



Centrifugal pump performance calculation for homogeneous and complex heterogeneous suspensions

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Synopsis

Centrifugal pumps are the workhorse of hydraulic conveying systems but their performance is derated when viscous non-Newtonian fluids, such as pastes and/or coarse solids are present. The reduced performance for purely viscous Newtonian fluids may be calculated using techniques such as the Hydraulic Institute method, but similar deration for non-Newtonian fluids is not so easy to apply. As part of the AMIRA P599 project investigating high concentration suspension pumping, centrifugal pump tests were systematically conducted on a wide range of non-Newtonian coarse particle suspensions. Suspensions up to 38% v/v of coarse particles with mean diameters in the range of $1.1 < d_{50} < 3.4$ mm suspended in carrier fluids with dynamic yield stresses of $0 < \tau_y < 17.2$ Pa and shear thinning indices in the range $0.35 < n < 0.79$ were examined. Techniques such as the Hydraulic Institute deration method require a characteristic shear rate to be assigned before application with a non-Newtonian fluid, and a method based on pump geometry, fluid rheology and flow rate is proposed. This method predicts the dramatic characteristic reduction in head at low flow rates that is often observed and explains why larger pumps are relatively insensitive to this form of head deration. Head deration due to the presence of coarse particles in non-Newtonian carrier fluids is seen to follow similar trends to those of Newtonian suspensions and often dominates the total deration procedure.

Introduction

Centrifugal pumps are widely used to transport fine particle suspensions, for example mineral tailings. These suspensions are, or may be approximated to be homogeneous non-Newtonian fluids and examples include clay suspensions and some food products. More complex suspensions, which include a non-Newtonian carrier fluid and non-interacting coarse solids, are also common, especially in tailings' disposal. Plant designers usually have only the water performance of the pump available for selection, with little data available in the public domain for fluids with more complex rheology or complex suspensions including coarse solids. An effective generalized method to allow calculation of pump deration for non-Newtonian fluids and complex suspensions is not currently available and pump tests using the actual fluids or suspensions usually need to be carried out.

For Newtonian fluids, the effect of viscosity on a centrifugal pump's performance has been well established and methods like the Hydraulic Institute method are used every day. Non-Newtonian deration, however, is more complex, as the fluid's viscosity is dependent upon the local shear rate, and determining a suitable representative value for geometry as complex as a centrifugal pump is a very difficult task. Furthermore, whereas Newtonian fluids are by definition all similar, non-Newtonian fluids vary widely in their nature. Few papers exist in the open literature addressing the problem of non-Newtonian pump deration. Probably the most cited is that of Walker and Goulas (1984) who were able to correlate their data for a range of visco-plastic fluids modelled as Bingham plastics, and using a Newtonian pump Reynolds number defined in terms of the plastic viscosity. The same approach was also followed by Sery and Slatter (2002) with some success for kaolin slurries. The use of the Bingham plastic viscosity has no fundamental rheological meaning per se, but this model parameter does approach the high shear viscosity of the suspension, and as such should be appropriate for high Reynolds number flows, i.e. high flow rates, low viscosities or large pumps. At lower Reynolds numbers Walker and Goulas suggested that, based on potential flow analysis through the rotor, a characteristic shear rate of 2Ω be used to determine the viscosity, where Ω is the rotational speed of the pump. This implies that the viscosity of the fluid is determined by the action of the rotor on a stationary fluid, similar to that of a mixer, whereas it is the belief of the authors that it is the flow of the material through the pump (i.e. the flow through the

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equivalent pipe of the passages in the pump) that dictates the fluid's viscosity. In this case the shear rates experienced by the bulk of the material would be rather less than that proposed by Walker and Goulas.

When shear thinning fluids are used at low flow rates the head deration becomes severe, producing a drop in the characteristic that can cross the system curve at a second (or third) lower flow rate than the desired operating point (see Figure 1). This is particularly true for yield pseudo-plastic fluids, and a method that predicts this behaviour is required.

The deration of centrifugal pump performance through the presence of coarse solids has been extensively studied for the case where the carrier fluid is water. Engin and Gur (2003) provide a recent review of the various methods available for derating pumps for the effect of coarse solids.

Literature reporting experimental results of the combined effect of non-Newtonian carrier fluids and coarse particle deration is rare, although some can be found in the literature, e.g. (Sellgren *et al.* 1999).

Recent developments in pump design have produced centrifugal pumps that are more tolerant of highly viscous non-Newtonian fluids and which can operate without any significant head de-ration, e.g. the Warman 3AHF froth pump (Xu *et al.* 2002), and which the authors of this paper have found can readily pump slurries with yield stresses in excess of 200 Pa.

As stated earlier, the complexity of predicting non-Newtonian suspension deration on a pump's performance is in part due to the plethora of forms that non-Newtonian fluids can take and the way in which any suspension might behave in them. The flow geometry for pumps is also very varied, with different manufacturers and pumps of different specific speeds having quite different designs. There is also the problem of determining which part of the wetted geometry is primarily responsible for the deration, e.g. is it the rotor passages, the casing or the back of rotor flow? Unlike equivalent studies in mixing, the geometry and rheology alone do not determine the flow in the system and so approaches such as Metzner and Otto (1957) cannot be successfully applied here. Instead the flow, from which the viscosity can be estimated, is governed by the pump's rotational speed and the system curve. The flow regime in the inlet and delivery pipes may be either laminar or turbulent,

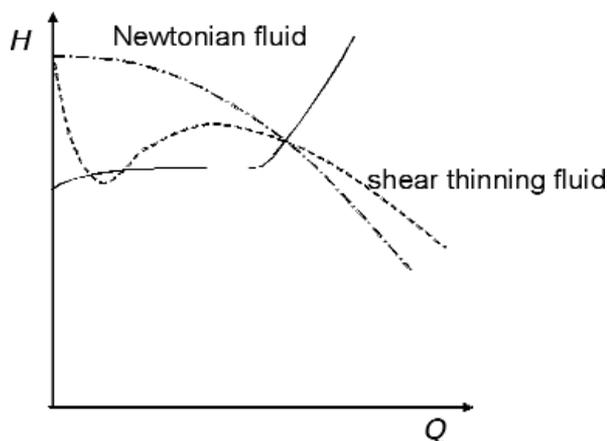


Figure 1—Shear thinning system curve (solid curve) and pump curves for Newtonian (chain line) and shear thinning fluids (dashed line)

but in the smaller passages of the pump, the flow is often laminar once a material with an appreciable viscosity is used. Consider Figure 2, where the flow rate at which two moderately viscous non-Newtonian slurries become turbulent is plotted against pipe size. For the purpose of illustration Herschel Bulkley model fluids have been used with the properties $\tau_y = 5$ Pa, $\tau_l = 10$ Pa, $k = 3$ Pa s n , $n = 0.6$ and $\rho_f = 1500$ kg m $^{-3}$. To estimate the flow regime inside the rotor passages, flow through a conduit with an equivalent hydraulic diameter, based on the rotor's geometric properties at mid radius, is used. Assuming geometric similarity, a gross estimate of the equivalent rotor passage diameters for other size pumps within a given pump series can be made. The normal flow rate range versus the equivalent rotor passage diameters for two common pump series is also plotted in this figure.

For most of the flow rates for the series the regime in the rotor passages is likely to be laminar. Since these passages have characteristic dimensions that are either larger or comparable to other features in the pumps, we can assume that the flow will be laminar throughout the pumps for these types of slurries. Consequently, to examine the pump's deration it will also be necessary to estimate the slurry viscosities for each flow value.

The method proposed in this paper is restricted to inelastic shear thinning fluids, with or without suspensions of narrowly graded coarse particles that are too large to modify the underlying carrier fluid's rheology. The variability of centrifugal pump's geometries and the necessarily small subset of pumps that could be tested here, have meant that the proposed method is not universal, rather that it points to a method that could be employed with a series of geometrically similar pumps. This method considers a modification of the Hydraulic Institute pump head deration method to use an apparent viscosity based on rotor geometry, flow rate and suspension rheology as detailed below. This technique is independent of the rheological model. At the time this work was done, the new Hydraulic Institute methods had not been released and so the methods described here use the earlier method embodied in their standards (1969).

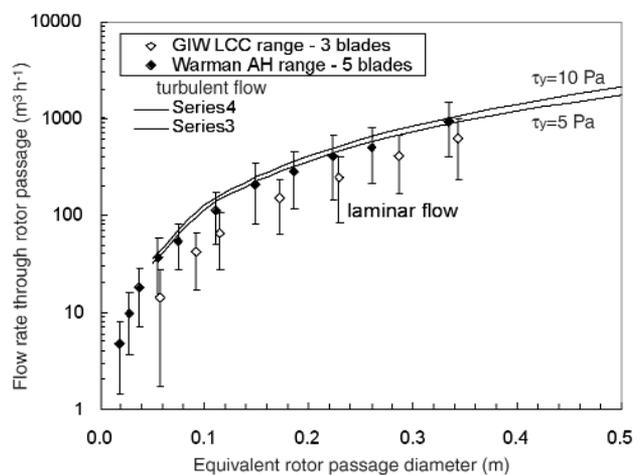


Figure 2—Turbulent transition flow rates for two fluids, $\rho_f = 1500$ kg m $^{-3}$, with yield stresses of 5 and 10 Pa versus flow passage diameter. Superimposed on this graph, assuming geometric similarity, are estimations of equivalent rotor passage diameters and flow rates for two commercially available pump series

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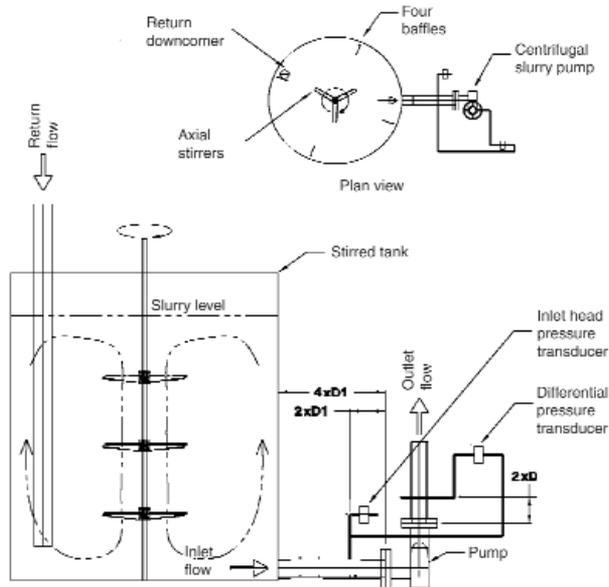


Figure 3—Schematic of pump test rig at CSIRO

Pump	Impeller diameter (m)	Number of blades
Warman 4x3 AH	0.245	5
GIW 4x3 LCC-M80-300	0.310	3

The effect of coarse particles on centrifugal pump performance when conveyed by a non-Newtonian fluid is also considered and recommendations for deration calculations are presented.

Experimental facility

The non-Newtonian fluids used were aqueous polymer solutions: CMC (approximating a power law fluid) and Ultrez 10 (approximating a Herschel-Bulkley fluid). These fluids were chosen as their rheologies are typical of many thickener underflows and carrier fluids found in industry. Complex suspensions were made up by adding crushed glass or sand to the carrier fluids. The characteristics of the suspension were determined by measuring the carrier fluid rheology and particle size distribution by screening where applicable. A Bohlin CVO 50 controlled stress viscometer was used to determine the rheological properties of the CMC and Ultrez carrier fluids.

Initial tests conducted using water showed that both pumps' head characteristics were essentially the same as their published clean water curves.

Modified Hydraulic Institute method

The Hydraulic Institute method, used to derate a centrifugal pump's performance, assumes that the viscosity of the fluid is constant, i.e. a Newtonian fluid. For non-Newtonian fluids,

where the viscosity is a function of the local shear rate, it is necessary to establish a representative shear rate, at the flow rate of interest, and from that determine a representative viscosity from the fluid's constitutive model or rheograms.

The approach adopted here is to determine an equivalent 'pipe' for the pump based on the pump's main dimensions. The flow through this pipe is considered to be a model of the far more complex flow through the pump. The flow rate of interest is then used to determine the shear rate at the wall of this pipe from which a viscosity can be deduced and applied using the Hydraulic Institute method. The pipe geometry is based on a rotor passage as follows:

The equivalent 'pipe' diameter

$$D_h = \frac{4w\pi D_{imp}}{2(\pi D_{imp} + w)} \quad [1]$$

where w is a characteristic dimension to be determined experimentally. Although it is tempting to use the actual rotor passage width, it is very unlikely that such a value would be correct. This is because the equivalent pipe must account for all flows within the pump and as such is purely fictitious—a characteristic dimension used for data reduction. Note this form also ignores the rotor blade thickness, but since a fictitious characteristic geometry is sought such refinement is not justified.

Velocity through the 'pipe'

$$V = \frac{4Q}{\pi D_h^2} \quad [2]$$

If the flow is laminar, the shear rate is then obtained from the Rabinowitsch-Mooney relationship

$$\dot{\gamma} = \left(\frac{3n^2 + 1}{4n^2} \right) \frac{8V}{D_h} \quad [3]$$

Where n is the gradient of the curve $\ln(\tau_0)/\ln(8V/D_h)$ obtained from the rheological model (or rheograms) of choice.

If the flow is turbulent a high shear rate viscosity η_∞ is used—at a shear rate typically in excess of 4 000 s⁻¹.

The resulting viscosity $\eta(\dot{\gamma})$ or η_∞ is then used with the Hydraulic Institute deration method as usual.

Viscous results and analysis

Viscous effects

Determination of the characteristic dimension, w , for a centrifugal pump, currently requires experimental pump head data for a non-Newtonian fluid of the expected range of rheologies. The correction procedure detailed above is applied to the experimental data and a global non-linear minimization procedure used to determine a value of the characteristic dimension, w , which minimizes the error between the actual non-Newtonian fluid data and that calculated with the deration method for these data-sets. This characteristic dimension can then be used for other non-Newtonian fluids being pumped by the same pump.

The details of the fluids' rheologies for the tests reported here are given in Table II.

For these fluids and pumps the head developed with the test fluids typically fell 25% below that produced with water, i.e. $HR = 0.75$.

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Table II

Details of the pump tests and fluids for determination of characteristic dimension

Pump	Fluid	τ_y	k	n
GIW 4x3	CMC	0	5.8	0.48
GIW 4x3	Ultrez 10	14.0	11.0	0.39
GIW 4x3	Ultrez 10	17.0	7.7	0.43
Warman 4x3	CMC	0	8.5	0.46
Warman 4x3	CMC	0	4.5	0.48
Warman 4x3	CMC	0	8.6	0.45
Warman 4x3	CMC	0	4.0	0.47
Warman 4x3	CMC	0	13.9	0.41
Warman 4x3	Ultrez 10	12.2	10.8	0.39
Warman 4x3	Ultrez 10	17.2	18.5	0.35

Table III

Characteristic dimension for Warman and GIW pumps

Pump	Characteristic dimension, w (m)	w/D_{imp}
Warman 4x3 AH	0.059	24.0%
GIW 4x3 LCC-M80-30	0.084	27.0%

Table III shows the pump characteristic dimensions as determined from the minimization procedure. It can be seen that the ratio of the characteristic dimension to the impeller diameter is similar for both pumps.

Results from the two pumps tested are shown in Figure 4 where the predicted head at the same flow rate is shown plotted against the actual head for the various non-Newtonian fluids.

As seen, the GIW 4x3 data and the Warman 4x3 data are predicted within $\pm 10\%$.

Typical results of the predicted head versus flow rate once the characteristic dimension, w , has been globally determined, are shown in Figure 5.

Using this method over the entire flow rate range of the pump produces the characteristic droop in the pump's performance at low flow rates as shown in Figure 6.

The extremely shear thinning nature of the fluid is shown in the rheogram displayed in the right-hand pane of the figure. Beneath this rheogram are estimates of the corresponding flow through the pump. As the flow rate reduces, so the viscosity of the fluid increases and hence the head reduction calculated by the Hydraulic Institute method increases. For this particular combination of pump geometry and fluid rheology the valid extent of the HI method extends to flow rates down to around 8 LS^{-1} . Beyond that, the deration still increases as indicated by the dashed line. At pump shut off, however, the pump will still deliver the full shut off head and so the curve must rise again as shown.

Similar deration is expected to occur for larger pumps, but depending upon the geometry and fluid rheology, the flow may become turbulent, since the Reynolds number is directly proportional to pump size. Under these conditions the flow is only a very weak function of the viscosity of the fluid and so head deration tends to zero.

Other pumps

To assess the independence of material type and fluid rheology, the characteristic dimension, w , obtained above for the GIW 4x3 pump, was used to predict the pump head characteristics for a similar GIW 4x3 pump that was pumping kaolin suspensions through the Cape Peninsular University of Technology's (formerly Cape Technikon) test loop. The predictions are shown in Figure 7 where the agreement is also within $\pm 10\%$ of the actual values as before. The rheological properties of these suspensions are given in Table IV.

The similarity in the value of w/D_{imp} found for these two different types of 4x3 pumps suggests, albeit tentatively, that the perhaps the average value may be usable as a general value for pump head predictions. As a test of the generality of the dimensions with size and geometry, the average w/D_{imp} value of 25% has also been used to predict the head for a

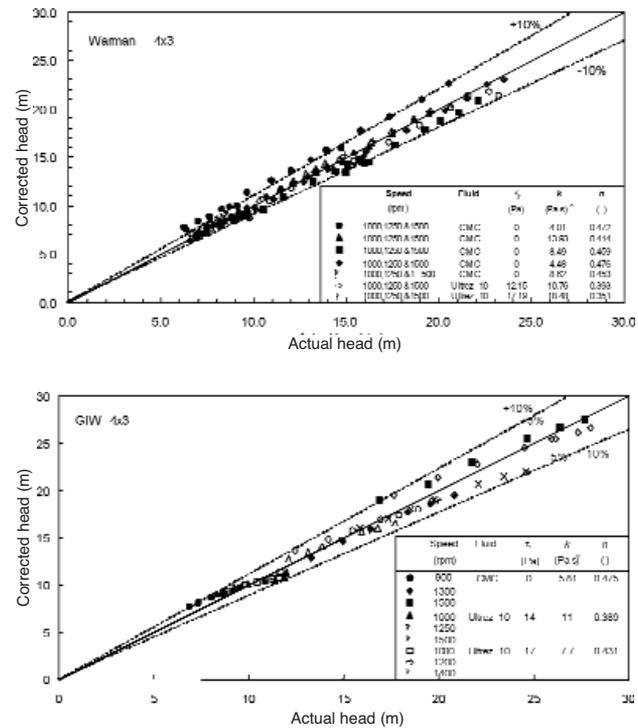


Figure 4—Corrected head versus the actual head for the Warman and GIW pumps

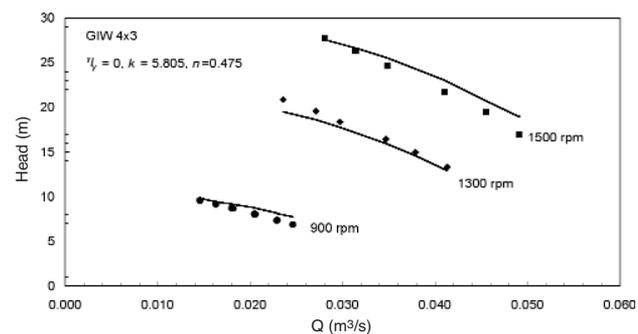


Figure 5—Head flow curves for GIW 4x3 pump and CMC fluid. Experimental data and predictions

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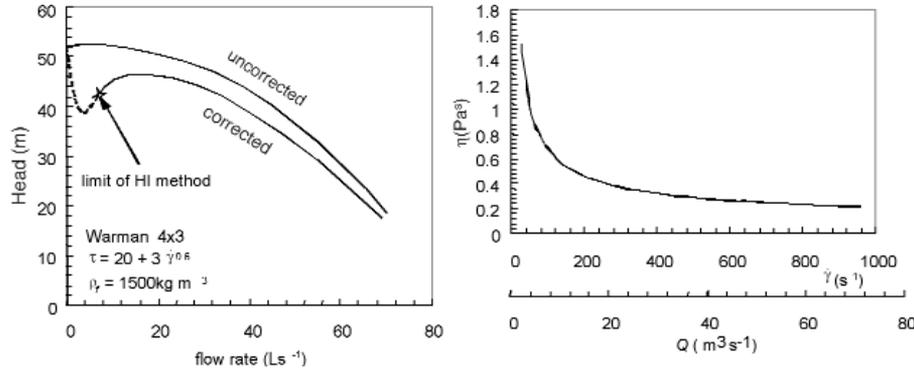


Figure 6—Predicted head flow curve for the Warman 4x3 for a yield visco-plastic fluid and the corresponding fluid rheograms

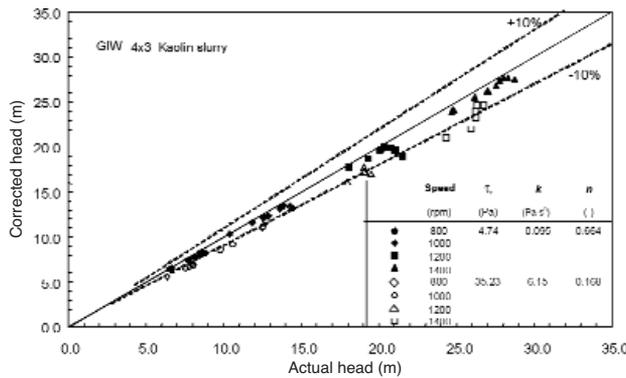


Figure 7—Corrected head versus actual head for a GIW pump pumping kaolin suspensions

Table IV
Rheological parameters for kaolin slurries

Fluid	τ_y (Pa)	k (Pa s ⁿ)	n (-)
5% kaolin	4.74	0.095	0.664
10% kaolin	35.23	6.15	0.168

GIW 18x18 LSA pumping phosphate slimes. Data obtained at the GIW laboratories (Addie 2005) while pumping these slimes through a 440NB pipe are shown in Figure 8 along with the predictions.

The phosphates slimes are modelled as Bingham plastic materials in this instance. As seen, the pump's performance is unaffected by the lower viscosity material with the lower yield stress, whereas the high viscosity fluid has derated the pump's performance in a similar manner to that shown in Figure 6. Estimates of the velocity through the impeller passages indicate that transitional flow rates for the lower and higher viscosity fluid are approximately 1.0 ms⁻¹ and 1.5 ms⁻¹ respectively. Consequently, the lower viscosity fluid can be considered turbulent for most of the data shown and so there would be little if any viscous deration, whereas the higher viscosity fluid is laminar for the entire range and so viscous deration does occur.

Comparison between the actual data and predictions based on $w/D_{imp} = 25\%$ are shown as dashed lines in Figure 8. The agreement is gratifying considering that this is a much larger pump and from a different pump series.

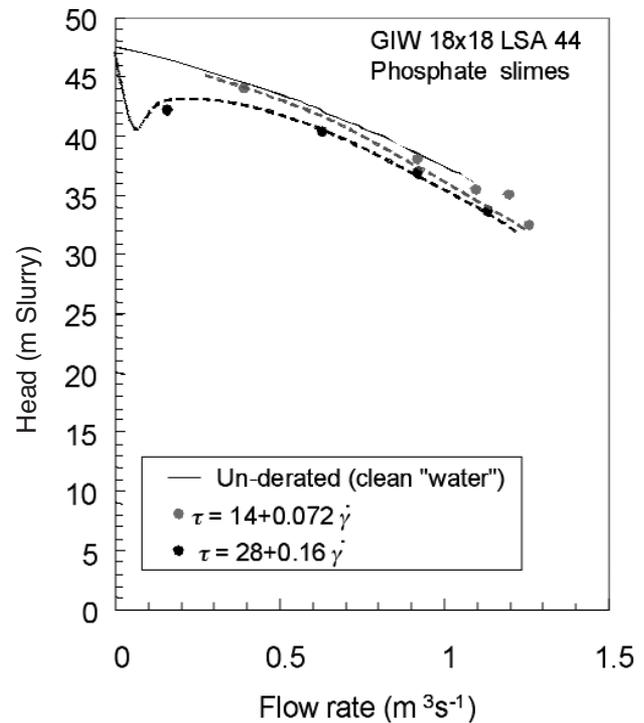


Figure 8—Comparison of the un-derated and actual head versus flow rate curves for a 18x18 LSA pump conveying two different phosphate slimes

Table V
Rheological parameters for phosphate slurries

ρ_m (kg m ⁻³)	τ_y (Pa)	k (Pa s ⁿ)	n (-)
1116	14	0.072	1
1142	28	0.16	1

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Coarse solids effects

Pump performance experiments were carried out to determine the effect of adding coarse solids to non-Newtonian carrier fluids. The suspension, detailed in Table VI, used either sand or crushed glass with sizes ranging from 1.1 mm to 7 mm.

The total head deration for the suspensions was from 10%–40% for all suspensions tested and incorporated the carrier fluid's viscous deration as well, which ranged from 5 to 25%.

Typical results for power law fluids are shown in Figure 9 where the head deration is seen to be a strong function of the solids concentration and follows similar behaviour to that found with water-based suspensions. Similar behaviour was observed for the visco-plastic fluids too.

To illustrate the similarities with water-borne suspension behaviour, the correlation due to Engin and Gur has been applied to these data and displayed in Figure 9. This correlation, based on the analysis of a large amount of experimental data taken from water-based suspensions, is only a function of the solid's properties and impeller diameter. Nevertheless, the agreement with the data obtained here for a range of non-Newtonian fluids is reasonable. Closer examination of the data, however, displays a trend, indicated by broken lines in Figure 9, where the water-based correlations systematically over-predict the low flow rate data. This is to be expected, as at these low flows the carrier fluid's viscosity is high (see Figure 6) and particle-wall interactions and consequently head reduction would be expected to be less. Sellgren, Addie and Yu (1999) have produced a solids-deration correlation that is cognisant of the carrier fluid's viscosity and a comparison between this and the Engin and Gur correlation is shown in Figure 10. Here the effect of the increased viscosity is seen to reduce the deration at low flows in a similar way.

Unfortunately, the variability of the carrier fluid's rheology in Figure 9 prevents the Sellgren *et al.* correlation from being applied in any meaningful way to these data. The

relative insensitivity to carrier fluids viscosity and form for this type of head deration is surprising, especially since all of the Ultrez suspensions were statically stable, i.e. the solids do not settle in unshered flows. In all cases, the suspensions

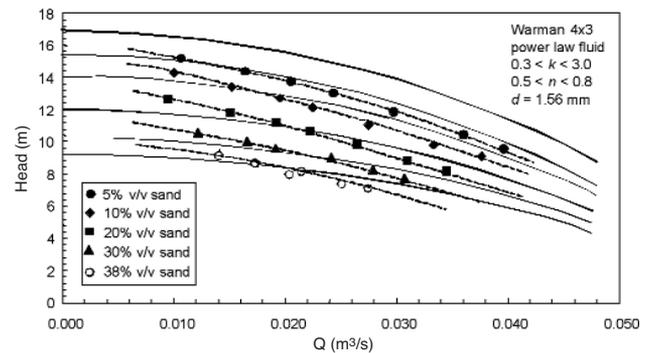


Figure 9—Typical coarse solids' deration characteristics. Solid lines are predictions based on the Engin and Gur correlation

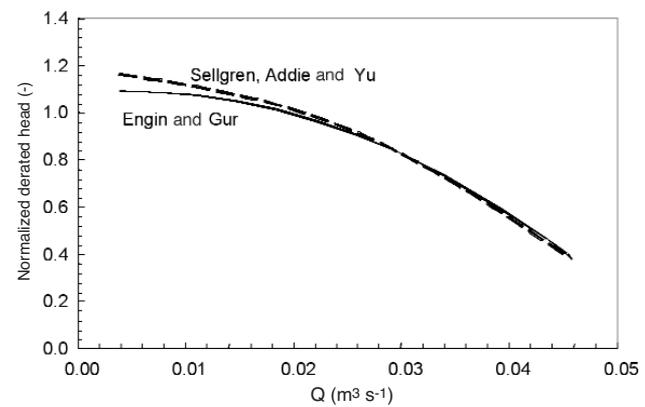


Figure 10—Engin and Gur and Sellgren *et al.* correlations showing the effect of fluid viscosity

Table VI

Parameters for Warman 4x3 pump tests

Fluid	τ_y	k	n	c_v (%)	Solid	d_{50} (mm)
CMC	0	3.38	0.579	30	Sand	1.56
CMC	0	1.10	0.695	30	Sand	1.56
CMC	0	5.33	0.535	5	Sand	2.24
CMC	0	4.55	0.556	10	Sand	2.24
CMC	0	2.09	0.637	20	Sand	2.24
CMC	0	3.06	0.531	5	Sand	1.56
CMC	0	2.56	0.559	10	Sand	1.56
CMC	0	0.86	0.658	20	Sand	1.56
CMC	0	0.50	0.713	30	Sand	1.56
CMC	0	0.26	0.789	38	Sand	1.56
CMC	0	3.51	0.494	5	Glass	1.12
CMC	0	3.27	0.499	10	Glass	1.12
CMC	0	2.37	0.524	20	Glass	1.41
CMC	0	1.87	0.552	30	Glass	1.43
Ultrez 10	8.79	8.57	0.406	10	Glass	1.84
Ultrez 10	8.25	3.58	0.487	20	Glass	1.85
Ultrez 10	6.91	11.05	0.377	20	Glass	1.85
Ultrez 10	4.86	2.37	0.542	30	Glass	1.87
Ultrez 10	3.61	1.10	0.615	30	Glass	1.87
Ultrez 10	14.56	14.40	0.369	10	Glass	3.41
Ultrez 10	12.59	10.34	0.399	20	Glass	3.33
Ultrez 10	6.50	4.07	0.474	30	Glass	3.33

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were only mildly agitated in the mixing tank prior to entry to the pump and the evidence that they are interfering with the pump's behaviour like this requires that the solids have high settling rates or large Stokes numbers comparable to water-based suspensions. Similarly enhanced settling rates have been observed in pipeline flows of these materials (Pullum and Graham 1999) and evidence is building to suggest that particle mobility through non-Newtonian fluids may be greater than that expected from the local fluid viscosity. Further examination of this phenomenon will be made in subsequent studies.

Conclusions

Some findings from pump tests using non-Newtonian fluids and suspensions, conducted during the AMIRA P599 project, have been presented. From these studies the following conclusions can be drawn:

- For moderate to high viscosity suspensions typical of high concentration tailings disposal lines, the flow regime inside a centrifugal pump's passages is probably laminar.
- A method to estimate the representative viscosity within the pump based on the flow through an equivalent 'pipe' passage has been developed. This viscosity is then used in conjunction with the Hydraulic Institute method to estimate the viscous deration.
- The exact dimension of the equivalent pipe is left to a fitting exercise for a given series of pumps, and as such is not a universal solution. However, such an analysis is relatively simple to do for a given pump series and does provide insight into the pump's behaviour.
- A non-linear minimization exercise conducted on the Warman and GIW 4x3 pumps for a variety of non-Newtonian fluids produced similar non-dimensional characteristic dimensions with an average value of (w/D_{imp}) of ~25%. This value was used to predict the performance of a similar pump pumping different non-Newtonian fluids and a much larger pump from a different series. The agreement was found to be satisfactory. However, further research and data comparison must be made before this value can be considered general.
- The sensitivity of the method to the local shear rate is used to explain the increased viscous deration at low flow rates.
- Centrifugal pump tests with complex suspensions consisting of non-Newtonian fluids and coarse particles showed that the head deration was primarily a function of the coarse solids concentration. Adequate predictions of head performance could be obtained by calculating the effect of solids deration alone.

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Nomenclature

D_h	Equivalent hydraulic diameter of pump	(m)
D_{imp}	Diameter of pump impeller	(m)
H	Pump head	(m)
HR	Head ratio	(-)
k	Consistency index	(Pa s ⁿ)
n	Flow behaviour index	
N	Pump speed	(RPS)
N_B	Number of blades	(-)
Q	Pump flow rate	(m ³ s ⁻¹)
V	Fluid velocity through pump passage	(m s ⁻¹)
w	Pump characteristic dimension	(m)
$\dot{\gamma}$	Shear rate	(s ⁻¹)
η	Apparent viscosity	(Pa s)
τ_y	Fluid yield stress	(Pa)