

# A viscoelastic approach to the modelling of the transient closure behaviour of tabular excavations after blasting

by D.F. Malan\*

## Contribution by G. Narsimhant

It was with much pleasure that I went through D.F. Malan's excellent paper. The following explanatory and critical note has been the result of my deep study of the paper.

Although by qualifications and experience I have been a chemical engineer, involved over four decades with problems in the modelling and simulation of transport-limited (heat and mass) chemical reactions in multi-phase systems, I have off and on interested myself in challenging problems in extractive metallurgy. This is the first time I have veered off to continuum mechanics, at the age of 71!

### Discrepancy between experimental and predicted profiles†

In his paper, Malan, for the first time, investigates the closure profiles of tabular excavations as a transient problem. He derives a closed-form solution for the viscoelastic convergence of a tabular stope based on the assumption that the rock mass behaves like a two-parameter Kelvin body under deformation and as an elastic mass under compression. Based on the correspondence principle, the transient solution is derived from a known form of the steady-state elastic solution. After investigating the effect of system parameters on the nature of the convergence, he compares the experimental stope closure data of Güler<sup>1</sup> with those predicted by his model equation (eq. [6]). While the prediction accuracy for times larger than 5 hours is of a high order, for times smaller than 5 hours, there are significant departures in the predicted data from experimental ones (Figure 11). In fact, the deviation is large at times close to the time of blast. Also, the prediction from the empirical equation of McGarr<sup>2</sup> strays away (Figure 10). Eq. [6] of Malan's paper thus under-estimates the closure data in this critical period and, as such, could present a risky predictive mode.

The defect in the modelling procedure can be overcome, and the discrepancy between the experimental and predicted profiles, especially in the primary phase, removed, if the three-parameter viscoelastic model, rather than the two-parameter Kelvin model, is adopted under appropriate modification. The modification relates to the removal of the discontinuity in the closure profile at  $t = 0^+$  and its replacement by a strain rate  $\dot{S}_z(x,0)$  of infinitely large magnitude, making the profile continuous. Not only does the experimental profile of Güler indicate the existence of a steep

gradient at  $t = 0^+$ , but also the observation of Leeman<sup>3</sup>, that the rate of closure suddenly increases just after blasting time. Therefore, the prediction of the primary-phase closure profile ought to be based on a modified form of the three-parameter viscoelastic model. I show later that the secondary-phase closure can be predicted from either model.

In eq. [A16] of Malan's Addendum, the inverse time constants  $a$ , and  $a_3$  are inter-related. An elementary algebraic exercise splits these constants into differentiable parameters, namely  $a_1$  and  $\bar{a}_2$ :

$$\begin{aligned} & (1 + c_3 e^{-a_1 t} + c_4 e^{-a_3 t}) \\ &= \left[ 1 + c_3 e^{-a_1 t} + c_4 \cdot e^{\frac{-(a_1 + \bar{a}_2) \cdot t}{\alpha}} \right] \\ &= f(t) \end{aligned} \quad [C1]$$

where

$$\begin{aligned} a_1 &= (q_0/q_1) \\ \bar{a}_2 &= (6K/q_1) \\ \alpha &= \left( 1 + \frac{6K \cdot p_1}{q_1} \right). \end{aligned} \quad [C2]$$

Based on eq. [C1], it is possible to differentiate rationally between a logical primary time constant and a secondary one. The inequality that one looks for is

$$\frac{(a_1 + \bar{a}_2)}{\alpha} \gg a_1. \quad [C3]$$

Under a conservative estimate with  $\alpha > 1$ , the inequality of eq. [C3] transforms to

$$q_1 \gg q_0 \cdot p_1, \quad [C4]$$

which asserts that the viscosity coefficient should be larger than the product of shear modulus,  $G$ , and the third parameter,  $p_1$ , in the model. In such a case, the third term in the right-hand side of eq. [C1] decays much faster than the second term, and thus dominates the nature of the primary

† Correction of Malan's eqs. [3] and [4] on p. 212 of the paper:

$t/\tau$  should read  $-t/\tau$ .

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phase of the closure profile. The effect of bulk modulus seems non-existent.

Incidentally, Malan's eq. [6] can be resolved into an independent parametric representation of closure profiles:

$$\begin{aligned} & (1 + c_1 e^{-a_1 t} + c_2 e^{-a_2 t}) \\ & = [1 + c_1 - (1 + c_1) e^{-\bar{a}_2 t}] e^{-a_1 t}. \end{aligned} \quad [C5]$$

We thus deal with two independent time constants,  $a_1$  and  $\bar{a}_2$ , as against  $a_1$  and  $a_2$ , which are not independent. Here, the condition  $\bar{a}_2 \gg a_1$  means that

$$K \gg G, \quad [C6]$$

which means that the bulk modulus has to be much larger than the shear modulus. The viscosity coefficient,  $q_1$ , seems to have no effect. For both the models, the secondary phase is influenced by the ratio of shear modulus to the viscosity coefficient.

Malan's eq. [A16] can be rewritten as

$$S_z(x, t) = g \cdot \phi(x) \cdot f(t), \quad [C7]$$

where

$$\phi(x) = -4W_z(l^2 - x^2)^{1/2} \cdot \left(1 + \frac{dx}{2}\right)$$

$f(t)$  = the right-hand side of eq. [C1]

$g$  = compliance function.

The imposition of a strain  $\epsilon_0$  instantaneously at  $t = 0^+$  can be expressed as

$$\epsilon_0 = g \cdot \phi(x) \cdot (1 + c_3 + c_4). \quad [C8]$$

From an inspection of Güler's experimental data, it can readily be inferred that  $\epsilon_0 = 0.001$ . Thus, the transient closure profile can be represented by

$$S_z(x, t) = \frac{0.001 \left\langle 1 + c_3 \cdot e^{-a_1 t} + c_4 \cdot e^{-\frac{-(a_1 + \bar{a}_2)}{\alpha} t} \right\rangle}{(1 + c_3 + c_4)}. \quad [C9]$$

### The three-parameter model

A more general treatment of the three-parameter model is attempted now. This involves the use of a second time-dependent function,  $\beta(t)$ , which needs to be coupled to a time-varying parameter. As a very preliminary choice, this is taken as the compliance function,  $g$ . Eq. [C7] of this contribution can be rewritten as

$$S_z(x, t) = g_0 \cdot \phi(x) \cdot f(t) \cdot \beta(t), \quad [C10]$$

where  $g_0$  is the asymptotic value of the compliance function as  $t \rightarrow \infty$ . The nature of  $\beta(t)$  should be such that

$$S_z(x, t) \rightarrow 0, \text{ as } t \rightarrow 0 \quad [C11]$$

$$\dot{S}_z(x, t) \rightarrow \infty, \text{ as } t \rightarrow 0$$

$$S_z(x, t) \rightarrow \text{constant}, \text{ as } t \rightarrow \infty.$$

The chemical-engineering research literature, especially in the realm of chemical-reaction engineering, cites several examples of industrial reactions, both catalytic and non-catalytic, that are characterized by a process of diffusion followed by instantaneous reaction. Two examples are cited:

- the chemisorption of hydrogen on palladium or platinum
- the absorption of carbon dioxide by ethanol-amine solutions.

For such cases, the rates of initial absorption tend to be very high and, in appropriate transient modelling exercises, the rate functions,  $\beta(t)$ , have the following forms (normalized):

$$\beta(t) = \text{erf} \sqrt{\beta t} \quad [C12]$$

$$\beta(t) = \frac{\sqrt{\beta t}}{1 + \sqrt{\beta t}}. \quad [C13]$$

The time derivatives are as follows:

$$\dot{\beta}(t) = \sqrt{\frac{\beta}{\pi t}} \cdot e^{-\beta t} \quad [C14]$$

$$\text{or } = \frac{\sqrt{\beta}}{2\sqrt{t}(1 + \sqrt{\beta t})^2}.$$

$$\text{As } t \rightarrow 0, \beta(t) \rightarrow \infty$$

$$\beta(t) \rightarrow 0$$

$$t \rightarrow \infty, \beta(t) \rightarrow 1. \quad [C15]$$

$\beta(t)$  thus acts as a suitable weighting function. The desirable form for the transient closure profile would include the error function

$$S_z(x, t) = g_0 \cdot \phi(x) \cdot \text{erf} \sqrt{\beta t} \cdot f(t). \quad [C16]$$

The time derivative is

$$\dot{S}_z(x, t) = g_0 \cdot \phi(x) \left[ \sqrt{\frac{\beta}{\pi t}} \cdot e^{-\beta t} \cdot f(t) + \text{erf} \sqrt{\beta t} \cdot \dot{f}(t) \right]$$

since as  $t \rightarrow 0$ ,  $\text{erf} \sqrt{\beta t} \rightarrow 0$  and

$$f(t) \rightarrow (1 + c_3 + c_4), \dot{S}_z(x, 0) \rightarrow \infty. \quad [C17]$$

The advantage of using the error function is that it provides a clean Laplace transform:

$$\alpha(\text{erf} \sqrt{\beta t}) = \frac{1}{S} \cdot \sqrt{\frac{\beta}{S + \beta}}. \quad [C18]$$

Perhaps, in an investigation of the applicability of the correspondence principle, the above transformation may emerge naturally:  $g = g(k, q_0)$  and constants  $c_3, c_4$ , are also functions of  $(k, q_0)$ ;  $a_1 = f(q_0, q_1)$  and  $a_2 = f(k, q_1)$ . Therefore, tampering with  $g$  will lead to tampering with other constants.  $\beta(t)$  ought to be associated with a new emerging parameter in the modified three-parameter model and, as such, would leave  $g$  alone. Can anyone throw some light?

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Would Figure C1 be the true representation of the modified three-parameter model? See my earlier inclusion of an additional drag.

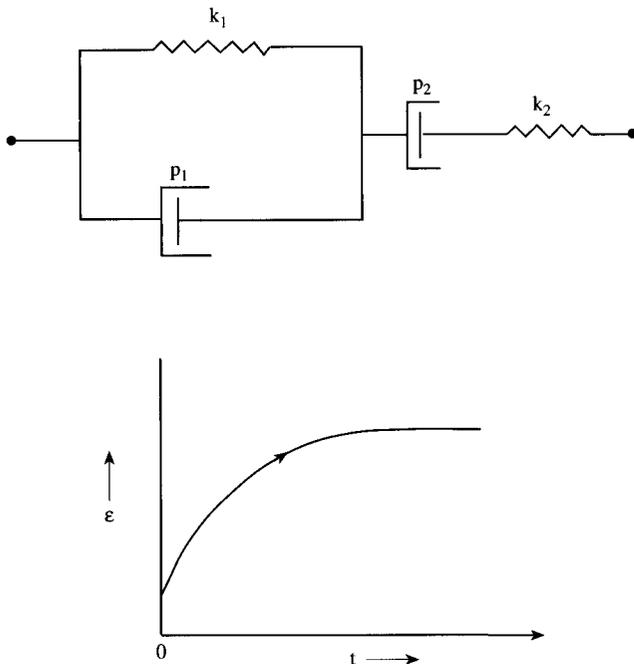


Figure C1—A suggested representation of the modified three-parameter model

### Compatibility of statements

The following two statements made by Malan (with my italics) are compatible *only, and only if*,

$S_z(x,0) = 0$  and  $\dot{S}_z(x,0) \rightarrow \infty$  at  $t = 0^+$ . Hence, a modified three-parameter model is the best bet!

- (1) p. 212, para. 2: Leeman 'observed profiles similar to Figure 1, in which the *rate of closure suddenly increases* just after blasting time ... similar closure profiles were observed at other mines in South Africa.'
- (2) p. 214, para 1: 'Data supplied by Güler and closure profiles published by Leeman confirm the statement that the *instantaneous closure at the time of face advance is negligible* compared with the eventual closure'.

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### Author's reply

I thank Professor Narsimhan for his extensive contribution to the paper. It provided me with the stimulus to derive an extended model. However, there are some inaccuracies in his arguments that need further discussion. Professor Narsimhan's main concern is that there is not a good enough fit between the derived viscoelastic Kelvin model and the

experimental data supplied by Güler<sup>1</sup> (see Figure 11 on page 218 of my paper). I will therefore explain the choice of a viscoelastic model in more detail and also make specific comments on the arguments given in his contribution.

My paper outlines a method to obtain a viscoelastic convergence solution for tabular stopes using a known elastic solution, the correspondence principle, and a viscoelastic model. The particular viscoelastic model, e.g. the Kelvin, Maxwell, or Burgers model, is one of the building blocks of the technique, and the method will stay the same no matter which model is used. However, the particular viscoelastic model will significantly affect the behaviour of the derived model and must therefore be chosen very carefully. The fundamental question that now remains is which viscoelastic model is best to describe time-dependent rock behaviour in the gold-mining industry. Unfortunately, very little experimental measurement of transient-closure behaviour is available. As pointed out correctly by Professor Narsimhan, Güler's data<sup>1</sup> indicate the existence of a steep gradient at  $t = 0^+$ , and a more complex model will give a better fit to the data. Some of the data published by Leeman<sup>2</sup>, however, show a less steep gradient at  $t = 0^+$  with no instantaneous components, and a simple Kelvin model will suffice. There is also some evidence of continuous steady-state closure in certain mines (Roberts and Jager<sup>3</sup>). The Kelvin model cannot model steady-state creep, and an additional dashpot in series with the other components is required. As different geotechnical areas exist in the mining industry, it may just be that different models will apply better to different areas. I am currently collecting data from different mines to obtain a possible correlation between transient-closure behaviour and geotechnical area. Before that study is completed, it is pointless to argue about the 'best' model. However, it is also true that a four-parameter Burgers model may be useful as a generalized model. Therefore, I later derive the closure equation for the Burgers model to supplement the models discussed in the paper.

For the three-parameter model, Professor Narsimhan is trying to differentiate between a primary and a secondary time constant (his eq. [C1]). This is indeed possible if  $a_3 \gg a_1$ .

Unfortunately,  $a_1$  and  $a_3$  (where  $a_3 = \frac{(a_1 + \bar{a}_2)}{\alpha}$ ) are not

independent (see eq. [R8]), and therefore it is not valid to enforce the inequality (given as eq. [C4] in his contribution) when calibrating the model. Incidentally, it is futile to split  $a_1$  and  $a_3$  into the parameters  $a_1$  and  $\bar{a}_2$  since they are also not independent (shown later). From his derived inequality  $q_1 \gg q_0 p_1$ , he implies that the viscosity coefficient must be much larger than the product of shear modulus and the third

### Corrigendum

The wrong equation was printed for parameter  $c_3$  in the Addendum to my paper (p. 220). The correct equation is as follows:

$$c_3 = \frac{q_0(p_1 q_0 - q_1)(6K + q_0)}{2q_0 q_1(3K + 2q_0)} \quad [A16b]$$

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parameter,  $p_1$ . This is also incorrect as explained below. The general differential equation for a viscoelastic model is as follows (Flügge<sup>4</sup>):

$$\sigma + p_1 \dot{\sigma} + p_2 \ddot{\sigma} + \dots = q_0 \varepsilon + q_1 \dot{\varepsilon} + q_2 \ddot{\varepsilon} + \dots \quad [R1]$$

Unfortunately, this normalized notation (which was also adopted in my paper) does not imply that the parameters  $p_1, p_2, \dots$  and  $q_1, q_2, \dots$  refer to the same material parameters for different viscoelastic models. It can be shown that, when it is assumed that the material behaves as an elastic solid in dilatation and as a Kelvin model in distortion,  $q_1$  is equal to the viscosity coefficient  $n$  and  $q_0 = 2G$ , where  $G$  is the shear modulus. This is not true for the three-parameter model. For the three-parameter model, the differential equation can be derived as follows (Flügge<sup>4</sup>):

$$\sigma + p_1 \dot{\sigma} = q_0 \varepsilon + q_1 \dot{\varepsilon}. \quad [R2]$$

If it is assumed that the model behaves like the three-parameter model in distortion, it follows that

$$p_1 = \frac{n_1}{2(G_1 + G_2)} \quad q_0 = \frac{2G_1 G_2}{G_1 + G_2} \quad q_1 = \frac{G_2 n_1}{G_1 + G_2}, \quad [R3]$$

where  $G_1$  is the shear modulus of the elastic element in parallel to the dashpot with viscosity coefficient  $n_1$ . The third elastic element (in series with the other components) has a shear modulus of  $G_2$ . For the time constants  $a_1, \bar{a}_2$ , and  $a_3$ , it therefore follows that

$$a_1 = \frac{q_0}{q_1} = \frac{2G_1}{n_1} \quad [R4]$$

$$\bar{a}_2 = \frac{6K}{q_1} = \frac{6K(G_1 + G_2)}{G_2 n_1} \quad [R5]$$

$$a_3 = \frac{6K + q_0}{6K p_1 + q_1} = \frac{6K(G_1 + G_2) + 2G_1 G_2}{n_1(3K + G_2)}. \quad [R6]$$

By elimination of  $n_1$  from [R4] and [R5],

$$\bar{a}_2 = a_1 \frac{6K(G_1 + G_2)}{2G_1 G_2}. \quad [R7]$$

Therefore,  $a_1$  and  $\bar{a}_2$  are not independent. Also, by elimination of  $n_1$  from [R4] and [R6], it follows that

$$a_3 = a_1 \frac{6K(G_1 + G_2) + 2G_1 G_2}{2G_1(3K + G_2)}, \quad [R8]$$

and  $a_1$  and  $a_3$  are not independent.

For the Kelvin model, Professor Narsimhan's eq. [C5] is incorrect and should read

$$\begin{aligned} & (1 + c_1 e^{-a_1 t} + c_2 e^{-a_2 t}) = \\ & [e^{a_1 t} + c_1 - (1 + c_1) e^{-\bar{a}_2 t}] e^{-a_1 t}. \end{aligned} \quad [R9]$$

He also claims that the time constants  $a_1$  and  $\bar{a}_2$  for the Kelvin model are independent. This is not true because the bulk modulus,  $K$ , and the shear modulus,  $G$ , for the

continuum elastic body are not independent. For a linear elastic medium, the bulk and shear modulus can be written in terms of Young's modulus,  $E$ , and Poisson's ratio,  $\nu$ , as follows (Jaeger and Cook<sup>5</sup>):

$$G = \frac{E}{2(1 + \nu)} \quad [R10]$$

$$K = \frac{E}{3(1 - 2\nu)}. \quad [R11]$$

For the Kelvin model, it follows that

$$a_1 = \frac{q_0}{q_1} = \frac{2G}{q_1} = \frac{E}{q_1(1 + \nu)} \quad [R12]$$

$$\bar{a}_2 = \frac{6K}{q_1} = \frac{2E}{q_1(1 - 2\nu)}. \quad [R13]$$

By elimination of  $E$  from equations [R12] and [R13], it follows that

$$a_1 = \bar{a}_2 \frac{(1 - 2\nu)}{2(1 + \nu)}, \quad [R14]$$

and therefore  $a_1$  and  $\bar{a}_2$  are not independent. The Kelvin model, like the three-parameter model, contains only one dashpot and therefore can have only one independent time constant.

To extend the three-parameter model, Professor Narsimhan uses an empirical approach to add a second time-dependent function to the solution. A more elegant solution can be obtained by the addition of another dashpot to the three-parameter model. The model suggested by Professor Narsimhan in his Figure C1 is well known in viscoelasticity theory as the Burgers model. Incidentally, the Burgers model will not fit Professor Narsimhan's requirement for  $S_2(x, t) \rightarrow$  constant as  $t \rightarrow \infty$  (which is a property of the Kelvin and three-parameter models) because the second dashpot causes steady-state creep. However, as mentioned earlier, there is some evidence of continuous steady-state closure in certain mines. The Burgers model can therefore still be useful as an improved three-parameter model.

### Viscoelastic convergence solution for a Burgers model

The Burgers model can be constructed by the combination of a Maxwell and a Kelvin model (my Figure R1). If it is assumed that the rock behaves like an elastic solid in dilatation and like a Burgers model in distortion, eq. [R23] can be derived for the distortional part.

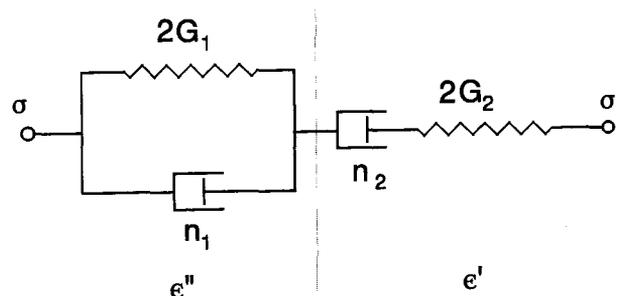


Figure R1—The combined Maxwell and Kelvin models

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For the Maxwell part, it follows that (Flügge<sup>4</sup>)

$$\sigma + p_1' \dot{\sigma} = q_1' \dot{\epsilon}' \quad [\text{R15}]$$

where

$$p_1' = \frac{n_2}{2G_2} \quad q_1' = n_2. \quad [\text{R16}]$$

For the Kelvin part, it follows that (Flügge<sup>4</sup>)

$$\sigma = q_0'' \epsilon'' + q_1'' \dot{\epsilon}'', \quad [\text{R17}]$$

where

$$q_0'' = 2G_1 \quad q_1'' = n_1. \quad [\text{R18}]$$

The Laplace transformation of eqs. [R15] and [R17] gives

$$(1 + p_1' s) \bar{\sigma} = q_1' s \bar{\epsilon}' \quad [\text{R19}]$$

$$\bar{\sigma} = (q_0'' + q_1'' s) \bar{\epsilon}''. \quad [\text{R20}]$$

Multiplying each equation with a suitable constant and adding give

$$(1 + p_1' s)(q_0'' + q_1'' s) \bar{\sigma} + q_1' s \bar{\sigma} = q_1' s (q_0'' + q_1'' s) \bar{\epsilon}', \quad [\text{R21}]$$

where

$$\bar{\epsilon} = \bar{\epsilon}' + \bar{\epsilon}''. \quad [\text{R22}]$$

The inverse Laplace transformation gives

$$\sigma + p_1 \dot{\sigma} + p_2 \ddot{\sigma} = q_1 \dot{\epsilon} + q_2 \ddot{\epsilon}, \quad [\text{R23}]$$

where

$$p_1 = \frac{q_1'' + p_1' q_0'' + q_1'}{q_0''} = \frac{n_1 G_2 + n_2 G_1 + n_2 G_2}{2G_1 G_2} \quad [\text{R24}]$$

$$p_2 = \frac{p_1' q_1''}{q_0''} = \frac{n_1 n_2}{4G_1 G_2} \quad [\text{R25}]$$

$$q_1 = q_1' = n_2 \quad [\text{R26}]$$

$$q_2 = \frac{q_1' q_1''}{q_0''} = \frac{n_1 n_2}{2G_1}. \quad [\text{R27}]$$

If the rock behaves elastically in dilatation and according to a Burgers model in distortion, it therefore follows, for the operators described in my paper, that

$$\tilde{P}' = 1 \quad \tilde{P}'' = 1 + p_1 s + p_2 s^2 \quad \tilde{Q}' = 3K \quad \tilde{Q}'' = q_1 s + q_2 s^2. \quad [\text{R28}]$$

When eq. [28] is substituted into [A9] of the paper,

$$F(t) = L^{-1} \left[ \frac{3K(1 + p_1 s + p_2 s^2)^2 + 2(1 + p_1 s + p_2 s^2)(q_1 s + q_2 s^2)}{s(q_1 s + q_2 s^2)[6K(1 + p_1 s + p_2 s^2) + (q_1 s + q_2 s^2)]} \right] \quad [\text{R29}]$$

Finding the inverse Laplace transformation by partial fraction expansion gives

$$F(t) = g_1 \left[ 1 + c_5 t + c_6 e^{-\beta t} + (c_7 \sinh bt + c_8 \cosh bt) e^{-\frac{ht}{2}} \right] \quad [\text{R30}]$$

where

$$b = \sqrt{\frac{(6Kp_1 + q_1)^2 - 24K(6Kp_2 + q_2)}{4(6Kp_2 + q_2)^2}} \quad [\text{R31}]$$

$$g_1 = \frac{2Kp_1 q_1 + q_1^2 - 2Kq_2}{4Kq_1^2} \quad [\text{R32}]$$

$$c_5 = \frac{2Kq_1}{2Kp_1 q_1 + q_1^2 - 2Kq_2} \quad [\text{R33}]$$

$$c_6 = \frac{2K(p_2 q_1^2 - p_1 q_1 q_2 + q_2^2)}{q_2(2Kp_1 q_1 + q_1^2 - 2Kq_2)} \quad [\text{R34}]$$

$$c_7 = \frac{q_1^2(-12Kp_2 q_1 + 6Kp_1 q_2 - q_1 q_2)}{2b(6Kp_2 + q_2)^2(2Kp_1 q_1 + q_1^2 - 2Kq_2)} \quad [\text{R35}]$$

$$c_8 = \frac{q_1^2 q_2}{(6Kp_2 + q_2)(-2Kp_1 q_1 - q_1^2 + 2Kq_2)} \quad [\text{R36}]$$

$$f = \frac{q_1}{q_2} \quad [\text{R37}]$$

$$h = \frac{6Kp_1 + q_1}{6Kp_2 + q_2}. \quad [\text{R38}]$$

Following the arguments in the paper, the incremental viscoelastic convergence for a slope with face advance  $\Delta l$  at one side can be written as

$$\Delta S_z(x, t) = -4W_z \left( 1 + \frac{dx}{2} \right) F(t) \quad [\text{R39}]$$

$$\left[ \sqrt{\left( L + \frac{\Delta l}{2} \right)^2 - \left( x - \frac{\Delta l}{2} \right)^2} - \sqrt{L^2 - x^2} \right]$$

This derived Burgers convergence model was fitted to the same data as given in my paper. Figure R2 gives the experimental data and the fitted model. It was assumed that all the elastic components have the same Poisson's ratio, namely 0.2. The calibrated values are  $E_1 = 190$  GPa,  $E_2 = 5000$  GPa,  $n_1 = 250$  GPa·h, and  $n_2 = 2250$  GPa·h. This model gives a better fit to the data for times less than 5 hours, and slightly worse for times longer than 5 hours, than the fit given in my paper. The

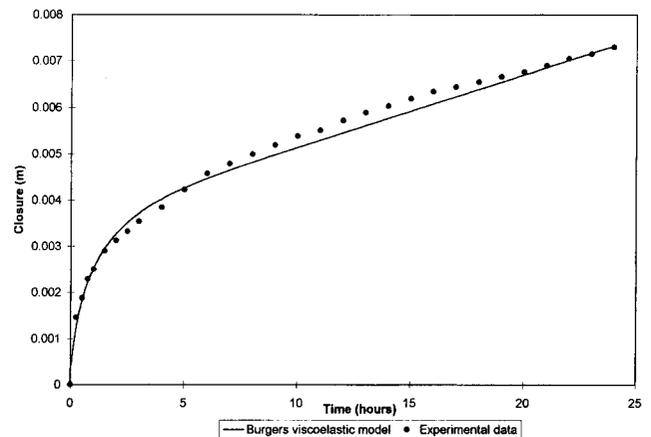


Figure R2—The Burgers convergence model fitted to experimental data

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experimental set needs to be extended beyond  $t = 24$  hours (for no changes in geometry) to show whether the closure undergoes steady-state movement or approaches an asymptotic value. As a concluding remark, it should be emphasized that, by the addition of enough elastic and viscous elements in series and parallel, virtually any time-dependent response can be modelled with high accuracy. However, as mentioned in my paper, the increased accuracy of more elaborate models must be weighed against the greater complexity of an increased number of parameters that need to be calibrated.

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## St. Barbara — Journey of a Saint through Time\*

by Rolfroderich Nemitz and Dieter Thierse

*Reviewed by:*  
*Professor Albert W. Davies*

This fascinating book about the patron saint of miners, metallurgists and others, was published on the 4th December, the Saint's day, in 1995.

The authors, who are prominent mining engineers in Germany and world-wide, have researched the history of St Barbara very thoroughly and produced a very interesting book, which is a first in its field. The very detailed research and the quality of the illustrations makes the book rank with the work of Georgius Agricola who produced the first well-illustrated text book of mining and metallurgy some 440 years ago.

Following years of intensive research by the authors and their assistants a picture emerges of the importance of St Barbara down the ages. Her presence is depicted in religious, historical and artistic spheres, manifested in her legendary role in ancient Christianity, or as an object of art. Her name has been given to countless churches and hospitals, and her influence has extended beyond the Christian faith. The authors discovered the unusual attraction of St Barbara and



became increasingly interested in her historical origins and the role she has played up to the present day.

The book deals with the legends and traditions surrounding St Barbara; the historical and spiritual background; religious rituals; the setting and locations of St Barbara's presence; the origin and means of worship and its dispersion world-wide; associated cults and customs, and the identification and portrayal of St Barbara. The influence of icons over several centuries is systematically considered and comprehensively described.

The book has 250 beautiful coloured illustrations which emphasise the strong widespread aura of this 3rd century legend and the message she symbolised.

Demand for the book in Europe is strong and an English version will be available soon from:

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