Life and design of bord-and-pillar workings affected by pillar scaling
by M.D.G. Salamon*, M.U. Ozbay#, and B.J. Madden†

Synopsis

It has been noted in recent years that the pillars in some bord-and-pillar workings scale or spall. In due course this process may result in the collapse of the affected panels. This paper presents a methodology for assessing the long-term behaviour of such workings, employing a simple model of pillar deterioration by scaling.

It is assumed that scaling causes a gradual reduction in pillar width, hence the safety factor of pillars reduces in time. It is also postulated that the process of scaling will be arrested if the coal rubble around the pillar reaches a critical height. The probability that the pillars will survive for a certain period of time, or indefinitely, can be estimated. This can be accomplished by using the log-normal distribution proposed by Salamon and Munro4 to describe the distribution of the apparent safety factor values at failure. If an estimate of the rate of scaling is known, it is also possible to quantify, using a Monte Carlo technique, the expected life of the pillars. The method also facilitates pillar design by specifying a probability of survival for a given number of years, or alternatively, for an indefinite period.

Using the limited pool of available data, analyses show that the proposed model does not appear to be inconsistent with observed behaviour of pillars in the Vaal Basin. The crucial parameter is the rate of scaling. Back calculations from eleven cases suggest that the rate of spalling in the Vaal Basin is about 0.2 m/year. It is noteworthy that two further cases in the same area give significantly higher rates (0.85 m/year). Some underground observations in the Witbank area indicate considerably lower rates in the order of 0.02-0.04 m/year. Further areas for study are recommended. These include a study of old workings to determine the extent and rate of scaling in the various coal fields, an investigation into the possible relationship between scaling and the chemical composition of the coal. In addition, model studies will be required to assess the practical impact of the research findings.

Introduction

Most South African bord-and-pillar workings are planned using a pillar design methodology developed more than 30 years ago. During the intervening years this approach to design has proved to be successful in most instances (Salamon and Wagner5, Madden1). However, during the past few years an increasing number of reports have been published which indicate that the behaviour of pillars does not always accord with the expectations arising from the original work of Salamon and Munro4. Madden1 shows that two of the failures he reported had particularly large safety factors. Van der Merwe7 goes considerably further and indicates that the ‘...Vaal Basin has long been recognised as a difficult area for coal mining in South Africa’. A number of pillar collapses have occurred in collieries situated in this Basin. For example, he goes on to say that Sigma Colliery experienced three unrelated pillar failures in 1991. In one of these cases neighbouring seams were mined and floor lifting was also practised. In the other two cases no unusual circumstances were known to exist.

Nevertheless, the panels collapsed. The safety factors of the pillars in these panels had been 1.7 and 2.8, respectively. According to the work of Salamon and Munro4 the probability of failure at a safety factor of 1.7 is about 0.0004. This is a very remote possibility. The probability of failure at 2.8 is just short of an impossibility. Not surprisingly, van der Merwe concluded that the traditional design procedure should be modified for the Vaal Basin.

There can be many reasons for this anomalous pillar behaviour. Perhaps the most obvious reason for such abnormal behaviour is an alteration in coal strength. For example, it has often been suggested that the strengths of coals encountered in different seams or in different coalfields are likely to differ from each other. Some recent publications appear to raise doubt concerning the practical significance of this observation. Mark and Barton3 have stated that the in situ coal strength remains remarkably constant in a wide variety of conditions. Salamon et al.6 have analysed both South African and Australian data concerning coal pillar behaviour and found that a single pillar strength formula can

* Colorado School of Mines, 1500 Illinois Street, Golden Colorado 80401, USA.
# University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg, South Africa
† CSIR Miningtech
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describe pillar behaviour remarkably well in both countries. Obviously these papers do not rule out the possibility of variation in coal strength from place to place. They merely suggest that in most instances these variations are relatively modest and, to paraphrase Mark and Barton\(^3\) freely, a strength formula based on field data is a better estimation of pillar strength than any other strength estimate based on laboratory or model pillar testing. However, as was said earlier, significant anomalies do occur. For example, Salamon et al.\(^6\) found that two out of their 177 case histories did not fit the general pattern. It would appear, therefore, that in the vast majority of cases pillar strength can be approximated well by a single formula. However, there are cases where this formula fails to yield an acceptable pillar strength.

The search for the explanation of abnormal behaviour must not stop at anomalously low coal strength. Pillars may be debilitated by some weakness in the floor or roof (i.e., foundation failure) or in the seam itself. Problems may also arise because of some anomaly at the contact surfaces between the seam and the surrounding strata. Madden and Canbulat\(^2\) in a recent survey of South African collieries have identified a number of pillar collapses that may be attributed to these somewhat abnormal conditions. Van der Merwe\(^7\) called attention to another mode of failure. Several of these phenomena can lead to the deterioration or even the collapse of pillars.

Van der Merwe argues that a given depth of scaling has a greater detrimental effect than the equivalent thickness of a roof fall. He reports that according to his observations ‘...scaling is much more common than roof falls. In most areas of the Vaal Basin, virtually all pillars scale, while roof falls tend to be restricted in extent and in occurrence’. In view of van der Merwe’s experience and because of the importance of designing safe and economical pillars in areas where scaling is prevalent, it has been decided to explore the influence of pillar side scaling on coal pillar design. The intention here is to present the results of a preliminary study.

The work described here is part of the SIMRAC (Safety in Mines Research and Advisory Committee) project COL337 ‘Coal Pillar Design Procedures’, and it focuses on situations where scaling represents the primary threat to the pillars. In this process coal strength remains unaltered and the weakening of the pillars is due entirely to a reduction in pillar width induced by spalling.

The investigation combines the statistical pillar strength model developed by Salamon and Munro\(^4\) with a simple set of hypotheses concerning the mechanism of scaling. First, a brief review of the relevant background is presented and then an attempt is made to review the basic concepts of coal pillar mechanics in conditions where side wall scaling occurs. Here concepts such as ‘pillar life’ and the ‘probability of survival’ for a given period of time are introduced. An initial attempt is also made to estimate possible rate of scaling by back-calculation from data collated for the Vaal Basin and the Witbank area. Finally, a tentative method of ‘pillar layout design’ is proposed. The method is not deterministic but recognises the essentially stochastic nature of pillar behaviour.

**Statistical model of pillar strength**

Salamon and Munro\(^4\) accepted that if the true strength of the pillar and the true mean stress (or load) acting on the pillar could be determined, failure would occur when these two quantities become equal. Unfortunately, it is not possible to determine these ‘true’ values and consequently the true safety factor of pillars cannot be calculated. This problem arises because no accurate method exists to compute the strength of coal pillars (or any other rock pillars for that matter). Pillar strength is estimated from empirical formulae which, at best, can be regarded only as approximate. Even the tributary area method of pillar stress calculation is imprecise.

The ratio of the approximate pillar strength and approximate pillar load can only yield an approximate value for the safety factor. These ratios may be referred to as apparent (or nominal) safety factors. It follows, therefore, that the apparent factors obtained for instances of pillar collapse, can be either larger or smaller than unity. Salamon and Munro postulated that the apparent safety factor arising from pillar collapses, that is \(S_A\), is a random variable and that the log-normal distribution is an acceptable approximation of its frequency distribution. Thus,

\[
f(S_A) = \frac{1}{\sqrt{2\pi} \sigma S_A} \exp\left[-\frac{1}{2} \left(\frac{\ln S_A}{\sigma}\right)^2\right]. \tag{1}
\]

This distribution has its median at \(S_A = 1\) and \(\sigma\) is its standard deviation in the logarithmic scale. (For the sake of simplicity, natural logarithm is used here, as opposed to the base 10 version employed in the original paper.)

The probability that failure will not occur, or the probability of survival, at a given apparent safety factor is given by the cumulative distribution function:

\[
p(S_A) = \int_0^{S_A} f(x)dx = \int_0^{\ln S_A} \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{1}{2} \left(\frac{\ln x}{\sigma}\right)^2} dx.
\]

The definitions employed here are as follows:

\[
Z(\alpha, \beta) = \int_{-\infty}^{\alpha} P(x) = \int_{-\infty}^{\alpha} Z(\varphi)d\varphi \tag{3}
\]

where \(Z(\cdot)\) is the standard normal distribution function and \(P(\cdot)\) is the standard cumulative normal probability distribution.

Using the method of maximum likelihood Salamon and Munro quantified, in the paper mentioned earlier, the parameters in the following empirical formula for pillar strength:

\[
\sigma_{p_{\alpha\beta}} = K \left(\frac{W}{W_0}\right)^{2\alpha} \left(\frac{M}{M_0}\right)^{2\beta} \tag{4}
\]

Note that \(W\) and \(M\) are the width and height of a pillar of a square cross-section. Parameters \(\alpha\) and \(\beta\) are dimensionless constants. The multiplier \(K\) is the compressive strength of a reference block of coal of height \(M_0\) and width \(W_0\). In the SI system \(M_0\) and \(W_0\) are taken to be 1.0 m. It should be noted that \(K\) is not a property of the material (i.e., of the coal), but the property of the involved system. The system referred to here consists of the union of the block and the loading conditions prevailing underground. The metric version of the constants found by Salamon and Munro\(^4\) are:

\[
K = 7.2 \text{ MPa} \quad \alpha = 0.46 \quad \beta = \pm 0.66 \quad \sigma = 0.159 \tag{5}
\]
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Subsequently, it became apparent that for squat pillars an alternative expression may give a better estimate of the pillar strength. This squat pillar strength formula can be put in the following form (Salamon and Wagner5):

$$\sigma_{squ} = K(W^2M)^\gamma R_0^\delta \left[ \frac{b}{\varepsilon} \left( \frac{W}{MR_0} \right)^{\varepsilon} \pm 1 \right] + 1. \quad [6]$$

The following definitions apply in this expression:

$$a = \frac{1}{3}(\alpha + \beta) \quad b = \frac{1}{3}(\alpha + 2\beta) \quad R_0 = 5 \quad \varepsilon = 2.5 \quad [7]$$

The original formula in [4] is used when \(W/M < R_0\). The squat formula in [6] is applied in the alternative case of \(W/M > R_0\). For the sake of brevity rectangular pillars are not dealt with in this paper. For the same reason the alternative linear pillar strength formula (where strength is a linear function of the width-to-height ratio) is omitted also. Both of these refinements can be incorporated without difficulty.

A model of pillar side scaling

Basic properties of the model

Little attention appears to have been devoted to the study of the mechanisms that cause time dependent deterioration of coal pillars. This deterioration may manifest itself in various ways, such as pillar side spalling, roof falls, floor heave and so on. Here the focus is placed on spalling. In general, the intensity of spalling may depend on environmental conditions, the composition of the coal and the stresses applied to the pillar. If the coal contains certain clay minerals, such as montmorillonite, a plausible mechanism may involve the swelling of the exposed coal skin in the presence of a humid environment. The pillar skin would swell in the direction perpendicular to the pillar side. The resulting tensile strain may cause the affected skin to peel off, exposing fresh coal and starting the process anew. Such a 'weathering' mechanism is the basic premise of the model to be proposed. Thus, in the model it is assumed that

- the time dependent pillar deterioration is solely due to pillar side scaling or spalling and
- this scaling is not due to excess stress, but to environmental conditions that would result in a time-dependent reduction of the widths of pillars.

It will also be assumed here that the scaling is uniform over the sides of the pillars and the pillar width at time \(t\) can be expressed in the following form:

$$W(t) = W_i \pm 2d(t). \quad [8]$$

where \(W_i\) is the initial pillar width, that is the width at time zero and function \(d(t)\) represents the thickness of coal peeled off one of the pillar sides between time zero and \(t\). Obviously, the decrease in pillar width is accompanied by a corresponding increase in bord widths, as the pillar centre distances remain unaltered. Function \(d(t)\) can be expressed as an integral of the scaling rate \(r(t)\):

$$d(t) = \int_0^t r(\xi) d\xi. \quad [9]$$

Since no information is available concerning the nature of the scaling function, the simplest assumption is used in this paper. It is postulated that the scaling rate is constant, thus

$$\dot{d}(t) = rt \quad [10]$$

where parameter \(r\) represents the uniform rate of scaling.

It follows from this development that in the presence of scaling the apparent safety factor of a pillar also becomes a function of time. If \(W(t)/M < R_0\), the safety factor can be put in the following form:

$$S_{app}(t) = \frac{KW(t)^\gamma M^\delta}{\sigma_m(t)} \quad [11]$$

and for a squat pillar \((W(t)/M > R_0)\) the safety factor becomes:

$$S_{app}(t) = \frac{K}{\sigma_m(t)} \left( W(t)^\gamma M^\delta \right) \left( \frac{W(t)}{MR_0} \right)^{\varepsilon} \pm 1 \right] + 1. \quad [12]$$

In these expressions \(\sigma_m(t)\) represents the tributary load (mean pillar stress) adapted for the present purpose:

$$\sigma_m(t) = \gamma H \left[ \frac{C}{W(t)^2} \right] \quad [13]$$

where \(\gamma = \rho g\) with \(\rho\) is the overburden rock density and \(g\) is the gravitational acceleration. Here, the pillar centre distance \(C = W_i + B = W(t) = 2d(t) + B\) remains unaffected. As usual, \(B\) is the original bord width in this relationship.

In the presence of scaling both the true and the apparent safety factors of pillars diminish with time. Time dependent deterioration of pillars diminishes with time. The apparent safety factor can be computed from [11] or [12] if the magnitude of scaling is known. The true factor must remain unknown right up to the moment of failure (if failure is to occur at all) when, by definition, it is equal to unity. In the absence of any contrary indication, it seems reasonable to postulate that the apparent safety factor at failure, or the critical safety factor, continues to be distributed in accordance with the original log-normal distribution defined in [1].

Maximum depth of scaling

According to the simple version of the proposed model, scaling proceeds at a steady rate. This seems to imply that in all cases where such deterioration occurs the pillars will eventually collapse. Simple considerations reveal, however, that failure is not inevitable. The piles of coal that have peeled off the pillar sides will gradually restrain the scaling process. As the piles grow in height this restriction becomes more significant. For simplicity, it is postulated here that scaling of the sides will proceed unimpeded, as illustrated in Figure 1, until the height of the peeled-off coal piles reaches the roof. At this stage scaling is arrested and, as long as this crumbled coal is left in place, the pillar will remain unchanged. This is a conservative assumption. In reality, pillar scaling will slow down and will even stop before the rubble reaches the roof. Of course, it will be realised that some pillars may never reach this condition, simply because they collapse before the scaling is choked in this manner.

Geometrically, two types of situation can arise. The bords could be sufficiently wide to accommodate the scaled material falling from the sides of neighbouring pillars without the resulting piles interfering with each other. In this situation...
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**Figure 1**—Plan (a) and section (b) illustrate maximum scaling that can be experienced by an isolated pillar (maximum scaling depth, \( d_m \); initial pillar width, \( W_i \); pillar height, \( M \); angle of repose of scaled coal, \( \rho \))

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**Figure 2**—Plan (a) and section (b) depict pillar scaling when the scaled coal piles from neighbouring pillars do not come into contact at the bord centres

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**Figure 3**—Plan (a) and section (b) show maximum pillar scaling when the scaled coal coalesces and forms a continuous pile in the bords

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The maximum depth to which scaling may penetrate before choking takes place, \( d_m \), can be estimated by equating two volumes. The volume of loose coal lying around a pillar must be equal to the bulked volume of the solid coal that has peeled off the pillar.

In the first case, that is when the piles of coal corresponding to each pillar are independent of each other, this criterion leads to a quadratic equation for \( d_m \) (see Appendix). If the bulking factor and the angle of repose are denoted by \( \delta \) and \( \rho \), respectively, then the solution of this equation results in the following maximum scaling depth \( d_m \) (see [A9] in the Appendix):

\[
d_m = \frac{M}{2\delta \mu} \left[ 1 + \delta \mu R_i \pm \sqrt{1 + (\delta \mu R_i)^2 + \frac{4}{3} \delta} \right]
\]

where

\[
\mu = \tan \rho \quad R_i = W_i / M.
\]

Here \( R \), represents the initial width-to-height ratio of the pillar. The result in [14] is valid as long as the piles do not come into contact at the bord centre. This requirement is fulfilled if the following inequality is satisfied (see [A12] in the Appendix):

\[
B \geq B_{co} = \frac{M}{\mu} \left[ 2 \pm \frac{1}{\delta} \sqrt{1 + (\delta \mu R_i)^2 + \frac{4}{3} \delta} \right]
\]

where \( B_{co} \) is that particular bord width at which the toes of the coal piles corresponding to neighbouring pillars first come into contact. If, however, \( B < B_{co} \) then the coal piles coalesce and the maximum scaling can no longer be calculated from [14], but an alternative value must be found. This is achieved by equating again the volume of the fragmented coal around a pillar with the bulked volume of the material that had scaled off the same pillar. This approach, when using the appropriate expressions for the volumes, leads to a cubic equation. This equation is given in the Appendix (see Equation [A19]) and can be solved for \( d_m \) by, for example, iteration.

It is simple to interpret the significance of \( d_m \), regardless of whether \( B \) is smaller or larger than \( B_{co} \). One of two possibilities may arise. A scaling pillar may or may not collapse before scaling has penetrated to this particular depth. If failure does not take place while \( d(t) < d_m \), then the progress of scaling will be arrested and the pillar will not fail. It is obvious, therefore, that a scaling pillar cannot stabilize and will collapse if \( W_i < 2d_m \). This statement can be put into a more useful form by defining a critical width-to-height ratio, \( R_{cr} \). All pillars with an initial width-to-height ratio smaller than or equal to \( R_{cr} \) will fail, if scaling occurs. (Obviously the converse is not necessarily true. Pillars with width-to-height ratios greater than \( R_{co} \) may or may not collapse.) This critical width–height ratio is defined by:

\[
R_{cr} = \frac{2}{\sqrt{3} \delta \mu}
\]

If reasonably typical values for the bulking factor \( \delta = 1.35 \) and the angle of repose \( \rho = 35^\circ \) are used, the value of this critical width–height ratio is 1.42.

**Life of pillars**

Perhaps the most important information concerning a panel supported by coal pillars, which are affected by scaling, is the period that elapses between the formation and collapse of the pillars. This period will be referred to as pillar life in the model. The expressions in [11] and [12] define safety factors that decrease with the passage of time. These results do not imply, however, that the life of a particular panel can be pre-calculated from the dimensions of the initial mining layout and the rate of scaling. This is understandable since the...
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Apparent safety factor corresponding to a specific failure is not known in advance. It was found, however, that the frequency of the apparent safety factor at failure could be represented by a log-normal distribution, where the variance \( \sigma \) had been determined earlier as given in [5].

To illustrate the problem, postulate that ten geometrically identical panels exist and the pillars in them are all subject to scaling at the same rate. In spite of these similarities, the panels cannot be expected to fail at the same time. This is because the apparent safety factor at failure is a random variable. This can be illustrated using the Monte Carlo method to draw a random sample of ten critical safety factors from the log-normal distribution. These values are tabulated in the first row of Table I.

The layout parameters used to construct the second and third rows of Table I are as follows:

\[
H = 150 \text{ m} \quad B = 5.5 \text{ m} \quad M = 3 \text{ m} \quad r = 0.4 \text{ m/year}.
\]  

[18]

These dimensions, when supplemented by a design safety factor of 1.6 and substituted into [11], yield a pillar width of 15.88 m. In practice, this value will be rounded. Hence, postulate that the initial pillar width is taken as \( W_i = 14 \text{ m} \).

As the initial pillars in this example are not squat \((R_i = 4.67)\), it is obvious that the scaled pillars will not be squat either. In this case, following the approach proposed by van der Merwe\(^7\), the expression in [11] can be solved for \( d_c \), that is, for the scaling at the time of failure. The result is:

\[
d_c(S_c) = \frac{1}{2} \left( W_i \pm \left( \frac{\gamma H S C}{K M^{0.5}} \right) ^{\frac{1}{2}} \right).
\]

[19]

This formula, when the already specified parameters and the tabulated critical safety factors are utilized, yields the second row of Table I. The third row of the same table is obtained from

\[
t_c(S_c) = \frac{d_c(S_c)}{r}
\]

which yields the time elapsed to failure, or the pillar life, in years. Next, for control purposes, the values of maximum scaling and the corresponding minimum bord width are calculated:

\[
d_m = 1.73 \text{ m} \quad B_{io} = 5.10 \text{ m}
\]

[21]

Since the bord width in [18] is 5.5 m, the requirement that \( B \geq B_{io} \) is satisfied and the coal piles from adjacent pillars will not come into contact.

It is important to note the variations, for ostensibly identical pillars, in the values of the parameters presented in Table I. In this particular sample of ten randomly-selected cases, the apparent safety factors at failure range from 0.76 to 1.42, the depth of scaling from 0.35 m to 1.85 m and most importantly, the pillar life from 0.89 years to 4.58 years. Moreover, it will be recalled that according to [21] scaling will cease when \( d_c \) reaches the choking value, that is, when \( d_c = d_m = 1.73 \text{ m} \). In the sixth column of Table I the depth of scaling is 1.85 m, which exceeds this limiting value. Thus, scaling will stop before the pillar width is reduced to the value pertaining to failure. Hence no pillar collapse will take place in this instance.

This example illustrates that no unique relationship exists between the initial mining geometry and pillar life. In Figure 4 an updated version of Madden's\(^{1}\) plot of apparent safety factors as a function of pillar life is shown. This illustration, which was prepared on the basis of field data, reinforces the conclusion that any relationship between design safety factor and pillar life is masked by the stochastic nature of the problem. Of course, there is an underlying trend, but this is blurred by the random behaviour of the apparent safety factors. This problem is especially serious when the sample is small.

### Pillar life expectancy and probability of survival

It is not possible, due to the probabilistic definition of pillar strength, to determine unequivocally whether a pillar will or will not fail. All we have is an estimated probability that failure will, or will not occur. If the composition of coal results in pillar side spalling, the situation becomes even more complex. As was illustrated in the previous section, the effect of spalling reduces the pillar width and, therefore, may cause the failure of a pillar at some later time. The Monte Carlo technique can be employed to gain a greater insight into this phenomenon.

Postulate that \( m \) panels have been constructed in accordance with an identical set of mining dimensions. Using the dimensions employed in the previous section, the layout of the panels is defined by:

\[
H = 150 \text{ m} \quad W = 14 \text{ m} \quad B = 5.5 \text{ m} \quad M = 3.0 \text{ m}
\]

[22]

In addition, the constants in [5], together with the following parameters are taken to represent the behaviour of scaling pillars:

\[
H = 150 \text{ m} \quad W = 14 \text{ m} \quad B = 5.5 \text{ m} \quad M = 3.0 \text{ m}
\]

[22]

\[
\gamma = 0.93 \quad \alpha = 0.03 \quad \beta = 0.03 \quad \sigma = 0.03
\]

[24]

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\[ \delta = 1.35 \quad \rho = 35^\circ \quad r = 0.2 \text{ m/year.} \]  \[23\]

The initial safety factor computed from the dimensions in [22] and using [11] has a value of \(S_i = 1.61\). In view of this safety factor, it can be said that these mining dimensions represent a conventionally designed layout. The probability that failure will not occur at a given initial safety factor is given in Equation [2]. Since, logically, the probability of failure is \(p^*(S_i) = 1 - p(S_i)\), it is simple to compute that \(p^* = 0.0014\). As before, the maximum scaling or the choking depth of scaling is \(d_m = 1.73\) m and the limiting bord width is \(B_{co} = 510\) m. Since \(B = 5.5\) m, the broken coal piles will not coalesce.

This case was simulated by using the results of 10 000 random draws taken from the distribution of the nominal safety factor. These safety factor values were subjected to the process described in the previous section. The resulting depths of scaling included eleven instances where the \(d_c\) values were non positive. This number approximates fairly closely the value of 14 that would arise from the probability of failure. These are the instances where the pillars will fail during their formation. Furthermore, there were 807 cases where the computed \(d_c\) value did reach or exceeded the choking depth of scaling. These instances represent the case where scaling is stopped by the surrounding rubble and the pillars are stabilized permanently.

The data resulting from the simulation are presented in two illustrations. In Figure 5 the histogram of the frequency of collapses is plotted as a function of time. This histogram gives a good indication of the distribution of pillar life. It is apparent that during the first year or so after the formation of the panels only a few collapse, but later the rate of failure increases significantly. The most frequently occurring life is about five to six years. From this time onwards, the rate of failure begins to diminish as fewer and fewer panels are left standing.

There are two apparently anomalous bars on this graph. The first at zero life is hardly noticeable and corresponds to the eleven cases where the failure is instantaneous. At the other end of the life scale, that is at \(t_{max} = 8.66\) years, there is...
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a conspicuous bar representing the 807 cases that remain standing permanently.

In Figure 6 the estimate of the survival probability is depicted. Here the ordinate \( p(t) \) approximates the probability that the pillars in this panel will survive without collapse to time \( t \). Clearly the probability of survival is very nearly unity initially, but it begins to diminish, first slowly and later quite rapidly. Finally, at 8.66 years the probability drops to 807/10000 = 0.0807. Since the remaining 807 panels remain standing permanently, this probability remains constant from then onwards.

It is worthwhile to examine the probability of survival curves for two more cases. The first example involves a panel where the design safety factor of the pillars is low. The other case relates to the situation where the pillars have an unusually high safety factor. In both cases the properties of the coal are assumed to be the same as specified earlier in [5] and [23]. Starting with the case of low safety factor and keeping all mining dimensions, with the exception of the pillar width, the same as those given in [22]. The new pillar width and the corresponding design safety factor are:

\[
W = 10 \text{ m} \quad S_f = 1.12. \quad [24]
\]

In this case 2444 cases failed instantaneously and none of them survived permanently. The probability of survival curve for this example is shown in Figure 7. It is noteworthy that the survival probability is just under 0.75 at the ‘instant’ of forming the panels and rapidly reduces with the passage of time. At five years the survival probability is negligible.

A very different situation prevails if the pillars are over-designed. In this case the pillar width and the safety factor are significantly higher than before:

\[
W = 18 \text{ m} \quad S_f = 2.06. \quad [25]
\]

The survival probability distribution corresponding to this case is illustrated in Figure 8. In this instance the probability remains virtually unity up to almost six years and then reduces only to 9065/10000 = 0.9065. After 8.50 years the probability remains unaltered indefinitely. Figure 8 appears to suggest that it is possible to design layouts, even in seriously spalling seams, where the probability that the pillars will remain unfailed is high.

As this discussion reveals, if exposed walls of a coal seam suffer from scaling induced by weathering, the mechanics of coal pillar behaviour become considerably more complex. An interesting insight into the deterioration of such pillars and their probability of survival can be gained by a close study of Figures 6, 7 and 8.

Back-calculation of scaling rates

No direct evidence appears to exist to substantiate the proposed model of pillar scaling. Thus, it is not possible to prove convincingly the validity or otherwise of the approach. However, some evidence can be marshalled which can be used to establish at least the plausibility of the assumptions put forward in the previous sections of the paper. This evidence consists of 13 documented cases of collapse that have occurred in collieries mining the No. 3 Seam in the Vaal Basin. No direct observation exists to prove that these failures were caused by scaling. However, in view of the earlier mentioned observations of van der Merwe7 concerning this geological region, it is not unreasonable to accept this supposition. These data contain sufficient information to perform some back-calculations. The aim of such analysis is to estimate the rate of scaling which could have prevailed in the recorded cases and seemingly caused the collapse of the pillars. If the depths of scaling, \( d_c \), were known for the 13 cases, the rate of scaling could be calculated from \( r = d_c / t_c \) (see [20]). Unfortunately, the scaling depths were not observed in these cases, therefore, this formula cannot be used in this simple manner. An alternative approach would have been to use the apparent safety factor at failure but, of course, these values were not available either. The only usable option involved the adoption of the Monte Carlo method to create a large number of apparent (critical) safety factors as was done earlier. From these values, through the use of the relationship in [19], a set of critical scaling depths was generated and stored as a vector, which was in turn sorted in ascending order.

Figure 9—Histogram of rate of scaling. The mean rate is 0.193 m/year

### Table II

Scaling rates and related data from case histories in the Vaal Basin, No. 3 Seam

<table>
<thead>
<tr>
<th>Case no.</th>
<th>Pillar life (years)</th>
<th>Mining depth (m)</th>
<th>Design safety factor</th>
<th>Pillar height (m)</th>
<th>Width-to-height ratio</th>
<th>Rate of scaling* (m/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>4.0</td>
<td>82.0</td>
<td>2.28</td>
<td>2.80</td>
<td>3.6</td>
<td>0.339</td>
</tr>
<tr>
<td>161</td>
<td>11.0</td>
<td>96.0</td>
<td>2.07</td>
<td>2.90</td>
<td>4.1</td>
<td>0.126</td>
</tr>
<tr>
<td>154</td>
<td>2.0</td>
<td>51.0</td>
<td>1.30</td>
<td>3.41</td>
<td>1.7</td>
<td>0.150</td>
</tr>
<tr>
<td>155</td>
<td>8.0</td>
<td>113.7</td>
<td>1.81</td>
<td>2.75</td>
<td>4.4</td>
<td>0.150</td>
</tr>
<tr>
<td>171</td>
<td>5.0</td>
<td>103.5</td>
<td>1.77</td>
<td>2.94</td>
<td>4.0</td>
<td>0.237</td>
</tr>
<tr>
<td>172</td>
<td>4.0</td>
<td>47.0</td>
<td>1.55</td>
<td>3.21</td>
<td>1.8</td>
<td>0.121</td>
</tr>
<tr>
<td>173</td>
<td>7.0</td>
<td>90.0</td>
<td>1.76</td>
<td>2.53</td>
<td>3.6</td>
<td>0.132</td>
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<td>174</td>
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<td>1.60</td>
<td>3.00</td>
<td>3.3</td>
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<td>59.6</td>
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<td>3.57</td>
<td>2.5</td>
<td>0.313</td>
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<tr>
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<td>6.0</td>
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<td>1.70</td>
<td>3.15</td>
<td>1.8</td>
<td>0.068</td>
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<tr>
<td>177a</td>
<td>3.0</td>
<td>103.7</td>
<td>1.63</td>
<td>3.05</td>
<td>3.6</td>
<td>0.332</td>
</tr>
<tr>
<td>178a</td>
<td>2.0</td>
<td>99.0</td>
<td>2.26</td>
<td>3.34</td>
<td>4.3</td>
<td>0.845</td>
</tr>
<tr>
<td>178b</td>
<td>2.0</td>
<td>99.0</td>
<td>2.44</td>
<td>3.30</td>
<td>4.4</td>
<td>0.862</td>
</tr>
</tbody>
</table>

* Mean rates estimated through Monte Carlo simulation.
Life and design of bord-and-pillar workings affected by pillar scaling

As mentioned earlier, non-positive depth of scaling values represent instantaneous failure, thus no scaling can be associated with them. At the other end of the scale, in other words at high values, scaling is restricted sometimes by choking, which occurs when \( d_c > d_{m} \). The cases satisfying this restriction correspond to that part of the distribution where the pillars do not fail. Hence, these instances again do not yield useful information concerning scaling that terminates in pillar collapse. Thus, using this approach and rejecting, when appropriate, the abnormal values at either end, the remaining elements of the vector can be used to calculate a series of scaling rates. This procedure was followed for all of the 13 cases.

Using all the usable values of scaling depth, scaling rates were computed with the aid of the earlier mentioned simple formula. An example of the scaling rate distribution obtained in this manner is illustrated in Figure 9. Once the distributions of scaling rates were obtained for all 13 cases, a corresponding number of mean scaling rates were calculated. The mean rates are summarized in Table II, together with the other relevant data concerning the 13 case histories.

It is apparent that the data in this table can be subdivided into two groups. The first eleven cases seem to belong to a geologically consistent region which can be represented by one set of parameters. The difference between this group and the group represented by the last two cases is so large that they must be treated separately. The second group consists of the last two cases, that is, 178a and 178b. In view of the small number of cases here, this group had been excluded from further analysis. The average of the eleven mean rates in the first group is 0.197 m/year. This value gives a good indication of the magnitude of scaling rates in mines where scaling appears to be a serious problem.

The correlation coefficients between rate of scaling and the other variables included in Table II were investigated next. The appropriate calculations show that the correlation coefficients between the mean rate of scaling and the pillar life, the design safety factor, the pillar height and the width-to-height ratio are -0.586, 0.369, 0.198 and 0.184, respectively. These figures indicate that there appear to be no significant correlations between the safety factor and the rate of scaling, the pillar height and the rate of scaling, or the width-to-height ratio and the rate of scaling. There is, however, a weak negative correlation between the rate of scaling and the pillar life. This correlation is explored in Figure 10 where the computed mean rates are plotted as a function of pillar life. Two further curves are shown in the same illustration. These curves are the linear, \( r_l(T) \), and exponential, \( r_e(T) \), regression functions fitted to the rate data.

\[
\begin{align*}
  r_l(T) &= -0.0198T + 0.3043 \text{ m/year} \\
  r_e(T) &= 0.3164e^{-0.1077T} \text{ m/year}
\end{align*}
\]

where \( T \) is the life of the pillar in years. It is not possible to express preference between the two functions on the basis of the illustration. Similarly, the correlation coefficients corresponding to the two expressions are almost identical and only slightly higher than the earlier mentioned coefficient. Perhaps on physical grounds the exponential form is to be preferred. However, in the light of the large scatter around the curves, no firm conclusion can be inferred.

It is important to put the results in Table II into proper perspective. It would appear that the simple model proposed in this paper provides an acceptable explanation of the first eleven cases in the Table. The obtained scaling rates are of reasonable magnitudes and tolerably consistent. So it seems that the predictions obtained from the model are not inconsistent with field observations. The model also provides a good explanation for the apparently inconsistent noted when pillar life is analysed as a function of the safety factor or other variables. It is tempting in this light to conclude that the model is ready for practical application. However, this eagerness must be tempered with caution since no underground observations are reported in conjunction with these case histories. Hence, no evidence is available to indicate conclusively whether the failures in the table are, in fact, the result of pillar side spalling.

In addition to the information back-calculated from the records obtained in the Vaal Basin, a few pieces of additional data concerning pillar scaling have come to light. These data were obtained by direct measurements in two collieries operating in the Witbank area and they relate to unfailed pillars. While the observations cannot claim to be of high precision, they do provide the opportunity of estimating scaling rates in regions where pillar side spalling has not proved to be a serious problem in the past. The pillars in these cases have not collapsed, therefore, the rates relate to

![Figure 10](image-url) — Plot of mean rates of scaling for eleven cases. The illustration also depicts the linear and exponential regression curves.
Life and design of bord-and-pillar workings affected by pillar scaling

piller scaling in progress. The basic data, together with the
derived rates, are presented in Table III. Collieries A and B
were mining in the No. 2 and No. 4 Seams, respectively.
The observed lives of the pillars in these examples are
long and the scaling rates are much lower than those
tabulated in Table II. The tentative estimate of the mean
scaling rates in the Nos. 2 and 4 seams in the Witbank area
are 0.019 m/year and 0.046 m/year, respectively.

**Design of layouts in scaling seams**

The next task involves the design of pillars in situations
where pillar deterioration due to scaling cannot be ignored.
Two design problems come to mind. The *first* of these entails
the determination of the pillar width which guarantees that
the probability of survival does not drop below *p* even after
the elapse of *t* years. The *second* problem involves
the computation of that width which ensures that the survival
probability of the pillars does not fall below *p* for an
*unlimited period*. Clearly, the solution of the second of these
problems represents the more cautious design and leads to
larger pillars.

To solve either of the problems it is necessary first to
determine the critical safety factor *S* corresponding to
the probability of survival *p*. This can be achieved with the aid of
the definition in [2]. In the *first problem* the pillar centre
distance is given by

\[ C = W(tp) + 2rt + B, \]

because

\[ d(tp) = rt \]

and

\[ W(tp) \]

is the residual pillar width at time *t* years. The next
step is to solve the following equations for the residual pillar
width:

\[ S_{pwr}(W(tp)) = S_c \quad \text{(a)} \]

\[ S_{squ}(W(tp)) = S_c \quad \text{(b)} \]

where the safety factor functions on the left-hand sides are
defined in [11], [12] and [13]. These equations represent the
requirement that the safety factor of the residual pillars
should be *S*.* Two equations are specified because it is not
known in advance whether or not the obtained pillar will be
squat. Let the solutions of Equations (a) and (b) be

\[ W_{pwr} \]

and

\[ W_{squ} \]

respectively. If the pillar width calculated from (a)

\[ \text{corresponds to a squat pillar, that is, if} \]

\[ W_{pwr} / M \]

then

\[ \text{the solution of the problem posed in (27) is a residual pillar} \]

\[ \text{width that accords} \]

\[ \text{with the probability } p. \]

\[ \text{The initial or the design pillar width is} \]

\[ W_l = W + 2rt. \]

\[ \text{Using this width, the safety factor safety factor of the} \]

\[ \text{pillar can be computed from either [11] or [12],} \]

\[ \text{depending on whether or not the pillar is squat.} \]

\[ \text{The solution obtained here is valid only if} \]

\[ \text{and} \]

\[ B \geq B_{co}, \]

\[ \text{where} \]

\[ \text{and} \]

\[ B_{co} \]

are given in [14] and [16],

\[ \text{respectively. Violation of the first inequality reveals that a solution} \]

\[ \text{was found where the scaling reached a depth which is greater} \]

\[ \text{than the maximum possible value, that is,} \]

\[ d_{co}. \]

\[ \text{This is an unacceptable result and the valid solution can be obtained} \]

\[ \text{by following the route prescribed for the second problem.} \]

\[ \text{Breaching of the second inequality conveys that the rubble} \]

\[ \text{piles around the pillars have merged and the correct solution} \]

\[ \text{can be found by solving the problem corresponding to that} \]

\[ \text{geometry.} \]

\[ \text{The basic premise of the second problem is that the depth} \]

\[ \text{of scaling is taken to be the value at which choking occurs,} \]

\[ \text{that is,} \]

\[ d_{co} \text{ in [14]. For the purposes of the present} \]

\[ \text{application, a version of} \]

\[ \text{is required which is expressed as a} \]

\[ \text{of the width of the residual pillar. The relevant} \]

form of the formula is (see [A7] in the Appendix):

\[ d_{co} = d(W) = \frac{M}{2\delta_d} \left[ \sqrt{\frac{\mu W}{M}} + 2\delta_d \frac{W}{M} - \frac{4}{3} \delta_d \mu W \frac{M}{W} \right]. \]

\[ [28] \]

On this occasion the distance between pillar centres can be
expressed as

\[ C(W) = W + 2d(W) + B, \]

where the dependence of the depth of scaling on *W* must be taken into account.

Again two solutions are obtained from the following equations:

\[ S_{pwr}(W) = S_c \quad \text{(a)} \]

\[ S_{squ}(W) = S_c \quad \text{(b)} \]

\[ [29] \]

One of the solutions corresponds to a non-squat pillar

\[ \text{(Equation (a)) and the other to a squat pillar (Equation (b)).} \]

\[ \text{As before, the appropriate choice picked from these values is} \]

\[ \text{the solution of the problem. The applicability of this solution} \]

\[ \text{is limited only by the inequality} \]

\[ B \geq B_{co}. \]

\[ \text{If this limitation is} \]

\[ \text{violated then the piles of fragmented coal are merged and the} \]

\[ \text{solution relevant to that case should be sought.} \]

\[ \text{To illustrate the capability of the design method, a few} \]

\[ \text{examples are discussed next. Assume that the mining site in} \]

\[ \text{question is in the Vaal Basin, therefore, let the rate of scaling} \]

\[ \text{be 0.2 m/year. The relevant mining dimensions are as} \]

\[ \text{follows:} \]

\[ H = 100 \text{ m} \quad B = 5.5 \text{ m} \quad M = 3.0 \text{ m.} \]

\[ [30] \]

Assume that the colliery is remote from populated areas

\[ \text{and the long-term survival of the pillars is not of great} \]

\[ \text{concern. In these conditions it may suffice to require that the} \]

\[ \text{probability of survival of the pillars for five years} \]

\[ (t_p = 5 \text{ years}) \]

\[ \text{should not be less than} \]

\[ p = 0.99. \]

\[ \text{This requirement} \]

\[ \text{yields} S_c = 1.4476. \]

\[ \text{Scaling, during the specified period of five} \]

\[ \text{years, reduces the pillar width by 2.0 m} \]

\[ \text{(d(t) = 1.0 m).} \]

\[ \text{The solution of the problem posed in [27] is a residual pillar} \]

\[ \text{width of 10.78 m. Hence, the initial pillar width that can} \]

\[ \text{accommodate 2.0 m of spalling is} \]

\[ W_l = 12.78 \text{ m.} \]

\[ \text{The design safety factor would have to be} \]

\[ S_c = 2.20 \text{ to obtain this pillar width} \]

\[ \text{using the conventional methodology. This layout} \]

\[ \text{would yield 51.1% extraction.} \]

\[ \text{It is noteworthy that if the design specification requires that the} \]

\[ \text{pillars should survive, with the same probability, for an} \]

\[ \text{unlimited period, the pillar width increases to 15.50 m} \]

\[ \text{and the extraction is reduced to 45.5%. The conventional} \]

\[ \text{method of design would yield this pillar size if a safety factor of} \]

\[ 2.68 \text{ was used.} \]

**Summary and recommendations**

The results presented in the paper establish, reasonably convincingly, the *plausibility* of the proposed model. No attempt is made to claim a stronger description to charac-
terize the current status of the research, because the field
evidence to support the formulation is meagre. The object of the
model is to describe the deterioration of pillars due to one
and only one particular cause, namely pillar side scaling or
spalling. The model deals with a time-dependent deterio-
ration which can be described loosely as weathering.

The attractiveness of the model lies in its simplicity and
versatility. It can readily explain the reason for the failure to
discrim, on the basis of a small field sample, sensible
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relationships between pillar life and other parameters such as safety factor. If an estimate of the rate of scaling is known, it is possible to quantify, using a Monte Carlo technique, the expected life of the pillars and the probability of survival (up to a specified age) of coal pillars. Moreover, the model permits the design of scaling pillars. Two approaches have been discussed. Pillars may be designed by specifying a probability of survival for a given number of years. Alternatively, it might be required that the probability of survival for an indefinite period should not be less than a specific value.

The model can be used for purposes of back-calculation as well. For example, if the initial mining dimensions and the life of the pillars are known, it is possible to estimate the rate of scaling that prevailed to cause the eventual failure of the pillar. Such back-calculations have suggested that the rate of spalling in the Vaal Basin, No. 5 Seam is usually in the order of 0.2 m/year (eleven case histories). However, two further cases in the same area have yielded significantly higher rates (0.85 m/year). Some underground observations in the Witbank area have provided considerably lower rates, in the order of 0.02–0.04 m/year.

The model is a straightforward extension of the ideas put forward by Salamon and Munro (1967). It is postulated that the underlying strength of pillars remains unaltered and the changes come about merely as a result of time-dependent reduction in pillar width. Moreover, the apparent safety factors at failure are distributed according to the same log-normal distribution obtained in the original derivation. For simplicity, it is taken that faces of pillars scale uniformly and, therefore, the changes in dimensions can be expressed merely as a reduction in pillar width. Furthermore, it is assumed that the span of the panel in question is sufficiently large to ensure that the full weight of the prism attributable to a particular pillar continues to load that pillar. In other words, it is a premise of the model that no load shedding takes place. In addition, it is postulated that once the coal rubble around a pillar reaches a critical height, the scaling is arrested.

There are subsidiary assumptions accepted in the paper. These can be modified fairly readily in the light of further studies. First, the rate of scaling is taken to be constant. Second, the rubble heap must reach the roof before scaling ceases and finally, the restraint provided by this pile of coal rubble around a pillar reaches a critical height, the scaling is arrested. It is important that the cause of abnormal collapses be investigated and explained as soon as possible. Such study is likely to find that abnormal behaviour is due to more than one cause. Van der Merwe's 57 observations imply that pillar scaling could be a reason for some of the premature failures. This deduction and the promising performance of the model proposed here provide a powerful basis for recommending that further study should be initiated to clarify the role of scaling or spalling in pillar mechanics. Three possible areas for additional research come to mind.

Study of scaling. Attempts should be made to establish the prevalence of scaling in South African collieries. Such investigations should cover mines that are suspected of being prone to spalling and those which hitherto were presumed to be free of time-dependent deterioration. This research would involve the study of old workings to determine the presence and the extent of scaling. Depending on the outcome of these investigations, the pillar strength formula, which is the kernel of this paper, may have to be re-examined.

The existence of scaled rubble at the foot of pillars is a good qualitative indication of pillar scaling. To estimate the magnitude and rate of spalling, an attempt should be made to compare old surveyor’s and current pillar offset data. This approach seems to have yielded reasonable data in the past.

Composition of coal. An attempt should be made to correlate the pervasiveness of scaling with the chemical composition of the coal. The analysis of the presence and composition of clay minerals may prove to be a fruitful direction of research in this respect.

Acknowledgement

The work reported here was carried out as part of the research programme of the Safety in Mines Research Advisory Committee under project COL 337.

Appendix

It is postulated in the main body of the paper that scaling is arrested when the pile of coal rubble accumulated at the foot of the pillar becomes sufficiently high to reach the roof. If the geometries in Figures 1, 2 and 3 are examined it becomes apparent that two situations can arise. At first the heaps of coal corresponding to each pillar are independent, Figures 1 and 2. Later, as scaling progresses, the piles may merge, Figure 3. These two situations are examined separately, starting with the case of independent piles.

Independent piles. In this case the aim is to ensure that the volume of the rubble equals the bulked volume of the coal that has scaled from the sides of the pillars. The critical depth of scaling, \(d_{cr}\), is reached when the height of the pile is equal to \(M\). The volume of the rubble, including the volume of intact coal at the heart of the pillar, can be expressed in terms of two other volumes. The first volume, \(V_1\), is the volume of a frustum of a square based pyramid of side length \(W + 2c\) and height \(M\), where

\[
\mu = \tan(\rho) \quad c = M/\mu
\]

Here \(\rho\) is the angle of repose, the length \(c\) is the distance (measured on the floor) from the toe of the pile to the edge of the residual pillar and \(W\) is the width of the residual pillar. These dimensions are illustrated in Figure 1. The required
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volume is

\[ V_i = \frac{1}{3} M(W + 2c)^2 \left[ 1 + \frac{W}{W + 2c} + \left( \frac{W}{W + 2c} \right)^2 \right]. \]  \[\text{[A2]}\]

The second volume, \( V_2 \), is the volume of coal in the centre of the pillar that remains intact:

\[ V_2 = MW^2. \]  \[\text{[A3]}\]

The volume of coal that has peeled off the pillar is:

\[ V_p = M \left( W^2 + W^2 \right) = 4Md_n \left( W + d_n \right). \]  \[\text{[A4]}\]

Clearly, the total volume of coal fragments, \( V_1 - V_2 \), must be equal to the dilated volume of the solid coal, \( \delta V_n \), that is,

\[ V_1 - V_2 = \delta V_s. \]  \[\text{[A5]}\]

The substitution into this equation leads to the following quadratic equation for \( d_n \):

\[ \delta d_n^2 + \delta Wd_n + \frac{1}{2} MW - \frac{1}{3} M^2 = 0. \]  \[\text{[A6]}\]

The relevant solution of this equation is:

\[ d_n = \frac{M}{2\delta W} \left[ \left( \frac{\delta W}{M} \right)^2 + 2\delta W + 4 \sqrt{\delta W} \frac{W}{M} + 3 \delta \right]. \]  \[\text{[A7]}\]

The solution in terms of the initial pillar width can be found by using the transformation \( W = W_i - 2dn \) in [A6]:

\[ \delta d_n^2 + \delta W + \frac{M}{\mu} d_n + \frac{1}{2\mu} \left( W + 2M \right) = 0 \]  \[\text{[A8]}\]

the appropriate solution of this quadratic equation is:

\[ d_n = \frac{M}{2\delta W} \left[ 1 + \frac{\delta W}{M} \left( \frac{W}{M} \right)^2 + \frac{4\delta}{3} \right]. \]  \[\text{[A9]}\]

The values of \( d_n \) calculated from [A7] and [A9] are identical.

These solutions are valid as long as the toes of the fragment piles do not come into contact with each other. This requirement will be fulfilled provided the original pillar centre distance is not exceeded by the sum of the residual pillar width and twice the base length of the coal pile at the side of the pillar. Mathematically this can be stated as an inequality:

\[ W + 2c = W_i + B, \]  \[\text{[A10]}\]

which, after some rearrangements, assumes the following form:

\[ 2(c - d_n) \leq B. \]  \[\text{[A11]}\]

Now introduce the notation

\[ B_{10} = 2(c - d_n) - \frac{M}{\mu} \left[ 2 + \frac{1}{1 + \frac{\delta W}{M}} \left( \frac{\delta W}{M} \right)^2 + \frac{4\delta}{3} \right]. \]  \[\text{[A12]}\]

Thus the coal piles are independent of each other as long as \( B_{10} \leq B \).

Continuous piles. Assume that \( B_{10} \leq B \). This postulate warrants that the adjacent coal piles have merged. This problem can be expressed in terms of four volumes: the volume of a parallelepiped of base area \((W_i+B)^2\) and height \(h\), \(V_1\), the volume of a frustum of a square-based pyramid of height \(M-h\), base width \(W_i+B\) and upper width \(W_i\), the volume of intact coal in the shape of a parallelepiped of width \(W\) and height \(M\), \(V_2\) and the volume of the coal that has peeled off the pillar, \(V_p\). These volumes are defined by the following expressions:

\[ V_1 = h(W_i + B), \]  \[\text{[A13]}\]

\[ V_2 = \frac{1}{3}(M + h)(W_i + B) \left[ 1 + \frac{W}{W + B} + \left( \frac{W}{W + B} \right)^2 \right]. \]  \[\text{[A14]}\]

\[ V_3 = MW^2. \]  \[\text{[A15]}\]

\[ V_p = M(W_i^2 + W^2) = 4Md_n(W_i + d_n). \]  \[\text{[A16]}\]

The equation of the volumes can be expressed on this occasion by a relationship that resembles the earlier one in [A5]:

\[ V_1 + V_2 - V_3 = \delta V_s. \]  \[\text{[A17]}\]

In addition, it will be observed that \( M - h = (B/2 + d_n)\), therefore, it follows that

\[ h = M \pm \left( \frac{1}{2} B + d_n \right) \mu. \]  \[\text{[A18]}\]

After this preparation it is possible to substitute into [A17] and derive a cubic equation for \( d_n \). The version presented here is expressed in terms of the initial pillar width:

\[ a_3 d_n^3 + a_2 d_n^2 + a_1 d_n + a_0 = 0. \]  \[\text{[A19]}\]

The coefficients in this equation are given next:

\[ a_0 = 2MBW_i + MB^2 \pm \frac{1}{2} \mu B W_i \pm \frac{1}{3} \mu B \]  \[\text{[A20]}\]

\[ a_1 = 4MW_i \pm 2\mu BW_i \pm \mu^2 \pm 4\delta MW_i \]  \[\text{[A20]}\]

\[ a_2 = 4\delta^2 M \pm 4\delta M \pm 2\mu W_i. \]  \[\text{[A20]}\]

Cubic equations have three roots which can be obtained by either analytical or numerical means. In this instance an iterative method is preferred, using the root in [A9] as an initial estimate.

References

3. MARK, C. and BARTON, T. The uniaxial compressive strength of coal: Should it be used to design pillars? Proc. 15th International Conf. on Ground Control in Mining, Golden, Colorado. 1996.
On February 14, 1998, after years of informal cooperation among the international community of minerals, metals, and materials societies and institutes, the IOMMMS was officially chartered during the 1998 Annual Meeting of The Minerals, Metals & Materials Society (TMS) in San Antonio, Texas.

The mission of the organization will be ‘to promote and facilitate communication and cooperation among the worldwide minerals, metals, and materials societies and institutes to the benefit of their members and the enhancement of the profession and industries associated with them’. Specifically, IOMMMS will:

➤ provide a forum through its annual meeting for the discussion and exchange of information on issues and opportunities among its member bodies
➤ provide a network for communication, visiting speakers, and exchange among its member bodies
➤ provide for the coordination of conferences, publications, electronic communication, continuing education, student activities and, where appropriate, membership affairs and services between and among the member societies and institutes in order to make optimal use of their resources
➤ provide reciprocal arrangements for advertising conferences and publications in each others’ journals
➤ provide for synergistic interaction between engineers, scientists, and educators within the minerals, metals, and materials field worldwide.

The following organizations comprise the founding membership of IOMMMS:

• The Australasian Institute of Mining and Metallurgy (AusIMM) - Australia
• Canadian Institute of Mining, Metallurgy and Petroleum (CIM) - Canada
• The Chinese Society for Metals (CSM) - China
• The Non-Ferrous Metals Society of China - China
• Federation of European Materials Societies (FEMS)
• Hungarian Mining and Metallurgical Society (OMBKE) - Hungary
• Japan Institute of Metals (JIM) - Japan
• The Korean Institute of Metals and Materials - Korea
• Mexican Academy of Materials (MAM) - Mexico
• Slovak Metallurgical Society (SHS) - Slovakia
• South African Institute of Mining and Metallurgy (SAIMM) - South Africa
• The Minerals, Metals & Materials Society (TMS) - USA
• ASM International - USA
• Iron and Steel Society (ISS) - USA
• Institute of Materials (IOM) - USA
• The Institution of Mining & Metallurgy (IMM) - United Kingdom

The first leadership group for the IOMMMS will include:

Chair: Judy Webber, AusIMM
Vice-Chairs: Michael DeHaemer, ASM International
Bernard Rickinson, IOM
Noriyoshi Taniguchi, JIM
Executive Committee: Peter Schepp, DGM
Zhong Zengyong, CSM
Secretary: Alexander Scott, TMS.

The IOMMMS future meetings schedule includes:

1999 Friday, May 28, 1999 in conjunction with the FEMS EUROMAT '99 Conference in Munich, Germany.
2000 Tuesday, September 12, 2000 in conjunction with the International Minerals Processing Congress (Minprex 2000) in Melbourne, Australia.

* Issued by: Gail Miller, Tel: 724/776-9000, ext 238
Alexander Scott, Tel: 724/776-9000, ext 211