

# Variability in the gold grade and spatial structures of Witwatersrand reefs: The implications for geostatistical ore valuations

by D.G. Krige\*

## SYNOPSIS

Recent publications have demonstrated the important role that can be played by *a priori* information of the grade levels, variability, and spatial patterns in adjacent or nearby areas in improving the quality of grade estimates for virgin or sparsely drilled areas. A research project was therefore conducted involving the collection and analysis of extensive sampling and borehole data from 7 reefs on some 30 Witwatersrand mines.

This paper shows that the 'equivalent' logvariance for the two-parameter lognormal model provides a good basis for the study and comparison of the levels and behaviour of relative variances. The trends of these against the corresponding logareas were found to form distinctive patterns, which are useful for these purposes.

The patterns for the Basal, Vaal, Carbon Leader, and Main Reef Series form a reasonably close group, whereas those for the V.C.R. (particularly where the facies are mixed), B Reef, and Kimberley-Elsburg Reefs (particularly for narrow channel widths) do not always conform. The patterns shown by underground chip samples are also compared with those obtained from borehole cores.

It is shown how this information can be used to improve the borehole valuation of new mines and of virgin areas in existing mines.

## SAMEVATTING

Die belangrike rol wat *a priori* inligting oor graadvlakke, variansies, en ruimtelike patrone in aangrensende of nabygeleë areas kan speel om die kwaliteit van graadskattings vir nuwe of ylgeboorde areas te verbeter is in onlangse publikasies benadruk. 'n Navorsingsprojek is gevolglik uitgevoer om wydverspreide monster- en boorgatdata te bekom vir 7 riewe op ongeveer 30 Witwatersrand myne.

Die 'ekwivalente' logaritmiëse variansie vir die twee-parameter lognormale model word aangedui as 'n goeie basis om die vlakke en gedrag van die relatiewe variansies te bestudeer en te vergelyk. Die tendense hiervan teenoor die korresponderende logaritmes van die betrokke areas vorm bepaalde patrone wat bruikbaar is vir hierdie doel.

Die patrone vir die Basaal, Vaal, Koolstof Leier en Hoofrif Series riewe vorm 'n redelike kompakte groep terwyl die Ventersdorp Kontak Rif (veral waar die facies gemeng is), B Rif en die Kimberley-Elsburg riewe (veral vir smal kanaalwydtes) nie altyd inpas nie. Die patrone vir ondergrondse kapmonsters word ook vergelyk met dié van boorgatkerns.

Daar word gewys dat hierdie inligting gebruik kan word om boorgatwaarderings van nuwe myne en van onontginde areas in bestaande myne aansienlik te verbeter.

## INTRODUCTION

Since 1984, considerable attention has been focused on the advantages to be gained in the borehole valuation of new mines from the use of grade and variance information relating to nearby properties<sup>1-5</sup>. The role played by such information has been shown to be of particular significance in the Bayesian-type macro-kriging techniques suggested. With the cooperation of the Anglo-American, Anglovaal, Genmin, J.C.I., and Rand Mines groups, extensive data were collected from some 30 Witwatersrand mines in the Evander, Rand, Far West Rand, Klerksdorp, and O.F.S. fields. Supplementary data were obtained from various published papers and from masters' and doctoral theses at the University of the Witwatersrand. The data covered 7 reefs and areas ranging in size from check chip samples and borehole-deflection areas, to mines and groups of mines.

Because of the very skew nature of the grade distributions and the high correlation between grade and variance, i.e.

the proportional effect, the variances of untransformed cm.g/t values from different areas and reefs cannot be compared directly. The relative variance, i.e. variance/mean, is a better measure but, when based on untransformed values, is extremely sensitive to one or more abnormally high values. The worst example was that from a V.C.R. area with 2924 values of which the highest value was 70 915 cm.g/t and the arithmetic mean 380 cm.g/t. The relative variance based on all the values was 12,29, whereas this was reduced to 0,545 when the obvious 'outlier' was omitted, i.e. a reduction of nearly 96 per cent.

The three-parameter lognormal model provides a much more stable estimate of the relative variance as follows:

$$\text{Relative variance (RV)} = \left[ \frac{\theta + \beta}{\theta} \right]^2 \left[ e^{\sigma_{\beta}^2} - 1 \right], \quad [1]$$

where  $\theta$  = arithmetic mean  
 $\beta$  = third parameter  
 $\sigma_{\beta}^2$  = variance of  $\ln(\text{cm.g/t} + \beta)$ .

In the above example, this approach gave an estimated relative variance of 0,632. The omission of the highest value in the case of every distribution would, of course,

\* University of the Witwatersrand, P.O. Wits, 2050 Transvaal.

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introduce a negative bias when the highest value is not a true outlier. Hence, to test whether the log approach is unbiased, all cases where the relative variance on untransformed values was reduced by more than 10 per cent following the omission of the highest value were eliminated in the analysis.

Figure 1 shows the correlation between the relative variances based on all untransformed and transformed values ( $y$  and  $x$  axes respectively) for each of 11 distributions where the reduction was less than 10 per cent. The correlation coefficient is 91 per cent, and the slope of the regression of  $y$  on  $x$  is 99 per cent. No global or conditional biases are indicated.

The logarithmic approach was therefore used throughout and, for convenience, the more generally known measure of the natural logarithmic variance was accepted. This was calculated in each case on the three-parameter lognormal model but, in order to eliminate the effects of variations in the third parameter ( $\beta$ ),  $\sigma_{\beta}^2$  was converted to the 'equivalent' logvariance ( $\sigma_o^2$ ) for a two-parameter lognormal model (i.e. for  $\beta = 0$ ) as follows:

$$\text{Population variance: Three-parameter: } (\theta + \beta)^2 (e^{\sigma_{\beta}^2} - 1)$$

$$\text{Two-parameter: } \theta^2 (e^{\sigma_o^2} - 1)$$

For these two variances to be equal,

$$\sigma_o^2 = \ln \left[ \left( \frac{\theta + \beta}{\theta} \right)^2 \left( e^{\sigma_{\beta}^2} - 1 \right) + 1 \right] \quad [2]$$

$$= \text{i.e., } \ln(RV + 1).$$

## ANALYSES

Statistics of more than 200 grade distributions on the 7 reefs were summarized for each distribution or group of distributions, as shown in the example for the underground chip samples from the Basal Reef (Table I and Figure 2).

Figure 2 shows the linear regression of the 'equivalent' logvariances against the sizes of the corresponding areas on a log scale. The slope of the regression line for this de-Wijsian model is a direct measure of the strength of the spatial structure of the grades; a zero slope will correspond to a random pattern, i.e. no spatial structure. Furthermore, the slope is also half of the slope of the corresponding lognormal de-Wijsian semivariogram. The residual variance around the regression line based on the individual distributions provides a measure of the variance of the actual variance of the data sets within the area(s) concerned.

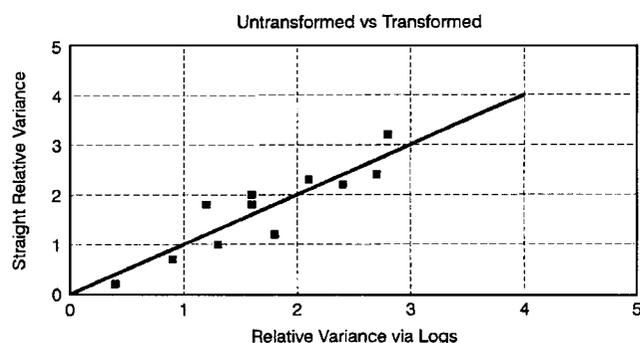


Figure 1—Regression of relative-variance estimates

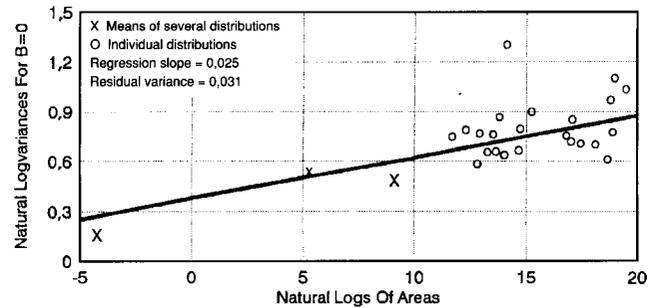


Figure 2—Regression of logvariance on logarea for chips from the Basal Reef (in cm.g/t)

The results of this procedure for the reefs concerned are summarized in Table II. Figure 3 shows the regression lines as fitted for chip samples and the indicated average variance levels for areas of 100 x 100 m, 1 x 1 km, and 18 x 10 km. It is clear that the variance levels for the B reef and on the V.C.R. (facies mixed) are much higher than for the other reefs and, to a lesser extent, this also applies to the Kimberley-Elsburg (narrow channel width) reefs. The slopes of the regression lines cover a fairly narrow range of 0,020 to 0,043, except for the V.C.R. mixed facies. Here, the slope is much lower but the spread of data, both for separate facies and for mixed facies, was not ideal, and the complicated facies patterns also present problems.

Figure 4 shows the borehole results for the Basal, Vaal, Kimberley-Elsburg, and V.C.R. Reefs. The level of the V.C.R. (mixed) regression line is again out of line, and the

Table I  
Example of analyses of grade distributions  
Basal Reef, O.F.S. Field—underground chip samples

Mine	Area m <sup>2</sup>	No. of values per area	No. of areas	Arith. mean cm.g/t	Third par. $\beta$	$\beta$ /A.M. %	Log. var. with $\beta$	Equiv. logvar. with $\beta=0$	ln of area
2	114289300	12238	1	5357	52	1,0	0,987	0,999	18,554
5	102490600	14590	1	4804	414	8,6	0,836	0,933	18,445
3	86762100	17726	1	4066	152	3,7	0,978	1,024	18,279
7	51960700	8436	1	2435	305	12,5	0,553	0,660	17,766
6	46358600	19687	1	2173	17	0,8	0,569	0,576	17,652
4	29598900	33031	1	1387	218	15,7	0,498	0,623	17,203
9	24266300	1000	1	1137	239	21,0	0,621	0,816	17,005
8	20382900	197	1	955	87	9,1	0,687	0,777	16,830
10	15210000	24606	1	759	200	26,4	0,431	0,621	16,537
10	7995000	12230	1	820	200	24,4	0,433	0,609	15,894
10	7410000	12376	1	698	200	28,7	0,421	0,624	15,818
1	3520000	2448	1	898	70	7,8	0,804	0,890	15,074
1	2640000	7257	1	746	105	14,1	0,617	0,747	14,786
1	1920000	3295	1	666	90	13,5	0,597	0,719	14,468
1	1440000	1418	1	629	5	0,8	1,355	1,367	14,180
1	1440000	1015	1	1240	130	10,5	0,653	0,754	14,180
1	720000	3962	1	812	125	15,4	0,601	0,741	13,487
1	960000	2209	1	1098	110	10,0	0,767	0,874	13,775
1	960000	2804	1	1149	120	10,4	0,664	0,765	13,775
10	800000	5398	1	712	200	28,1	0,447	0,654	13,592
10	585000	6832	1	905	200	22,1	0,420	0,576	13,279
10	562500	6202	1	750	200	26,7	0,426	0,617	13,240
1	480000	1433	1	586	115	19,6	0,579	0,752	13,082
10	390625	6174	1	645	190	29,5	0,423	0,633	12,876
1	240000	2525	1	940	131	13,9	0,580	0,703	12,388
10	10000	135	180	759	200	26,4	0,360	0,556	9,210
8	0,012	2	*	955	87	9,1	0,090	0,106	-4,423

\* Large no. of check sample pairs

**Table II**  
**Summary of regression results for all 7 reefs (in cm.g/t)**

Underground chip samples										
Reef	Basal Reef	Vaal Reef	Carbon Ldr	V.C.R.	VCR mixed	Main R.S.	Kim.Elsb-N	Kim. Elsb-W	B Reef	Average
	1	2	3	4	4a	5	6a	6b	7	1/5
<i>No. of mines:</i>										
O.F.S.	11	—	—	—	—	—	1	—	1	11
Klerksdorp	—	4	—	1	—	—	—	—	—	4
Far W. Rand	—	—	2	3	1	—	2	1	—	5
Rand	—	—	—	—	—	6	2	—	—	6
<i>Regression of logvariances on logareas:</i>										
No. of areas	25	16	8	33	3	7	5	29	10	89
Constant	0,379	0,361	0,551	0,587	2,617	0,459	0,686	0,501	2,108	0,530
Slope	0,025	0,043	0,026	0,025	0,011	0,035	0,043	0,020	0,031	0,026
<i>Average logvariance from regression:</i>										
(a) Large mines	0,854	1,178	1,045	1,053	2,836	1,124	1,503	0,889	2,697	1,024
(b) 1 km <sup>2</sup>	0,724	0,955	0,910	0,926	2,776	0,943	1,280	0,783	2,536	0,889
(c) 100 x 100 m	0,609	0,757	0,790	0,813	2,723	0,781	1,082	0,689	2,394	0,769
<i>Variance of logvariances:</i>										
Regr. Resids.	0,03	0,05	0,04	0,08	1,456	0,04	0,04	0,040	—	0,05
Indiv. Dists.	80–100x100m	21–1x1 km	8–(b) & (c)	—	—	—	—	—	4–(a)	—
	0,01	0,04	0,03	—	—	—	—	—	0,12	—
Surface borehole intersections										
Reef	Basal Reef	Vaal Reef	Carbon Ldr	V.C.R.	VCR mixed	Main R.S.	Kim.Elsb-N	B Reef		
	1	2	3	4	4a	5	6a	7		
<i>No. of mines:</i>										
O.F.S.	10	—	—	—	—	—	1	—		
Klerksdorp	—	4	—	1	1	—	—	—		
Far W. Rand	—	—	—	3	3	—	—	—		
Rand	—	—	—	—	—	—	—	—		
<i>Regression of logvariances on logareas:</i>										
No. of areas	11	12	—	4	9	—	130	—		
Constant	0,660	0,517	—	0,747	1,281	—	–0,011	—		
Slope	0,054	0,033	—	0,034	0,081	—	0,058	—		
<i>Average logvariance from regression:</i>										
(a) Large mines	1,686	1,136	—	1,393	2,812	—	1,100	—		
(b) 1 km <sup>2</sup>	1,406	0,967	—	1,217	2,394	—	0,796	—		
(c) 100 x 100 m	1,157	0,817	—	0,060	2,023	—	0,527	—		
<i>Variance of logvariances:</i>										
Regr. Resids.	—	(a)	—	(a), (b)	(a), (b)	—	—	—		
	—	0,30	—	0,04	0,16	—	—	—		

regression lines for the Basal and Kimberley–Elsburg Reefs are, in turn, higher than for the corresponding patterns based on chip samples. However, the values for the Basal Reef all relate to pre-1952 boreholes, when core recoveries were poorer, and are thus not necessarily indicative of present conditions. On the Vaal Reef, the levels of borehole and chip variances are very similar, and the slopes are also fairly close. The variance levels for the Kimberley–Elsburg Reef are lower than for the chip samples but do not cover the same mining areas.

The semivariograms corresponding to the variance patterns in Figures 3 and 4 are shown in Figure 5 standardized in each case to a population variance of 1,0.

Except for the chips from the B Reef and the borehole samples from the Kimberley–Elsburg Reef, the semivariograms all fall in a relatively narrow band. This is a

*priori* information of significance since it implies that, for most Witwatersrand Reefs, an average standardized semivariogram can be used where specific information from the area or adjacent areas is not available or is too limited.

### CONFIRMATION OF THE MODEL

If the lognormal de-Wijsian model used in the previous analyses is to be applied in macro-kriging techniques as proposed by Krige and Assibey-Bonsu<sup>5</sup>, the semi-variograms found for the actual grades of large 1 km x 1 km areas on the Vaal and Carbon Leader Reefs by these authors should be comparable with those which can be derived from the present analyses. Krige and Assibey-Bonsu's semivariograms<sup>5</sup> were corrected for  $\beta = 0$  and replotted on a logscale for lag. The fitted straight lines

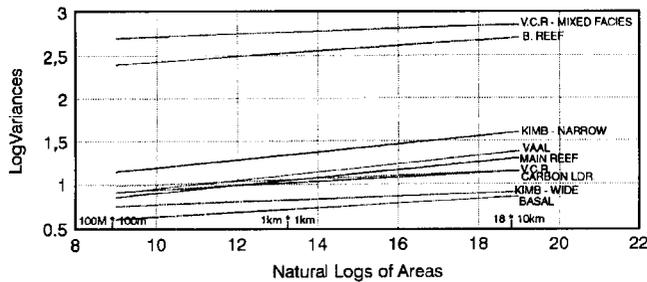


Figure 3—Regressed logvariances versus logareas for underground chip values (in cm.g/t)

have the slopes recorded in Table III compared with those indicated by the variance-size of the area analyses in the previous section of this paper (i.e. twice the slopes shown in Table II).

Table III confirms that the model is reasonable and can be used in practice.

### APPLICATIONS

The implications of these analyses cover two main aspects: firstly, the effects of *a priori* variance information on estimates of mean grade based on a limited set of borehole or underground chip samples; and, secondly, the effects of *a priori* evidence of variance patterns and spatial structures on macro-kriging via the introduction of outside information for such estimates.

#### Improved Mean Estimates

The arithmetic mean, although the basic orthodox estimate, is a very inefficient estimator for highly skewed distributions when the data are limited, e.g. as in a borehole estimate. Where the lognormal model can be used, Sichel's *t* estimator is more efficient, but it relies on a relatively poor estimate of logvariance based on the limited borehole data

Table III  
Slopes of de-Wijsian semivariograms for chip-sample grades of 1 km x 1 km areas

Based on variance/size of area analyses			Based on direct semivariogram analyses of 1 km x 1 km areas	
Reef	No. of mines	Slope	Mine	Slope
Vaal Reef	4	0,086	Hartebeestfontein	0,089
Carbon Ldr	2	0,052	Blyvooruitzicht	0,055

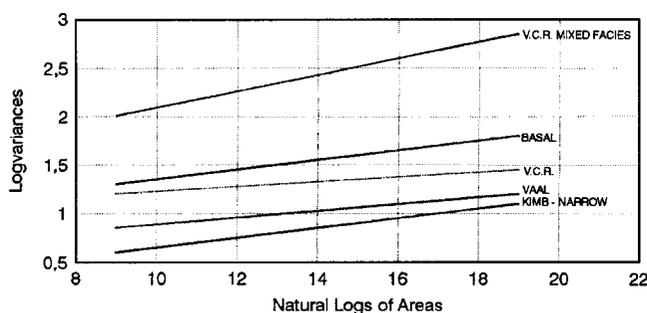


Figure 4—Regressed logvariances versus logareas for borehole samples (in cm.g/t)

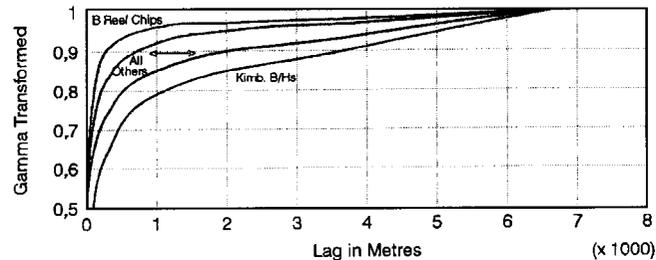


Figure 5—Standardized semivariograms

used<sup>4,5</sup>. If this variance is known *a priori*, the *t*" estimator will be much more efficient and, even where this *a priori* variance information is subject to error, this error could be much smaller than that inherent in the variance estimate based on a limited number of boreholes<sup>4,5</sup>. The following example, based on the evidence shown in Figures 3 and 4, demonstrates this point.

- Logvariance of borehole grades within mine: Say 1,0
- Logvariance of borehole grades within deflection areas: Say 0,5
- Log error variance<sup>6</sup> of arithmetic mean of 4 intersections per deflection area ( $\sigma_m^2$ ): 0,15\*
- Variance of these means within mine area (1,0 - 0,5 + 0,15) = 0,65
- Error variance of arithmetic mean of 10 borehole means = 0,088\*

$$* \sigma_m^2 = \ln \left[ \frac{(e^{\sigma^2} - 1)}{n} + 1 \right]$$

Based on the *a priori* mine variance of 1,0 subject to an error variance (or residual variance as in Figure 3) of say 0,04, the error variance of the 10 borehole means using the *t*" would be as follows:

- Error variance of *t*" of 4 intersections per borehole: 0,125
- Variance of borehole means within mine: 0,625
- Error variance of mean of 10 borehole means: 0,625/10 + 0,04/4 = 0,072

On average, the improvement is in line with that recorded by Krige and Assibey-Bonsu<sup>5</sup> for the Carbon Leader. In practice, a set of 10 boreholes could show a calculated variance as high as, or higher than, 2,0, which not only would show a final error variance for the arithmetic mean of 0,35 or higher (instead of 0,088), but would also introduce a significant positive conditional bias<sup>4</sup> in the grade estimate.

The introduction of *a priori* variance information can therefore be of substantial benefit.

#### Macro-kriging

The spatial structures indicated for the various reefs in Figures 3 and 4 enable macro-kriging to be performed on any size of area up to that of a large mine. If the available outside information covers areas already mined out and adequately sampled, and if there are no borehole or other data available inside the virgin area to be valued, the nugget effect for the mined-out areas will be zero and all

the covariances required for the matrix equation can be obtained from one parameter, i.e. the slope of the graph showing variance-size of area or of the corresponding de-Wijsian semivariogram (twice the former).

If the field is accepted at, say, the equivalent of 25 mines and, if the configuration of known data from 3 mines (X1, X2, and X3) and the virgin mine (Y) is as shown in Figure 6, the position would be as follows:

$$\begin{aligned} \text{Lag 1-2, 2-3, 2-Y} &= 1; \ln(\text{lag}) = 0,000 \\ \text{Lag 1-Y, 3-Y} &= \sqrt{2}; \ln(\text{lag}) = 0,347 \\ \text{Lag 1-3} &= 2; \ln(\text{lag}) = 0,693 \\ \sigma_y^2 = \sigma_x^2 &= 3,219(S); \quad (S = \text{Slope}) \\ \gamma(h) = 2 S(\ln h/0,446) &= S(2 \ln(h) + 1,615) \\ \text{Cov (1)} = \sigma_y^2 - \gamma(1) &= 3,219(S) - 1,615(S) = 1,604(S) \\ \text{Cov}(\sqrt{2}) = \sigma_y^2 - \gamma(\sqrt{2}) &= 3,219(S) - 2,308(S) = 0,911(S) \\ \text{Cov (2)} = \sigma_y^2 - \gamma(2) &= 3,219(S) - 3,001(S) = 0,218(S) \end{aligned}$$

As (S) is common throughout, the elements of the matrix equation will be as follows:

$$\begin{bmatrix} 3,219 & 1,604 & 0,218 \\ 1,604 & 3,219 & 1,604 \\ 0,218 & 1,604 & 3,219 \end{bmatrix} \begin{bmatrix} 0,911 \\ 1,604 \\ 0,911 \end{bmatrix}$$

The solution gives the following weights:

$$\begin{aligned} \text{Mines 1 and 3} &= 6,1\% \\ \text{Mine 2} &= 43,8\% \\ \text{Field} &= 44\%. \end{aligned}$$

If use is made of the  $t''$  approach as described in the previous sub-section with 4 intersections per hole and 10 boreholes inside Y, the position is as follows where

$$\begin{aligned} \text{borehole estimate} &= Z \\ \ln \text{ of mine area} &= 16 \\ \text{deflection area} &= 10 \text{ m} \times 10 \text{ m} \\ \ln \text{ of deflection area} &= 2,303 \\ \sigma_z^2 = \sigma_y^2 + \text{error variance (Z)} &= 3,219(S) + \text{error variance (Z)} \\ \text{variance (deflection area} = DA) \text{ inside mine} &= \\ &= (S)(\ln(M/DA)) = 13,697(S) \\ \text{variance of intersections inside } DA = D: & \\ \text{Error variance of mean of 4 intersections} & \\ & \text{inside } DA = D/4 \\ \text{Variance of mean of 4 intersections inside mine area} &= \\ &= 13,697(S) + D/4 \\ \text{Error variance of 10 borehole } t'' \text{ means (Z)} &= \\ &= 1,370(S) + D/40 \end{aligned}$$

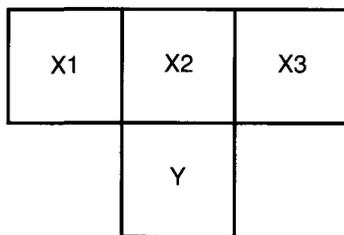


Figure 6—Configuration of 3 known mines (X1, X2, and X3) and the virgin mine Y

$$\begin{aligned} \sigma_z^2 &= 3,219(S) + 1,37(S) + D/40 \\ &= 4,589(S) + D/40 \\ &= S(4,589 + 0,025 D/S). \end{aligned}$$

Figure 7 demonstrates the logic followed.

The elements of the matrix equation will still have (S) throughout except for the covariance of Z with itself, where  $0,025(D/S)$  appears, i.e. the solution will now no longer be fixed as above but will depend on the relationship of D (i.e. the variance inside the deflection area) to S (i.e. the slope of the basic spatial structure). From all the analyses done, the value of D/S varies between 8 and 32. With D/S at, say, 8, the elements of the matrix equation become

$$\begin{bmatrix} 3,219 & 1,604 & 0,218 & 0,911 \\ 1,604 & 3,219 & 1,604 & 1,604 \\ 0,218 & 1,604 & 3,219 & 0,911 \\ 0,911 & 1,604 & 0,911 & 4,789 \end{bmatrix} \begin{bmatrix} 0,911 \\ 1,604 \\ 0,911 \\ 3,219 \end{bmatrix}$$

The weights become

$$\begin{aligned} \text{Mines 1 and 3} & 2,4\% \\ \text{Mine 2} & 17,3\% \\ \text{Field} & 17,4\% \\ \text{Borehole mean (Z)} & 60,5\% \end{aligned} \left. \vphantom{\begin{aligned} \text{Mines 1 and 3} \\ \text{Mine 2} \\ \text{Field} \\ \text{Borehole mean (Z)} \end{aligned}} \right\} 39,5\%$$

(Field = 44% of 39,5%).

The error variance of such a macro-kriged estimate is also a function of slope S and the variance inside deflection areas D, and can be calculated by the summation of the products of the weights and the corresponding elements of the right-hand matrix. In this case, it is 0,95(S).

Table IV shows the weights and kriging error variances for values of D/S of 8, 20, and 40, i.e. for a somewhat wider range than that mentioned above. The weights and the kriging variances were obtained from the solution of the matrix equation.

With only one adjacent mine, the weights for mines X1 and X3 are shared equally between mine X2 and the field and the kriging errors remain effectively the same.

For the example given earlier for which the slope S was 0,0365 and D/S was 13,7, the macro-kriging error variance would be 1,0(S), i.e. 0,0365 compared with the error variances for the orthodox arithmetic mean of 0,088, and for the straight  $t''$  of 0,062.

For the range of D/S values as in Table IV, the improvement in the kriging error variance from the orthodox arithmetic mean through the use of the  $t''$  estimator, to the use of the macro-kriged estimator with three outside mines is shown in Table V.

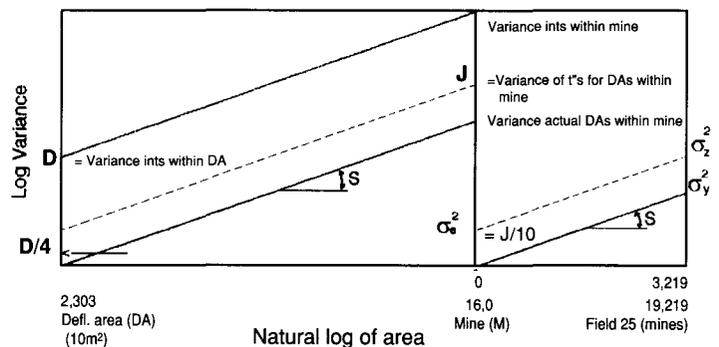


Figure 7—The relationship between S, D,  $\sigma_y^2$ , and  $\sigma_z^2$

**Table IV**

The weights for three adjacent mines X1, X2, and X3, the field and the borehole  $t''$  estimates, and the kriging error variances in terms of the value of slope/deflection variance

Item	D/S		
	8	20	40
Weight, %: X1 and X3	2,4	2,7	3,0
X2	17,3	19,1	21,7
Field	17,4	19,3	21,9
Borehole $t''$	60,5	56,3	50,4
Total outside, %	39,5	43,7	49,6
Field / total outside, %	44,0	44,1	44,1
(Field + 2) / outside %	87,8	87,9	87,9
Kriging variance / S	0,95	1,05	1,19

From Table V it is clear that, within the practical range of D/S values (8 to 32), substantial advantages can be gained from the use of the models suggested, and that the advantage of macro-kriging over orthodox methods improves as the ratio D/S increases.

In practice, the third parameter  $\beta$  will have to be used to avoid biased  $t''$  borehole estimates, and the *a priori* variance must be adjusted accordingly. However, in the macro-kriging process, the variances and covariances are relatively small, and the two-parameter lognormal method could be suitable. Furthermore, as recorded by Krige and Assibey-Bonsu<sup>5</sup>, the weights calculated on the transformed basis can be applied directly to the untransformed grades; the result will effectively be the same and will avoid any possible biases on back transformation.

The use of the analyses in comparisons between chip samples and borehole cores is not conclusive, and requires further study.

In cases where the lognormal model is not suitable even after the area(s) have been split into geologically homogeneous subareas, the logarithmic variance will form an essential parameter for alternative models<sup>7</sup>, and *a priori* knowledge of the variance and spatial structures can also be of substantial benefit.

**Table V**

Error variances in units of slope S for various techniques

Technique	D/S		
	8	20	40
Arithmetic mean*	1,83 to 2,32	2,33 to 3,37	3,47 to 6,94
$t''$ borehole estimator	1,57	1,87	2,37
Macro-kriged estimator	0,95	1,05	1,19

\* for slopes of 0,02 to 0,05

## CONCLUSIONS

This study has shown that the relative variances of gold grades for areas of similar size and, for either underground chip samples or borehole cores, as well as the spatial structures, can be established within reasonable limits for a particular reef horizon in a mining field. They also vary within narrow ranges for most of the common Witwatersrand reef horizons. This knowledge can be used as *a priori* information with substantial advantages in the valuation of new mines or large virgin areas in existing mines. The study is not yet exhaustive, and the results warrant further analysis, particularly of more borehole values, and the comparison of these with underground chip values.

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## School's Quiz\*

Minquiz, Mintek's popular annual schools' science quiz, will this year be held on Tuesday, 4th May.

The main objective of the quiz is to stimulate scholars' interest in science and technology. It is an important element in Mintek's promotion of careers in chemistry, metallurgy, and mineralogy, and provides an insight into Mintek's pivotal role in mineral technology in South Africa.

This year a similar quiz will be held at the University of Pretoria on the same day as Minquiz, and it is planned to

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expand the concept to other venues in the future, with the ultimate object of upgrading the quiz to a national event.

Minquiz is well supported by some 60 local high schools, and the teams are entertained to a buffet lunch and an educational video before undertaking the written quiz. This is followed by conducted tours of Mintek's extensive facilities.

The oral quiz, which is held in Mintek's 400-seat auditorium, culminates in the presentation of prizes, which total R6000 this year.