Performance evaluation of a new stochastic network flow approach to optimal open pit mine design—application at a gold mine

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Synopsis

The optimal design of production phases and ultimate pit limit for an open pit mining operation may be generated using conventional or stochastic approaches. Unlike the conventional approach, the stochastic framework accounts for expected variability and uncertainty in metal content by considering a set of equally probable realizations (models) of the orebody. This paper evaluates the performance of a new stochastic network flow approach for the development of optimal phase design and ultimate pit limit using a gold deposit as the case study. The stochastic and conventional frameworks as considered here utilize the maximum flow and Lerchs-Grossman (LG) algorithms, respectively. The LG algorithm is restricted to considering an estimated (average-type) orebody model, while the stochastic maximum flow algorithm is developed to simultaneously use a set of simulated orebody realizations as an input. The case study demonstrates that, when compared to the conventional LG algorithm as used in the industry, the stochastic approach generates a 30 per cent increase in discounted cash flow, a 21 per cent larger ultimate pit limit, and about 7 per cent more metal, while it maintains a consistency in phase size.

Keywords

open pit mine optimization, maximum flow algorithm, Lerchs-Grossman algorithm.

Introduction

An orebody model constitutes a three-dimensional accumulation of mining units (blocks). The available metal content in a mining unit and the economic parameters, such as metal price, mining cost, processing cost, marketing or selling cost, as well as the metallurgical recovery, are used to calculate the economic value of a particular mining block. A mining block with a positive value is classified as an ore block, while a mining block with a negative value is identified as a waste block. Usually, an orebody model consists of millions of blocks, and the economic value of these blocks becomes the basic input for the development of the optimal production phase design and determination of the optimal ultimate pit limit[1].

An optimal phase design refers to the sequence of extraction that reaches the ultimate pit limit or the extent of extraction[2]. It is well established that the phase design and ultimate pit limit may be accomplished using conventional and stochastic approaches[3,4]. The conventional optimization approaches may not account for the expected variations or uncertainty in all pertinent inputs, assuming that inputs (grade or metal content) of mining blocks and all other parameters are constant and in fact the actual values. More specifically, conventional optimization methods consider input from an estimated 'average'-type representation of the orebody model[5,6]. However, the utilization of the estimated block values may result in an unrealistic mine design and production schedules, leading to the possible failure of capital-intensive mining projects[7,8]. The stochastic approach, on the other hand, quantifies the inherent uncertainty in geological parameters (grade or metal content), anticipating the possibility of a block incorrectly identified as an ore block, thus accounting for and managing risk in the possible economic value at the time of extraction. Stochastic optimization in mine planning considers a set of simulated realizations of the orebody as an input[9–12], and each of these realizations constitutes economic values of mining blocks of its own.

Generally, optimization of production phase design and ultimate pit limit may be described by a directed graph[13], represented as $G = (N, A)$. A mining block i carrying a value v_i is represented as a node in N. Also, an arc directed from node i to node i' exists in A, representing the extraction precedence of block i' over block i. The solution to this graph problem maximizes $\sum_{i \in N} v_i$ by finding a set $N' \subseteq N$ that contains all predecessors related to the nodes with the maximum value. Here, N' is referred as the maximum closure on the

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graph, which in practice corresponds to the ultimate pit limit or a phase within the ultimate pit limit[14]. The mathematical formulation of this graph problem is given as[15,16]

$$\text{Maximize}: \ Z = \sum_{i \in N} v_i x_i \qquad [1]$$

$$\text{Subject to}: \ x_i - x_{i'} \leq 0, \ i' \in \Pi_i, \ i \in N \qquad [2]$$

$$x_i \in [0,1], \ i \in N \qquad [3]$$

where the binary variable x_i represents a block i. Its value is equal to 1 if block i is inside the pit (or closure) and 0 otherwise. Also Π_i represents the set of predecessors of block i.

Conventional optimization models such as the Whittle software[17] solve the graph problem (Equations [1–3]) using the Lerchs-Grossman (LG) algorithm[18]. Johnson[19] proposes the maximum flow algorithm as an alternative to the LG algorithm for finding the optimal ultimate pit limit. Unlike the LG algorithm, the structure of the maximum flow algorithm permits the consideration of a set of orebody realizations as an input[20]. As such, the stochastic optimization models[20,21] implement the maximum flow algorithm to solve the graph problem. A review of other stochastic optimization models is given in Dimitrakopoulos[22].

This paper describes the comparative analysis of conventional open pit mine optimization implementing the LG algorithm and the stochastic optimization based on the maximum flow algorithm. More specifically, the paper establishes the difference in structure of the LG and maximum flow algorithms, and then uses a gold deposit to demonstrate the value of the stochastic network flow approach that incorporates uncertainty in metal content into open pit mine design.

We first present an overview of pertinent conventional and stochastic optimization models; continue with the application of both the LG and maximum flow algorithms at gold deposit; and discuss the performance of both approaches using phase-wise risk profiles for forecasting cash flows, metal content, and stripping ratio. Conclusions are then presented.

Overview of pertinent conventional and stochastic approaches

Conventional approach

The conventional approach considered herein implements the most widely used LG algorithm. Hustrulid and Kuchta[23] explain the details of the LG algorithm through step-by-step examples. The calculation of the economic value of mining blocks is prerequisite to the application of the LG algorithm and the value of a particular block i may be calculated as follows:

$$v_i = \left[\left[(P - s) g_i y - c - m\right] Q_i\right] \qquad [4]$$

where P is the metal price, s is the selling cost, g is the grade of block i, y is the metallurgical recovery, c is the processing cost, m is the mining cost, and Q_i is the quantity of material in block i. Subsequent to calculation of v_i in Equation [4], to cater for the blocks that are mined, but classified as waste and incur mining cost only, v_i may be adjusted as follows:

$$v_i = \begin{cases} v_i, \ \text{if } v_i > 0; \\ -mQ_i, \ \text{if } v_i \leq 0. \end{cases} \qquad [5]$$

The LG algorithm takes the orebody model containing the block location and corresponding value v_i as an input and develops an optimal ultimate pit limit. However, a parametric implementation of the LG algorithm develops phase design, which requires a change in the block value by increasing or decreasing a parameter of interest, λ, usually metal price[24]. As such, the block value may be modified using $v_i(\lambda) = \alpha\lambda - \beta$, where α represents revenue parameters such as metal price and β represents cost parameters like mining or processing cost. It is evident that a particular value of λ increases or decreases metal price, which results in an ultimate increase or decrease in the block value v_i. Equations [1–3] may be modified to incorporate parametric implementation of the LG algorithm

$$\text{Maximize}: \ Z = \sum_{i \in N} v_i(\lambda) x_i \qquad [6]$$

$$\text{Subject to}: \ x_i - x_{i'} \leq 0, \ i' \in \Pi_i, \ i \in N \qquad [7]$$

$$x_i \in [0,1], \ i \in N. \qquad [8]$$

Equations [6–8] generate 'nested pit shells' that are used to develop production phases within the ultimate pit limit. Therefore, the conventional approach achieves the phase design and ultimate pit limit in two separate steps. The first step requires repeat computation with a range of λ values, selected through trial and error, resulting in a series of nested pits. The second step utilizes these nested pit shells to develop a number of phases within the ultimate pit. The approaches of Francois-Bongarcon and Guibal[25] and Wang and Sevim[26] are amongst the well-known variations of pit parameterization addressing the generation of an optimal phase design and ultimate pit limit.

Stochastic approach

The stochastic approach considered herein is based on extending the maximum flow algorithm[27]. Hochbaum[28] indicates that the graph problem (Equations [1–3]) is a dual of the maximum flow problem i.e. minimum cut problem. Picard[29] suggests the conversion of the graph problem into a related graph by adding a source node s and a sink node t and implementing the minimum cut algorithm to solve this related graph for developing the maximum closure i.e. the ultimate pit limit. Meagher *et al.*[20] propose a theoretical background to utilize the minimum cut algorithm in a stochastic framework incorporating multiple realizations of the simulated orebody models for developing both phase design and the ultimate pit limit. Asad and Dimitrakopoulos[21] propose an extension in Meagher *et al.*[20] and augment the graph problem with a pit production capacity constraint, which is given as follows:

$$\sum_{i \in N} Q_i x_i \leq M \qquad [9]$$

where M is the mining or pit production capacity.

It is evident that the inclusion of Equation [9] violates the classical structure of the graph problem (Equations [1]–[3]). Therefore, Asad and Dimitrakopoulos[21] integrate the

Lagrangian relaxation of the production capacity constraint as suggested in Tachefine and Soumis[15]. The modified graph problem is presented as follows:

$$\text{Maximize}: \; Z = \sum_{i \in N} \left[\left[v_i - \lambda Q_i \right] x_i \right] + \lambda M \qquad [10]$$

$$\text{Subject to}: \; x_i - x_{i'} \le 0, \; i' \in \Pi_i, \; i \in N \qquad [11]$$

$$x_i \in [0,1], \; i \in N. \qquad [12]$$

This procedure allows systematic selection of the λ values leading to the development of a number of consistent size phases within the ultimate pit limit in a single step, as opposed to the conventional LG approach. As reflected in the modified graph problem (Equations [10]–[12]), the maximum flow algorithm also requires economic values of mining blocks as an input. Therefore, the calculation of economic values of mining blocks for a stochastic framework entertaining multiple simulated realizations of the orebody may be achieved using a modified Equation [5] as follows:

$$v_{\gamma i} = \left[\left[(P - s) g_{\gamma i} y - c - m \right] Q_{\gamma i} \right] \qquad [13]$$

where $v_{\gamma i}$ is the economic value of block i in realization γ that is derived from $g_{\gamma i}$ and $Q_{\gamma i}$ i.e. the grade and amount of material of block i in realization γ, respectively. Accordingly, $v_{\gamma i}$ may be adjusted as follows:

$$v_{\gamma i} = \begin{cases} v_{\gamma i}, & \text{if } v_{\gamma i} > 0; \\ -mQ_{\gamma i}, & \text{if } v_{\gamma i} \le 0. \end{cases} \qquad [14]$$

Equation [14] reflects the notion that, unlike the conventional approach, the uncertainty in metal content allows a block to be an ore block in one realization and a waste block in another realization. An update of the problem (Equations [10–12]) for a stochastic framework gives

$$\text{Maximize}: \; Z = \sum_{i \in N} \sum_{\gamma} \left[\left[v_{\gamma i} - \lambda Q_{\gamma i} \right] x_{\gamma i} \right] + \lambda M \qquad [15]$$

$$\text{Subject to}: \; x_{\gamma i} - x_{\gamma i'} \le 0, \; i' \in \Pi_{\gamma i}, \; i \in N \qquad [16]$$

$$\sum_{\gamma} x_{\gamma i} \le 1, \; \forall i \qquad [17]$$

$$x_{\gamma i} \in [0,1], \; i \in N. \qquad [18]$$

where binary variable $x_{\gamma i}$ is equal to 1 if block i in orebody model γ is inside the pit, 0 otherwise. Equation [17] supports this argument by maintaining an arc from a particular block in one orebody model to the same block in another orebody model, ensuring that if this block is included inside the pit, it must remain inside the pit in all orebody realizations. The mathematical formulation in Equations [15]–[18] may be translated into construction of a graph containing nodes and arcs. The related graph suggested in Picard[29] consists of five elements:

1. The set of nodes with positive (+v) and negative (–v) values, representing ore and waste blocks, respectively
2. The set of arcs from source s to the nodes with positive value (Equation [15]). The capacity of these arcs is equal to the absolute value ($|+v|$) of the node
3. The set of arcs from a node with a negative value to the sink t. The capacity of these arcs is equal to the absolute value ($|+v|$) of the node (Equation [15])
4. The set of arcs from a particular block i to blocks i' in $\Pi_{\gamma i}$, maintaining precedence or slope requirement (Equation [16]) in all realizations of the orebody models. The capacity of these arcs is ∞, ensuring that if block i is inside the pit, then blocks i' in $\Pi_{\gamma i}$ are also inside the pit
5. The set of arcs from block i in one realization to block i in another realization (Equation [17]), ensuring that if block i is inside the pit in one realization, it remains inside the pit in all realizations.

Figure 1 explains the structure of this related graph.

It is evident in the mathematical formulation of the graph problem (Equations [15–18]) and its presentation in Figure 1

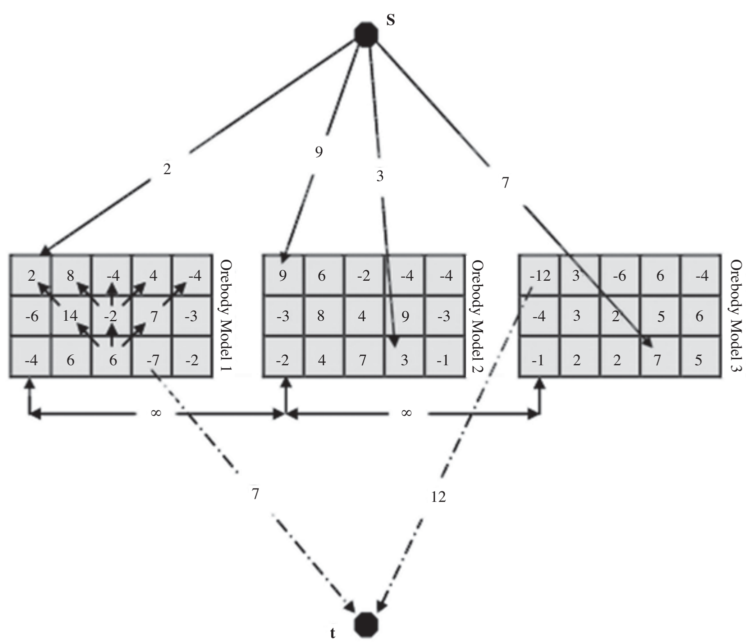

Figure 1—The structure of the related graph in the stochastic approach

that the structure of the graph in the stochastic network flow approach is different from the problem (Equations [6–8] in the conventional LG approach. The following section presents solution to the problems (Equations [6–8] and [15–18]), i.e., the stochastic and conventional frameworks respectively, in an application at a gold deposit.

Case study – gold deposit

The application demonstrates the practical aspects and performance analysis of the maximum flow and LG algorithms in stochastic and conventional frameworks respectively.

Description of the deposit and orebody models

The geological database of the deposit contains 421 drillholes. The holes are placed in an irregular grid of 1161 × 930 m with a spacing varying from 16 to 50 m. The sample spacing varies from 30 cm to several metres leading to 4 046 lithology records and 23 585 gold assay records. This data is then set to 5 m long composites with an average gold (Au) content of 0.61 g/t and a standard deviation of 1.47 g/t.

Given the composite data, a wireframe is developed using an Au cutoff grade equal to 1.00 g/t, which leads to an average grade of 1.13 g/t and a standard deviation of 3.84 g/t for the samples within this wireframe. The wireframe corresponds to a rock type that is consistent over its entire volume. As such, the density of the material is 2.94 t/m^3. With block size of 15 × 15 × 10 m, the wireframe constitutes 10 988 mining blocks on an irregular grid, averaging 6615 t of material per block. Using the data within this wireframe, the direct block simulation method[11] is employed to generate 20 simulated grade realizations of the orebody. The uncertainty in metal content is reflected by a variation of the gold grade from one realization of the simulated orebody model to the next. These simulated orebody models become an input for calculating the block economic values to be used for stochastic optimization with network flow algorithm. To develop an estimated or 'average' (e-type model), the grades in simulated orebody models are averaged by block. This e-type model is used for the conventional optimization with the LG algorithm.

Application of the stochastic network flow approach

The stochastic approach utilizes the open-source Goldberg Algorithm[27] available from Microsoft Research[30] that solves the graph problem (Equations [15–18]) using the maximum flow algorithm.

Given a set of simulated orebody models and economic parameters, the application in stochastic approach commences with calculation of the economic values of mining blocks. Keeping the quantity of material in each block (Q) at 6 615 t, the metal price (P) at \$10.00 per gram of Au, the mining cost (m) at \$1.00 per ton of material, the processing cost (c) at \$9.00 per ton of ore, the refining or selling cost (r) at \$0.05 per gram of Au, and the metallurgical recovery (y) at 95 per cent, the block economic values are calculated using Equations [13] and [14].

Knowing economic values of mining blocks, steps outlined in Asad and Dimitrakopoulos[21] develop a related graph (as given in Figure 1) and implement the maximum flow algorithm to solve this related graph. Maintaining the slope angle at 45° in all (0° to 360°) azimuths, the mining capacity at 16 Mt, and the processing capacity at 8 Mt, the optimal phase design constitutes five phases leading to the ultimate pit limit. Table I shows a summary of results in terms of quantity of ore, waste, and metal, stripping ratio, cash flows, and the number of blocks inside each phase. Figure 2 presents the horizontal and vertical sections of the optimal phase design and ultimate pit limit with the stochastic network flow approach.

Application of the conventional LG approach

The application in the conventional approach utilizes Whittle 4.3.1[17,31] (commercial mine design software, referred herein as Whittle) in developing the optimal phase design and ultimate pit limit, in two distinct steps. The first step implements the LG algorithm for the development of a range of nested pit shells. The second step employs the commercially available Milawa Scheduler[17,31] to find a combination of nested pit shells for the development of a number of phases within the ultimate pit limit. Knowing the quantity of material in each block and the economic parameters as defined in the stochastic network flow approach, the block economic values are calculated using Equations [4] and [5].

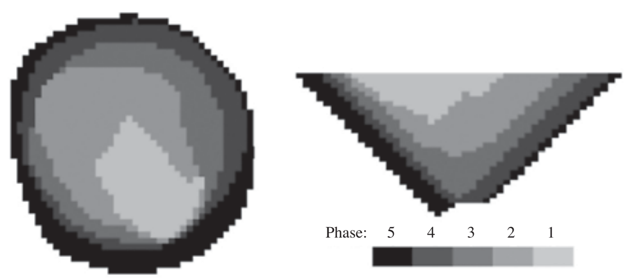

Figure 2—Horizontal plane and cross section of the stochastically optimal phase design (stochastic network flow approach)

Table I

Phase design and ultimate pit limit in the stochastic network flow approach

Phase	Quantity (t) Ore	Waste	Gold	Stripping ratio	Cash flow (\$) Discounted at 8%	Undiscounted	Number of blocks
1	2 652 284	7 852 336	6	2.96	20 042 479	21 645 878	1 588
2	7 107 818	21 951 878	13	3.09	30 197 899	38 597 087	4 393
3	5 446 130	22 211 186	9	4.08	12 616 653	19 331 257	4 181
4	4 128 752	24 858 178	8	6.02	5 462 196	9 774 662	4 382
5	4 446 272	29 568 058	8	6.65	1 222 721	4 660 862	5 142
Ultimate pit	23 781 256	106 441 634	44	4.48	69 541 948	94 009 746	19 686

During the first step, the process (i) maintains a pit slope angle of 45° along all (0° to 360°) azimuths; (ii) varies the λ value from 0.3 to 2.0 for modifying the metal price and block economic values from one pit shell to the next; and (iii) generates 65 nested pit shells, where the 19th pit shell corresponds to the ultimate pit limit at $\lambda = 1$.

During the second step, the process: (i) maintains an input of mining and processing capacities at 16 Mt and 8 Mt, respectively; (ii) considers pit shells 1 to 19 to find the optimal combination of pit shells merged into five phases within the ultimate pit limit; and (iii) identifies optimal phase design by merging pit shells 1 to 5 in phase 1, pit shells 6 to 11 in phase 2, pit shells 7 to 15 in phase 3, pit shell 16 into phase 4, and pit shells 17 to 19 in phase 5. Table II shows a summary of results in terms of quantity of ore, waste, and metal, stripping ratio, cash flows, and the number of blocks inside each phase. Figure 3 presents a horizontal and vertical section of the optimal phase design and ultimate pit limit.

Comparative analysis

Figures 2 and 3 present the horizontal and vertical section views (at identical planes and cross-sections) for stochastic and conventional approaches, respectively. It is evident in Figures 2 and 3 that the optimal phase design and ultimate pit limit are physically different in both stochastic and conventional approaches, used herein. Similarly, Tables I and II show that the stochastic approach targets high-value ore blocks and generates significantly higher worth by mining relatively less valuable material with a high metal content as compared to the conventional approach. Table II also shows that phase 5 generates a negative discounted cash flow. It demonstrates that mining activity should be terminated upon completion of phase 4, which further reduces the quantity of metal from 41 t to 36 t. The optimal phase design with the stochastic approach produces relatively consistently-sized phases due to the systematic selection of λ values as compared to the trial–and-error method employed in the conventional approach. The mining and processing capacities have been used as a guideline for maintaining consistency in developing an optimal phase design in both approaches. These capacities do not serve to develop a yearly optimal production schedule; instead, in order to accomplish a valid comparison with the LG approach in Whittle, a mining capacity based phase-wise mining schedule is used to facilitate calculating the discounted value.

Figures 4, 5, and 6 present the comparison of the two approaches in terms of risk profiles for yearly cash flows,

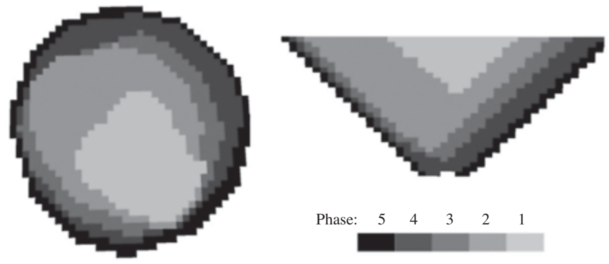

Figure 3—Horizontal plane and cross section of the conventional (LG approach) phase design (Note the physical difference for the same section in Figure 2)

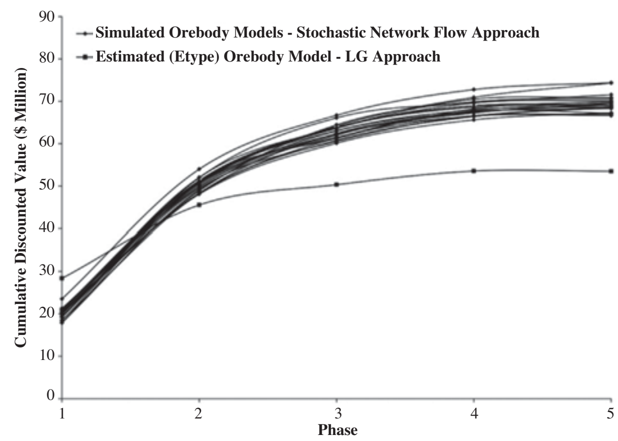

Figure 4—Risk profiles for discounted cash flows in the stochastic network flow and the conventional LG approaches

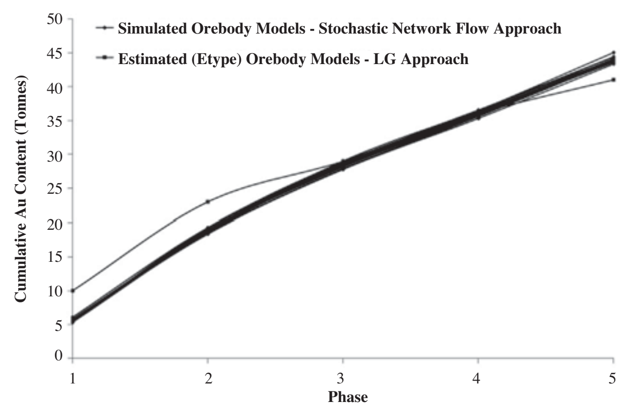

Figure 5—Risk profiles for metal content in the stochastic network flow and the conventional LG approaches

Table II

Phase design and ultimate pit limit in the conventional LG approach

Phase	Quantity (t)			Stripping	Cash flow ($)		Number of
	Ore	Waste	Gold	ratio	Discounted at 8%	Undiscounted	blocks
1	6 039 495	11 437 335	10	1.89	28 375 735	30 784 055	2 642
2	8 030 610	20 824 020	13	2.59	17 173 427	23 000 111	4 362
3	3 843 315	12 925 710	6	3.36	4 779 974	6 887 446	2 535
4	4 888 485	20 618 955	7	4.22	3 223 296	5 653 023	3 856
5	3 340 575	15 670 935	5	4.69	-63 279	1 078 280	2 874
Ultimate pit	26 142 480	81 476 955	41	3.12	53 489 153	67 402 915	16 269

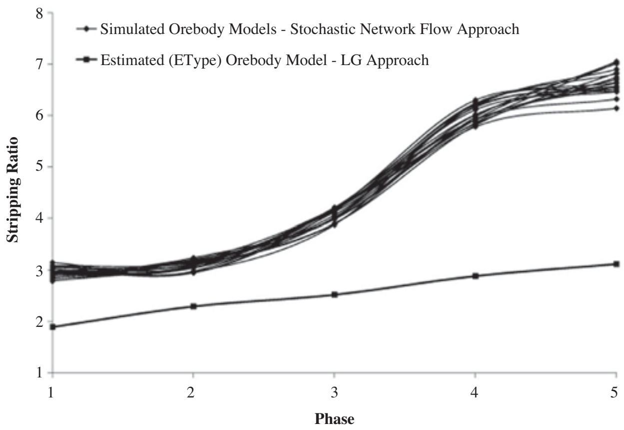

Figure 6—Risk profiles for stripping ratios in the stochastic network flow and the conventional LG approaches

metal content, and stripping ratios. It is evident that the conventional approach commits higher returns during initial years due mainly to the size of phase 1. However, the stochastic network flow approach promises improvement in quality of solutions in terms of an increase in total discounted as well as undiscounted value and total metal content as compared to the conventional LG approach. The structure of the graph in the stochastic approach ensures that more waste blocks may be supported to generate higher overall returns through a large ultimate pit limit. The risk profiles for stripping ratios show that the stochastic network flow approach maintains relatively higher incremental and overall stripping ratios for exposing more valuable material as compared to the conventional LG approach.

This case study verifies that the valuable information contained in multiple realizations of the simulated orebody models, that is, the local joint uncertainty of metal content, is lost by merely averaging the metal content for producing e-type or conventionally estimated single orebody models, which is used subsequently in the conventional optimization approach. The observations herein using stochastic network flow approach, that is, larger ultimate pit limit, and more metal, are consistent with the results of stochastic scheduling approaches based on simulated annealing[8,32] and Stochastic Integer Programming (SIP)[32–34]. Additionally, to establish the relevance of accounting uncertainty in metal content and the effect of using multiple simulated orebody models, we utilized the e-type model in maximum flow algorithm and experienced a 12.41 per cent decrease in the size of the ultimate pit limit (17 243 blocks inside the ultimate pit limit as opposed to 19 686 blocks in stochastic network flow approach).

Conclusions

This paper presents a performance analysis of the stochastic network flow and conventional LG approaches for developing the optimal phase design and ultimate pit limit for an open pit mine. The outcomes in the case study at a gold deposit show that the stochastic network flow approach performs substantially better by forecasting a 39 per cent and 30 per cent increase in total undiscounted and discounted cash flows, respectively; along with a 21 per cent larger pit (stochastically optimal pit limit), and 7 per cent increase in total metal content as compared to the conventional approach widely practiced in the mining industry.

The case study also (i) highlights the importance of accounting for the expected variability and uncertainty in the available metal content for the optimal pit design; and (ii) demonstrates that the structure of the graph and the Lagrangian relaxation of the production capacity constraints in the stochastic network flow approach generate relatively consistently-sized phases and higher value.

The optimization models used in the case study do not consider uncertainty in the economic parameters, such as metal price, mining and processing cost, and the discount rate. However, a modification in the structure of the related graph in the stochastic network flow approach can further accommodate uncertainty in economic parameters and enable investigation of its implications for optimal pit design.

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