



Determination of optimal cut-off grade policy to optimize NPV using a new approach with optimization factor

by A. Bascetin,* and A. Nieto†

Synopsis

One very important aspect of mining is deciding what material in a deposit is worth mining and processing, versus what material should be considered waste. This decision is summarized by the cut-off grade policy. The cut-off grade strategy for an open pit mine influences the annual cash flows and affects the net present value of a project. The optimization of cut-off grade strategy requires the knowledge of detailed operational costs, mining sequence, mining operation, processing, and product constraints, as well as the grade distribution of a given deposit defined by pit, phase, and different material types. Cut-off grade optimization algorithms have been developed in the past. The most common criteria used in cut-off grade optimization is to maximize the net present value (NPV). This paper describes the determination of a cut-off grade strategy based on Lane's algorithm adding an optimization factor based on the generalized reduced gradient (GRG) algorithm to maximize the project's NPV. The introduced algorithm is a windows based program developed at Virginia Tech that is based on a series of Visual Basic routines within a spreadsheet environment (Excel). The benefits of the methodology are demonstrated using a hypothetical case study.

Keywords: Cut-off grade optimization, net present value maximization, generalized reduced gradient algorithm method.

Summary

One of the most difficult problems in mining operation is how to determine optimum cut-off grades of ores at different periods over the lifespan of the mine that will maximize the net present value (NPV) of the mine. Maximizing the NPV of a mining operation, subject to different constraints is a nonlinear programming problem. Cut-off grade optimization is used to arrive at an operating strategy that maximizes the value of a mine. Cut-off optimization as a concept has been known for many years, particularly since Lane's latest revolution, but it is still not widely practised.

Cut-off grade optimization maximizes the NPV of a project subject to capacity constraints in the mine, mill and the market. These are usually expressed as annual limits to the tonnage mined, tonnage milled and product sold. At any point in time at least one limit,

and possibly two or all three, will be constraining the system. For cut-off optimization to work correctly, capacity constraints must be independent of the cut-off grade. The cut-off grade is used to distinguish ore from waste materials. If the cut-off grade is too high, much of the mined material will go to the waste dump area. If the cut-off is too low, then the input capacity of the entire mining and mineral processing operations will be fully stretched, while revenues do not necessarily increase. The optimal strategy is one that strikes the correct balance between these two limits.

This paper describes the determination of a cut-off grade strategy based on Lane's (Lane, 1964), algorithm adding an optimization factor based on the generalized reduced gradient (GRG) algorithm to maximize the project's NPV. In this study, a computer program using Excel and Visual Basic has been developed to implement the algorithm. An interactive user-friendly cut-off grade program was coded using Visual Basic in a Windows based spreadsheet application (Excel), a well-known application commonly used by mining professionals.

The cut-off grade program developed is used to incorporate the optimization factor (σ) which is included in the ultimate cut-off grade equation, which considers the mining (m) cost,

$$g_u(i) = \frac{c + m + f + F_i + \sigma}{(P - s)xy}$$

to further maximize the total NPV of the mine project. σ is the optimization factor and m is the mining cost per ton. The program solves

* Istanbul University, Engineering Faculty, Mining Engineering Department, Istanbul, Turkey.

† Virginia Polytechnic Institute and State University, Mining and Minerals Engineering, Blacksburg, Virginia, USA.

© The Southern African Institute of Mining and Metallurgy, 2007. SA ISSN 0038-223X/3.00 + 0.00. Paper received Mar. 2006; revised paper received Jan 2007.

Determination of optimal cut-off grade policy to optimize NPV

for the optimization factor (σ), which is a nonlinear problem (NLP), based on the generalized reduced gradient (GRG) algorithm by maximizing the project's NPV.

The algorithm was tested using first the optimization factor to find the maximum the NPV then it was tested by removing the optimization factor. The results were examined and presented in the paper. As a result of the study, the cut-off grade policy determined by the optimization approach using the (σ) factor gives a higher NPV than the cut-off grade policy estimated without the optimization factor. The results given by the used case study indicate that the impact of the optimization factor (σ) on the objective function (NPV) is significant at a \$17,889,613.00 increase which is equivalent to a 5% NPV increment. A NPV sensitivity analysis, which consisted in varying the optimization factor (σ) from 0.0 to 12.0 in 1.0 unit increments, indicates—as calculated by the introduced algorithm—that the maximum NPV value is at $\sigma = 5.84$. This approach provides great flexibility at the mine planning stage for evaluation of various economic and grade/ton alternatives.

Introduction

Cut-off grade is a geologic/technical measure that embodies important economic aspects of mineral production within a mineral deposit. In other words, it is defined not only by the deposit's geologic characteristics and the technological limits of extraction and processing, but also by costs and mineral prices. Taylor (1972) presents one of the best definitions of cut-off grade. He defined cut-off grade as any grade that, for any specific reason, is used to separate two sources of action, e.g. to mine or to leave, mill or to dump.

Some researchers have developed new optimization techniques based on Lane's (Lane, 1964) algorithm to determine a cut-off grade policy. Cut-off grade optimization is used to derive an operating strategy that maximizes the value of a mine. Where the mine's capacity allows, sacrificing low-grade material enables the mill to process ore that delivers a higher cash flow. Hence, the cut-off grade policy has a significant influence on the overall economics of the mining operation. The determination of cut-off grade policy, which maximizes the NPV, is already established in the industry. It has been realized that, as opposed to constant breakeven cut-off grade, the optimum/dynamic cut-off grades, which change due to the declining effect of NPV during mine life, not only honour the metal price and cash costs of mining, milling, and refining stages, but also take into account the limiting capacities of these stages and grade-tonnage distribution of the deposit (Dagdelen, 1992; Lane, 1988; Dagdelen and Mohammad, 1997). In other words, the techniques that determine the optimum cut-off grade policy consider the opportunity cost of not receiving the future cash flows earlier during the mine life due to limiting capacities present in the stages of mining, milling, or refining.

The optimization factor indicates the level of optimization achieved for the cut-off grade strategy calculated by the algorithm. This paper describes the use of an optimization factor calculated based on the generalized reduced gradient (GRG) algorithm to improve optimum cut-off grades.

Methods of cut-off grade optimization

As discussed earlier cut-off grade is the criterion normally used in mining operation to discriminate between ore and waste within a mineral deposit. Waste may either be left in place or sent to waste dumps. Ore is sent to the treatment plant for further processing. Optimizing mine cut-off grades means to maximize the net present value (NPV) of mining projects. Cut-off grade directly affects the cash flows of a mining operation, based on the fact that higher cut-off grades lead to higher grades per ton of ore; hence, higher revenues are realized depending upon the grade distribution of the deposit (Dagdelen, 1993). In any given period, the allocation of material sent to the mill and the product created in the refinery for sale are also dependent on the cut-off grade. It is generally accepted that the cut-off grade policy that gives higher NPVs is a policy that uses declining cut-off grades throughout the life of the project. Lane (1964, 1988) has developed a comprehensive theory of cut-off grade calculation. Table I shows the notation used in the algorithm.

In his approach K. Lane demonstrates that a cut-off grade calculation which maximizes NPV has to include the fixed cost associated with not receiving the future cash flows quicker due to the cut-off grade decision taken now. The cut-off grade when the concentrator is the constraint is given below.

$$g_m = \frac{c + f + F_i}{(P - s)y} \quad [1]$$

Where g_m (i): milling cut-off grade, f : fixed cost, F_i : opportunity cost per ton of material milled in Year i , P : profit (\$), s : selling price (\$/unit of product), y : recovery (%).

The opportunity cost is determined as:

$$F_i = \frac{d NPV_i}{C} \quad [2]$$

$$f = \frac{f_a}{c} \quad [3]$$

Where f_a is the annual fixed costs,

Notation	Explanation	Unit
i	Year	—
N	Mine life	Years
P	Metal price	\$/oz
s	Sales cost	\$/oz
m	Mining cost	\$/ton
c	Processing cost	\$/ton ore
f_a	Fixed costs	\$/year
f	Fixed cost	\$/ton
y	Recovery	%
d	Discount rate	%
CC	Capital costs	\$
M	Mining capacity	Tons/year
C	Milling capacity	Tons/year
R	Refining capacity	Tons/year
Q_m	Material mined	Tons/year
Q_c	Ore processed	Tons/year
Q_r	Concentrate refined	Tons/year

Determination of optimal cut-off grade policy to optimize NPV

d is the discount rate, NPV_i is the NPV of the future cash flows of the years (i) to the end mine life N , and the C is the total milling capacity in Year i .

The cut-off grade $g_m(i)$ depends on the NPV_i and NPV_{i+1} cannot be determined until the optimum cut-off grades have been decided. The solution to this type of interdependency problem is obtained by an iterative approach. In this study, a computer program using Excel and Visual Basic has been developed to implement the algorithm. An interactive user-friendly cut-off grade program, based on K. Lane's algorithm was coded using Visual Basic in a Windows based spreadsheet application (Excel), a well-known application commonly used by mining professionals (Figure 1).

The cut-off grade program developed is used to incorporate the optimization factor (σ) which is included in the ultimate cut-off grade equation, which considers the mining (m) cost,

$$g_u(i) = \frac{c + m + f + F_i + \sigma}{(P - s)y} \quad [4]$$

to further maximize the total NPV of the mine project. Where σ is the optimization factor and m is the mining cost per ton

The program solves for the optimization factor (σ), which is a nonlinear problem (NLP), based on the generalized reduced gradient (GRG) algorithm by maximizing the project's NPV.

A typical difficulty associated with nonlinear optimization is that in most cases it is only possible to determine a locally optimal solution but not the global optimum. Loosely

speaking, the global optimum is the best of all possible values while a local optimum is only the best in the neighborhood. One of the most powerful nonlinear optimization algorithms is the generalized reduced gradient algorithm method. The GRG algorithm was first developed by Ladson *et al.* (1978). This procedure is one of a class of techniques called reduced-gradient or gradient projection methods, which are based on nonlinear constraints.

The development of the procedure begins with the nonlinear optimization problem written with equality constraints. The idea of generalized reduced gradient is to convert the constrained problem into an unconstrained one by using direct substitution. If direct substitution were possible it would reduce the number of independent variables to $(n-m)$ and eliminate the constraint equations. However, with nonlinear constraint equations, it is not feasible to solve the m constraint equations for m of the independent variables in terms of the remaining $(n-m)$ variables and then to substitute to these equations into the economic model. Therefore, the procedures of constrained variation and Lagrange multipliers in the classical theory of maxima and minima are required. There, the economic model and constraint equations were expanded in a Taylor series, and only the first order terms were retained. Then with these linear equations, the constraint equations could be used to reduce the number of independent variables. This led to the Jacobian determinants of the method of constrained variation and the definition of the Lagrange multiplier being a ratio of partial derivatives.

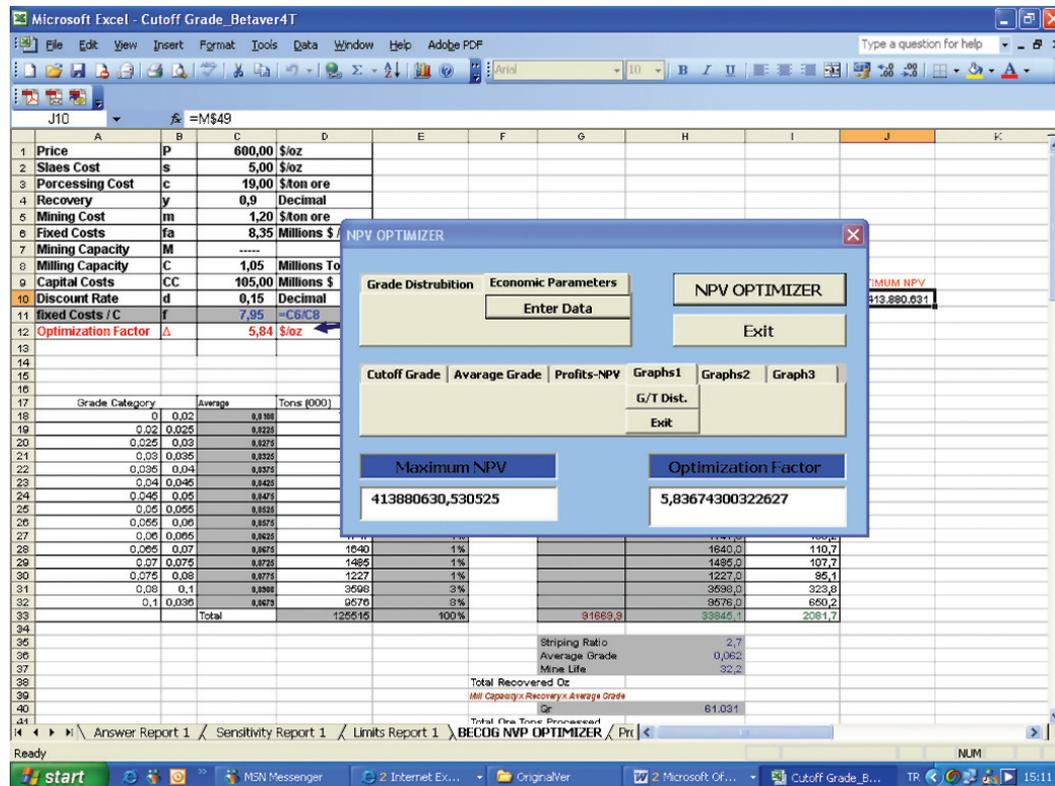


Figure 1—The Windows based program for maximization of NPV

Determination of optimal cut-off grade policy to optimize NPV

The development of the generalized reduced gradient method follows that of constrained variation. The case of two independent variables and one constraint equation will be used to demonstrate the concept, and then the general case will be described. Consider the following problem.

$$\begin{aligned} \text{Optimize : } & y(x_1, x_2) \\ \text{Subjectto : } & f(x_1, x_2) = 0 \end{aligned} \quad [5]$$

Expanding the above in a Taylor series about a feasible point $x_k(x_{1k}, x_{2k})$ gives:

$$y = y(x_k) + \frac{dy}{dx_1}(x_k)(x_1 - x_{1k}) + \frac{dy}{dx_2}(x_k)(x_2 - x_{2k}) \quad [6]$$

$$0 = f(x_k) + \frac{df(x_k)(x_1 - x_{1k})}{dx_1} + \frac{df(x_k)(x_2 - x_{2k})}{dx_2} \quad [7]$$

Substituting Equation [6] into Equation [7] to eliminate x_2 gives, after some rearrangement:

$$\begin{aligned} y(x) = & y(x_k) - \left(\frac{dy(x_k)}{dx_2} \frac{df(x_k)}{dx_2} \right)^{-1} f(x_k) + \left(\frac{df(x_k)}{dx_2} \right)^{-1} \\ & \left[\frac{dy(x_k)}{dx_1} \frac{df(x_k)}{dx_2} - \frac{dy(x_k)}{dx_2} \frac{df(x_k)}{dx_1} \right] (x_1 - x_{1k}) \end{aligned} \quad [8]$$

In Equation [8] the first two terms are known constants being evaluated at point x_k , and the coefficient of $(x_1 - x_{1k})$ is also a known constant and gives the x_1 the direction to move. Thus, to compute the stationary point for this equation, $dy/dx_1 = 0$; and the result is the same as for constrained variation which is the term in the brackets of Equation [8] that is solved together with the constraint equation for the stationary point. However, the term in the bracket also can be viewed as giving the direction to move away from x_k to obtain improved values of the economic model and satisfy the constraint equation.

For a better explanation consider;

$$\min f(\mathbf{x}) \quad [9]$$

$$\text{s.t. } \mathbf{Ax} = \mathbf{b}, \mathbf{x} \geq \mathbf{0} \quad [10]$$

Partition variables into two groups $\mathbf{x} = (\mathbf{y}, \mathbf{z})$ where \mathbf{y} has dimension m and \mathbf{z} has dimension $n-m$. This partition is formed such that all variables in \mathbf{y} are strictly positive.

Now, the original problem can be expressed as:

$$\min f(\mathbf{y}, \mathbf{z}) \quad [11]$$

$$\text{s.t. } \mathbf{By} + \mathbf{Cz} = \mathbf{b}, \mathbf{y} \geq \mathbf{0}, \mathbf{z} \geq \mathbf{0} \quad [12]$$

$$(\text{with, of course, } \mathbf{A} = [\mathbf{B.C}])$$

Key notion is that if \mathbf{z} is specified (independent variables), then \mathbf{y} (the dependent variables) can be uniquely solved. NOTE: \mathbf{y} and \mathbf{z} are dependent. Because of this dependency, if we move \mathbf{z} along the line $\mathbf{z} + \mathbf{aDz}$, then \mathbf{y} will have to move along a corresponding line $\mathbf{y} + \mathbf{aDy}$. Dependent variables \mathbf{y} are also referred to as basic variables. Independent variables \mathbf{z} are also referred to as non-basic variables.

The basic idea of the reduced gradient method is to consider, at each stage, the problem only in terms of the independent variables. Since \mathbf{y} can be obtained from \mathbf{z} , the objective function f can be considered as a function of \mathbf{z} only.

The gradient of f with respect to the independent variables \mathbf{z} is found by evaluating the gradient of $f(\mathbf{B}^{-1}\mathbf{b} - \mathbf{B}^{-1}\mathbf{Cz}, \mathbf{z})$. The resulting gradient

$$\mathbf{R}^T = \nabla_{\mathbf{z}} f(\mathbf{y}, \mathbf{z}) - \nabla_{\mathbf{y}} f(\mathbf{y}, \mathbf{z}) \mathbf{B}^{-1} \mathbf{C} \quad [13]$$

is called the reduced gradient.

The generalized reduced gradient solves nonlinear programming problems in the standard form

$$\text{minimize } f(\mathbf{x}) \quad [14]$$

$$\text{subject to } \mathbf{h}(\mathbf{x}) = \mathbf{0}, \mathbf{a} \leq \mathbf{x} \leq \mathbf{b} \quad [15]$$

where $\mathbf{h}(\mathbf{x})$ is of dimension m .

The generalized reduced gradient is

$$\begin{aligned} \mathbf{r}^T = & \nabla_{\mathbf{z}} f(\mathbf{y}, \mathbf{z}) - \nabla_{\mathbf{y}} f(\mathbf{y}, \mathbf{z}) \\ & [\nabla_{\mathbf{y}} \mathbf{h}(\mathbf{y}, \mathbf{z})]^{-1} \nabla_{\mathbf{z}} \mathbf{h}(\mathbf{y}, \mathbf{z}) \end{aligned} \quad [16]$$

GRG algorithm works similarly to linear constraints. However, it is also plagued with similar problems as gradient projection methods regarding maintaining feasibility.

Probably, the most important role of this approach is that it calculates the optimization factor σ in an iterative approach updating the remaining reserves, thus the mine life, at every year, in each iteration, in order to maximize the NPV of the project. This new approach using a variable optimization factor basis resulted in an improved total NPV as shown later in this paper. The program solves for the optimization factor σ by maximizing the project NPV, which is based on the ore tonnage-grade distribution and economic parameters of the mine (see Table IV and Figure 4). The program was developed at Virginia Tech (Nieto and Bascetin, 2006).

The cut-off grade dictates the quantity mined, processed and refined in a given period ' t ', and accordingly, the profits becomes dependent upon the definition of cut-off grade. Therefore, the solution of the problem is in the determination of an optimum cut-off grade in a given period, which ultimately maximizes the objective function (Asad, 2005). In addition, using the introduced program, the degree of precision can be determined, the tolerance, and the degree of convergence of the results. All of these precision parameters can be reported in the program as an Excel sheet.

Algorithm procedure of the new cut-off grade optimization method

The cut-off grade $g_m(t)$ depends on the NPV_i , and the NPV_i cannot be determined until the optimum cut-off grades have been found. The solution to this type of interdependency problem is obtained by an iterative approach. This iteration process belongs to the developed package, which is based on Excell and can be seen as a flowchart demonstrated in Figure 2.

The steps of the algorithm as mentioned above (Equation [1]) are as follows:

- (i) read the input files:
 - a. Economic parameters (price, selling cost, capacities, etc.)

Determination of optimal cut-off grade policy to optimize NPV

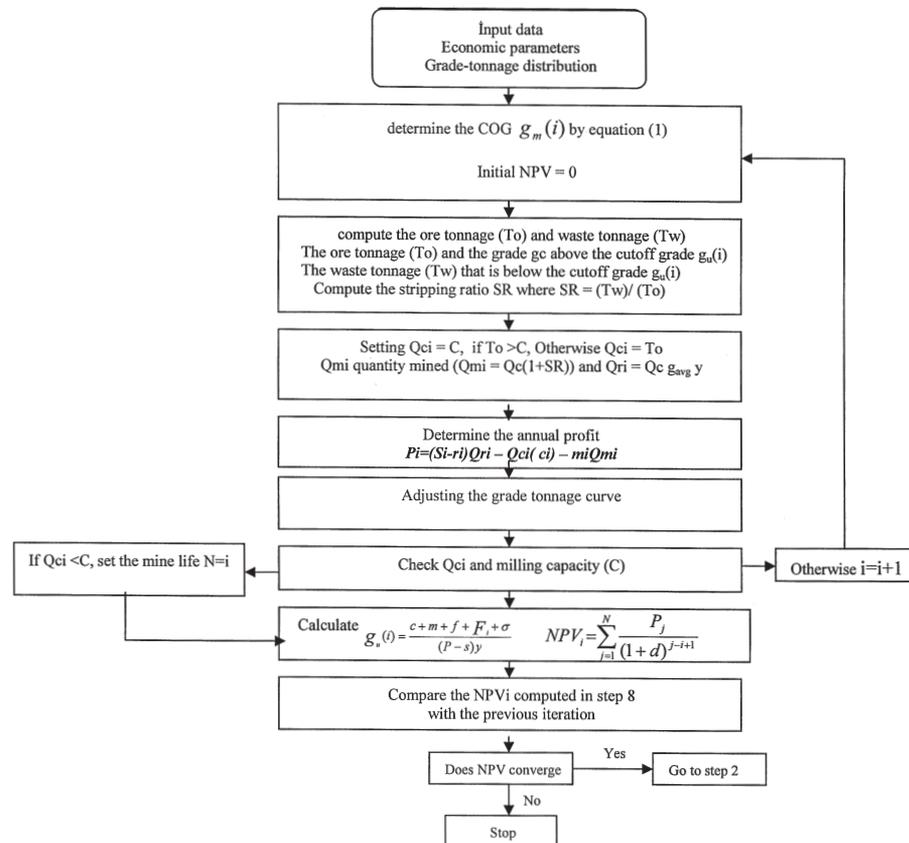


Figure 2—Flowchart of new algorithm for cut-off grade optimization

- (ii) determine the COG $g_m(i)$ by Equation [1];
Setting $V = NPV_i$, the initial $NPV_i = 0$
- (iii) compute the ore tonnage (T_o) and waste tonnage (T_w) from the grade-tonnage curve of the deposit:
 - a. the ore tonnage (T_o) and the grade g_c above the COG $g_m(i)$
 - b. the waste tonnage (T_w) that is below the COG $g_m(i)$
 - c. Also, compute the stripping ratio SR where $SR = (T_w)/(T_o)$
- (iv) Set $Q_{ci} = C$, if T_o is greater than the milling capacity, otherwise, $Q_{ci} = T_o$. Also, set the Q_{mi} quantity mined ($Q_{mi} = Q_{ci}(1+SR)$) and $Q_{ri} = Q_{ci}g_{avg}$
- (v) determine the annual profit by using the following equation:
 $P_i = (S_i - r_i)Q_{ri} - Q_{ci}(c_i + f) - m_i Q_{mi}$
- (vi) adjusting the grade tonnage curve of the deposit by subtracting ore tons Q_{ci} from the grade distribution intervals above optimum COG $g_m(i)$ and the waste tons $Q_{mi} - Q_{ci}$ from the intervals below optimum COG $g_m(i)$ in proportionate among such that the distribution is not changed.
- (vii) check, if Q_{ci} less than the milling capacity (C), then set mine life $N=i$ and go to step (viii); otherwise set the year indicator $i = i + 1$ and go to step (ii).
- (viii) calculate the accumulated future NPV_i 's based on the profits P_i calculated in step (v) for each year from i to N by the following equation

$$NPV_i = \sum_{j=1}^N \frac{P_j}{(1+d)^{j-i+1}}$$

for

each year $i = 1, N$ where N is total mine life in years.

Total NPV is calculated at this stage considering a optimization factor equal to zero.

$$g_u(i) = \frac{c + m + f + F_i + \sigma}{(P - s)y}$$

where σ is the optimization factor equal to zero.

- (ix) the first iteration is performed by adjusting the optimization factor s using the GRG method maximizing the NPV_i computed in step (viii). If the computed NPV_i does not converge, the algorithm goes to step (ii) and calculates the COG by varying the embedded optimization factor s . If the computed NPV converges, the program stops. The COG g_i s for years $i=1, N$ is the optimum policy that maximizes the NPV. Table IV shows COG calculated for each year using σ .

The algorithm was tested using first the optimization factor to find the maximum the NPV then it was tested by removing the optimization factor. The results were examined and are presented in the next section.

Hypothetical case study and results

Consider the following hypothetical case of an open pit gold mine (Dagdelen, 1992). The tonnage grade distribution and

Determination of optimal cut-off grade policy to optimize NPV

mine design parameters are shown in Tables II and Figure 3, respectively. The values in Table II give assumed capacities and accepted costs to mine this deposit at 2 857 ton/day milling rate. The mine will be worked at the rate of seven days a week for 350 days per year.

Table III presents the optimum cut-off grade policy without the optimization factor. As indicated in Table III, this optimizing approach gives a total NPV of \$354.6 million and \$676.5 million of non-discounted profit.

On the other hand, Table IV presents the results of the optimum cut-off grade approach using the optimization factor ($\sigma=5.84$). The cut-off grade policy that is determined by this optimizing approach using the optimization factor (σ) gives a total NPV of \$372.5 million and \$638.9 million of non-discounted profit.

According to the values that are shown in Tables III–IV, the cut-off grade policy determined by the optimization approach using the (σ) factor gives a higher NPV than the cut-off grade policy estimated without the optimization factor. The results are given graphically in Figure 4. An NPV sensitivity analysis, which consisted in varying the optimization factor (σ) from 0.0 to 12.0 in 1.0 unit increments, indicates—as calculated by the introduced algorithm—that the maximum NPV value is at $\sigma = 5.84$ which can be seen clearly in Figure 5.

Acknowledgement

This work was supported by the Scientific and Technological Research Council of Turkey and also by the Research Fund of Istanbul University (Project number: UDP- 724/26042006). The software can be requested from: atac@istanbul.edu.tr.

Table II
Mine design parameters

Parameters	Values
Price	500 \$/oz
Sales cost	4 \$/oz
Processing cost	17 \$/ton
Mining cost	1.3 \$/ton
Capital costs	\$154 M
Fixed costs (f_a)	\$9.2 M/year
Fixed cost (f)	9.2 \$/ton
Mining capacity	--
Milling capacity	1.00 Mt/year
Discount rate	14%
Recovery	95%

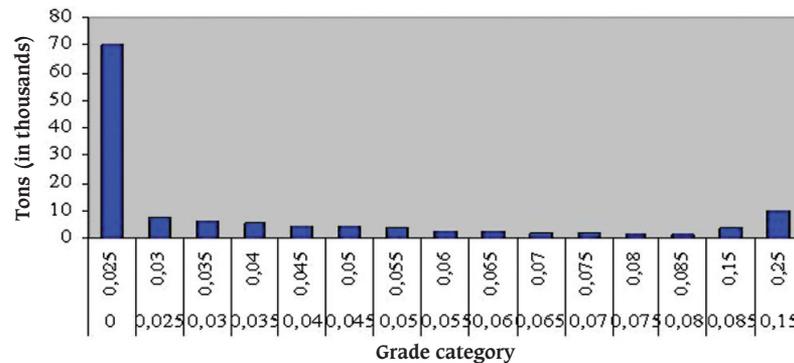


Figure 3—Grade-tonnage distribution of the deposit

Table III

Optimum cut-off grade policy of the gold mine (without σ)

Year	Optimum cut-off grade (oz/ton)	Quantity mined	Quantity concentrated	Quantity refined	Profit (\$M)	NPV (\$M)
1	0.096	12.274.288	1.000.000	209158	61.585.935	354.674.647
2	0.092	11.336.481	1.000.000	199710	58.118.871	293.088.712
3	0.088	10.624.419	1.000.000	192537	55.486.385	242.107.246
4	0.084	10.069.606	1.000.000	186947	53.435.245	199.412.274
5	0.081	9.628.546	1.000.000	182504	51.804.650	163.345.006
6	0.078	9.182.735	1.000.000	177583	49.943.365	132.672.494
7	0.076	8.797.020	1.000.000	173216	48.278.930	106.733.475
8	0.073	8.437.322	1.000.000	168992	46.651.502	84.738.244
9	0.072	8.132.220	1.000.000	165372	45.252.454	66.094.563
10	0.070	7.880.241	1.000.000	162369	44.090.812	50.230.905
11	0.068	7.652.378	1.000.000	159528	42.977.995	36.672.630
12	0.067	7.652.378	1.000.000	159528	42.977.995	25.079.582
13	0.066	7.652.378	1.000.000	159528	42.977.995	14.910.242
14	0.065	7.652.378	809551	129146	32.898.270	5.989.768
Total		126.972.390	13.809.551	2.426.118	676.480.404	354.674.647

Determination of optimal cut-off grade policy to optimize NPV

Table IV
Optimum cut-off grade policy of the gold mine (with σ)

Year	Optimum cut-off grade (oz/ton)	Quantity mined	Quantity concentrated	Quantity refined	Profit (\$M)	NPV (\$M)
1	0.109	13.568.726	1.000.000	217550	64.065.456	372.564.260
2	0.104	13.317.461	1.000.000	217550	64.392.101	308.498.804
3	0.100	13.094.386	1.000.000	217420	64.617.832	252.014.505
4	0.096	12.261.729	1.000.000	209032	61.539.507	202.293.181
5	0.093	11.613.909	1.000.000	202505	59.144.521	160.755.767
6	0.090	11.099.508	1.000.000	197323	57.242.785	125.737.463
7	0.088	10.684.393	1.000.000	193141	55.708.109	96.007.354
8	0.086	10.345.010	1.000.000	189722	54.453.412	70.627.489
9	0.084	10.064.576	1.000.000	186896	53.416.648	48.865.873
10	0.082	9.830.808	1.000.000	184541	52.552.410	30.140.183
11	0.081	9.634.511	999.998	182563	51.826.583	13.979.900
Total		125.515.017	10.999.998	2.198.243	638.959.363	372.564.260

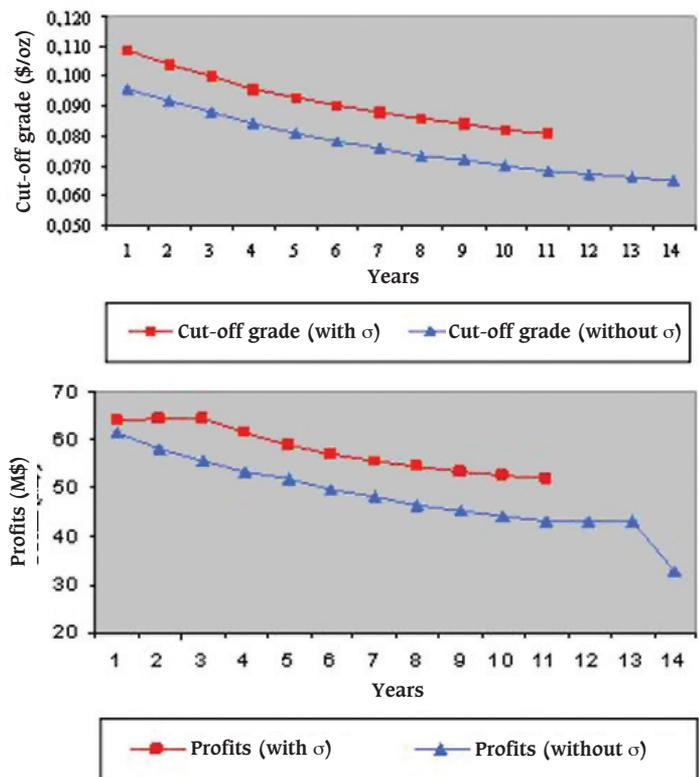


Figure 4—Comparison of optimization with and without σ , (a) cut-off grade vs. years, (b) profits vs. years

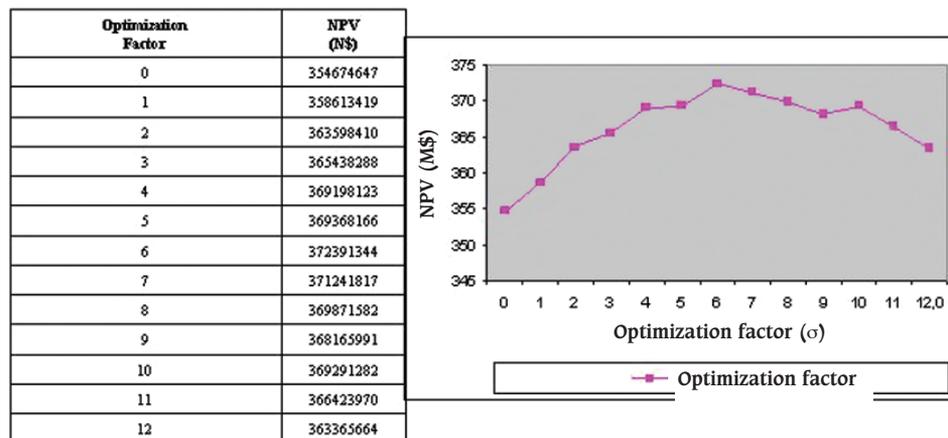


Figure 5—Sensitivity analysis of total NPV vs. different σ values

Determination of optimal cut-off grade policy to optimize NPV

Conclusions

The results given by the case study indicate that the impact of the optimization factor (σ) on the objective function (NPV) is significant at a \$17,889,613.00 increase, which is equivalent to a 5% NPV increment. Therefore, the cut-off grade optimization algorithm presented here is a tool that improves the cut-off grade policy and serves as a user-friendly platform for eventual algorithm adaptations such as the use of cost escalation and simulation for risk analysis. This approach provides great flexibility at the mine planning stage for evaluation of various economic and grade/ton alternatives. The program has been developed within a Windows environment which is a user-friendly tool, used in this case, to calculate interactively different mining cut-off grade scenarios. The other potential benefit of this user-friendly application is that can be adapted to handle multiple sources/grades of ore and to incorporate cost escalation factors based on striping ratios. The research introduced here uses what we have called the generalized reduced gradient (GRG) factor, which further maximizes the total project's NPV.

References

ABADIE, J. and CARPENTIER, J. Generalization of the Wolfe Reduced Gradient Method to the case of nonlinear constraints in optimization, Gletcher, R. (ed.), *Academic Press, New York*. 1969.

- ASAD, M.W.A. Cut-off grade optimization algorithm for open pit mining operations with consideration of dynamic metal price and cost escalation during mine life, *Application of Computers and Operations Research in the Mineral Industry, (APCOM 2005)*, Tuscon, USA. 2005. pp. 273–277.
- DAGDELEN, K. Cut-off grade optimization, *23rd Application of Computers and Operations Research in the Mineral Industry, SME*, Littleton, Colorado, USA. 1992. pp. 157–165.
- DAGDELEN, K. An NPV optimization algorithm for open pit mine design, *24th Application of Computers and Operations Research in the Mineral Industry*, Montreal, Quebec, Canada. 1993. pp. 257–263.
- DAGDELEN, K. and MOHAMMAD, W.A. Multi-mineral cut-off grade optimization with option to stockpile, *SME annual meeting*, Denver, Colorado, USA. 1997.
- LANE, K.F. Choosing the optimum cut-off grade, *Colorado School of Mines Quarterly*, vol. 59, 1964. pp. 485–492.
- LANE K.F. The economic definition of ore, cut-off grade in theory and practice, *Mining Journal*, Books Limited, London. 1988.
- LASDON, L.S., WAREN, A.D., JAIN, A., and RATNER, M. 1978, Design and testing of a generalized reduced gradient code for nonlinear programming *ACM Trans. Math. Software*, vol. 4, 1978. pp. 34–50.
- NIETO, A. and BASCETIN, A. Mining cut-off grade strategy to optimise NPV based on multiyear GRG iterative factor, *Transactions of the Institutions of Mining and Metallurgy (AusIMM), Section A Mining Technology*, vol. 115, no. 2. 2006. pp.59–63.
- TAYLOR, H.K. General background theory of cut-off grades, *Transactions of the Institutions of Mining and Metallurgy-(AusIMM), Section A—Mining Technology*, 1972. pp.160–179. ◆

Mintek commissions new grinding roll for comminution research

As part of its research into more cost-effective comminution technologies, Mintek, specialists in minerals and metallurgical technology and beneficiation, has commissioned a pilot-scale high pressure grinding roll (HPGR).

The unit, supplied by Polysius, is powered by dual 11 kW motors and can operate at pressures up to 200 bar, with a nominal throughput of between 0.5 and 2 t/h.

The HPGR is a relatively new technology, which became commercially available in the mid-1980s. Initially adopted by the cement industry for replacing a conventional ball mill for fine grinding, it is now also commonly used in iron ore processing, for grinding primary ore, as well as in pellet feed preparation, and for processing diamond ores. Adoption by the non-ferrous metals sector has been slower, but in recent years a number of precious- and base-metal producers have evaluated the HPGR, particularly for ores that are not ideally suited for semi-autogenous grinding. The technology is now viewed as having the potential to affect comminution technology to the same extent as autogenous and semi-autogenous grinding.

The HPGR uses the principle of interparticle crushing between two counter-rotating rolls, one of which is fixed and the other 'floating' by means of hydraulic pressure. The almost pure compressive forces generate a high proportion of fines, along with extensive micro-cracking in the larger particles.

'As well as being more energy-efficient than conventional technology, the HPGR results in a product with very favourable characteristics for further downstream processing,' explained Jan Lagendijk, Mintek's Chief Engineer: Comminution Services. 'An HPGR in place of secondary or tertiary crushers can increase the capacity of a ball mill considerably. Further major benefits are that the fines produced may result in better liberation, while the micro-cracking in the coarser product particles may result in improved mineral and metal recovery in downstream processes such as flotation and leaching.'

'The HPGR test work at Mintek will initially focus on PGM ore processing, and later be extended to other commodities,' said Agit Singh, Manager of Mintek's Mineral Processing Division. 'We will be running extensive trials on different ore types to look at the possibility of reducing the specific energy consumption in comminution circuits. In addition, using the HPGR in conjunction with our other facilities, we will investigate the effect of the process on downstream unit operations such as dense-media separation, flotation, and leaching.'

For further information, contact Jan Lagendijk at Tel.: (011) 709-4550 or e-mail janl@mintek.co.za. ◆