) denote the vector space of real measures in \(\mathbb{R}^n\) with finite supports. A second-order random function (RF) \(Z: \mathbb{R}^n \rightarrow L^2(\Omega, A, P)\) admits a linear extension \(Z: G \rightarrow L^2(\Omega, A, P)\) defined by

\[
Z(h) = \int h(dx) Z(x) \quad \text{for } h \in A
\]

which implies the strict positive definiteness of the covariance matrix \(<Z(x_1), Z(x_2)\>\) for any finite set of distinct points \(x_1\) and \(x_2\) in \(\mathbb{R}^n\). As an example, Wiener’s linear estimator is such a type (Wiener, 1949). An IRF-\(k\) is defined in a more restrictive way. A continuous function \(p(x)\) is chosen in a way that a subspace \(G\) is defined on the space \(A\) by

\[
G = \{h: h \in A, \int h(dx) p_j(x) = 0\} \quad \text{for } j=0, ..., k \text{ and } h(0) = -1
\]

As such, the linear mapping \(Z: G \rightarrow L^2(\Omega, A, P)\) is a generalized RF on the space \(G\).

For a nonstationary process, the covariance calculated from sample data can cause a serious bias in the prediction (Serra, 1984). Matheron defined a generalized covariance for IRF-\(k\) using the distribution theory (Matheron, 1973). It is generally difficult to characterize and construct an effective generalized covariance function in practice (Chauvet, 1989), but in most applications, the variance of the first order, called the variogram, suffices.

Synopsis

This paper presents a matrix formulation of factorial kriging, and its relationships with simple and ordinary kriging. Similar to other kriging methods, factorial kriging can be applied to both stationary and intrinsic stochastic processes, and is often used as a local operator. Therefore, the concepts of intrinsic random function and local stationarity are first briefly reviewed. Kriging is presented in a block matrix form in which the kriging solution is useful not only for understanding the relationships between simple and ordinary kriging methods, but also the relationships between interpolative kriging and factorial kriging. When used as a signal/noise-filtering method, factorial kriging is especially useful for multiscale modelling. Examples for general signal analysis and geophysical data signal filtering are given to illustrate the method.

Keywords

kriging in matrix form, locally stationary, block matrix, multiscale modeling, relationship between simple and ordinary kriging, filtering.

Factorial kriging for multiscale modelling

by Y.Z. Ma*, J.-J. Royer†, H. Wang‡, Y. Wang*, and T. Zhang§

Synopsis

This paper presents a matrix formulation of factorial kriging, and its relationships with simple and ordinary kriging. Similar to other kriging methods, factorial kriging can be applied to both stationary and intrinsic stochastic processes, and is often used as a local operator. Therefore, the concepts of intrinsic random function and local stationarity are first briefly reviewed. Kriging is presented in a block matrix form in which the kriging solution is useful not only for understanding the relationships between simple and ordinary kriging methods, but also the relationships between interpolative kriging and factorial kriging. When used as a signal/noise-filtering method, factorial kriging is especially useful for multiscale modelling. Examples for general signal analysis and geophysical data signal filtering are given to illustrate the method.

Keywords

kriging in matrix form, locally stationary, block matrix, multiscale modeling, relationship between simple and ordinary kriging, filtering.
### Factorial kriging for multiscale modelling

Stationarity of a stochastic process in a strict sense implies translation invariant of the probability density function in space or time (Papoulis 1965, p. 300). A wide sense or weak stationarity of a stochastic process assumes a constant expected value or mean and translation invariant of correlation function or variogram in space or time (Papoulis 1965; Matheron 1989). When applying kriging or factorial kriging with a local moving neighborhood, global stationarity is not required, but a local stationarity suffices. Local stationarity is a weaker assumption by definition (Matheron 1989). When applying kriging or factorial kriging, we generally recommend using a local operator. An example is presented in a later section.

### Matrix formulation of kriging

Several kriging methods have been proposed according to stationarity or non-stationarity assumptions, including simple kriging, ordinary kriging, universal kriging, and intrinsic kriging (Chiles and Delfiner, 2012). The applicability of each method depends on the physical problem of concern, and the availability of data.

Consider a random variable, \( Z(x) \), defined in a spatial domain such as:

\[
\{ \, Z(x) : x \in D \subseteq \mathbb{R}^n \}
\]

where \( x \) is the sampling location of the variable \( Z(x) \) within the defined domain \( D \), which is a bounded subset of the \( n \)-dimensional real space, \( \mathbb{R}^n \).

#### Simple kriging

Simple kriging uses an affine linear equation for spatial prediction, such as

\[
Z^*(x) = m + \sum_{j=1}^{n} \lambda_j (Z(x_j) - m)
\]

where \(\lambda_j\) is the vector for the simple kriging weights.

Because of the assumption of constant mean that can be estimated from the data, the kriging system can be obtained by minimizing the sum of the squared errors (the least-square method), and it can be expressed in the following matrix form:

\[
C_{zz} \Lambda_k = c_z \quad \text{or} \quad \Lambda_k = C_{zz}^{-1} c_z \quad \text{(4a)}
\]

where \( C(\cdot) \) represents the covariance; \( C_{zz} \) is the \( n \times n \) matrix of the spatial covariance of the data used for prediction, \( Z(x_i) \) to \( Z(x_j) \); \( c_z \) is the \( n \times 1 \) vector of the spatial covariance between \( Z(x) \) and the data \( Z(x_j) \) used for prediction; and \( \Lambda_k \) is the vector for the simple kriging weights. In an expanded form, Equation [4a] is written as

\[
\begin{pmatrix}
C_{11} & \ldots & C_{1k} \\
C_{21} & \ldots & C_{2k} \\
\vdots & \ddots & \vdots \\
C_{nk} & \ldots & C_{nk}
\end{pmatrix}
\begin{pmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_k
\end{pmatrix}
= 
\begin{pmatrix}
c_{z1} \\
c_{z2} \\
\vdots \\
c_{zk}
\end{pmatrix}
\]

The variance, \( \sigma^2_{0k} \), of the error, \( \varepsilon = Z(x) - Z^*(x) \), between the estimation and the truth can be expressed as follows:

\[
\sigma^2_{0k} = \sigma^2_{Z} - C_{zz}^{-1} \Lambda_k = \sigma^2_{Z} - C_{zz}^{-1} c_z
\]

with \( C_{zz}^{-1} \) the inverse matrix of the covariance matrix \( C_{zz} \), and \( \sigma^2_Z \) the variance of \( Z(x) \).

Simple kriging is applicable to phenomena with a strong assumption of global stationarity, which implies that the global mean can be calculated from the samples. Simple kriging is especially useful for spatial stochastic simulation.

#### Ordinary kriging

For an IRF-0, or locally stationary random fields, the mean is not known or cannot be estimated globally by averaging the sample values. The affine Equation [3] cannot be used for estimation; an ordinary linear combination is used instead:

\[
Z^*(x) = \sum_{j=1}^{n} \lambda_j Z(x_j) \quad \text{with} \quad \sum_{j=1}^{n} \lambda_j = 1
\]

where the constraint on the kriging weights \( \sum \lambda_j = 1 \) is imposed to the estimator \( Z^*(x) \) (i.e. the estimation errors being null on average, \( E(\varepsilon) = 0 \), with \( \varepsilon = Z(x) - Z^*(x) \)).

The ordinary kriging system can be obtained by minimizing the sum of the square errors under the constraint in Equation [5] by using the least-square and Lagrange methods, and can be expressed in the following block matrix equation:

\[
\begin{pmatrix}
C_{zz} & u \\
u^t & 0
\end{pmatrix}
\begin{pmatrix}
\Lambda_{ok} \\
\mu_{ok}
\end{pmatrix}
= 
\begin{pmatrix}
c_z \\
l
\end{pmatrix}
\]

The error variance, \( \sigma^2_{ok} \), is equal to:

\[
\sigma^2_{ok} = \sigma^2_{Z} - C_{zz}^{-1} \Lambda_{ok} \cdot \mu_{ok}
\]

where \( C_{zz} \) represents the sample covariance matrix, \( \Lambda_{ok} \) is the vector of the ordinary kriging weights, \( u \) is a unit vector with all the entries equal to 1, such as \( u = [1 \ 1 \ \ldots \ 1]^t \), superscript \( t \) is the vector transpose, \( \mu_{ok} \) is the Lagrange multiplier due to the constraint on the kriging weights which sum to 1, and \( c_z \) is the vector of covariance between the estimation point, \( x \), and each of the sample points, \( x_j \).

#### Relationship between simple kriging and ordinary kriging

Equation [6a] is an expanded matrix formulation from simple kriging (see Equation [4]), taking into account the constraint on the ordinary kriging weights. The block-matrix inversion formula (Appendix 1) is used to rewrite the square matrix on the left side of Equation [6a]. The solution is thus as follows:

\[
\begin{align}
\Lambda_{ok} &= C_{zz}^{-1} c_z - C_{zz}^{-1} u (u' C_{zz}^{-1} u)^{-1} u' C_{zz}^{-1} c_z + \\
C_{zz}^{-1} u (u' C_{zz}^{-1} u)^{-1} &= A_{sk} - A_m u' A_{sk} + \\
A_{m} &= A_{sk} + (1 - u' A_{sk}) A_{mm} = A_{sk} + \lambda_m A_m + \\
A_{m} &= C_{zz}^{-1} u (u' C_{zz}^{-1} u)^{-1} = C_{zz}^{-1} u \mu_m + \\
\lambda_m &= 1 - u' A_{sk} + \\
\mu_m &= -(u' C_{zz}^{-1} u)^{-1} + \\
\mu_{ok} &= (u' C_{zz}^{-1} u)^{-1} u' C_{zz}^{-1} c_z - (u' C_{zz}^{-1} u)^{-1} = \\
&- \lambda_m (u' C_{zz}^{-1} u)^{-1} = \lambda_m \mu_m
\end{align}
\]
Factorial kriging for multiscale modelling

where \( \mathbf{A}_k \) is the vector of simple kriging weights defined in 
Equation [4], \( \mathbf{A}_m \) is the vector of kriging weights for estimation of 
the mean, and \( \mu_m \) the corresponding Lagrange multiplier. The latter 
is obtained by replacing the vector \( \mathbf{c}_s \) with a zero vector of the same 
size in Equation [6a], because of the nil correlation between a random variable and its 
mean. \( \lambda_m \) is the weight of the mean in simple kriging, because Equation [3] can be expressed as

\[
Z^*(x) = (I - \sum_{j=1}^{n} \lambda_j)_m m + \sum_{j=1}^{n} \lambda_j Z(x_j) = \lambda_m m + \sum_{j=1}^{n} \lambda_j Z(x_j) \tag{8a}
\]

Equations [7a-d] can also be used to deduce an additive relationship among the error variances of the ordinary kriging, the local mean, and simple kriging. As the error variance \( \sigma^2_m \) in estimating the mean is equal to \( -\mu_m \) (Ma and Myers, 1994), the ordinary kriging variance \( \sigma^2_{ok} \) (Equation [6b]) can be expressed as a function of the simple kriging error variance and the estimation variance of the local mean:

\[
\sigma^2_{ok} = \sigma^2_{sk} + \lambda^2_m \sigma^2_m \tag{8b}
\]

Equations [7a-d] describe the relationship between the simple kriging weights and ordinary kriging weights, and Equation [8b] the relationship between the corresponding error variances. The ordinary kriging weights are expressed as the sum of the simple kriging weights and the ordinary kriging weights for the local mean multiplied by the weight for the mean in simple kriging. One obvious advantage of this formulation is the explicit expression of the impact of the constraint on the kriging weights. The error variance increases when using ordinary kriging because it is necessary to estimate both the local mean and the residuals.

Another advantage of Equations [7a-d] is the use of the weighting vector for the mean. By using a local operator under the local stationarity assumption, the local mean is not assumed to be known, and the estimator given by Equation [5] is equivalent to (see e.g., Matheron, 1971):

\[
Z^*(x) = m^*(x) + \sum_{j=1}^{n} \lambda_j[Z(x_j) - m^*(x)] \tag{9}
\]

with

\[
m^*(x) = \sum_{j=1}^{n} \beta_j Z(x_j) \tag{10}
\]

The kriging solution for the local mean is given in 
Equation [7b]. Furthermore, the estimation for the random function, \( Z(x) \), can be done with simple kriging using Equation [9], which yields the same solution as in Equations [7a-d]. In other words, the estimation of the random function by ordinary kriging is identical to the combination of the estimation of the mean using ordinary kriging and the estimation of the residual by simple kriging. This is termed the additivity theorem, which is also valid for universal kriging (Matheron, 1971; Ma, 1987). In practice, as a result of using a sliding window, \( m^*(x) \) describes a low-frequency, large-scale component of the multiscale RF \( Z(x) \), which is related to factorial kriging.

Factorial kriging for multiple-scale modeling

Although kriging has been used most commonly for spatial interpolation, it can also be used for filtering. Typically, filtering is a decomposition based on the multiple scales of variations in a physical process. The filtering method in geostatistics is termed factorial kriging (Matheron, 1982; Ma and Royer, 1988). This method has been used in a variety of scientific applications, including signal and image processing (Ma and Royer, 1988; Wen and Sinding-Larsen, 1997; Van Meirvenne and Goovaerts, 2002), petroleum exploration (Jaquet, 1989; Du et al., 2011), soil description (Goovaerts and Webster, 1994; Bochi et al., 2000), geochemistry (Reis et al., 2004), water resource monitoring (Yeh et al., 2006), seismic data analysis (Yao et al., 1999; Abreu et al., 2005), ecology (Lin et al., 2008), crime risk pattern analysis (Kerry et al., 2010), and health risk analysis (Goovaerts et al., 2005, 2009; Dubois et al., 2007). It assumes that the observed physical process can be interpreted as a linear combination of several sub-processes that generally exhibit different scales with different spatial dependencies, such as

\[
Z(x) = \sum_{i=1}^{q} a_i Y_i(x) + T(x) \tag{11}
\]

where \( Z(x) \) is the RF representing the observed (though often only partially observed) physical process, \( Y_i(x) \) represents a component RF or a sub-process at a certain scale of variation, \( a_i \) are normalization coefficients, and \( T(x) \) is a trend function which can be approximated using orthogonal or trigonometric polynomials (Royer, 2008). The number of components \( q \) can be chosen according to the number of nested terms used in this decomposition.

As such, factorial kriging can decompose the random process, \( Z(x) \), into several sub-processes of different scales linked with the spatial correlation structures. In theory, all the RFs, \( Z(x) \) and \( Y_i(x) \), can be an IRF-k (Matheron, 1982; Ma, 1987). Kriging prediction of these RFs can then use a generalized covariance, defined as conditionally positive definite. The predictor of the component RFs \( Y_i(x) \), is formed as a linear combination of known data of the original (composite) RF, such as

\[
Y_i^*(x) = \sum_{j=1}^{n} \lambda_j[Z(x_j) - T^*(x_j)] \tag{12}
\]

The trend function is estimated by the following linear combination:

\[
T^*(x) = \sum_{j=1}^{n} \beta_j Z(x_j) \tag{13}
\]

The kriging systems used to estimate the components and the trend can be obtained by minimizing the sum of the squared errors under the constraint using the least-square and Lagrange methods, and they can be expressed in the following block matrix equations (Ma, 1987, 1993):

\[
\begin{pmatrix}
C_{zz} & P \\
P_{1} & 0 \\
\end{pmatrix}
\begin{pmatrix}
\mathbf{A}_{zf} \\
\mathbf{L} \\
\end{pmatrix} = \begin{pmatrix}
c_{ze} \\
p \\
\end{pmatrix} \tag{14}
\]
where $C(.)$ represents the generalized covariance; $C_{zz}$ is the $n 	imes n$ matrix of the spatial covariance of the data used for prediction; $C_{zy}$ is the $n 	imes 1$ vector of the spatial covariance between $Y(x)$ and the data $Z(x_i)$; $P$ and $p$ are, respectively, the $nk 	imes k$ matrix and $k 	imes 1$ vector of a chosen analytical function $p(x)$ for fitting the nonstationary component; $A_y$ is the vector for the kriging weights; and $L$ is a vector of Lagrange multipliers. The zero vector on the right-hand side of Equation [15] is a result of the non-randomness of the trend, $T(x)$.

Equations [14] and [15] are the kriging systems for estimating the zero-mean component $Y(x)$ and the trend $T(x)$, respectively. Similar to the ordinary kriging system discussed earlier, the weighting vector can be obtained by using the block matrix inversion method (Appendix 1), and the solutions are

$$A_y = C_{zz}^{-1}C_{zy}P(C_{yy}^{-1}P)^{-1}$$

$$A_T = C_{zz}^{-1}P(C_{yy}^{-1}P)^{-1}p$$

$$L_y = (C_{zz}^{-1}P(C_{yy}^{-1}P)^{-1})^+C_{zy}$$

$$L_T = (C_{zz}^{-1}P)^+p$$

[15]

The estimation variances of the components, $Y_i$, and the trend, $T$, are given by:

$$\sigma^2_{Y_i} = \sigma^2_Z - \langle c_{y_i}A_y \rangle + p^T L_y$$

$$\sigma^2_T = p^T L_T$$

[16a] and [16b]

For interpolation of the original RF, $Z(x)$, the counterpart to Equation [16a] is expressed as

$$A_z = C_{zz}^{-1}c - C_{zz}^{-1}P(C_{yy}^{-1}P)^{-1}P C_{zz}^{-1}c_z + A_T$$

[16c]

As the components, $Y_i(x)$, are assumed to be orthogonal in factorial kriging, the additive relationship of covariances is such that $c_z = \Sigma c_{y_i}$ (Ma and Myers, 1994). Thus, Equations (16a) and (16b) verify the coherence condition in factorial universal kriging and factorial kriging for IRF-k:

$$A_z = A_T + \Sigma_i A_{y_i}$$

These equations are valid for ordinary kriging but with a simplified formulation (Ma, 1993). For most applications, these components can be considered to be locally stationary. The local stationarity eases the ergodicity hypothesis, and makes the spatial prediction suitable to a local operation (Matheron, 1989).

Moreover, assuming local stationarity, all the decomposed sub-processes are estimated from the observed composite process, such as:

$$Y_i(x) = m_{yi}(x) + \Sigma_j p_{ij}[Z(x)-m_z(x)]$$

or in matrix form:

$$Y_i(x) = m_{yi}(x) + A_{yi}[Z - m_z(x)]$$

[17] and [18]

Where $m_{yi}(x)$ is the locally varying mean for the component $Y_i(x)$, $Z$ is the data vector, $m_z$ is the locally varying mean of $Z(x)$, and $u$ is a unit vector with all the entries equal to 1. Because the mean is estimated, Equation [17] is not a true affine linear combination, despite its affine-like form.

**Discussion**

It is noteworthy that kriging is an exact interpolator, meaning that the kriging estimator is equal to the known value if the latter is estimated, or the sample data are all honoured (Armstrong, 1998, pp. 97–98). This is not the case for linear regression. Because factorial kriging is a probabilistic decomposition, not an interpolation by design, it is purely a filtering process at the known locations; but in the unknown locations it is also an interpolation, inherited from kriging (Ma, 1993).

The estimation of the components by factorial kriging can be considered as an ecological inference (Robinson, 1950; Wakefield, 2004), since the component processes are estimated using data from the composite or total process. It is well known that ecological inference can cause a bias in estimation (Gotway and Young, 2002; Ma, 2009). However, it is possible to objectively identify sub-processes based on the specific application by using the contextual information. For example, in image processing it is commonly useful to filter the noise and enhance the signal. The noise and signal represent different scales of variations, or different frequency contents from a viewpoint of spectral theory. Spatial filtering by factorial kriging is generally based on a nested covariance model. Some researchers have questioned the tenability of nested models (Stein, 1999, pp. 13–14). A nested covariance model may not gain much for the purpose of spatial interpolation, because of the inherent uncertainty in empirical spatial covariance or variogram for most applications. However, this is an important step for random field decomposition and signal filtering. It is tenable when combined with the contextual information, especially if the sample data are large enough. Two examples are discussed in the next section.

**Application to signal analysis and noise filtering**

In image processing, decomposition by factorial kriging is sometimes used for visual interpretation, either for filtering noise or extracting a specific feature (Ma and Royer, 1988; Wen and Sinding-Larsen, 1997). Here, an example of filtering noise and extracting signal illustrates how to simultaneously model two differently-scaled spatial heterogeneities. The factorial kriging works for any number of components for different scales as shown in Equation [11], but the examples presented here include two components only.

Kriging with an unknown mean, either stationary or nonstationary, for a two-component model can be represented by a signal plus an additive noise model:

$$Z(x) = S(x) + N(x)$$

[19]

where $S(x)$ represents a larger-scale-component random function, $N(x)$ represents a smaller-scale-component random function, and $Z(x)$ represents the composite random process.
Factorial kriging for multiscale modelling

Modelling the three random functions can be performed simultaneously using the sampling data of \( Z(x) \).

\[
Z^* (x) = \sum_{j=1}^{n} \lambda_j Z(x_j), \quad \sum_{j=1}^{n} \lambda_j = 1 \tag{20}
\]

\[
S^* (x) = \sum_{j=1}^{n} w_j Z(x_j), \quad \sum_{j=1}^{n} w_j = 1 \tag{21}
\]

\[
N^* (x) = \sum_{j=1}^{n} h_j [Z(x_j) - m_z(x_j)], \quad \sum_{j=1}^{n} h_j = 0 \tag{22}
\]

Note that as a result of non-bias constraint on the estimators, the sum of the weights for all sample points and the mean is equal to unity, whereas the sum of the weights for the noise is equal to zero. The weighting function can be considered as a transfer function.

Figure 1 shows an example of filtering the noise in image processing. The Lena picture is widely used in 2D signal analysis for noise filtering and feature detection. First, the variogram of the noisy digital picture (Figure 1a) was calculated, and then the calculated variogram was fitted into a theoretical model (Figure 1b), including a nugget effect of 365 intensity in square (IIS), an exponential variogram with the sill equal to 570 IIS and the range equal to 10 pixels, an exponential variogram with the sill equal to 1270 IIS and the range equal to 27 pixels, and a spherical variogram with the sill equal to 620 IIS and the range equal to 36 pixels. The nugget effect component in the variogram at the origin is due to the presence of white noise. Factorial kriging is used to filter out the noise by eliminating the nugget effect. Specifically, using the matrix notation under the local stationary model (Equation [18]), the signal weighting vector

\[
W_s = C_{zz}^{-1} c_{sz} - C_{zz}^{-1} u (u'C_{zz}^{-1} u)^{-1}
\]

\[
u' C_{zz}^{-1} c_{sz} + C_{zz}^{-1} u (u'C_{zz}^{-1} u)^{-1}
\]

where \( C_{zz} \) represents the spatial covariance matrix for the noisy picture, and \( c_{sz} \) is the vector of spatial covariance between the noisy picture and the signal. All other terms are defined in Equations [7a-d]. The spatial covariance or correlation terms are calculated from the variogram in Figure 2b using the following relations (Journel and Huijbregts, 1978):

\[
C(h) = C(0) - \gamma(h)
\]

where \( C(h) \) is the covariance, \( C(0) = \sigma^2 \) the variance, and \( \gamma(h) \) the variogram of the spatial variable \( Z(x) \).

Figure 1c shows the denoised picture (i.e., the signal component of the noisy picture in Figure 1a). In this application, the noise is not of interest, and thus is not shown. The signal contains several different scales of information related to not only the two exponential variograms and one spherical variogram stated above, but also the locally varying mean or the trend that is expressed by Equation [10] or [13]. In other words, the signal itself still has several scales of information.

Another example of filtering noise, in a seismic attribute, is shown in Figure 2. The noisy attribute had a correlation coefficient of 0.53 with the measured porosity. After filtering out the noise using factorial kriging by eliminating the nugget-effect component, the correlation was improved to 0.79. This is because the noise component had no correlation to the porosity as the noise represents the smallest-scaled
Factorial kriging for multiscale modelling

component, and removing it is equivalent to matching the more correlated parts of the two variables. In this sense, factorial kriging is a disaggregation that enables the identification of the scale of information and thus provides a means for matching the scale between the two variables.

Concluding remarks
Kriging was initially developed as an interpolation method dealing with stationary or mild non-stationary (IRF-0) processes. With many extensions, there are now a variety of kriging methods to deal with multiscale problems of natural phenomena, including universal kriging, IRF-k, and factorial kriging. Although IRF-k is theoretically elegant, it is often difficult to use, and does not always give good results, often because of an inadequate identification of the complexity of heterogeneities (Ma, 2010). Defining a multilevel model based on the hierarchy of scales can deal with multiscale problems more effectively for many applications (Ma et al., 2009). In such a framework, factorial kriging offers a filtering technique that explicitly decomposes the composite phenomenon of multiple scales into component processes. In some cases, even ordinary kriging, if a local neighborhood is utilized, can be used to deal with two-scale problems as shown by Equation [7]. These methods are also applicable to multivariate geostatistics that involves multiple physical variables (Wackernagel, 2003).

Acknowledgements
The authors thank Schlumberger Ltd for permission to publish this work, and Dr David Psaila for reviewing an early version of the draft. Dr. Hongliang Wang is the correspondence author, and his work was partly funded by the National Natural Science Foundation (Grant No. 91114203) and the National Science and Technology Major Projects framework.

References


Figure 2—Example of filtering noise in a seismic attribute using factorial kriging. (a) Noisy attribute (8 × 8 km), (b) varigrams of the noisy attribute (symbol ‘-’) and denoised attribute (symbol ‘*’); the lag distance is in km, (c) attribute after noise filtered by factorial kriging, (d) scatter plot between the porosity and noisy seismic attribute, (e) scatter plot between the porosity and seismic attribute denoised by factorial kriging.
Factorial kriging for multiscale modelling


Appendix 1. Block Matrix Inversion

Consider a block matrix, such as

\[
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\]

Where \(A_{11}\) and \(A_{22}\) are square matrices, and \(A_{12}\) and \(A_{21}\) are matrices or vectors.

Its inverse is

\[
\begin{pmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{pmatrix}
\]

Where \(B_{11}\) and \(B_{22}\) are square matrices, and \(B_{12}\) and \(B_{21}\) are matrices or vectors. They are of the same sizes as their corresponding \(A_{ij}\).

Two solutions exist. The first solution is:

\[
B_{22} = (A_{22} - A_{12}A_{11}^{-1}A_{12})^{-1}
\]

\[
B_{12} = (A_{12} - A_{12}A_{11}^{-1}A_{12})A_{22}A_{11}^{-1}A_{12}^{-1} = B_{22}A_{11}^{-1}A_{12}^{-1}
\]

\[
B_{11} = A_{11}^{-1} - A_{11}^{-1}A_{12}A_{22}A_{12}^{-1}A_{11}^{-1} = A_{11}^{-1} + B_{22}A_{11}^{-1}
\]

The second solution is

\[
B_{11} = (A_{11} - A_{12}A_{22}^{-1}A_{12})^{-1}
\]

\[
B_{12} = A_{12}A_{22}^{-1}A_{11} - A_{12}A_{22}^{-1}A_{12}A_{22}^{-1}A_{11} = A_{12}A_{22}^{-1}A_{11}^{-1}A_{12}
\]

\[
B_{22} = A_{22}^{-1} - A_{12}A_{22}^{-1}A_{11} - A_{12}A_{22}^{-1}A_{12}A_{22}^{-1} - A_{12}A_{22}^{-1}A_{11}^{-1}A_{12}
\]

The matrices \(A_{11} - A_{12}A_{22}^{-1}A_{12}\) and \(A_{22} - A_{21}A_{11}^{-1}A_{12}\) are sometimes referred as the Schur complements (Haynesworth, 1968). In ordinary kriging, \(A_{22}\) is the scalar 0. Therefore, its inverse does not exist and the first solution is used.
HazardAvert®

Strata Products Worldwide, LLC is the OEM for the proximity detection system used to warn personal and stop the underground Inby vehicles in their tracks. Recently this system proved itself by saving a life.

Event

The Continuous Miner machine operator was standing in an intersection aligning the Continuous Miner with the laser beam when he felt dizzy and fell down. A Colleague saw what happened and rushed to his aid as he had fallen in the direct path of the approaching Schuttle Car. Only after the Schuttle Car tripped due to the activated PDS system of the two persons in the danger zone (3m from the machine) the shuttle car operator saw them and realized that the PDS prevented a serious accident.

Coal Mining

HazardAvert® components are enclosed in explosion-resistant armour and have local test authority approval (MASC). When installed on continuous miners, HazardAvert® works to prevent crushing- and pinning-type accidents that can be prevalent with these machines. The presence of these safety fields may, however, force the operator to stand in dangerous positions during operation. To prevent this, Strata configured optional dynamic field configurations to enable the fields to be reduced when cutter heads are in coal or rock, allowing the operator to work normally while remaining protected.

HazardAvert® on haulage vehicles protects workers when these machines are in motion. Fields are at their largest during tramming, ensuring the protection of individuals in the vehicle’s path or those who may unintentionally enter its path. Miners working in a blind spot or around a corner are also protected by HazardAvert®’s ability to penetrate substrates such as coal or rock. Since haulage vehicles are required to interact with other machinery and with miners, dynamic configurations enable the fields to shrink as the vehicle approaches a machine and permits interaction without jeopardising safety.

Surface Mining

On the surface, HazardAvert® works to reduce accidents and collisions between mobile vehicles and machinery. Fields are designed to cover all blind spots and can be customised to specific operations and needs. Compared with other proximity detection products, the HazardAvert® system offers the highest precision and fewest nuisance alarms for near field applications, states Strata.

Data Retrieval

HazardAvert® monitors and logs all activities and interactions. It registers personnel outside of its warning zone even when no alarms have been triggered. The collected data is stored on board the machine and is identifiable by individual. This creates a useful tool in location tracking, productivity measurements and even in assisting in incident reports and investigations. The data can be retrieved using Bluetooth® or by transmission over a wireless communications.

For more information

SP Mine Safety (Pty) Ltd SA GM Jonathan Metcalf
Tel +27 (0)12 450 0960
Email: info@strata-safety.co.za
www.strataworldwide.com
Application of a localized direct conditioning mineral resource modelling technique for medium- and long-term planning of underground mining operations

by W. Assibey-Bonsu*, C. Muller†, and H. Pretorius‡

Synopsis
At the exploration stage for new mining projects or for medium- and long-term areas of existing mines (typical of South African gold mines), drilling data is on a relatively large grid. This grid is normally larger than the selective mining units (SMUs). Direct estimates for the SMUs and also of much larger block units will then be smoothed due to the information effect and the high error variance. Any capital-intensive project decisions made on the basis of any of the smoothed estimates will tend to misrepresent the economic value of the project or operation, i.e. the average grade of blocks above cut-off will be underestimated and the tonnage overestimated for cut-off grades below the mean grade of the orebody.

This paper presents a direct approach technique for deriving recoverable resources, referred to as localized direct conditioning (LDC), designed to correct smoothing effects and also to provide support corrections.

Keywords
direct conditioning, localized direct conditioning, smoothing effect, conditional biases, information effect, mine call factor, post-processing.

Introduction
For new mining projects or for medium- to long-term areas of existing mines, drilling data is on a relatively large grid. This grid is normally larger than the block size of a selective mining interest, i.e. the selective mining units (SMUs). Direct estimates for SMUs and also of much larger block units will then be smoothed due to the information effect and the high error variance of the estimate.

During actual mining, selection will be based on the estimated grades of SMUs utilizing the final close-spaced data grid, which will be available at that time. However, this ultimate position is unknown at the project feasibility stage or for medium- to long-term planning purposes, and therefore has to be estimated. Any capital-intensive project decisions made on the basis of any of the smoothed estimates will tend to misrepresent the economic value of the project or operation, i.e. the average grade of blocks above cut-off will be underestimated and the tonnage overestimated for cut-off grades below the average grade of the orebody.

Various post-processing techniques have been proposed to correct for this smoothing feature in estimates, such as spectral post-processor (Journel et al., 2000), conditional simulations, as well as indirect post-processing techniques (Assibey-Bonsu and Kriège, 1999; Marcotte and David, 1985). The general indirect approach to the problem above entails deriving the unknown SMU distribution from the observed distribution of relative large kriged blocks. Alternatives for the indirect approach are uniform conditioning (UC) and the assumed lognormal distribution of SMUs within large planning blocks.

This paper presents a direct approach correction technique developed within the Gold Fields Group. It uses the recoverable functions on direct estimates of individual selective mining units within the orebody (instead of indirect SMU distribution within large kriged blocks as used for UC). In essence, the direct conditioning approach is based on the principle of replacing these direct smoothed SMU kriged estimates with probability distributions representing the expected follow-up ‘actual’ grades. The probability distribution uses each smoothed SMU kriged estimate as the mean of the distribution at an uncertainty variance level that is equal to that expected for the ‘actual’ SMU grades (after correcting for the expected final production information effect). The direct approach and its equivalent, referred to as localized direct conditioning (LDC) estimates in this paper, provides a solution to certain practical difficulties of using indirect probability recoverable estimates for underground mine planning and design, before more data become available.

* Gold Fields Ltd, Perth, Australia.
† Minesoft Group, Johannesburg, South Africa.
‡ South Deep Mine, Gold Fields Ltd, Johannesburg, South Africa.
© The Southern African Institute of Mining and Metallurgy, 2014. ISSN 2225-6253.
Application of a localized direct conditioning mineral resource modelling technique

Description of methodology
The direct approach based on direct estimates of SMUs within the population area (Assibey-Bonsu and Krige, 1999) as used in the paper is summarized as follows.

Definitions:

- $S$ = Selective mining unit (SMU) of interest
- $P$ = Entire population or global area being estimated
- $BV_j$ = Variance of ‘actual’ SMUs in $P$ (the theoretical dispersion variance of $S$ in $P$)
- $\sigma^2_{w1}$, $\sigma^2_{w2}$ = Error variances of conditionally unbiased estimates at exploration (or for long-term estimates) and final production stages for SMUs respectively.

Dispersion variances of conditionally unbiased estimates for direct SMUs

\[
BV_j - \sigma^2_{w1} = [1]
\]

\[
BV_j - \sigma^2_{w2} = [2]
\]

Dispersion variance at exploration stage from Equation [1] has to be adjusted up to that of the final production stage, i.e. Equation [2] by:

\[
BV_j - \sigma^2_{w2} - (BV_j - \sigma^2_{w1}) = \sigma^2_{w1} - \sigma^2_{w2} = [3]
\]

Thus, the LDC estimates are derived from the initial direct smoothed kriged estimates of the SMUs. The initial direct conditioning (DC) is derived by superimposing on the kriged estimate for each SMU block, a simulated lognormal distribution of expected ‘actual’ values with a variance equal to the difference in variability between the smoothed and ‘actual’ grades (Equation [3]). The simulated probability distributions have been derived using lognormal distribution of SMUs based on observed lognormal distribution of the point data (see Krige, 2003). The end result is an estimated unsmoothed grade-tonnage curve to replace the smoothed kriged equivalent.

Localization of estimates
The LDC methodology referred to in this paper extends the localized UC technique as proposed by Abzalov (2006). Abzalov uses the grade-tonnage functions from the large panel indirect estimates (derived from UC) and then decomposes the panel-specific grade-tonnage data into a suite of individual SMU grades within their respective panels. Abzalov suggested that the individual parcel grades of the SMUs derived from the decomposed UC approach be assigned to the SMU-size blocks within the respective panels based on a ranking of direct SMU grade estimates.

The DC post-processed grade-tonnage curves (Abzalov proposed use of the indirect equivalent) were summed within 120 m x 120 m areas to derive DC estimation results that are representative of the larger panels. These panel results were then decomposed as discussed above and used to assign non-smoothed individual SMU grade values to the respective SMU-size blocks. In essence, the LDC is derived by ranking of the original smoothed SMU simple kriging (SK) block estimates within the 120 m x 120 m areas in increasing order, and these are subsequently assigned the mean grade of a parcel (or a grade class) from the post-processed DC to the SMU blocks that matches the grade class in terms of rank. The LDC approach provides some advantages (including spatially located SMUs) particularly for underground mine design, where stope designs using indirect probability recoverable estimates have practical limitations.

Summary description of the LDC technique used in this paper
The basic process for LDC consists of the following steps:

➤ Derive simple kriging (SK) estimates of SMUs
➤ Estimate the local conditional probability distribution (lcpd) of non-smoothed grade estimates for each SMU (direct conditioning). The lcpd uses each smoothed SMU kriged estimate as the mean of the distribution and an associated variance parameter that is equal to that expected for the ‘actual’ SMU grades, (after correcting for the expected final production information effect). The lcpd process provides probability distributions of exceeding cut-off and grade above cut-off values for the respective SMUs (essentially a grade-tonnage curve per SMU)
➤ Agglomerate individual SMU grade-tonnage results into a ‘panel’ (in this case study a 120 m x 120 m panel is used). The aggregated probabilities of exceeding cut-off are then re-interpreted as ‘proportions’ of the panel that exceed the cut-off
➤ Decompose the resultant ‘panel’ grade-tonnage function to yield unsmoothed SMU grades; the linkage of these grades to SMU is obtained through ranking of the original smoothed SMU estimates within respective ‘panels’.

The direct post-processing techniques effectively provide (for a set of blocks) the average proportion of ore and the corresponding grade that can be expected to be selected for mining when grade control data becomes available. The direct post-processing technique also provides for each SMU a probability that the SMU will exceed cut-off and the corresponding conditional grade estimate for that SMU. Note that such techniques can be applied only to block estimates that are conditionally unbiased. As demonstrated by Krige (1996), conditionally biased block/SMU estimates could display negative block efficiencies, leading to unreliable SMU estimates and inefficient inputs into the lcpds. These unsmoothed post-processed estimates (DC and LDC) can then be used in the medium term, as well as life-of-mine plan or for feasibility studies. However, the method does not provide (and no technique can ever fully provide, at an early stage) a
practical detailed definition of the grades of the in situ individual SMU, which will eventually be selected or discarded. This is due to the limited drilling data and wide data spacing characteristic of the exploration stage for new mining projects or for medium- and long-term areas of existing mines.

**Geology**

The LDC methodology has been developed for Gold Fields’ South Deep underground mine, where stope designs using probability recoverable estimates on indirect bases have practical limitations. South Deep is situated in the geologically unique and renowned Witwatersrand Basin, which is the world’s premier gold region.

The reefs are widely considered to represent extensive fluvial deposits into a yoked basin, some 350 km long in an east-northeasterly direction and 200 km wide in a north-northwesterly direction. The gold is mainly of detrital origin, deposited syngenetically with the conglomerates and associated with sedimentary features such as unconformities and fluvial channels (Pretorius, 1991).

Locally, the reef horizons that are currently mined at South Deep include the Ventersdorp Contact Reef (VCR) and the Upper Elsburg reefs of the Mondeor Formation. In the western half of the mining lease area the VCR occurs as a single reef horizon that overlies footwall lithologies of the Turffontein Subgroup.

The Upper Elsburgs, which suboutcrop beneath the VCR in a north-northeast trend, comprise multiple stacked reef horizons that form part of an easterly-divergent clastic wedge as illustrated in Figure 2. The wedge attains a thickness of about 120 m to 130 m in the vicinity of the eastern boundary of the mining right area. The Upper Elsburg Reefs constitute 99% of the South Deep mineral reserve ounces, while the VCR makes up the remaining 1% (Osburn et al., 2014).

**Database and summary of mineral resource assessment process used on the mine**

The database consists of surface and underground grade-control diamond drilling originating from exploration, and resource and grade-control drilling data. For sample support requirement, the gold data-set is composited on one-metre intervals.

Sampling of the surface diamond core is done on half-core basis, while underground grade-control holes are sampled on a full-core basis. Every tenth sample is a control sample and alternates between a blank sample or a certified dual standard (both gold and uranium). For thinner intersections, which contain less than ten samples, a blank sample is inserted at the start of the sampling sequence and a certified reference material at the end.

Surface diamond core QA/QC control is based on two standard deviations from the mean expected value of the certified reference material, while that of the in-stope grade-control core samples is based on three standard deviations. Blank material QA/QC control is based on three times the detection limit.

Batches of 20 samples are dispatched to the laboratory containing one blank and one certified standard reference sample. It is also required that at least 90% of the pulverized material should pass 75 μm.

If either of the two control samples in a batch fails the control criteria, a new reference material is placed in the batch and the whole batch is then re-assayed. If a batch fails the second round, a further new reference material is included in the batch. The batch will then be submitted to a third-party laboratory (umpire) for re-assay. Batches that further fail at an umpire laboratory level are not authorized for any resource estimation work. Blind repeat pulp samples are re-submitted under new sample identifiers for precision analyses. These blind samples constitute a total of 5% of the total samples submitted in a quarterly period.

**Geological modelling**

Geological models are based on all available structural, grade, and sedimentological data. The structural data is used to generate 3D models, while the sedimentological, gold accumulation, gold grade, and channel width data is used to delineate geologically homogeneous local facies zones or geozones. These geozones or domains are used to constrain the statistical and geostatistical analyses on a soft domain basis, taking into account the drilling density in respective domains. The geology and the resource estimation models are updated on an ongoing basis as new data becomes available.

Various geology modelling software packages are used to develop the geological models and generate resource envelopes, taking cognisance of the geological models and
Application of a localized direct conditioning mineral resource modelling technique

geozones. Cross-referencing of the relevant plans and detailed checks ensure integrity of the defined envelopes. The resource estimation, including the development of LDC recoverable estimates, was derived using Gold Fields’ in-house software (Resource Evaluation System – RES).

Statistical and variogram analyses
Descriptive statistics are applied to develop an understanding of the statistical characteristics and sample population distribution relationships. Descriptive statistics in the form of histograms and probability plots (Figure 3) are analysed to evaluate the lognormality of the respective domain distributions, since the post-processing phase of the analysis assumes lognormality of the distribution. All other relevant analyses, including variograms and kriged grades, are derived in the untransformed space.

The statistical analysis also facilitates the application of top-cut values for the variography and kriging processes. Some of the project areas and domains require top-cutting at the variography stage. These top-cuts are applied to minimize excessive variance of the experimental variogram caused by grade outliers. The top-cut values are derived from review and calculation from the normal and log probability plots. High nugget effects are common and are usually in the order of 40% to 50% of the total sill. High nugget effects contribute to poor initial SMU grade estimates and also impact negatively on the LDC recoverable estimates when limited data is available (especially for long-term areas of existing mines).

Selective mining units
All SMU sizes are defined according to possible equipment, mining method, and mining selectivity, together with the geology of the orebody. At South Deep, different SMU sizes are used taking into account different mining methods, including longhole stoping.

Block estimation methods
Both ordinary kriging (OK) and simple kriging (SK) techniques are applied in developing the pseudo SMU blocks (30 m × 30 m × 1 m) estimation grade models. However, for post-processing of the recoverable resources which are used for mine planning, the SK panel grades are used. This is as a result of the relatively efficient SK estimates as reflected by higher kriging efficiencies observed for the SK panel estimates in most relevant areas of the operation, and especially in areas of limited data. Theoretically, the slope of regression of an SK estimate is unity. In this regard, the SK panel estimates are conditionally unbiased. The SK process uses a local or global mean in the kriging process depending on availability of data. If insufficient samples exist to support the local mean then the ‘global’ mean of the respective geozone is used in the SK process. In providing the mean value used in the SK estimation, historical mined-out information is taken into account for the respective reef horizons, to ensure that the input SK mean values are efficient. It is critical that the SK input means are analysed for robustness as inappropriate choice of this value may propagate biases, particularly in the presence of a drift. In addition, the estimation error associated with the SK input means can be incorporated in the SK estimation process. The weight assigned to the mean can also be used as an indicator of the quality of the estimates.

Kriged block model checks
Various checks are performed on the kriged block models to ensure grades are assigned correctly to model cells. The first check entails viewing the composite data with block model cells to check that the grade values in the boreholes relate to block model cell values. Other checks performed include the number of samples used in estimation, kriging efficiency, regression slopes, block distance from samples, and search.

Figure 3—Typical histograms and log-probability plots
volumes. The estimation process also produces a log file to check outputs, including average raw data versus average kriged values, minimum and maximum values, and SK versus OK values.

**Post-processing to derive LDC recoverable estimates**

The kriged blocks (30 m x 30 m x 1 m) are subjected to LDC post-processing as discussed under methodology, and each SMU block is assigned a non-smoothed individual grade value (see Assibey-Bonsu and Krige, 1999; Krige, Assibey-Bonsu and Tolmay, 2008). These non-smoothed post-processed LDC estimates are used for mine planning. A final production grade-control grid of 30 m x 30 m has also been assumed in deriving the recoverable resources, which is based on the expected final production grade-control drilling on the mine.

Figure 4 compares the smoothed SK panel estimates with the unsmoothed LDC estimates for the ECMC economic reef horizon (the *in situ* case study shown later is for the MBB reef). The grade-tonnage curves based on the non-post-processed smoothed estimates generally show that the average grade of blocks above cut-off will be underestimated and the tonnage overestimated. Thus, any capital-intensive project decisions or mine planning made on the basis of any of the smoothed estimates will tend to misrepresent the economic value of the project. In certain cases some deviations have been observed between the LDC and the original recoverable functions at very high cut-offs. This could be associated with the relatively small number of SMUs within the panels and also at these high cut-offs.

**Resource classification**

The Mineral Resource classification is a function of the confidence of the whole process from drilling, sampling, geological understanding, and geostatistical relationships. The following aspects or parameters are considered for Mineral Resource classification:

- **Sampling – quality assurance and quality control (QA/QC)**
- **Geological confidence**
- **Slope of regression and kriging efficiency**
- **Deviation based on the lower 90% confidence limit estimates.**

**Summary of production reconciliation results following the introduction of LDC on the mine**

**In situ reconciliation of LDC resource estimates and grade control estimates**

The LDC resource estimation technique aims at providing medium- to long-term estimates. As a result, the mine has embarked on a planned 30 m x 30 m grade-control drilling programme, aimed at improving local estimate for short-term production planning/forecasting. Some progress has been made in certain areas of the mine, however, there have been various practical problems in keeping abreast with the GC drilling campaign, including non-availability of drilling platforms.

Table I shows results from one of the areas where the LDC resource model estimates (based on an average data spacing of approximately 80 m x 80 m), have been reconciled with the follow-up grade control (GC) model post the GC drilling on 30 x 30 m grid over a 2-year period. For the purpose of this paper, the *in situ* reconciliation case study was done in a selected area measuring approximately 300 m x 200 m for the MBB reef, which is one of the main economic reefs at the mine. Within the case study area the total MBB tonnage is 158 kt.

Table I shows the result of aggregating the LDC recoverable resources for individual SMUs over the ‘global’ GC drilled area. Although the case study represents a small area of the mine, the table shows good results for the ‘global’ grade and tonnage reconciliations for the respective cut-offs, but some underestimation for the LDC technique. For all practical purposes this is not serious, as at the mine’s current economic cut-off of about 4 g/t, the ‘global’ LDC percentage errors are within 3% for tons and grade. However, conditional biases were still present on individual LDC SMU

**Table I**

**In situ LDC resource recoverable estimates compared with follow-up grade control (GC) estimate for the MBB reef horizon based on 2-year GC drilling (case study covers approx. 300 m x 200 m area, with MBB tonnage of 158 kt)**

<table>
<thead>
<tr>
<th>Cut-off (g/t)</th>
<th>Mean grade above cut-off (g/t)</th>
<th>Proportion above cut-off (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LDC</strong></td>
<td><strong>Grade control</strong></td>
<td><strong>LDC</strong></td>
</tr>
<tr>
<td>0</td>
<td>16.89</td>
<td>17.85</td>
</tr>
<tr>
<td>3</td>
<td>17.39</td>
<td>17.76</td>
</tr>
<tr>
<td>4</td>
<td>17.39</td>
<td>17.76</td>
</tr>
<tr>
<td>5</td>
<td>17.43</td>
<td>17.76</td>
</tr>
<tr>
<td>6</td>
<td>17.62</td>
<td>17.86</td>
</tr>
<tr>
<td>7</td>
<td>17.78</td>
<td>17.91</td>
</tr>
<tr>
<td>8</td>
<td>17.98</td>
<td>17.96</td>
</tr>
</tbody>
</table>

Figure 4—Typical comparison between the initial smoothed SK tonnage and grade (tonnage, SK and AUSK), the direct post-processed tonnage and grade (tonnage, TPGP and Ave AU), and the final LDC tonnage and grade (tonnage, LDC and AU_LDC) at respective cut-off grades for the ECMC reef.
Application of a localized direct conditioning mineral resource modelling technique

estimates, as reflected by a slope of regression of 0.62, which could present local-scale inefficient estimates due to limited data available for the initial LDC estimates.

Analysis of MCF at South Deep Following the introduction of LDC resource estimation

The mine call factor (MCF) reconciles the gold called for at the planning stage with the gold recovered at the end of a period. The International standard definition of MCF is as follows (AMIRA, 2007):

The ratio, expressed as a percentage, which the specific product accounted for in the product recovered from a plant, plus that in tailings/residues bears to the corresponding product called for (claim) by the measurement and evaluation methods used at the mine.

This section of the paper provides a summary analysis of the MCF observed at South Deep since the introduction of the LDC technique in June 2010.

Although there are several direct and indirect production and modifying factors that affect the MCF (including planned/unplanned dilution, planned/unplanned mining losses, introduction of new additional mining methods etc.), the paper presents the observed MCF changes before and after the introduction of LDC and provides indirect comments in this regard. These comments are provided taking into account the good in situ reconciliations of LDC estimates compared to that of GCs, as observed in several scattered areas of the mine where GC drilling has taken place; also as demonstrated in this paper for the case study area. The observed improved correlation of the MCF following the introduction of LDC does not necessarily provide conclusive evidence of the performance of LDC. However, as highlighted above, the good LDC and GC model in situ reconciliations observed so far tend to support this assertion.

Although the mine is systematically introducing GC drilling (on a 30 m x 30 m grid), most of the areas mined to date are planned on the basis of the LDC estimates. In this regard, any impact on the MCF cannot be attributed to increased drill data density.

The introduction of the LDC technique also coincides with the launching of the longhole mining method. Although longhole stopes are planned in specific months, mining and final cleaning of ore from these stopes takes place typically over a period of three to six months. As a result, short-term MCF numbers represent only indications of what the respective stopes will finally deliver. In this regard and in order to provide a more reliable MCF for South Deep during the period when the LDC technique was introduced, a 6-month average of MCF production results is provided in this paper.

Table II shows the 6-month MCFs over the 3 years and 6-monthly periods Mine call factor before LDC Mine call factor after LDC

<table>
<thead>
<tr>
<th>6-monthly periods</th>
<th>Mine call factor before LDC</th>
<th>Mine call factor after LDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul-08 to Dec-08</td>
<td>105%</td>
<td>-</td>
</tr>
<tr>
<td>Jan-09 to Jun-09</td>
<td>124%</td>
<td>-</td>
</tr>
<tr>
<td>Jul-09 to Dec-09</td>
<td>120%</td>
<td>-</td>
</tr>
<tr>
<td>Jan-10 to May-10</td>
<td>117%</td>
<td>-</td>
</tr>
<tr>
<td>Jun-10 to Nov-10</td>
<td>-</td>
<td>106%</td>
</tr>
<tr>
<td>Dec-10 to May-11</td>
<td>-</td>
<td>101%</td>
</tr>
<tr>
<td>Jun-11 to Nov-11</td>
<td>-</td>
<td>102%</td>
</tr>
<tr>
<td>Dec-11 to May-12</td>
<td>-</td>
<td>97%</td>
</tr>
<tr>
<td>Jun-12 to Nov-12</td>
<td>-</td>
<td>105%</td>
</tr>
<tr>
<td>Dec-12 to Jun-13</td>
<td>-</td>
<td>91%</td>
</tr>
<tr>
<td>Jul-13 to Dec-13</td>
<td>-</td>
<td>105%</td>
</tr>
<tr>
<td>Average over the period</td>
<td>116.3%</td>
<td>101.0%</td>
</tr>
</tbody>
</table>

Thus, the pre-LDC and post-LDC MCFs seem to indicate an improvement of the MCF (post introduction of LDC). As emphasized previously, the observed improved correlation of the MCF following the introduction of LDC does not necessarily provide conclusive evidence of the performance of LDC (due to other production and modifying factors that affect MCFs). However, the good LDC and GC model in situ reconciliations observed so far, and as discussed in this paper, tend to support this assertion.

Conclusions

Reconciliations to date at South Deep show encouraging results following the introduction of a simple kriging-based localized direct conditioning (LDC) methodology to the resource estimation procedure at the mine. However, conditional biases have been observed at a small-scale level for the LDC estimates, indicating that caution is required when using LDC for short-term mine planning. It is important that the mine strives to maintain and keep abreast of the necessary grade-control drilling programmes, which will improve local-scale estimation, as the LDC technique aims to improve medium- to long-term recoverable resource estimates.
Application of a localized direct conditioning mineral resource modelling technique

Although there are other indirect production and modifying factors affecting the MCF, the general observed improvement in the MCF over a period of more than three years since the introduction of LDC could be attributed partly to the improved LDC estimates.

References


The Southern African Institute of Mining and Metallurgy are hosting the 6th International Platinum Conference

‘Platinum— Metal for the Future’

20–22 October 2014 — Conference
23–24 October 2014 — Technical Visits

BACKGROUND
The 6th International Platinum Conference is to be held in the North West province, Sun City, South Africa on 20–22 October 2014 with technical visits planned for 23–24 October 2014.

This bi-annual Platinum conference has covered a range of themes since inception in 2004, and traditionally addresses the opportunities and challenges facing the platinum industry.

This prestigious event attracts key role players and industry leaders through:
- High quality technical papers and presentations
- Facilitating industry networking
- Having large, knowledgeable audiences
- Global participation, and
- Comprehensive support from industry role players.

The 2014 event will, under the guidance of the organising committee, structure a programme which covers critical aspects of this continually evolving and exciting industry.

However the success and relevance of this event to the industry really depends on your participation and support.

You can participate in this event as an organising committee member, author/presenter, delegate or sponsor.

We look forward to your support and engagement in the 6th International Platinum Conference.

Thank you.

Dr Gordon Smith
Chair—Organising Committee

WHO SHOULD ATTEND
- Academics
- Business development managers
- Concentrator managers • Consultants • Engineers
- Exploration managers • Explosives engineers
- Fund managers • Geologists • Hydrogeologists
- Innovation managers • Investment managers
- Market researchers and surveyors • Marketing managers
- Mechanical engineers • Metallurgical managers
- Metallurgical consultants • Metallurgists • Mine managers
- Mining engineers • New business development managers
- Planning managers • Process engineers • Product developers
- Production managers • Project managers • Pyrometallurgists
- Researchers • Rock engineers • Scientists • Strategy analysts
- Ventilation managers

ABOUT THE VENUE
Nestled in the rolling hills of the Pilanesberg, one of South Africa’s most scenic locations, Sun City is a world unto itself and has earned its reputation as Africa’s Kingdom of Pleasure.

Finally re-discovered and now part of Sun City, the Lost City and the Valley of Waves, fabled to be the Ruins of a glorious ancient civilisation, celebrate and bring to life the legends of this mystical city.
On the reduction of algorithmic smoothing of kriged estimates

by L. Tolmay*

Introduction

Smoothing of kriged local estimates on an operating mine that uses selective mining is undesirable. Smoothing reduces the resolution of individual estimates, thus inhibiting the selection process and leading to sub-optimal mining practices. Although the overall average grade may be correct, only selected portions of the orebody are mined. Our concern is not with how much pay or unpay ground exists within a certain area, but rather where it exists, and where to drill and blast. The problem is exacerbated on marginal mines, where smoothing obscures small differences in grade and could result in incorrect allocation of pay ground to waste, or vice versa, and lead to substantial financial losses. Although various post-processing techniques are used to correct smoothing, such as spectral post-processor, indirect post-processing techniques (Assibey-Bonsu and Krige, 1999), and localized direct conditioning (Assibey-Bonsu and Krige 1999; Krige, Assibey-Bonsu, and Tolmay, 2008), these have dealt with smoothing as a single phenomenon and not identified algorithmic smoothing on its own. Thus the aim of this research was to devise a methodology that reduces the algorithmic smoothing of individual estimates.

The method has not yet been integrated into the kriging process, but can easily be integrated into the underlying mathematical formalism in the future.

Why algorithmic smoothing occurs

In the case of simple kriging, the weight assigned to the mean increases as estimates move further away from the data source, but in the case of ordinary kriging the criterion is that the sum of the weights should add up to unity, and this is exactly the reason for the occurrence of algorithmic smoothing in this case. If relatively few samples are included in the estimation process there are likely to be large differences between weightings. However, as the number of samples used in estimation increases, the differences between weightings decreases until these differences become so small that the weights to all intents and purposes are considered to be equal. At this stage the estimate simply becomes the arithmetic mean of the samples used in the estimate. This effect is demonstrated in Figure 1.

Figure 1 illustrates how the maximum difference in weights decreases as the number of samples used in the kriging estimates increases, in this case the difference being close to zero when approximately 68 samples are used. At this stage the estimate is a totally smooth estimate of all samples used and will, to all intents and purposes, be equal to the simple arithmetic mean.

Synopsis

Utilizing a very large database from a mined-out area on a South African gold mine, the relative efficacy of a method to mitigate the smoothing effect introduced by the algorithmic constraints imposed by kriging was investigated. Smoothing effects arising from limited data availability are differentiated from the smoothing arising from the application of estimation algorithms. Very little can be done to ameliorate smoothing of estimates because of too little data, barring additional drilling or sampling. However, the smoothing effects resulting from the kriging process are shown to be mitigated by use of an alternative algorithm. The primary criterion in the development of the new algorithm was to avoid re-introducing conditional bias. This paper examines firstly the smoothing effects introduced into estimates via the kriging covariance matrix, secondly the process for ameliorating the smoothing effect, and finally it uses a case study to demonstrate the effectiveness of the new algorithm on a very large database. The database was used to introduce a 60 m by 60 m drilling pattern which in turn was used to model the semi-variogram and produce 30 m by 30 m kriged block estimates. The follow-up database was then re-introduced (roughly 5 m by 5 m grid spacing) and averaged into 30 m by 30 m blocks to provide a direct comparison with the initial estimates. In this way the extent of smoothing and accuracy of the estimates before and after the corrections was tested.

Keywords

kriging, smoothing, direct conditioning, kriging weights, algorithmic smoothing.
On the reduction of algorithmic smoothing of kriged estimates

Figure 1—Graph showing a decrease in the maximum difference in weights as the number of samples used in kriging estimates increases

Methodology

The premise for applying a new methodology is that at least some of the smoothing of kriged estimates is a direct result of the estimation algorithm. The smoothing results in kriged estimates moving toward the mean, the movement being negative if the estimate is above the mean and positive if the estimate is below the mean. The smoothing effect is zero when the difference between the location of the point being estimated and the location of the samples is zero. In addition, if the weight attributable to a sample is zero the smoothing effect on the estimate is also zero. Similar estimation weights result in similar smoothing effects (with the proviso that they are in the same direction and have the same distance); hence the circumstances relevant to an estimate may be different from one estimate to another. This means that in order to obtain an unbiased estimate, the amount by which an estimate must move away from the average value of samples on one side of an estimate is equivalent to the amount by which it must move towards the average value of the samples on the other side of the estimate. Thus any attempt to remove the effects of smoothing must take into account:

- The distance and direction of the sample from the estimate
- The estimation weighting of each sample
- The value of the sample relative to the mean of all samples
- Whether or not the sample used is above or below the average of all samples used
- Each estimate must be considered on its own merit
- The amount by which an estimate must move away from the average value of samples on the one side of an estimate to get to the actual value is equivalent to the amount by which it moves towards the average value on the alternative side of the estimate.

If the kriged estimate is considered to be the best linear unbiased estimate, given the sampling data and the associated semivariogram model, one can reasonably assume that the difference between the estimate and its corresponding actual value is attributable to the smoothing effects of both information and algorithmic. From the basic premises outlined above, the following algorithm represents the proposed solution for a unsmoothed, unbiased value of a kriged estimate:

$$\sum WA.IA.SA + \sum WB.IB.SB = \text{Actual}$$

Substituting SB with 1/SA gives:

$$\sum WA.IA.SA + \sum WB.IB.1SA = \text{Actual}$$

Re-arranging and multiplying by SA;

$$\sum WA.IA.SA^2 - \text{Actual}.SA + \sum WB.IB = 0$$

where

- WA a kriging weight for a value above the estimate
- WB a kriging weight for a value below the estimate
- SA the smoothing effect attributable to the sampling above the estimate
- SB the smoothing effect attributable to the sampling below the estimate
- IA samples used for estimation whose values are above the estimate
- IB samples used for estimation whose values are below the estimate
- SB 1/SA. This is inferred from the last point in underlying premises above, and is a pre-condition and axiomatic.

Solving Equation [3] provides a correction for the smoothing effect on an individual estimate. For any quadratic equation such as that shown in Equation [3], two solutions exist, one of which is correct and the other extraneous, so due caution must be exercised as only the solution which increases the weighting on the side of the estimate opposite to the smoothing effect will provide a corrected estimate. The point is illustrated in the example presented in Table I.

Table I shows the estimated value, 12.80 compared to the actual follow-up value of 13.1. Substituting the coefficients from Table I into Equation [3] yields:

$$9.64SA^2 - 13.1SA + 3.16 = 0$$

and solving for SA we obtain:

$$SA = (13.1 + \sqrt{13.1^2 - 4 \times 9.64 \times 3.16}) / 2 \times 9.64 = 1.0457$$

or

$$SA=(13.1 - \sqrt{13.1^2 - 4 \times 9.64 \times 3.16}) / 2 \times 9.64 = 0.3136$$

In this case the estimated value must increase in order to move from the estimate to the actual value. Thus the correction factor must increase the weighting of the samples above the estimate and decrease it for those below the estimate. An example of choosing the wrong root as a solution for our quadratic equation is shown in Table II.

Table I

<table>
<thead>
<tr>
<th>Sample description</th>
<th>Samples</th>
<th>Old weight</th>
<th>Weight x sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>1.8</td>
<td>0.050</td>
<td>0.09</td>
</tr>
<tr>
<td>2B</td>
<td>3.5</td>
<td>0.100</td>
<td>0.35</td>
</tr>
<tr>
<td>3B</td>
<td>6</td>
<td>0.180</td>
<td>1.08</td>
</tr>
<tr>
<td>4B</td>
<td>8</td>
<td>0.100</td>
<td>0.80</td>
</tr>
<tr>
<td>5B</td>
<td>12</td>
<td>0.070</td>
<td>0.84</td>
</tr>
<tr>
<td>1A</td>
<td>15</td>
<td>0.120</td>
<td>1.80</td>
</tr>
<tr>
<td>2A</td>
<td>17.6</td>
<td>0.180</td>
<td>3.17</td>
</tr>
<tr>
<td>3A</td>
<td>23</td>
<td>0.100</td>
<td>2.30</td>
</tr>
<tr>
<td>4A</td>
<td>23.7</td>
<td>0.100</td>
<td>2.37</td>
</tr>
<tr>
<td>Sums and averages</td>
<td>12.29</td>
<td>1.000</td>
<td>12.80</td>
</tr>
<tr>
<td>Actual</td>
<td>13.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>13.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
On the reduction of algorithmic smoothing of kriged estimates

As can be observed in Table II, although the correction factor produces the correct value, it does not increase the weights of the samples above the estimate and hence is the extraneous solution. An example of the choice of the correct quadratic root is illustrated in Table III.

In Table III, the selection of the correct quadratic root provides a correction factor that increases the weights of the values above the estimate and is therefore the correct factor.

We should now consider the case in which the smoothing has increased the value of the estimate, as shown in Table IV.

The value of the estimate shown in Table IV is 12.8, whereas the actual follow-up value is 11.1. Again, substituting the appropriate coefficients from Table V gives:

\[ 9.64SA^2 - 11.1SA + 3.16 = 0 \]

Solving for \( SA \) gives:

\[ SA = \frac{11.1 \pm \sqrt{11.1^2 - 4 \times 9.64 \times 3.16}}{2 \times 9.64} = 0.6369 \]

or

\[ SA = \frac{11.1 - \sqrt{11.1^2 - 4 \times 9.64 \times 3.16}}{2 \times 9.64} = 0.5148 \]

In this case, in order to move from the estimate to the actual value, the value must decrease. Therefore the correction factor must decrease the weighting of samples above the estimate, and increase that of those below the estimate. An example of choosing the correct root as a solution for our quadratic equation in this case is shown in Table V.

### Table II
Incorrect choice of quadratic root will produce an incorrect factor

<table>
<thead>
<tr>
<th>Sample description</th>
<th>Samples</th>
<th>Old weight</th>
<th>Weight x sample</th>
<th>New weight</th>
<th>New weight x sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>1.8</td>
<td>0.050</td>
<td>0.09</td>
<td>0.159</td>
<td>0.29</td>
</tr>
<tr>
<td>2B</td>
<td>3.5</td>
<td>0.100</td>
<td>0.35</td>
<td>0.319</td>
<td>1.12</td>
</tr>
<tr>
<td>3B</td>
<td>6</td>
<td>0.180</td>
<td>1.08</td>
<td>0.574</td>
<td>3.44</td>
</tr>
<tr>
<td>4B</td>
<td>8</td>
<td>0.100</td>
<td>0.80</td>
<td>0.319</td>
<td>2.55</td>
</tr>
<tr>
<td>5B</td>
<td>12</td>
<td>0.070</td>
<td>0.84</td>
<td>0.223</td>
<td>2.68</td>
</tr>
<tr>
<td>1A</td>
<td>15</td>
<td>0.120</td>
<td>1.80</td>
<td>0.038</td>
<td>0.56</td>
</tr>
<tr>
<td>2A</td>
<td>17.6</td>
<td>0.180</td>
<td>3.17</td>
<td>0.086</td>
<td>0.99</td>
</tr>
<tr>
<td>3A</td>
<td>23</td>
<td>0.100</td>
<td>2.30</td>
<td>0.031</td>
<td>0.72</td>
</tr>
<tr>
<td>4A</td>
<td>23.7</td>
<td>0.100</td>
<td>2.37</td>
<td>0.031</td>
<td>0.74</td>
</tr>
<tr>
<td>Sums and averages</td>
<td>12.29</td>
<td>1.000</td>
<td>12.80</td>
<td>1.751</td>
<td>13.10</td>
</tr>
<tr>
<td>Actual value</td>
<td>13.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>13.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor below</td>
<td>3.135559</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor above</td>
<td>0.3182242</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table III
Choice of the correct quadratic root provides a correct factor for correction of the estimate

<table>
<thead>
<tr>
<th>Sample description</th>
<th>Samples</th>
<th>Old weight</th>
<th>Weight x sample</th>
<th>New weight</th>
<th>New weight x sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>1.8</td>
<td>0.050</td>
<td>0.09</td>
<td>0.048</td>
<td>0.09</td>
</tr>
<tr>
<td>2B</td>
<td>3.5</td>
<td>0.100</td>
<td>0.35</td>
<td>0.096</td>
<td>0.33</td>
</tr>
<tr>
<td>3B</td>
<td>6</td>
<td>0.180</td>
<td>1.08</td>
<td>0.172</td>
<td>1.03</td>
</tr>
<tr>
<td>4B</td>
<td>8</td>
<td>0.100</td>
<td>0.80</td>
<td>0.096</td>
<td>0.77</td>
</tr>
<tr>
<td>5B</td>
<td>12</td>
<td>0.070</td>
<td>0.84</td>
<td>0.067</td>
<td>0.80</td>
</tr>
<tr>
<td>1A</td>
<td>15</td>
<td>0.120</td>
<td>1.80</td>
<td>0.125</td>
<td>1.88</td>
</tr>
<tr>
<td>2A</td>
<td>17.6</td>
<td>0.180</td>
<td>3.17</td>
<td>0.188</td>
<td>3.31</td>
</tr>
<tr>
<td>3A</td>
<td>23</td>
<td>0.100</td>
<td>2.30</td>
<td>0.105</td>
<td>2.40</td>
</tr>
<tr>
<td>4A</td>
<td>23.7</td>
<td>0.100</td>
<td>2.37</td>
<td>0.105</td>
<td>2.48</td>
</tr>
<tr>
<td>Sums and averages</td>
<td>12.29</td>
<td>1.000</td>
<td>12.80</td>
<td>1.001</td>
<td>13.10</td>
</tr>
<tr>
<td>Actual value</td>
<td>13.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>13.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor below</td>
<td>0.5663454</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor above</td>
<td>1.0456473</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
On the reduction of algorithmic smoothing of kriged estimates

As can be observed from Table V, the correction factor produces the correct value, and increases the weights of the samples below the estimate and hence is a valid solution.

Consider Table VI, where the correction factor increases the weights of the values below the estimate and hence is a valid solution, however not the optimal.

In the second example, shown in Tables V and VI, we are presented with two valid solutions. The choice of which one to use should be based on the rational decision not to re-introduce conditional bias, and it is therefore the solution having the lowest factor for increasing the weights below the mean.

One may ask why smoothing above the estimate was solved for in both cases, rather than above and below. The answer is that under certain conditions when solving for the smoothing below, \( b^2 - 4ac \) provides a negative value and hence is not solvable; however, this does not occur in the case of solving for the smoothing factor above the estimate. Thus it is possible, using a cross-validation routine, to calculate the smoothing correction for various distances and directions. These can then be modelled and used in a corresponding correction for smoothing routine.

### The process

The process for reduction of algorithmic smoothing for each estimate with a follow-up actual can be summarized as follows:
- Utilizing the samples used for the estimate, calculate and save the correction factor for each sample distance and direction
- Average the correction factors into directions and distances using a directional and lag tolerance.
- Although in an entirely homoscedastic environment those correction factors for increasing and decreasing grades of estimates should theoretically be similar, the spatial relationships surrounding these may not be so. It is therefore prudent to model the factors for increasing grade estimates separately from those for decreasing grade estimates
- Model the correction factors
- For each estimate, apply model of correction factors to each sample in order to obtain the corrected weight, taking due cognisance of where the individual sample is in relation to the estimate and the mean of all samples used, *i.e.*

### Table V

<table>
<thead>
<tr>
<th>Sample description</th>
<th>Samples</th>
<th>Old weight</th>
<th>Weight x sample</th>
<th>New weight</th>
<th>New weight x sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>1.8</td>
<td>0.050</td>
<td>0.09</td>
<td>0.079</td>
<td>0.14</td>
</tr>
<tr>
<td>2B</td>
<td>3.5</td>
<td>0.100</td>
<td>0.35</td>
<td>0.157</td>
<td>0.55</td>
</tr>
<tr>
<td>3B</td>
<td>6</td>
<td>0.180</td>
<td>1.08</td>
<td>0.283</td>
<td>1.70</td>
</tr>
<tr>
<td>4B</td>
<td>8</td>
<td>0.100</td>
<td>0.80</td>
<td>0.157</td>
<td>1.26</td>
</tr>
<tr>
<td>5B</td>
<td>12</td>
<td>0.070</td>
<td>0.84</td>
<td>0.110</td>
<td>1.32</td>
</tr>
<tr>
<td>1A</td>
<td>15</td>
<td>0.120</td>
<td>1.80</td>
<td>0.076</td>
<td>1.15</td>
</tr>
<tr>
<td>2A</td>
<td>17.6</td>
<td>0.180</td>
<td>3.17</td>
<td>0.115</td>
<td>2.02</td>
</tr>
<tr>
<td>3A</td>
<td>23</td>
<td>0.100</td>
<td>2.30</td>
<td>0.084</td>
<td>1.46</td>
</tr>
<tr>
<td>4A</td>
<td>23.7</td>
<td>0.100</td>
<td>2.37</td>
<td>0.064</td>
<td>1.51</td>
</tr>
<tr>
<td>Sums and averages</td>
<td></td>
<td>12.29</td>
<td>1.000</td>
<td>12.80</td>
<td>11.00</td>
</tr>
<tr>
<td>Actual value</td>
<td></td>
<td>11.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>11.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor below</td>
<td>1.5700713</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor above</td>
<td>0.6369137</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table VI

<table>
<thead>
<tr>
<th>Sample description</th>
<th>Samples</th>
<th>Old weight</th>
<th>Weight x sample</th>
<th>New weight</th>
<th>New weight x sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>1.8</td>
<td>0.050</td>
<td>0.09</td>
<td>0.097</td>
<td>0.17</td>
</tr>
<tr>
<td>2B</td>
<td>3.5</td>
<td>0.100</td>
<td>0.35</td>
<td>0.194</td>
<td>0.68</td>
</tr>
<tr>
<td>3B</td>
<td>6</td>
<td>0.180</td>
<td>1.08</td>
<td>0.350</td>
<td>2.10</td>
</tr>
<tr>
<td>4B</td>
<td>8</td>
<td>0.100</td>
<td>0.80</td>
<td>0.194</td>
<td>1.55</td>
</tr>
<tr>
<td>5B</td>
<td>12</td>
<td>0.070</td>
<td>0.84</td>
<td>0.136</td>
<td>1.63</td>
</tr>
<tr>
<td>1A</td>
<td>15</td>
<td>0.120</td>
<td>1.80</td>
<td>0.062</td>
<td>0.93</td>
</tr>
<tr>
<td>2A</td>
<td>17.6</td>
<td>0.180</td>
<td>3.17</td>
<td>0.093</td>
<td>1.63</td>
</tr>
<tr>
<td>3A</td>
<td>23</td>
<td>0.100</td>
<td>2.30</td>
<td>0.051</td>
<td>1.18</td>
</tr>
<tr>
<td>4A</td>
<td>23.7</td>
<td>0.100</td>
<td>2.37</td>
<td>0.051</td>
<td>1.22</td>
</tr>
<tr>
<td>Sums and averages</td>
<td></td>
<td>12.29</td>
<td>1.000</td>
<td>12.80</td>
<td>11.10</td>
</tr>
<tr>
<td>Actual value</td>
<td></td>
<td>11.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>9.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>11.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor below</td>
<td>1.9425869</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor above</td>
<td>0.5147775</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
On the reduction of algorithmic smoothing of kriged estimates

- If the estimate is below the mean then the smoothing will have dragged the estimate upwards towards the mean. Hence values below the estimate will have a weight correction factor greater than unity, and values above the estimate will have the weight correction factor less than unity.
- If the estimate is above the mean then the smoothing will have dragged the estimate downwards towards the mean. In this case values above the estimate will have a weight correction factor greater than unity, whereas those values below the estimate will have the weight correction factor less than unity.

Use the corrected weights to obtain new estimate.

An outstanding question, for which there is no immediate answer, is ‘does the kriging variance include the error made in the smoothing of the estimate?’ If the answer is yes, as the author suspects is the case, the distribution of errors will no longer be symmetrical around the adjusted estimate and will have to be corrected for by the amount of the adjustment.

The case study (or doing it Danie’s way)

A sampling database of 61 834 individual channel samples was used for the follow-up. The database used for estimation was obtained by overlaying a 60 by 60 m grid on the channel samples, and using the sample closest to the centre of each 60 by 60 m block as a drill point. In addition, 3 years of data was removed in order to check on the effects of extrapolation as well as interpolation.

Using the abovementioned data a 30 by 30 m grid was kriged and used for the analysis. The follow-up was obtained by utilizing all the data in 30 by 30 m averaged blocks that had a minimum of 14 samples within the block in order to ensure representative means for each block. This methodology was employed to obtain robust, smoothed estimates from a regular grid underlying the area of interest. These estimates were used to reduce the levels of smoothing and to test the efficiency of the newly proposed method, rather than improving the estimates per se. Nevertheless, a certain amount of reduction in error would of necessity be observed due to the averaging process, the results being shown in Table VII.

Table VII indicates that the dispersion variance of the adjusted values (post-processed dispersion variance) is improved relative to that of the ordinary kriging, thus rendering a greater resolution of estimates.

A regression plot of the unmodified ordinary kriged estimates can be compared with the modified estimates in Figure 2a and 2b. The unmodified estimates have a multiplicative bias of 1.188, while the modified estimates only show a bias of 1.037. The question as to whether the modified estimates are actually of greater quality must be answered in the affirmative, with only a marginal conditional bias and seemingly less error.

Although a large proportion of the smoothing is removed (Figure 2b compared with Figure 2a), a minor amount of smoothing, reflected as a 1.037 multiplicative bias due to the information effect, remains.

When the data is plotted in grade-tonnage curves as shown in Figure 3, it is evident that the curves for the modified data are closer to the actual curves than the unmodified ordinary kriging estimates.

![Table VII](image-url)

<table>
<thead>
<tr>
<th>Minimum of 14 follow-up samples</th>
<th>Variance</th>
<th>% of actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual dispersion variance</td>
<td>3 370 157</td>
<td>66%</td>
</tr>
<tr>
<td>OK dispersion variance</td>
<td>2 219 681</td>
<td></td>
</tr>
<tr>
<td>Post processed dispersion variance</td>
<td>2 984 705</td>
<td>89%</td>
</tr>
</tbody>
</table>

![Figure 2](image-url)

![Figure 3](image-url)
On the reduction of algorithmic smoothing of kriged estimates

Conclusion
Comparison of the modified and unmodified kriging estimates on regression curves and grade-tonnage curves indicates that the newly proposed algorithm for eliminating the effects of algorithmic smoothing through the ordinary kriging process achieves this end in an effective manner without reintroducing conditional bias. This method can be effectively applied in areas with limited sampling coverage, and where the effects of smoothing could adversely impact current and future mining plans.

Acknowledgement
To Daniel Gerhardus Krige, for showing me the benefit of plugging along even when it appears that no-one is listening.

References


Multivariate block simulations of a lateritic nickel deposit and post-processing of a representative subset

by J. Deraisme*, O. Bertoli*, and P. Epinoux†

Introduction
Société le Nickel (SLN) exploits the Dôme lateritic nickel orebody at its Tiébaghi operations located 385 km north of Nouméa on the western coastline of the Northern Province of New Caledonia (see Figure 1). Tiébaghi is SLN’s largest active mining complex and relies on a fairly advanced UTM (usine de traitement minerai: ore treatment plant) to increase the nickel grade of marginal ore in order to augment production of traditional ore (‘tradi’) for shipment to the Doniambo pyrometallurgical processing plant in Nouméa. The geochemistry of the ore also impacts the nickel smelting and recovery processes at the Doniambo plant.

Owing to the confidential nature of the UTM design, all economic and technical parameters relevant to the ore resource characterisation process have been altered or disguised prior to publication of this paper. The deposit geology significantly affects the recovery and metallurgical performance at the UTM, with controls that are extremely complex at all scales: lithology, oxidation-reduction conditions, mineralogy, and multivariate geochemistry.

It is within that context that SLN decided in 2011 to improve their recoverable resource estimation method to ensure robust models that would be produced capable of handling:

➤ The traditional support effect—the exploration data-set can only warrant the estimation of large panels 20×20×3 m³, which are much larger than the selective mining units (SMUs) of 5×5×3 m³
➤ The information effect—the selection at production stage uses estimates based on information much denser than that available for mine planning.

The problem is rendered more complex in this type of deposit by the fact that the additive variables used for any estimation process (metal accumulation and ore tonnage) are not the ones used to establish the selection at mining stage (i.e., the nickel grade of SMUs, which is the ratio of accumulation on tonnage). Eventually, the selection criteria involve not only Ni grades but also other constituents such as Al₂O₃, Fe₂O₃, MgO, and SiO₂.

The solution presented in this paper is the construction of a platform of SMU multivariate simulations in the saprolitic horizon, together with efficient post-processing aimed at producing the multivariate recoverable resource estimates. The efficiency constraints imposed by the number of blocks to be simulated (1.5 million) lead to resorting to direct block simulations. The underlying multivariate discrete Gaussian model (DGM), the validity of which is tested beforehand, is put to good use to mimic the selection process at the mining stage, by offering the ability to simulate in each block a composite value located at random within the block.

Finally, the paper presents the application of a scenario reduction algorithm to pick a representative subset of a few simulations to help appraise the risk attached to the downstream phases (reserve optimization, mine sequencing) of the project. The implementation presented here is for the first generation of the Scenario Reduction plug-in built in Isatis software, where measuring the distance between the initial set of scenarios and the reduced set is based on Ni quantities only. Research is under way to adapt the measure to the true multivariate nature of the problem.

Keywords
direct block simulations, discrete Gaussian model, multivariate, scenario reduction.

Synopsis
Société le Nickel (SLN) exploits the Dôme lateritic nickel orebody at its Tiébaghi operations in New Caledonia. The site geology has an obvious bearing on recovery and metallurgical performance, and the controls are extremely complex at all scales: lithology, oxidation-reduction conditions, mineralogy, and multivariate geochemistry. In that context, establishing an adapted recoverable resource estimation method that is efficient and transparent proves an interesting challenge. The method must address the key notions of:

➤ Support effect: the exploration data-set can only warrant the estimation of large panels 20×20×3 m³, which are much larger than the selective mining units (SMUs) of 5×5×3 m³
➤ Information effect: the selection at production stage uses estimates based on information much denser than that available for mine planning.

The problem is rendered more complex in this type of deposit by the fact that the additive variables used for any estimation process (metal accumulation and ore tonnage) are not the ones used to establish the selection at mining stage (i.e., the nickel grade of SMUs, which is the ratio of accumulation on tonnage). Eventually, the selection criteria involve not only Ni grades but also other constituents such as Al₂O₃, Fe₂O₃, MgO, and SiO₂.

The solution presented in this paper is the construction of a platform of SMU multivariate simulations in the saprolitic horizon, together with efficient post-processing aimed at producing the multivariate recoverable resource estimates. The efficiency constraints imposed by the number of blocks to be simulated (1.5 million) lead to resorting to direct block simulations. The underlying multivariate discrete Gaussian model (DGM), the validity of which is tested beforehand, is put to good use to mimic the selection process at the mining stage, by offering the ability to simulate in each block a composite value located at random within the block.

Finally, the paper presents the application of a scenario reduction algorithm to pick a representative subset of a few simulations to help appraise the risk attached to the downstream phases (reserve optimization, mine sequencing) of the project. The implementation presented here is for the first generation of the Scenario Reduction plug-in built in Isatis software, where measuring the distance between the initial set of scenarios and the reduced set is based on Ni quantities only. Research is under way to adapt the measure to the true multivariate nature of the problem.

Keywords
direct block simulations, discrete Gaussian model, multivariate, scenario reduction.
Multivariate block simulations of a lateritic nickel deposit

The problem of non-additivity of the selection variable—the additive variables used for estimation (metal accumulation and ore tonnage) are not the ones used to establish the selection at the mining stage (i.e. the SMU nickel grade, which is the ratio of accumulation on tonnage)

The multivariate nature of the ore control selection criteria, which involve not only Ni but also other constituents such as Al₂O₃, Fe₂O₃, MgO, and SiO₂.

The solution presented in this paper is to construct a platform of SMU multivariate simulations in the saprolitic horizon (which represents 90% of the recoverable resource at Dôme) and construct an efficient post-processing method aimed at producing the multivariate recoverable resource estimates used as inputs to the pit optimization and mine sequencing phases.

Regional and mine geology at Tiébaghi

Nickel laterites represent nearly 60% of the world’s Ni resources. Tiébaghi mine is located in the north of New Caledonia in the South Pacific. This island is known to contain about 25% of the world’s nickel resources, developed from lateritic weathering profiles.

About one-third of New Caledonia’s is covered by ultramafic rocks, mainly peridotites, emplaced as an ophiolite nappe by obduction of the oceanic mantle over the New Caledonia – Norfolk continental ridge during the late Eocene orogenic event (Cluzel et al., 2001). A lateritic profile overlies the ultramafic rocks (see Figure 2). The distribution of ultramafic rocks (a large southeastern massif and smaller ones along the northwestern coast, with various elevations and a stepped lateritic surface) is the result of the combined effects of isostatic post-orogenic uplift, tectonics, and erosion.

The Tiébaghi massif is a klippe of peridotite overlying Cretaceous basalts and Eocene-Paleocene sedimentary rocks. The massif forms a gently west-dipping plateau culminating at 584 m. The study area, called Dôme, is located in the central-western part of the plateau and displays a complete and thick (30–80 m) lateritic weathering mantle.

Four main horizons make up the lateritic regoliths (see Figure 3), from the base to the top: rocky saprolites (isovolumic weathering), saprolites (isovolumic weathering), limonite (oxide of iron, closed porosity), and ferruginous/aluminous duricrust (Eggleton, 2001). Nickel is concentrated in secondary oxides and hydrous phyllosilicates.
Mineralization is controlled by the interplay of lithology, tectonics, climate, and geomorphology, and developed in several phases (Pelletier, 2003). Mining operations at Tiébaghiaim are conducted to selectively extract three types of ore:

➤ The Ni-Co rich limonites, which are stockpiled for subsequent processing
➤ The intrinsically nickel-rich saprolites and saprolites that cannot be treated in the UTM because of low enrichment capacity or mineralogy preventing the solid-liquid separation (‘Tradi’ ore)
➤ The saprolite beneficiated by wet treatment in the UTM. These saprolites have undergone a sufficient degree of weathering to enable economic recovery and removal of gangue mineral by the UTM (UTM ore).

The paper focuses on the saprolitic ores: ‘Tradi’ and UTM. The beneficiation of UTM ore is impacted by numerous complex geological controls:

➤ The lateritic leaching processes induce intense chemical dissolution of the peridotite, as magnesia and part of silica are progressively leached while Al, Fe, Cr, Co, and Ni accumulate in the weathering mantles (Tardy, 1997; Nahon et al., 1992; Anand and Paine, 2002). The ratio between iron-alumina and magnesia therefore indicates the intensity of weathering.
➤ In addition to a high variability of peridotites with different compositions (plagioclase lherzolites, spinel dunite, harzburgite), the Tiébaghi massif exhibits a high degree of serpentinization, a hydrothermal phenomenon probably related to felsic and gabbro intrusions. These rocks introduce a significant amount of aluminum into the weathering profile. The presence of secondary aluminous phyllosilicates such as kaolinite in the saprolites also affects the beneficiation characteristics at UTM.
➤ Secondary silica may be present (chalcedony and quartz), resulting in the dilution of in situ nickel. Such silicification is due to the weathering of peridotite in which the low alumina content has restricted the formation of clays. Removal of silica by the UTM is an essential beneficiation step, increasing the recovered grade.
➤ Structural phenomena and alteration have also led to the formation of a karst system, with alteration by groundwater having induced the formation of reduced horizons (gley) with specific mineralogical and chemical signatures (presence of carbonate and sulphides). These minerals affect the value and potential treatment of saprolites in the UTM.

Mineral processing and mineralogical studies have thus helped define the geochemical classification criteria of the nickel ore for Dome:

➤ The ratio \( \frac{Fe_2O_3 + Al_2O_3}{MgO} \) characterizes the degree of weathering, which affects the processing capacity and mass efficiency of the UTM.
➤ Anomalous concentrations of alumina (\( Al_2O_3 \)), iron (\( Fe_2O_3 \)), and silica (\( SiO_2 \)) are used to understand the mineralogy and indirectly predict nickel output from the UTM and subsequently the pyrometallurgical processing performance of the Doniambo plant.

It is therefore essential to estimate the nickel grade, together with all the secondary variables involved in the above calculations. These grades being non-additive, the variables to consider in any estimation process are \( Ts/m^3 \) (proportion of altered material \( x \) density) and accumulated variables i.e. \( Ni'Ts/m^3, Al_2O_3'Ts/m^3, Fe_2O_3'Ts/m^3, \)
Multivariate block simulations of a lateritic nickel deposit

MgO*Ts/m³, and SiO₂*Ts/m³. The raw variables are then deduced by the ratios of the estimated accumulations on the estimated Ts/m³. The grades are non-additive because they apply to the proportion of altered material for each composite, and even if these composites are of constant length (3 m), the proportion affected by alteration is variable. Consequently, the support over which these grades are measured becomes variable (Journel and Huijbregts, 1978).

The objective of the procedure developed for Tiébaghi is to estimate the recoverable proportions of 5x5x3 m³ SMUs within each 20x20x3 m³ panel when a cut-off is applied on Ni, and to attach to that proportion the grade of all five relevant elements being studied. An original procedure, based on direct block simulations and which is capable of handling the impacts of information and support effects as well as the non-additivity of the input variables, is presented below.

Direct block simulation

Block simulation may be performed either by averaging of point simulations or by a direct method. Direct block simulations are obtained by first generating block values that reproduce a predefined block variogram model, and then conditioning the block values directly from the point information. The obvious advantage of direct simulation is the gain in CPU performance and disk space. Owing to the efficiency constraints imposed by the number of blocks to be simulated at Tiébaghi (1.5 million), direct block simulation is preferable to re-blocking of point simulations.

The direct implementation can be performed in the framework of the discrete Gaussian model (DGM), where the deposit is partitioned into blocks of size \( n \) (SMUs), and each sample point \( x \) is considered random within each block, allowing a statistical characterization of the link between point and block information to be utilized for nonlinear estimation and simulations (Deraisme et al., 2008).

In DGM, the variogram model attached to the block Gaussian equivalent values \( Y_x \) may be deduced from the variogram of point Gaussian values \( Y_r \) regularized on blocks (Emery and Ortiz, 2005). The conditioning step is then achieved by simple kriging (SK) in the DGM framework, where \( Y_x \) is obtained by linear regression from \( Y_r \) (the correlation coefficient being the change of support coefficient \( r \) that is linked to the physical loss of variability when changing the support of the distributions from point to block). This property of DGM can also be used to obtain a point simulated value within each simulated block value. The simulated points may be viewed as samples of the simulated reality, which can be used to mimic the grade control process by re-estimating the SMU values and comparing the resulting estimates to the simulated values. The only loss of optimality in that process concerns the location of the simulated point value, which is considered random within the block. The extension of the above framework to the multivariate setting is presented in Deraisme et al. (2008).

One must note that the conditioning by SK imposes a very strict adherence to the stationarity hypothesis for the different grade distributions. The stationarity assumptions are well respected horizontally for all distributions, but obvious departures are observed along the vertical direction (see Figure 4), in keeping with the differential leaching of material in lateritic deposits.

This prompted tests for whether the conditioning step in the simulation algorithm (see the following section on Production and validation of multivariate simulations for the Dôme orebody) would benefit from using ordinary kriging (OK), which is less sensitive to departures from ideal stationarity conditions, instead of the theoretically sound choice of SK. The tests concluded that simulations obtained via the two conditioning strategies were highly correlated realization-per-realization (0.95), and that SK-based realizations offer a better reproduction of the input variogram model as well as a better reproduction of the input statistical profile. SK was thus vindicated as the algorithm of choice for conditioning the simulations.

As for testing the applicability of the DGM, checking ratios of grade indicator variograms can theoretically test the applicability of the diffusive model (Rivoirard, 1994; Chilès et al., 1999). Tests carried out on metric passes in the saprolitic horizon show that the ratios of cross-variograms to variograms of indicators increase with distance, confirming the applicability of the DGM. Moreover, the traditional test involving the ratio of the square root of the variogram to the first-order variogram (madogram) for Gaussian Ni grades further supports the plausibility of the bi-Gaussianity assumptions at play in the model (see Figure 5).

![Figure 4](https://example.com/figure4.png) — Vertical swath plots for Ni, MgO, and Fe₂O₃ of the 3 m composites within the saprolitic horizon
Production and validation of multivariate simulations for the Dôme orebody

Several tests were first carried out on bivariate simulations of Ts/m^3 and Ni*Ts/m^3 to help calibrate some key simulation parameters (number of bands in the turning band algorithm used for producing the non-conditional simulations, definition of the neighbourhood search parameters), and define the overall strategy to simulate the complete variable set. The number of simulations to be produced was a particular concern from a pragmatic viewpoint, as the implications of producing 50, 100, or 200 simulations are certainly not trivial from a performance viewpoint.

Initial tests were performed in the central area (see Figure 6) of the deposit by producing 100 simulations and testing how post-processing statistics of the first lot of 50 realizations compared to the statistics of the second lot of 50 simulations.

One of the comparisons performed is illustrated in Table I, which shows the average estimation error incurred in estimating the portion of the deposit above the economic cut-off for the different lot of simulations sampled at different drill spacings. The fact that the first lot of 50 realizations strictly reproduces the statistics for the second lot of 50 suggests that for the global characterization of the resource at Dôme, working with 50 realizations is acceptable.

The initial step for the simulation of the six studied variables is to implement their histogram modelling via Gaussian anamorphosis. The histograms reconstructed by the anamorphosis functions offer a satisfactory fit of the weighted experimental histograms. The weighting of the distribution is mandatory as the data-set mixes different sampling grids (58x26 m^2 for exploration, 20x20 m^2 centred for mine planning, and 5x5 m^2 for grade control) obtained via a cell declustering technique using a window of 20x20 m rotated at 45 degrees in keeping with the orientation of drilling (corresponding to step 4 in Figure 7).
Multivariate block simulations of a lateritic nickel deposit

Experimental variography is then performed on the Gaussian transforms in the two main directions of continuity in the horizontal plane (azimuth 135° and azimuth 45°) and along the vertical direction. A model is fitted using the linear model of coregionalization (LMC). The LMC is a generalization of the nested variogram model to the multivariate case. In this model, all simple and cross-variograms are linear combinations of the same elementary components. This can be interpreted by a decomposition of the variables themselves into linear combinations of independent random functions. The quality of the fitting obtained for the main variables (Ts/m³ and Ni*Ts/m³) is shown in Figure 8.

Following the regularization of the variogram on 5x5x3 m³ support, the DGM change of support parameters are determined (see Deraisme et al., 2008).

Fifty direct 5x5x3 m³ block co-simulations of Ts/m³, Ni*Ts/m³, Al₂O₃*Ts/m³, Fe₂O₃*Ts/m³, MgO*Ts/m³, and SiO₂*Ts/m³ are then produced within the saprolites. The number of turning bands used for the non-conditional simulations is set at 1000 to avoid potential streaking artefacts. The conditioning by SK is performed within a (300x200x12 m³) neighbourhood rotated at azimuth 135° using eight sectors and seven 3 m composites per sector (total of 56 composites per neighbourhood).

To allow the incorporation of information effect incurred during grade control from a 10 m centred (10 m C) pre-exploitation (‘pre-ex’) drilling grid, a kriging of the variables of interest from the pre-ex sampling grid extracted from each realization is done following the production of the direct block simulation values and associated 3 m point values. The ranges of the neighbourhood utilized for that kriging are adapted (40x30x12 m³) to reflect the density of information available at pre-ex stage.

A post-processing (utilizing all the realizations from the simulation platform) is then implemented to estimate the tonnage and metal quantities recoverable for each variable within 20x20x3 m³ panels. The same post-processing will be applied ultimately to each realization retained by scenario reduction.

In summary, the estimation of multivariate recoverable resources at 20x20x3 m³ panel level is performed for the series of economic cut-offs in the following manner:

1. For each 5x5x3 m³ SMU simulated, 3 m composite values are extracted at random
2. A selection corresponding to the 10 m centered pre-ex sampling grid (10 m C) is made on the simulated points
3. 5x5x3 m³ SMU values are re-estimated from the 10 m C sampling set
4. Then, for each cut-off grade:
   - For each realization of the simulation platform, select the SMUs such that the estimated values kriged at step 3 are above the cut-off grade

Figure 8—Extract of the multivariate variogram model of the Gaussian transforms for Ts/m³ and Ni*Ts/m³ (main direction azimuth 135°, 3 m composites – saprolites)
Multivariate block simulations of a lateritic nickel deposit

- For each 20x20x3 m³, calculate tonnage and metal contents for all variables and all 5x5x3 m³ smu making it past the cut-off
- Looping on all realizations, calculate the recoverable estimates as the average of the values obtained at step b for all the realizations

5. Repeat for all cut-offs.

Scenario reduction
The above procedure allowed production of a robust estimate of the recoverable resources for the six variables of interest, which can be fed into the reserve calculation process. We present here the application of a scenario reduction algorithm that selects a representative subset of a few simulations out of the original platform to help appraise the risk attached to the downstream phases (reserve optimization, mine sequencing) of the project.

The implementation presented here is for the first generation of the scenario reduction plug-in built in Isatis® software (see Bleines et al., 2012), where measurement of the distance between the initial set of scenarios and the reduced set is based on Ni quantities only. Research is under way to adapt the measure to the true multivariate nature of the problem.

The central idea to the algorithm used is to characterize the difference between any two simulations by integrating, over all the panels, the difference in recoverable metal quantities between the two simulations for all the cut-offs. Once a dissimilarity matrix has been established for the 50 simulations in the platform, a combinatorial based on k-mean clustering is used to select the k (here, k=5) simulations that best capture the space of uncertainty as characterized by the 50 realizations. A detailed presentation of the underlying concepts is given by Armstrong et al. (2010), who base the minimization of the distance between the selected subset of simulations and the remainder set on a random sampling procedure of the combinatorial to be treated. The current selection procedure is based on a genetic sampling algorithm, which is presented in a companion paper in this volume. The parameters of the genetic sampling are optimized for the prior selection of three simulations out of 50 for which it is possible to perform an exhaustive sampling of all possibilities.

The set of simulations selected (29, 49, 2, 31, 32) comes with associated probabilities (76%, 18%, 2%, 2%, 2%). As can be seen, realization 29 can be viewed as a representative case, while realization 49 represents more marginal situations, and realizations 2, 31, and 32 can be considered as representative of the extremes.

Post-processing of each one of these five realizations is then possible, and will help qualify the answer given by the resource estimate.

Impact on economic evaluation
Several pit optimizations (using Whittle software) are then implemented using as an input to the process either:

➤ The traditional estimates based on the independent ordinary kriging of the accumulated variables panel by panel (leading to what is referred to in the following tables as the ‘Traditional Pit’), or
➤ The recoverable resource estimation based on the full multivariate conditional simulation platform (leading to what is referred to in the following tables as the ‘Pit based on the CS platform’).

To help assess the risk and sensitivity to the uncertainty on the recoverable resources, the reserves are then calculated for both pits using the above resource models as well as the five simulations selected by scenario reduction.

Tables II and III give the ore tonnage for the seven input models (actual = traditional estimates, recov = recoverable resource based on CS platform, simu29, simu49, simu2, simu31, simu32 for the simulations selected by scenario reduction) in the two pits selected. The total tons are split resource based on CS platform, simu29, simu49, simu2, simu31, simu32 for the simulations selected by scenario reduction).

The results highlight the following:

➤ The risk of underachieving actual ore tons mining the current pit geometry
➤ The higher value, longer LOM within the pit optimized using the conditional co-simulation (CS) platform

Table II
Ore tons in pit based on traditional estimate for alternative input models

<table>
<thead>
<tr>
<th>Esti</th>
<th>R</th>
<th>Tradi ore</th>
<th>UTM concentrate ore</th>
<th>TOT</th>
<th>% ts</th>
<th>% Cashflow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Kth</td>
<td>% Distribution</td>
<td>Kth</td>
<td>% Distribution</td>
<td>Kth</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>100</td>
<td></td>
<td>-18.8%</td>
<td>-36.5%</td>
</tr>
<tr>
<td>Actual</td>
<td>10.8</td>
<td>26</td>
<td>26%</td>
<td>74</td>
<td>74%</td>
<td>100</td>
</tr>
<tr>
<td>Recup 20 x 20</td>
<td>13.2</td>
<td>31</td>
<td>37%</td>
<td>51</td>
<td>63%</td>
<td>82</td>
</tr>
<tr>
<td>Simu2</td>
<td>13.0</td>
<td>31</td>
<td>37%</td>
<td>53</td>
<td>63%</td>
<td>84</td>
</tr>
<tr>
<td>Simu29</td>
<td>12.7</td>
<td>31</td>
<td>37%</td>
<td>54</td>
<td>63%</td>
<td>85</td>
</tr>
<tr>
<td>Simu31</td>
<td>12.8</td>
<td>32</td>
<td>37%</td>
<td>53</td>
<td>63%</td>
<td>85</td>
</tr>
<tr>
<td>Simu32</td>
<td>12.8</td>
<td>31</td>
<td>37%</td>
<td>53</td>
<td>63%</td>
<td>85</td>
</tr>
<tr>
<td>Simu49</td>
<td>13.0</td>
<td>31</td>
<td>37%</td>
<td>53</td>
<td>63%</td>
<td>84</td>
</tr>
</tbody>
</table>

Actual = ‘Ordinary kriging estimation’, R = ‘ratio mining extracted (handled) to ore production’ Kth (wet tons) and Tni (Ni grade) are expressed in relative terms to the figures based on the ordinary kriged estimate within the traditional pit. %ts and %cashflow represents the relative differences in recoverable dry tonnes and generated cashflow when compared to the results for the ordinary kriged estimate within the traditional pit.
Multivariate block simulations of a lateritic nickel deposit

Table III

<table>
<thead>
<tr>
<th>Esti</th>
<th>R</th>
<th>Tradi ore</th>
<th>UTM concentrate ore</th>
<th>TOT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Kth % Distribution</td>
<td>Kth % Distribution</td>
<td>Kth Tni</td>
</tr>
<tr>
<td>Actual</td>
<td>13.4</td>
<td>26 25%</td>
<td>75 75%</td>
<td>101 101</td>
</tr>
<tr>
<td>Recup 20 x 20</td>
<td>12.5</td>
<td>38 35%</td>
<td>70 65%</td>
<td>108 104</td>
</tr>
<tr>
<td>Simu2</td>
<td>12.5</td>
<td>38 35%</td>
<td>70 65%</td>
<td>108 104</td>
</tr>
<tr>
<td>Simu29</td>
<td>12.1</td>
<td>39 35%</td>
<td>73 65%</td>
<td>111 108</td>
</tr>
<tr>
<td>Simu31</td>
<td>12.5</td>
<td>38 35%</td>
<td>71 65%</td>
<td>109 106</td>
</tr>
<tr>
<td>Simu32</td>
<td>12.2</td>
<td>38 35%</td>
<td>72 65%</td>
<td>111 107</td>
</tr>
<tr>
<td>Simu49</td>
<td>12.5</td>
<td>37 34%</td>
<td>71 66%</td>
<td>108 104</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% ts</th>
<th>% Cashflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>0.2%</td>
<td>0.1%</td>
</tr>
<tr>
<td>0.2%</td>
<td>0.1%</td>
</tr>
<tr>
<td>0.2%</td>
<td>0.1%</td>
</tr>
<tr>
<td>3.4%</td>
<td>6.5%</td>
</tr>
<tr>
<td>2.6%</td>
<td>4.2%</td>
</tr>
<tr>
<td>0.2%</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

Actual = ‘Ordinary kriging estimation’. R = ‘ratio mining extracted (handled) to ore production’ Kth (wet tons) and Tni(Ni grade) are expressed in relative terms to the figures based on the ordinary kriged estimate within the traditional pit. %ts and %cashflow represents the relative differences in recoverable dry tonnes and generated cashflow when compared to the results for the ordinary kriged estimate within the traditional pit.

Conclusions

The scenario reduction procedure that is implemented on direct block simulations, which is capable of handling the impacts of information and support effects as well as the non-additivity of the input variables has been developed. The improved product mix within the CS pit (35% of TRADI (i.e. rich) ore instead of 25% for the actual pit), meaning a reduction in cost and time lost remobilizing material.

References

List of papers published by Danie Krige from 1951–2001


10. The application of correlation and regression techniques in the selective mining of gold ores - 2nd APCOM Symposium, University of Arizona, Tucson, April 1962. (Vol. 3…3)

11. The significance of a limited number of borehole results in the exploration of new South African gold fields - 2nd APCOM Symposium, University of Arizona, Tucson, April 1962. (Vol. 3…2)


15. Recent developments in South Africa in the application of trend surface and multiple regression techniques to gold ore valuation - 4th APCOM Symposium, Colorado School of Mines Quarterly Vol. 59, No. 4, October 1964. (Vol. 2…4)


19. Ore value trend surfaces for the South African gold mines based on a weighted moving average - 6th APCOM Symposium, Penn. State University, April 1966. (Vol. 3…4)


22. The development of statistical models for gold ore valuation in South Africa - Sixth International Mining Congress, Madrid, 1970. (Vol. 3…9)


26. Some basic considerations in the application of geostatistics to gold ore valuation J.S.A.I.M.M, April 1976, pp 38/53. (Vol. 3…14)


33. Analysis of the potential benefits to the State of realistic adjustments to the mining tax structure, J.S.A.I.M.M, July 1979. (Vol. 4…19)

34. Some novel features and implications of a general risk analysis model for new mining ventures - J.S.A.I.M.M Colloquium, 4 May 1979; J.S.A.I.M.M, October 1979. (Vol. 4…23)

35. Gold production trends of South Africa - Analysis of the potential benefits to the State of realistic adjustments to the mining tax structure, J.S.A.I.M.M, July 1979. (Vol. 4…19)


38. Geostatistical case studies of the advantages of lognormal-de Wijksian kriging with mean for a base metal mine and a gold mine (jointly with E.J. Magri) - Mathematical Geology, Vol. 14, No. 6, 1982, pp. 547/555. (Vol. 6…8)

39. Gold production productivity as affected by the average wet bulb temperature of the underground working places (jointly with C. P. Barnard) - J.S.A.I.M.M, May 1981, pp. 117/121. (Vol. 6…9)


44. Factors affecting the long-term prospects of South Africa's gold production - Long-term Prospects for Gold Mining, Queen's Univ., Kingston, Canada, Proceedings No. 20 of Seminar, 1987. (Vol. 6…14)

45. Letter on 'Matheronian Geostatistics' - 1986. (Vol. 6…15)

46. Capital Investment in New Mining Projects-Taiper, 1986. (Vol. 6…11a)
INTERNATIONAL ACTIVITIES

2014

20-22 August 2014 — MineSafe Conference 2014
Technical Conference and Industry day
Emperors Palace, Hotel Casino Convention Resort, Johannesburg
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

1-2 September 2014 — Drilling and Blasting
Swakopmund Hotel & Entertainment Centre,
Swakopmund, Namibia
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

9-11 September 2014 — 3rd Mineral Project Valuation School
Mine Design Lab, Chamber of Mines Building,
The University of the Witwatersrand
Contact: Camielah Jardine
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: camielah@saimm.co.za, Website: http://www.saimm.co.za

16-17 September 2014 — Surface Mining 2014
The Black Eagle Room, Nasrec Expo Centre
Contact: Camielah Jardine,
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: camielah@saimm.co.za, Website: http://www.saimm.co.za

20-24 October 2014 — 6th International Platinum Conference
Sun City, South Africa
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za,Website: http://www.saimm.co.za

12 November 2014 — 12th Annual Southern African Student Colloquium
Mintek, Randburg
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

Shahid Beheshti International Conference Center,
Tehran, Iran
Contact: Kourosh Hamidi, E-mail: info@imatconf.com

19-20 November 2014 — Accessing Africa’s Mineral Wealth: Mining Transport Logistics
Emperors Palace, Hotel Casino Convention Resort, Johannesburg
Contact: Camielah Jardine
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: camielah@saimm.co.za, Website: http://www.saimm.co.za

2015

11-13 March 2015 — Diamonds Conference 2015
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

8-10 April 2015 — 5th Sulphur and Sulphuric Acid 2015 Conference
Southern Sun Elangeni Maharani KwaZulu-Natal,
South Africa
Contact: Camielah Jardine
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: camielah@saimm.co.za, Website: http://www.saimm.co.za

12-13 May 2015 — Mining, Environment and Society Conference: Beyond sustainability—Building resilience
Johannesburg, South Africa
Contact: Yolanda Ramokgadi
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: yolanda@saimm.co.za, Website: http://www.saimm.co.za

14-17 June 2015 — European Metallurgical Conference
Dusseldorf, Germany, Website: http://www.emc.gdmb.de

14-17 June 2015 — Lead Zinc Symposium 2015
Dusseldorf, Germany, Website: http://www.pb-zn.gdmb.de

Dusseldorf, Germany, Website: http://www.metec-tradefair.com

6-8 July 2015 — Copper Cobalt Africa Incorporating The 8th Southern African Base Metals Conference
Zambezi Sun Hotel, Victoria Falls, Livingstone, Zambia
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

12-14 August 2015 — The Seventh International Heavy Minerals Conference ‘Expanding the horizon’
Sun City, South Africa
Contact: Camielah Jardine,
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: camielah@saimm.co.za, Website: http://www.saimm.co.za

1–2 September 2014

6–8 July 2015 — Copper Cobalt Africa Incorporating The 8th Southern African Base Metals Conference
Zambezi Sun Hotel, Victoria Falls, Livingstone, Zambia
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

28 September-2 October 2015 — WorldGold Conference 2015
Misty Hills Country Hotel and Conference Centre,
Cradle of Humankind
Gauteng, South Africa
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

12-14 October 2015 — Slope Stability 2015: International Symposium on slope stability in open pit mining and civil engineering
Cape Town Convention Centre, Cape Town
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

12-13 May 2015 — Mining, Environment and Society Conference: Beyond sustainability—Building resilience
Johannesburg, South Africa
Contact: Yolanda Ramokgadi
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: yolanda@saimm.co.za, Website: http://www.saimm.co.za

28-30 October 2015 — AMI: Nuclear Materials Development Network Conference
Nelson Mandela Metropolitan University, North Campus
Conference Centre, Port Elizabeth
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

8–10 April 2015 — 5th Sulphur and Sulphuric Acid 2015 Conference
Southern Sun Elangeni Maharani KwaZulu-Natal,
South Africa
Contact: Camielah Jardine
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: camielah@saimm.co.za, Website: http://www.saimm.co.za

14–17 June 2015 — European Metallurgical Conference
Dusseldorf, Germany, Website: http://www.emc.gdmb.de

Dusseldorf, Germany, Website: http://www.pb-zn.gdmb.de

Dusseldorf, Germany, Website: http://www.metec-tradefair.com

6–8 July 2015 — Copper Cobalt Africa Incorporating The 8th Southern African Base Metals Conference
Zambezi Sun Hotel, Victoria Falls, Livingstone, Zambia
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

28 September-2 October 2015 — WorldGold Conference 2015
Misty Hills Country Hotel and Conference Centre,
Cradle of Humankind
Gauteng, South Africa
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

12-14 August 2015 — The Seventh International Heavy Minerals Conference ‘Expanding the horizon’
Sun City, South Africa
Contact: Camielah Jardine,
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: camielah@saimm.co.za, Website: http://www.saimm.co.za

28-30 October 2015 — AMI: Nuclear Materials Development Network Conference
Nelson Mandela Metropolitan University, North Campus
Conference Centre, Port Elizabeth
Contact: Raymond van der Berg
Tel: +27 11 834-1273/7, Fax: +27 11 838-5923/833-8156
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za

8–10 April 2015 — 5th Sulphur and Sulphuric Acid 2015 Conference
Southern Sun Elangeni Maharani KwaZulu-Natal,
South Africa
Contact: Raj Singhal, E-mail: singhal@shaw.ca or
E-mail: raymond@saimm.co.za, Website: http://www.saimm.co.za
### Company Affiliates

The following organizations have been admitted to the Institute as Company Affiliates:

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Country/Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>AECOM SA (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>AEL Mining Services Limited</td>
<td></td>
</tr>
<tr>
<td>Air Liquide (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>AMEC GRD SA</td>
<td></td>
</tr>
<tr>
<td>AMIRA International Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>ANDRITZ Delkor (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Anglo Operations (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Anglogold Ashanti Ltd</td>
<td></td>
</tr>
<tr>
<td>Arcus Gibb (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Atlas Copco Holdings South Africa (Pty) Limited</td>
<td></td>
</tr>
<tr>
<td>Ameva Mining Shafts and Underground</td>
<td></td>
</tr>
<tr>
<td>Aveng Moolmans (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Axis House (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Bafokeng Rasimone Platinum Mine</td>
<td></td>
</tr>
<tr>
<td>Barloworld Equipment -Mining</td>
<td></td>
</tr>
<tr>
<td>BASF Holdings SA (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Bateman Minerals and Metals (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>BCL Limited (BCL001)</td>
<td></td>
</tr>
<tr>
<td>Becker Mining (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>BedRock Mining Support Pty Ltd</td>
<td></td>
</tr>
<tr>
<td>Bell Equipment Limited</td>
<td></td>
</tr>
<tr>
<td>BHP Billiton Energy Coal SA Ltd</td>
<td></td>
</tr>
<tr>
<td>Blue Cube Systems (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Bluhm Burton Engineering Pty Ltd</td>
<td></td>
</tr>
<tr>
<td>Blyvooruitzicht Gold Mining Company Ltd</td>
<td></td>
</tr>
<tr>
<td>BSC Resources Ltd</td>
<td></td>
</tr>
<tr>
<td>CAE Mining (Pty) Limited</td>
<td></td>
</tr>
<tr>
<td>Caledonia Mining Corporation</td>
<td></td>
</tr>
<tr>
<td>CDM Group</td>
<td></td>
</tr>
<tr>
<td>CGG Services SA</td>
<td></td>
</tr>
<tr>
<td>Chamber of Mines</td>
<td></td>
</tr>
<tr>
<td>Concor Mining</td>
<td></td>
</tr>
<tr>
<td>Concor Technicrete</td>
<td></td>
</tr>
<tr>
<td>Council for Geoscience</td>
<td></td>
</tr>
<tr>
<td>CSIR Natural Resources and the Environment</td>
<td></td>
</tr>
<tr>
<td>Department of Water Affairs and Forestry</td>
<td></td>
</tr>
<tr>
<td>Deutsche Securities (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Digby Wells and Associates</td>
<td></td>
</tr>
<tr>
<td>DMS Powders</td>
<td></td>
</tr>
<tr>
<td>Downer EDI Mining</td>
<td></td>
</tr>
<tr>
<td>DRA Mineral Projects (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Duraset</td>
<td></td>
</tr>
<tr>
<td>E+PC Engineering and Projects</td>
<td></td>
</tr>
<tr>
<td>Elbrowc Ltd</td>
<td></td>
</tr>
<tr>
<td>Elbroc Mining Products (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>eThekwini Municipality</td>
<td></td>
</tr>
<tr>
<td>Evraz Highveld Steel and Vanadium Limited</td>
<td></td>
</tr>
<tr>
<td>Exxaro Coal (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Exxaro Resources Limited</td>
<td></td>
</tr>
<tr>
<td>Fasken Martineau</td>
<td></td>
</tr>
<tr>
<td>FLSmidth Minerals (Pty) Ltd (FFE001)</td>
<td></td>
</tr>
<tr>
<td>Fluor Daniel SA (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Franki Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Fraser Alexander Group</td>
<td></td>
</tr>
<tr>
<td>Goba (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Hall Core Drilling (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Hatch (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Herrenknecht AG</td>
<td></td>
</tr>
<tr>
<td>HPE Hydro Power Equipment (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Impala Platinum Holdings Limited</td>
<td></td>
</tr>
<tr>
<td>IMS Engineering (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>JENNMAR South Africa</td>
<td></td>
</tr>
<tr>
<td>Joy Global Inc. (Africa)</td>
<td></td>
</tr>
<tr>
<td>Leco Africa (Pty) Limited</td>
<td></td>
</tr>
<tr>
<td>Longyear South Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Lonmin Plc</td>
<td></td>
</tr>
<tr>
<td>Ludwici Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Wekaba Engineering (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Magnetech (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>MAGOTTEAUX (PTY) LTD</td>
<td></td>
</tr>
<tr>
<td>MBE Minerals SA Pty Ltd</td>
<td></td>
</tr>
<tr>
<td>MCC Contracts (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>MDM Technical Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Metalock Industrial Services Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Metorex Limited</td>
<td></td>
</tr>
<tr>
<td>Metso Minerals (South Africa) (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Minerals Operations Executive (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>MineRP</td>
<td></td>
</tr>
<tr>
<td>Mintek</td>
<td></td>
</tr>
<tr>
<td>Modular Mining Systems Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>MSA Group (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Multotec (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Murray and Roberts Cementation</td>
<td></td>
</tr>
<tr>
<td>NaIco Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Namakwa Sands (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>New Concept Mining (Pty) Limited</td>
<td></td>
</tr>
<tr>
<td>Northam Platinum Ltd - Zondereinde</td>
<td></td>
</tr>
<tr>
<td>Osborn Engineered Products SA (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Outotec (RSA) (Proprietary) Limited</td>
<td></td>
</tr>
<tr>
<td>PANalytical (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Paterson and Cooke Consulting Engineers</td>
<td></td>
</tr>
<tr>
<td>Paul Wurth International SA</td>
<td></td>
</tr>
<tr>
<td>Polysius A Division Of Thyssenkrupp Engineering</td>
<td></td>
</tr>
<tr>
<td>Precious Metals Refiners</td>
<td></td>
</tr>
<tr>
<td>Rand Refinery Limited</td>
<td></td>
</tr>
<tr>
<td>Redpath Mining South Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Rosond (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Royal Bafokeng Platinum</td>
<td></td>
</tr>
<tr>
<td>Roymecc Technologies (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>RSV Misym Engineering Service (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>RungePinockingarco Limited</td>
<td></td>
</tr>
<tr>
<td>Rustenburg Platinum Mines Limited</td>
<td></td>
</tr>
<tr>
<td>SAIEG</td>
<td></td>
</tr>
<tr>
<td>Salene Mining (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Sandvik Mining and Construction Delmas (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Sandvik Mining and Construction RSA(Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>SANIRE</td>
<td></td>
</tr>
<tr>
<td>Sasol Mining (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Scanimin Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Sebilo Resources (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>SENET (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Senmin International (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Shaft Sinkers (Pty) Limited</td>
<td></td>
</tr>
<tr>
<td>Sibanye Gold Limited</td>
<td></td>
</tr>
<tr>
<td>Smeq SA</td>
<td></td>
</tr>
<tr>
<td>SMS Siemag</td>
<td></td>
</tr>
<tr>
<td>SNC Lavalin (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Sound Mining Solution (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>SRK Consulting SA (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Time Mining and Processing (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Tomra Sorting Solutions Mining (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>TWP Projects (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Ukwazi Mining Solutions (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Umgeni Water</td>
<td></td>
</tr>
<tr>
<td>VBJKOM Consulting Engineers</td>
<td></td>
</tr>
<tr>
<td>Webber Wentzel</td>
<td></td>
</tr>
<tr>
<td>Weir Minerals Africa (Pty) Ltd</td>
<td></td>
</tr>
<tr>
<td>Xstrata Coal South Africa (Pty) Ltd</td>
<td></td>
</tr>
</tbody>
</table>
For the past 120 years, the Southern African Institute of Mining and Metallurgy, has promoted technical excellence in the minerals industry. We strive to continuously stay at the cutting edge of new developments in the mining and metallurgy industry. The SAIMM acts as the corporate voice for the mining and metallurgy industry in the South African economy. We actively encourage contact and networking between members and the strengthening of ties. The SAIMM offers a variety of conferences that are designed to bring you technical knowledge and information of interest for the good of the industry. Here is a glimpse of the events we have lined up for 2014. Visit our website for more information.

SAIMM DIARY

2014

◆ CONFERENCE
MineSafe Conference 2014
Technical Conference and Industry day
20–22 August 2014, Emperors Palace, Hotel Casino Convention Resort, Johannesburg

◆ SCHOOL
Drilling and Blasting
1–2 September 2014, Swakopmund Hotel & Entertainment Centre, Swakopmund, Namibia

◆ SCHOOL
3rd Mineral Project Valuation School
9–11 September 2014, Mine Design Lab, Chamber of Mines Building, The University of the Witwaterstand

◆ CONFERENCE
Surface Mining 2014
16–17 September 2014, The Black Eagle Room, Nasrec Expo Centre

◆ CONFERENCE
6th International Platinum Conference
20–24 October 2014, Sun City, South Africa

◆ COLLOQUIUM
12th Annual Southern African Student Colloquium
12 November 2014, Mintek, Randburg

◆ CONFERENCE
Accessing Africa’s Mineral Wealth: Mining Transport Logistics
19–20 November 2014, Emperors Palace, Hotel Casino Convention Resort, Johannesburg

2015

◆ CONFERENCE
Diamonds 2015 Conference
11–13 March 2015

◆ CONFERENCE
5th Sulphur and Sulphuric Acid 2015 Conference
8–10 April 2015, Southern Sun Elangeni Maharani KwaZulu-Natal, South Africa

◆ CONFERENCE
Mining, Environment and Society Conference
12–15 May 2015, Johannesburg, South Africa

◆ CONFERENCE
Copper Cobalt Africa Incorporating The 8th Southern African Base Metals Conference
6–8 July 2015, Zambesi Sun Hotel, Victoria Falls, Livingstone, Zambia

◆ CONFERENCE
The Seventh International Heavy Minerals Conference
12–14 August 2015, Sun City South Africa

For further information contact:
Confincing, SAIMM
P O Box 61127, Marshalltown 2107
Tel: (011) 834-1273/7
Fax: (011) 833-8156 or (011) 838-5923
E-mail: raymond@saimm.co.za

Website: http://www.saimm.co.za
Applying Poka Yokes in the mining industry

New Concept Mining
Integrated systems of support

Jackpot®
Pre-Stressing Units

Hydrabolt®
Roofbolts

Welded Mesh

Jackpack®
Pre-Stressing Plates

FOGLight®
Ground Movement Monitor

Rockstop®

Accessories

011 494 6000
www.ncm.co.za