

Determination of the *in situ* modulus of the rockmass by the use of backfill measurements

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SYNOPSIS

In situ measurements and numerical modelling based on elastic theory showed that backfill stresses are considerably higher than originally thought. This has led to a change in understanding of rockmass behaviour. After describing previous work on the behaviour of the rockmass, this paper discusses possible reasons for the high stresses measured in backfill, and identifies the rockmass mechanisms possibly generating these stresses. It is concluded that the joints and the geological stratigraphy can lead to increased elastic convergence, and that the inelastic behaviour of the rockmass is also partly responsible for the increased closure measured in backfill. The use of an *in situ* modulus is suggested as an interim solution until a new computer model that allows for joints, different layers, and inelastic closures becomes available.

SAMEVATTING

Metings ter plaatse en numeriese modellering wat op elastiese teorie gebaseer is, het getoon dat terugvulsel-spannings heelwat hoër is as wat aanvanklik gereken is. Dit het gelei tot 'n verandering in die begrip van die gedrag van die rotsmassa. Na 'n beskrywing van vorige werk oor die gedrag van die rotsmassa, bespreek die referaat moontlike redes vir die hoë spannings wat in terugvulsel gemeet is en identifiseer die rotsmassameganismes wat moontlik hierdie spannings genereer. Die gevolgtrekking word gemaak dat die mate en die geologiese stratigrafie tot groter elastiese konvergensie kan lei en dat die onelastiese gedrag van die rotsmassa ook deels verantwoordelik is vir die groter sluiting wat in terugvulsel gemeet is. Die gebruik van 'n *in situ*-modulus word aan die hand gedoen as 'n tussentydse oplossing totdat 'n nuwe rekenaarmodel wat vir nate, verskillende lae en onelastiese sluitings voorsiening maak, beskikbaar word.

Introduction

The *in situ* performance of various backfill materials and the surrounding rockmass has been monitored at a number of mines for more than three years. Some of the results obtained from these measurements have revealed that the rockmass behaviour is somewhat different from what had been thought.

The stress-strain behaviour of backfill measured underground gives an indication of the behaviour of the rockmass surrounding a backfilled stope. The *in situ* measurements and the numerical modelling based on elastic theory carried out to date¹ have shown that backfill stresses are considerably higher than originally expected. It is believed that the only possible mechanism that could induce such high stresses in backfill is the behaviour of the rockmass itself. The inelastic component of closure can account for only a limited portion (i.e. 3 to 5 MPa maximum) of the high stresses in backfill¹. If it is assumed that the elastic convergence induced by the rockmass is responsible for the generation of the major portion of the high stresses in backfill, then an elastic-stress analysis program such as MINSIM-D should be able to simulate the stresses measured in the backfill with reasonable accuracy.

Since the terms *convergence* and *closure* are used frequently in this paper, these terms are defined here. *Convergence* refers to the elastic component of closure measured underground, and inelastic closure with elastic convergence together form the *closure*.

Previous Work

In the early 1960s, field measurements were made to gain an understanding of the overall behaviour of the rockmass, and the results obtained were compared with theoretical simulations. These results were published in two papers^{2,3}.

Ortlepp and Nicol², comparing field measurements carried out at ERPM and Harmony mines with the results obtained from an electrical resistance analogue, reported a good correlation between the measured and the theoretical results. It was concluded that this good correlation confirmed the results of earlier simplified analyses and validated the concept of elastic behaviour for Witwatersrand-type rock masses.

Fig. 1 shows the measurements taken along a haulage situated in the hangingwall of two longwall stopes. There is very good agreement between the measured and the theoretical results above the pillar (i.e. between benchmarks 7 and 19), but a difference of more than 20 per cent at the benchmarks above the mined areas.

Two important points emerge from Ortlepp and Nicol's paper².

The first is that the levelling measurements along the haulage were not related to a stable benchmark. Benchmark 15 was selected as the reference point, and all the observed elevation changes were related to that benchmark. However, it was also found that benchmark 15 was not a stable point, and the displacement was therefore determined by use of the electrical resistance analogue. From Fig. 1 the amount of the displacement at benchmark 15 is calculated as about 3 cm. This means that about 3 cm of correction was made to each benchmark along the haulage. As will be discussed later, joints and other geological features affect the overall behaviour of

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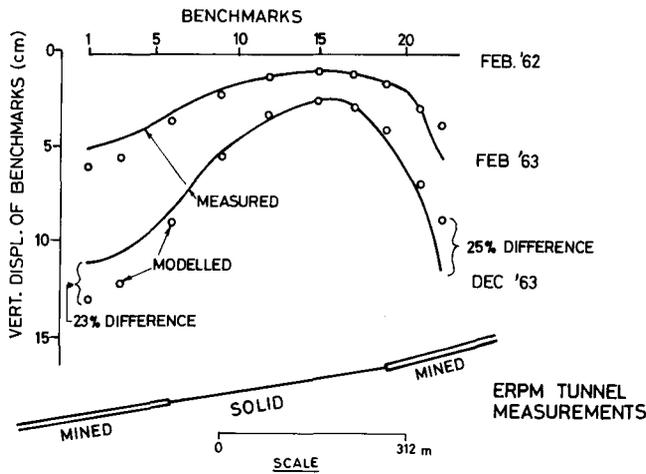


Fig. 1—A comparison of elastic theory with closure measurements (after Ortlepp and Nicol²)

the rockmass. Therefore, it can be assumed that the displacement of 3 cm is an underestimate by the electrical resistance analogue of the real situation, and that the actual displacement of benchmark 15 should be higher than 3 cm. This would displace the measured curve in Fig. 1 downwards and away from the modelled curve. However, since benchmark 15 is in a strong compressive environment, the required correction is likely to be rather small (as argued later).

The second point of importance is shown in Fig. 2, where a comparison is made between the measured and the predicted subsidence profiles. The subsidence measurements were made above the longwalls of a gold mine at a depth of about 1500 m. The diagram shows that the agreement between the predicted and the measured subsidence profiles is poor. Ortlepp and Nicol argued that the existence of many small pillars throughout the mined-out area, which had possibly been crushed and were therefore not providing the same complete resistance to closure as the unmined ground, introduced an ambiguity in the theoretical displacements. Therefore, they removed all the remnants with minimum dimensions of 15 to 16 m from their modelling.

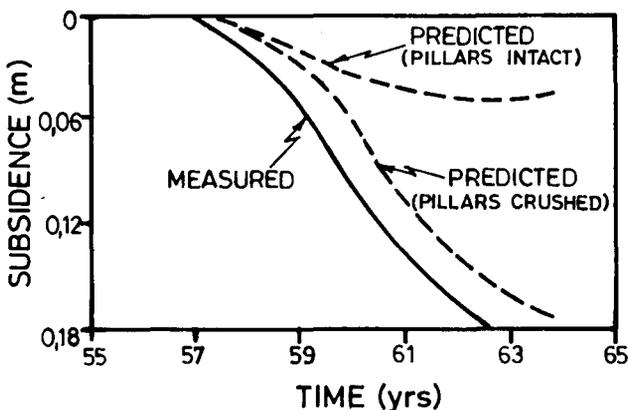


Fig. 2—A comparison of predicted and measured subsidence profiles (after Ortlepp and Nicol²)

The predicted subsidence curve that was then obtained shows better agreement (Fig. 2). However, it is known that a crushed pillar of 3 m width can easily provide a

support resistance of 20 to 25 MPa to the strata⁴. A pillar 15 m wide would become crushed at the edges, but would be carrying considerable load towards the centre of the pillar.

In a more recent paper, Budavari and Croeser⁵ used field measurements obtained from the Harmony gold mine in the Orange Free State to determine an *in situ* modulus for MINSIM-D simulations. The computed rise of the footwall close to the shaft was compared with the corresponding measured value. The choice of the footwall rise as a measure of the rockmass response was made because the footwall was thought to be free from strata separation and other inelastic deformations. Budavari and Croeser found that the correlation between the measured and the calculated displacements was not close when the widely accepted 70 GPa was used as the modulus value. They reported a reasonable correlation when using a modulus value of 50 GPa.

In a study carried out by More O'Ferrall⁶, the *in situ* modulus was calculated as 27 GPa for a shaft-pillar extraction in the Klerksdorp area. He found by trial and error that a strain profile corresponding to that of an elastic modulus of 27 GPa fitted more accurately the actual strain profile along the shaft.

It has long been accepted by rock-mechanics researchers from overseas that the deformability or *in situ* modulus of the rockmass differs from the Young's modulus of intact rock mainly owing to the existence of joints, faults, and stress-induced fractures within the rockmass. It has also been recognized that different geological layers influence the *in situ* modulus of the rockmass in different ways.

There are many examples from the overseas literature in which the *in situ* modulus for the rockmass has been used. Here, only two interesting papers will be discussed.

The first of these papers⁷ demonstrates the relationship between the modulus reduction factors and the discontinuity spacing, as shown in Fig. 3. A reduction factor (α_c), which is the ratio of the *in situ* modulus of the rockmass (E_{mass}) to the Young's modulus of an intact rock specimen (E_{rock}), is calculated as follows:

$$\alpha_c = \frac{E_{mass}}{E_{rock}} = \frac{1}{1 + \frac{E_{rock}}{t_a k_n}} \quad (1)$$

where t_a = Discontinuity spacing (m)
 k_n = Normal stiffness of joints (MPa).

Equation (1) is based on the assumption that the rockmass is a transversely isotropic medium. The rockmass is treated as being composed of rock material of conventional elastic properties with discontinuities such as joints. Further details about the derivation of equation (1) are given by Stephansson⁸.

In Fig. 3, if the joint spacing is 0,6 m and the normal joint stiffness is equal to the Young's modulus of intact rock (i.e. $E_{rock}/k_n = 1$), the reduction factor (α_c) becomes 0,38. This means that, if the Young's modulus of the rock is 70 GPa, the *in situ* modulus of the rockmass becomes 27 GPa. In the method illustrated in Fig. 3, the joint orientation is not considered; this would introduce some error since the strength of two joints at different orientations within the same stress environment would

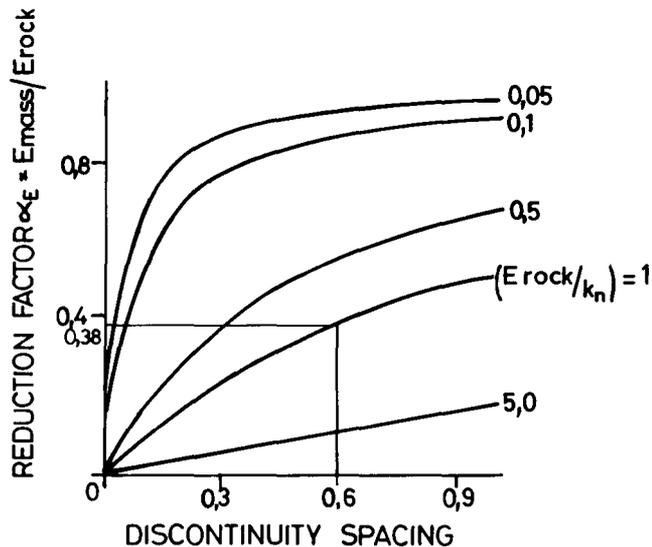


Fig. 3—The relationship between joint spacing and reduction factor for the *in situ* modulus of rockmass (after Kulhawy⁷)

differ.

In the second paper, Fossum⁹ derived a constitutive model for an elastic randomly jointed rockmass. The joints were modelled as one-dimensional deforming elements, while the intact rock was modelled as a three-dimensional continuum. Fig. 4 shows the change of *in situ* shear and bulk moduli of a rockmass with respect to the intact moduli, with varying joint spacing. The diagram illustrates that the influence of joints on the *in situ* modulus of the rockmass is reduced as the joint spacing becomes large.

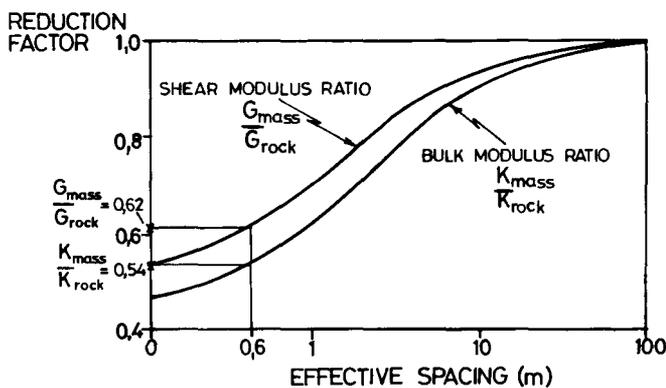


Fig. 4—The behaviour of *in situ* shear and bulk modulus with respect to joint spacing (after Fossum⁹)

If the effective joint spacing is 0,6 m, the *in situ* modulus calculated by Fossum's technique is 42,5 GPa for a rock with an intact modulus of 70 GPa. The effective *in situ* modulus is calculated as follows:

$$E_{mass} = \frac{9K_{mass} G_{mass}}{3K_{mass} + G_{mass}}, \dots \dots \dots (2)$$

where K_{mass} = *in situ* bulk modulus
 G_{mass} = *in situ* shear modulus.

However, the *in situ* modulus of the rockmass was calculated as 27 GPa according to Kulhawy's method⁷ for the same joint spacing of 0,6 m. This shows that,

although these two approaches demonstrate the influence of joints and discontinuities on the rockmass, the results obtained are different. The simple reason for this difference is that the methods are empirical and should be used carefully if they are used for different conditions.

In Situ Modulus of Rockmass

For modelling purposes in South Africa, the rockmass has generally been assumed to be a homogeneous isotropic elastic body, and the influence of joints, geological discontinuities, and different layers on rockmass behaviour has been thought not to be significant. For example, a Young's modulus of 70 GPa (i.e. Young's modulus for a specimen of intact quartzite rock) has been used for most of the MINSIM-D simulations carried out on the gold mines. However, it is also known that the geological stratigraphy differs in the central Witwatersrand, the West Rand, and the Orange Free State goldfields, and that the rockmass is intersected by joints at various orientations and spacings¹⁰.

A typical *in situ* stress-strain graph for dewatered tailings backfill material is shown in Fig. 5. The diagram displays the confined compression behaviour of the material measured in the vertical, strike, and dip directions, and indicates that the backfill provides about 45 MPa of vertical stress resistance after a compression of 18 per cent at about 55 m behind the face. However, if the mining geometry of the area from which the measurement in Fig. 5 was obtained is modelled by MINSIM-D for an elastic modulus of 70 GPa in the rockmass, a vertical stress of 10 MPa is predicted at the position where the measured stress was 45 MPa. This discrepancy is accounted for by the difference between the theoretical elastic convergence and the measured closure.

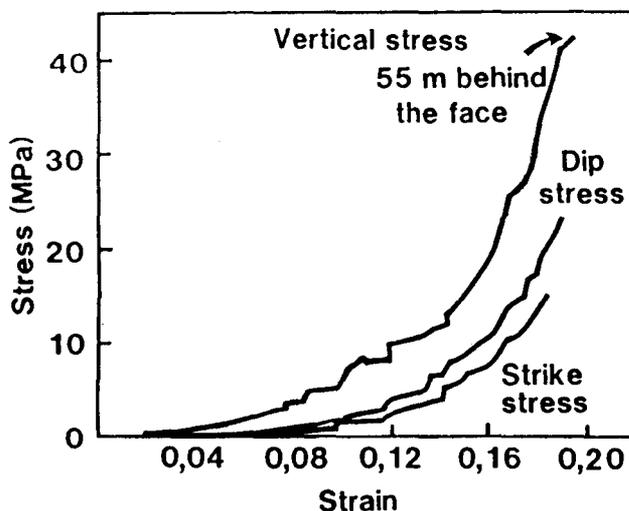


Fig. 5—The *in situ* confined compression behaviour of dewatered tailings

In order to demonstrate the difference between the profiles for theoretical elastic convergence and measured closure, numerical modelling was carried out for three of the backfill sites. The results are illustrated in Fig. 6. The profiles for elastic convergence assuming a Young's modulus of 70 GPa are given for comparison with the profiles for measured closure. As shown in the diagram,

the measured closure is significantly higher than the elastic convergence. This difference in closure is the reason for the discrepancy between the computed and the measured vertical backfill stresses. MINSIM-D uses constitutive equations that approximate the stress-strain behaviour of backfill, and calculate the backfill stresses according to the induced closure and, hence, the strain at that point. If the convergence estimated by the program is less than the measured closure, then the corresponding stress value will also be lower than the measured stress in the fill. The mechanisms of rockmass behaviour possibly causing these differences between the theoretical and the measured closure are discussed later.

An equivalent *in situ* modulus for the rockmass can be determined from the stress and closure measurements recorded in the backfill. MINSIM-D models are then run with different Young's moduli until agreement is achieved between the profiles for the theoretical and measured closure. For example, as shown in Fig. 7, if the Young's modulus of the rock is reduced to 40 GPa from 70 GPa, the profile for backfill stress agrees well with that for the measured stress.

A second possible approach to the modelling of measured backfill stress is to assume that the rockmass modulus remains at 70 GPa, but that irreversible bulking takes place in the immediate hangingwall and foot-wall strata because of inelastic fracturing and sliding processes ahead of and behind the highly stressed mining face¹¹. If this inelastic bulking amounts to, say, 0,15 m, the effective closures are all increased by 0,15 m and the effective stoping width to be input to MINSIM-D falls to 0,85 m. The resulting theoretical profile again gives a reasonable fit to the measured profile (Fig. 7).

Fig. 8 shows the closure profiles corresponding to the stress profiles depicted in Fig. 7. The accuracy in the first 10 m of the measured closure profile is somewhat poor because the closure station was installed at about 8 m from the face. Therefore, the closure for the first 8 m of the profile was estimated and added to the measured closure. There is reasonable agreement between the measured and the 40 GPa theoretical closure profiles. The reduced stoping width and the 70 GPa closure profiles show a slightly less good correlation with the measured profile.

The modelling of the correct backfill stresses in MINSIM-D simulations is important for the calculation of energy release rates (ERR) to assess the regional support potential of backfills. The amount of convergence and the stress, immediately behind and ahead of the face respectively, are the important parameters that determine ERR. If the correct backfill stresses and closures are not simulated by MINSIM-D, then the ERR values calculated by the program can also be expected to be incorrect.

The closure profile of reduced modulus (i.e. 40 GPa) simulates the profile for *in situ* backfill closure with reasonable accuracy, but both profiles have components of elastic convergence and inelastic closure. However, only the elastic component of closure should be considered for the correct calculation of ERR. Therefore, the inelastic component of the closure needs to be determined and subtracted from the measured and computed closure, although the differentiation between the inelastic and the elastic components is difficult. In this way, a new

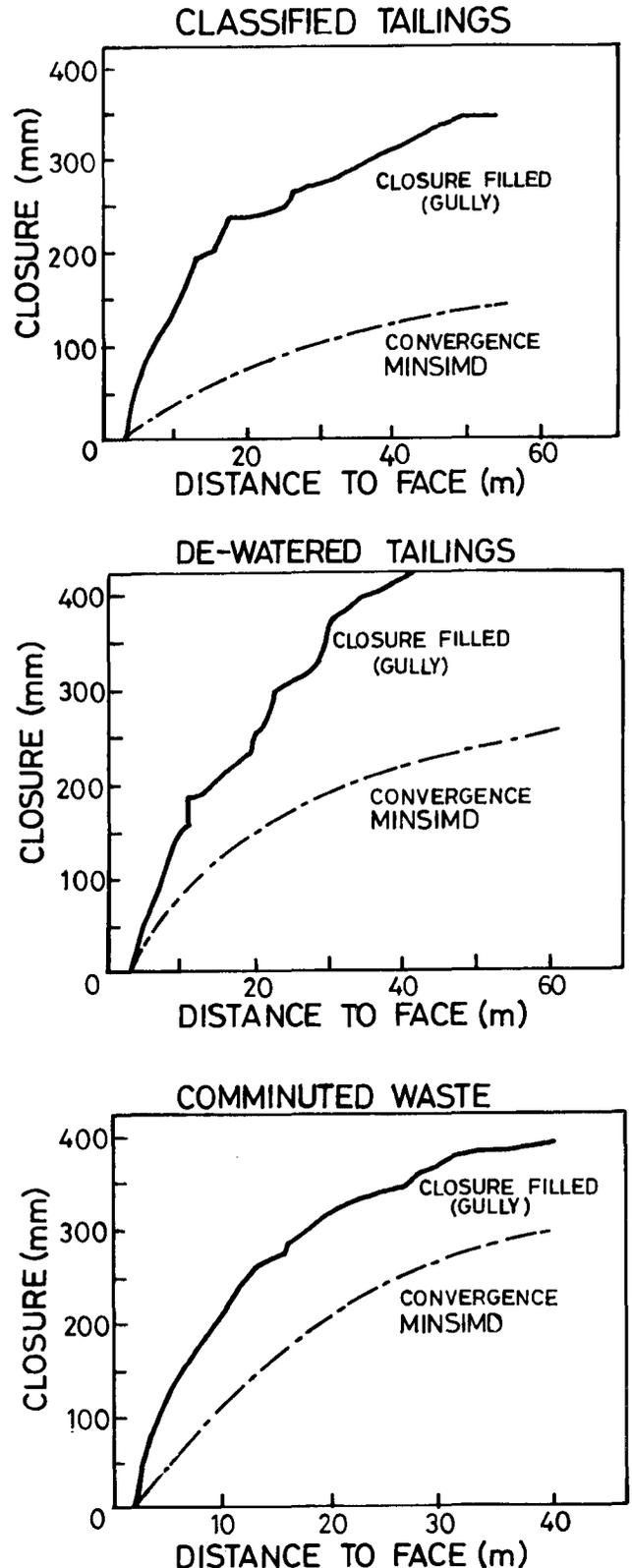
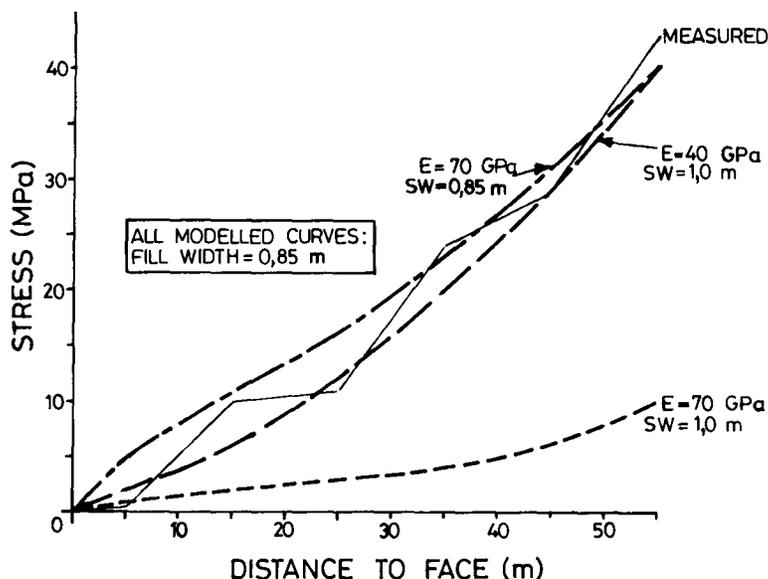


Fig. 6—A comparison of theoretical and measured closure profiles

in situ modulus, somewhere between 70 GPa and 40 GPa, can be calculated. This newly calculated modulus should then be used to determine more realistic ERR values.

In the approach using reduced stoping width, the predicted closure profile probably under-estimates the

Fig. 7—A comparison of profiles for measured and calculated backfill stress



'elastic' convergence, and hence the ERR, because the influence of the joints and different layers in the rockmass is ignored. However, since the magnitudes of the elastic and inelastic components of closure are hard to establish, as in the case of the reduced modulus concept, it becomes difficult to estimate the magnitude of the reduced stoving width under different mining conditions.

It is believed that the final solution lies somewhere between these two extreme approaches, and that some reduction of both the effective stoving width and the Young's modulus are required for more realistic answers. The reduced stoving width and *in situ* modulus would account for the inelastic and elastic components of closure respectively. However, in the elastic component of closure, the influence of joints and different geological layers still needs to be considered. Until a new computer model that realistically allows for joints, different layers, and inelastic closures becomes readily available, a suggested interim solution is to continue to use MINSIM-D but simply to reduce the equivalent rockmass modulus to appropriate values.

A number of MINSIM-D simulations have been carried

out to determine these *in situ* moduli for different regions in South African gold mines on the assumption that there are no irreversible bulking closure components. The results suggested that suitable values¹² of E_{mass} for strata in the central Witwatersrand, Klerksdorp, and the Orange Free State are 52 GPa, 35 to 40 GPa, and 30 to 35 GPa respectively.

Factors Affecting Rockmass Behaviour

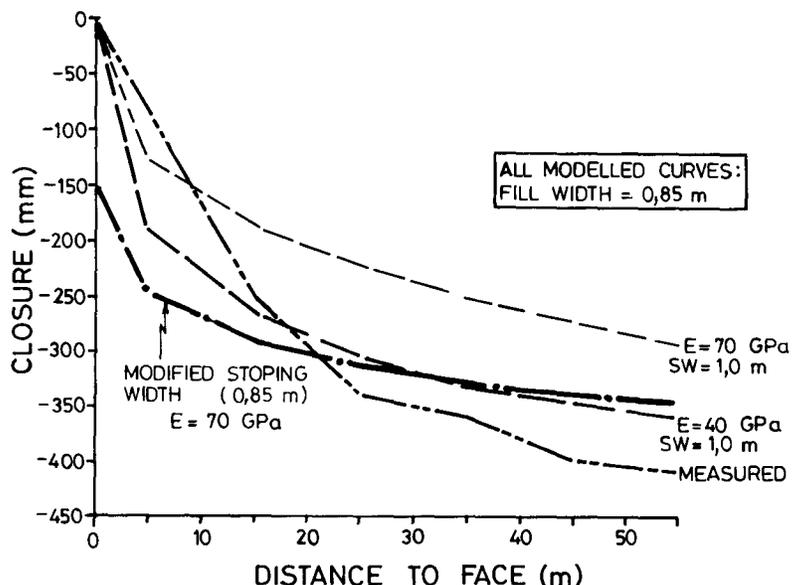
Based on field observations and various modelling exercises, it appears that there are three major factors that affect the overall response of the rockmass to mining and cause the difference between the profiles for measured and modelled closure. These are as follows:

- (i) joints in the rockmass,
- (ii) different geological layers, and
- (iii) inelastic behaviour such as fracture, dilation, bed separation, etc.

Influence of Joints

Fig. 9 shows the stress-deformation behaviour of

Fig. 8—A comparison of closure profiles obtained by different modelling approaches



specimens of intact rock and jointed rock. It is important to observe that the stiffness of both specimens at a normal stress of around 40 to 50 MPa becomes almost equal. However, the amount of hysteresis is different and implies that, in jointed rock, there is considerably more inelastic strain. Furthermore, the jointed rock is subjected to more deformation than the intact rock to reach the same stiffness.

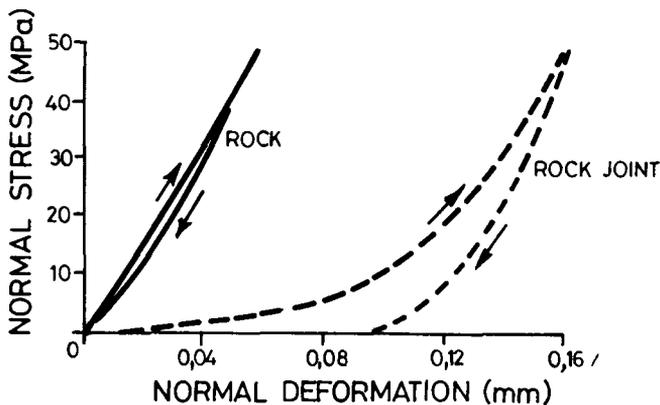


Fig. 9—The stress–deformation behaviour of intact and jointed rock (after Bandis, Lumsden, and Barton¹³)

Fig. 9 can be related to conditions in South African deep-level gold mines as follows. It is known that the stresses ahead of a longwall face are very high (i.e. 300 to 400 MPa). In that case, the joints within the rockmass would be clamped and the stiffness of the rockmass would be similar to that of an intact rockmass. However, in mined-out areas, the stresses on these joints would have been released owing to the relaxation of the rockmass. At that stage, the total deformation of the jointed rock (similar to the hysteresis of the specimen of jointed rock) would be greater than that of the intact rock.

To further quantify the influence of joints, a conventional stope with a half span of 100 m at a depth of 1000 m was modelled as shown in Fig. 10 using the FLAC modelling program, which allows for both elastic and inelastic behaviour. The computer runs were carried out on the assumption of a homogeneous isotropic rockmass with and without joints. Ubiquitous joints with an inclination of 60 degrees were modelled. The joint properties used were an angle of friction of 30 degrees and a cohesion of 100 kPa.

Fig. 11 shows the results of the modelling in terms of the closure profiles obtained from different model solutions. A closure profile calculated according to Salamon's analytical method¹⁴ is also included. The agreement between Salamon's method and the FLAC run with homogeneous isotropic solution is reasonable. This verifies the acceptability of the mesh used for the modelled geometry. The same geometry was also modelled using the two-dimensional computer program MINAP, and the resultant closure profile was also fairly close to those obtained from the first two solutions.

The effect of jointing on the closure is also shown in Fig. 11, and it is clear that the closure increases considerably when compared with the homogeneous isotropic solutions. The difference between the maximum closures calculated by the jointed version and by the homogeneous

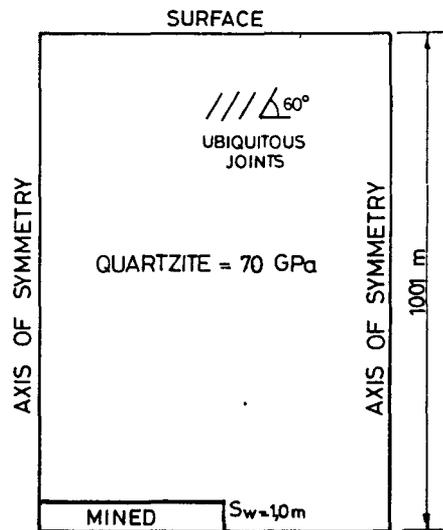


Fig. 10—The modelled geometry for the FLAC program

isotropic program was about 17 per cent. This difference will probably vary with the inclination of the joints and the primitive stress ratio.

The joint orientations and the spacing between the joints are usually determined from observations carried out in stopes or from borehole cores. However, this limited information is not enough to show the distribution of joints within the rockmass. For instance, the joints might be dipping at 60 degrees in the stope, but their orientation, the spacing between them, and even their continuity could be entirely different away from the stope. Therefore, the use of FLAC to determine the amount of joint influence on closure does not give an accurate answer.

As discussed earlier, if back analyses were carried out on backfill measurements, an *in situ* modulus could be obtained. This modulus, however, would contain all the effects induced by joints, different geological layers, and inelastic behaviour within the rockmass.

Influence of Different Geological Layers

To assess the influence of different geological horizons on rockmass behaviour, a layered model, which is typical of the strata overlying the gold reefs in the Orange Free State, was modelled using FLAC. Fig. 12 shows the modelled geometry, and the same mesh as for the homogeneous isotropic and jointed rockmass solutions was used in the modelling. The input modulus parameters for quartzite and shale layers were 70 GPa and 25 GPa respectively.

Fig. 13 shows that the calculated maximum closure in the centre of the panel for the layered model is 90 mm. This is about 17 per cent higher than the maximum closure given by the homogeneous isotropic solution. However, if the joints and the shale layers are included in the mesh, the maximum closure becomes about 39 per cent greater than for the homogeneous elastic isotropic solution, indicating that a model allowing for unhomogeneities and inelastic behaviour is probably much more appropriate.

Inelastic Behaviour of Rockmass

The closure measured underground has three com-

Fig. 11—Closure profiles obtained by the use of different modelling techniques

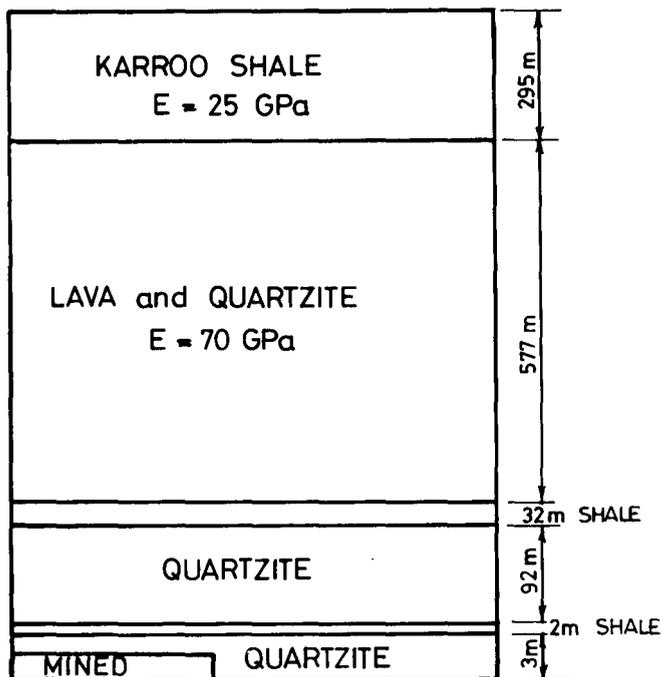
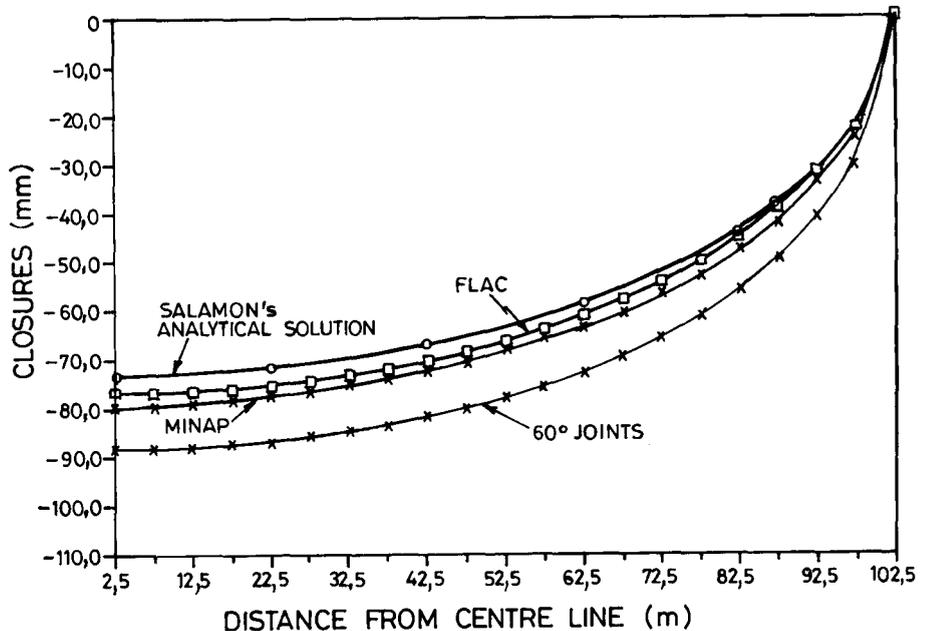


Fig. 12—The modelled geometry for a layered rockmass

ponents. These are elastic convergence, inelastic bed separation, and closure due to dilation. The elastic convergence is the elastic relaxation of the rockmass in response to mining and, as shown above, it is influenced by the joints in the rockmass.

Inelastic bed separation takes place very close to the face, where the first few layers in the hangingwall are displaced downwards owing to their own mass. Dilation in the rockmass ahead of the face, owing to shear movements and extension fracturing, induces closure as a result of the generation of horizontal compressive forces that cause the strata to buckle into the stope. It is also found that some vertical dilation takes place on fractures in the plane of bedding in response to horizontal shearing move-

ments. This causes the strata to ride up irregularities on bedding surfaces and to be forced apart by gouge and thus down into the excavation¹. It has been further advocated by Napier¹⁵ that some inelastic closure also occurs in stopes as a result of slip along vertical fractures. This type of inelastic slip along vertical fractures has been modelled successfully, but no underground measurements have yet been made to support the argument.

The amount of closure induced by each component of inelastic closure is not known. However, Gurtunca *et al.*¹ showed that the high rates of inelastic closure in backfilled panels occur only until a vertical reaction stress of 3 to 4 MPa is built up in the backfill. The required closure to induce this stress is about 100 to 150 mm. It should be noted that a part of this 100 to 150 mm of closure is contributed by the elastic components of the rockmass.

Conclusions

The recently available *in situ* measurements made in various backfills used on the gold mines indicate that there is considerable difference between the measured and the theoretical closures and stresses in the backfill. As an explanation of this difference, the following conclusions were reached from the work described in this paper.

- (i) The influence of joints in the rockmass should not be ignored since it was found that the presence of joints can increase the elastic convergence significantly.
- (ii) The geological stratigraphy is different in various gold-mining districts, and the presence of different layers in the rockmass can increase the elastic convergence.
- (iii) Although it is difficult to separate the magnitudes of inelastic closure from the elastic convergence, the inelastic behaviour of the rockmass is partly responsible for the unexpectedly large closure measured in backfill.
- (iv) It is very difficult to quantify the amount of closure contributed by each of the closure components.

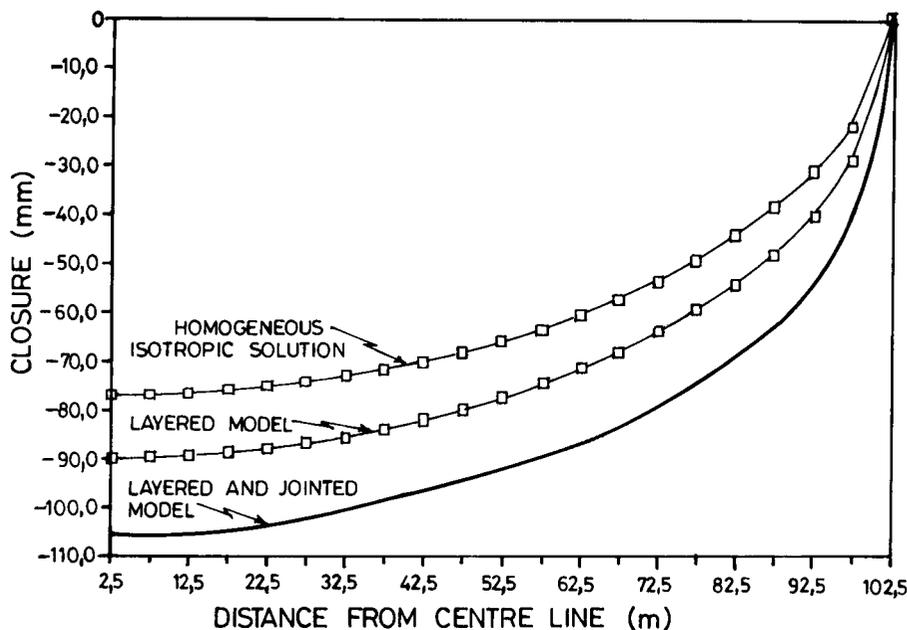


Fig. 13—A comparison of closure profiles obtained from different models

Therefore, an *in situ* modulus for the entire rockmass needs to be derived for more realistic modelling. Backfill measurements will provide an important input to these derivations.

- (v) The use of an *in situ* modulus provides an effective interim solution until a new computer model that allows for joints, different layers, and inelastic closures becomes generally available.

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References

1. GÜRTUNCA, R.G., JAGER, A.J., ADAMS, D.J., and GONLAG, M.D. The *in situ* behaviour of backfill materials and the surrounding rockmass in South African gold mines. *Proc. 4th Int. Symp. of Mining with Backfill*. Montreal (Canada), 1989.
2. ORTLEPP, W.D., and NICOL, A. The elastic analysis of observed strata movement by means of an electrical analogue. *Proc. Symp. on Rock Mechanics and Strata Control in Mines*. Johannesburg, 1963. pp. 469–490.
3. RYDER, J.A., and OFFICER, N.C. An elastic analysis of strata movement observed in the vicinity of inclined excavations. *Ibid.*, pp. 169–193.
4. JAGER, A.J., and ROBERTS, M.K.C. Support systems in productive excavations. *GOLD 100*. Fivaz, C.E., and King, R.P. (eds.). Johannesburg, South African Institute of Mining and Metallurgy, 1986. vol. 1, pp. 289–300.
5. BUDAVARI, S., and CROESER, R.W. Effects of the extraction of tabular deposits around vertical shafts in deep-level mines. *Proc. 6th Int. Congress on Rock Mechanics*. Montreal (Canada), 1987. pp. 825–829.
6. MORE O'FERRALL, R.C. Shaft pillar extraction experience at Stillfontein Gold Mining Company Limited. *Rock Mechanics and Other Aspects of Shaft Protection Systems Seminar, COMRO Research Report*, no. 49/87. 1987.
7. KULHAWY, F.M. Geomechanical model for rock foundation settlement. *Journal of the Geotechnical Engineering Division, ASCE*, vol. 104, no. GT2. Feb. 1978. pp. 211–227.
8. STEPHANSSON, O. The Näsliden Project—rock mass investigations. *Conference on Application of Rock Mechanics to Cut and Fill Mining*. Lulea (Sweden), 1980. pp. 22–71.
9. FOSSUM, A.F. Effective elastic properties for a randomly jointed rock mass. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, vol. 22, no. 6. 1985. pp. 467–470.
10. VAN DER HEEVER, P.K. The influence of geological structure on seismicity and rockbursts in the Klerksdorp Goldfield. M.Sc. thesis (unpubl.), Rand Afrikaans University, Johannesburg, 1982.
11. RYDER, J.A. Personal communications, Johannesburg, 1990.
12. ADAMS, D.J., GÜRTUNCA, R.G., JAGER, A.J., and GAY, N.C. Assessment of a new mine layout incorporating concrete pillars as regional support. *Proc. 4th Int. Symp. of Mining with Backfill*. Montreal (Canada), 1989.
13. BANDIS, S., LUMSDEN, A.C., and BARTON, N. Fundamentals of rock joint deformation. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* vol. 20. 1983. pp. 249–268.
14. SALAMON, M.D.G. Elastic analysis of displacements and stresses induced by the mining of seam or reef deposits, Part I. *J. S. Afr. Inst. Min. Metall.*, vol. 64, no. 4. Nov. 1963.
15. NAPIER, J.A.L. Personal communications, Johannesburg, 1990.