Introduction
At the exploration stage, kriged block estimates with a proper search routine will be conditionally unbiased and will have the lowest level of uncertainty, but will, unavoidably, be smoothed because of the level of data then available. This means they will have a lower dispersion variance than that of the final selective mining unit (SMU) distribution at the production stage, when more information will be available.

The smooth estimates will generally overestimate the tonnage above the economic cut-off and underestimate the corresponding grade, i.e. for cut-off grades below the mean grade of the orebody. The reason for this smoothing effect is that proper kriging is, in fact, a regression estimate and it is well known in classical statistics that regressed estimates have a variance equal to the variance of the dependent variable 'y' less the conditional variance of the 'y' values (or error variance of the regressed estimates). This error variance reduces as more data become available; at the same time, the smoothing effect will decrease, i.e., the dispersion variance of the estimates will increase and the efficiency of the estimates will improve.

At the eventual production stage, more data will be available for acceptable estimation of large planning blocks as well as SMUs. However, this ultimate mining position is unknown at the project feasibility stage and therefore has to be estimated. This estimation is required because any cash flow calculations made on the basis of the smoothed estimates will obviously misrepresent the overall economic value of the project, i.e. the average grade of blocks above cut-off will be underestimated and the tonnage overestimated for cut-off grades below the average grade of the orebody. Similarly, unsmoothed estimates will be conditionally biased and will give even worse results, particularly in local areas of short- and medium-term planning or mining.

This paper presents a case study of indirect post-processing and proportional modelling of recoverable resources designed for medium- and long-term mining planning at the Gold Fields' Tarkwa Mine in Ghana. The case study compares long-term indirect recoverable mineral resource estimates based on typical widely spaced feasibility data to the corresponding production grade control model as well as the mine production. The paper also proposes certain critical regression slope and kriging efficiency control limits to avoid inefficient medium- to long-term recoverable estimates, and highlights the danger of accepting block estimates that have a slope of regression less than 0.5.

Keywords
indirect conditioning, smoothing effect, conditional biases, post-processing, kriging efficiency, regression slope, information effect.

Synopsis
For new mining projects or for medium- to long-term areas of existing mines, drilling data is invariably on a relatively large grid. Direct estimates for selective mining units (SMUs), and also for much larger block units, will then be smoothed due to the information effects and the high error variance.

The difficulty is that ultimately, during mining, selection will be done on the basis of SMUs on the final close-spaced data grid (grade control), which will then be available, i.e. the actual selection will be more efficient, with fewer misclassifications. However, this ultimate mining position is unknown at the project feasibility stage and therefore has to be estimated. This estimation is required because any cash flow calculations made on the basis of the smoothed estimates will obviously misrepresent the overall economic value of the project, i.e. the average grade of blocks above cut-off will be underestimated and the tonnage overestimated for cut-off grades below the average grade of the orebody. Similarly, unsmoothed estimates will be conditionally biased and will give even worse results, particularly in local areas of short- and medium-term planning or mining.

This paper presents a case study of indirect post-processing and proportional modelling of recoverable resources designed for medium- and long-term mine planning at the Gold Fields’ Tarkwa Mine in Ghana. The case study compares long-term indirect recoverable mineral resource estimates based on typical widely spaced feasibility data to the corresponding production grade control model as well as the mine production. The paper also proposes certain critical regression slope and kriging efficiency control limits to avoid inefficient medium- to long-term recoverable estimates, and highlights the danger of accepting block estimates that have a slope of regression less than 0.5.

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The use of indirect distributions of selective mining units for assessment of recoverable mineral resources designed for mine planning at Gold Fields’ Tarkwa Mine, Ghana
by W. Assibey-Bonsu*, J. Scarra†, and M. Aboagye†
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(SGS) have also been suggested (Deutsch and Journel, 1992). Several post-processing techniques have been researched and published elsewhere (Assibey-Bonsu and Krige, 1999; Krige et al., 2004; Deraisme and Assibey-Bonsu, 2011; 2012). The general indirect approach to the problem above is to derive the unknown SMU distribution from the observed distribution of relatively large kriged blocks (panels). The alternatives for the indirect approach as used in previous publications are uniform conditioning (Rivoirard, 1994, Assibey-Bonsu, 1998), multivariate uniform conditioning (Deraisme, Rivoirard, and Garrasco, 2008), and the assumed lognormal distribution of SMUs within large planning blocks (Assibey-Bonsu and Krige, 1999; Marcotte and David, 1985). These techniques are indirect in the sense that the SMU distributions are inferred indirectly from initial estimated large kriged blocks/panels.

Regardless of the theoretical soundness, from a practical point of view, actual follow-up comparisons are absolutely essential. It is worth noting that geostatistics started on that approach, and for the benefit of the discipline this should continue. From the authors’ perspective, such practical follow-up comparisons with real-world ‘actual’ production data for various post-processing techniques in the mining industry (at least in the gold mining context) are not readily found in the literature. It should be realized that whatever post-processing technique is used, the result will depend on the efficiency of the technique and parameters used in the execution of the process. Hence the estimate will always depend on how close the parameters are to the optimal choice, and thus the need for confirmation by comparisons with mined-out follow-up data.

The paper presents a case study, including production reconciliations, of an indirect post-processing and computational model of recoverable mineral resources, developed for medium and long-term mine planning at the Gold Fields Tarkwa mine in Ghana. The case study compares long-term indirect recoverable mineral resource estimates based on typical widely spaced feasibility data to the corresponding production grade control model as well as the mine production. The paper also proposes certain critical regression slope and kriging efficiency control limits to avoid inefficient production. The paper also proposes certain critical regression production grade control model as well as the mine typical widely spaced feasibility data to the corresponding indirect recoverable mineral resource estimates based on Tarkwa Mine in Ghana. The case study compares long-term for medium’ and long-term mine planning at the Gold Fields Tarkwa operation exploits narrow auriferous conglomerates, similar to those mined in the Witwatersrand Basin of South Africa. Mining is currently conducted at four open pits – Pepe-Mantraim, Teberebie, Akontansi, and Kottuvarethy.

\[ \sigma_{LE1}^2, \sigma_{LE2}^2 \] Error variances of conditionally unbiased estimates at exploration and final production stages for the large planning blocks \( \text{(BVL)} \) is assumed to be zero.

From Krige’s relationship; variances of actual SMU grades:

\[ BV_s = B^{(L)}_s + BV_{L} \]

That is, \( B^{(L)}_s = BV_s - BV_{L} \)

Dispersion variances of conditionally unbiased estimates within \( P \) for \( L \) \( (D(LE/P)) \)

\[ D(LE/P) = BV_{L} - \sigma_{LE1}^2 \quad \text{(at exploration stage)} \]

\[ = BV_{L} - 0 \quad \text{(at final production stage)} \]

Similarly, dispersion variances of conditionally unbiased estimates within \( L \) for \( S \) \( (D(L/E)) \) i.e., the required variance for long-term estimates:

\[ D(L/E) = B^{(L)}_s - \sigma_{LE1}^2 \quad \text{(at exploration stage)} \]

\[ = BV_{L} - \sigma_{LE2}^2 \quad \text{(at final production stage)} \]

Dispersion variance within \( P \) for \( S \) \( (D(L/E/L)) \):

\[ D(L/E/L) = B^{(L)}_s - \sigma_{LE2}^2 \]

To arrive at this required variance for long-term estimates (Equation [2]), the variance of \( L \) estimates at exploration (Equation [1]) must be adjusted or increased by:

\[ B^{(L)}_s + BV_L - \sigma_{LE2}^2 - (BV_L - \sigma_{LE1}) = BV^{(L)}_s - BV_L - \sigma_{LE2}^2 + \sigma_{LE1} \]

In all cases, the large panels must be conditionally unbiased and are adjusted accordingly (as per above) to reflect the average grade improvements and tonnage reductions expected on the basis of the additional information that will become available at the later production stage. Although all analyses (including variograms and panel kriging) are done in the untransformed space, the required variance adjustments assume lognormal distribution of SMUs within the large planning blocks.

Case study

Geology

Gold Fields’ Tarkwa operation exploits narrow auriferous conglomerates, similar to those mined in the Witwatersrand Basin of South Africa. Mining is currently conducted at four open pits – Pepe-Mantraim, Teberebie, Akontansi, and Kottuvarethy.
The Tarkwa orebodies are located within the Tarkwaian System, which forms a significant portion of the stratigraphy of the Ashanti Belt in southwest Ghana. The Ashanti Belt is a north-easterly striking, broadly synclinal structure made up of Lower Proterozoic sediments and volcanics underlain by the metavolcanics and metasediments of the Birimian System. The contact between the Birimian and the Tarkwaian is commonly marked by zones of intense shearing and is host to a number of significant shear-hosted gold deposits.

The Tarkwaian unconformably overlies the Birimian and is characterized by lower intensity metamorphism and the predominance of coarse-grained, immature sedimentary units, which from oldest to youngest are shown in Figure 2.

The Banket Series, which hosts the Tarkwa orebody, varies in thickness from 32 m in the east at Pepe to 270 m in the west at Kottraverchy. Figure 3 shows the mineralized and potentially economic reefs in the Banket Series from the base upwards. Sedimentological studies of the detailed stratigraphy within individual footwall reef units have led to the recognition of both lateral and vertical facies variations. The potentially economic reefs are locally named the AFC, A, A1, A3, B2, CDE, F2, and G reefs.

The sedimentological data, gold accumulation, gold grade, and channel width data is further used to delineate geologically homogeneous local facies zones or geozones. These geozones or domains are used to constrain the statistical and geostatistical analyses on a soft domain basis. These homogeneous domains are particularly important for the simple kriging panel estimates, which are used for conditioning of the recoverable resource estimates.

**Database and summary of mineral resources assessment process used on the mine**

The data-set consists of reverse circulation (RC), grade control (GC), and diamond drilling (DD) information originating from exploration, resource, and grade control drilling data. For sample support requirements the gold data-set is composited on 1 m intervals.

**Statistical and variographic analyses**

Descriptive statistics are applied to develop an understanding of the statistical characteristics and sample population distribution relationships. Descriptive statistics in the form of histograms and probability plots (to evaluate the normality...
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and lognormality of the distribution) are used to develop an understanding of the statistical relationships within each domain. These also provide useful inputs with regard to selection of a post-processing technique, which is required for recoverable resource modelling. The statistical analysis also facilitates the application of top-cut values for kriging and variography processes. The top-cut values are derived from review and calculation from the normal and log probability plots.

Proportional block model derivation

The indirect post-processing methodology as developed for the mine requires the in situ modelling of mineralized tonnage proportions within respective panels. These are derived using Gold Fields’ in-house software (Resource Evaluation System – RES’). The in situ mineralized proportions per panel are cross-checked against the physical wireframe volume as well as against the block model produced. The results are typically within 0.01% of each other and are considered accurate. Each panel is assigned a proportion of ore and waste (in situ), which are based essentially on the 3D geology wireframe of the orebody.

Selective mining units

All SMU sizes are as defined according to possible equipment, mining method, and mining selectivity, together with the geology of the orebody. At Tarkwa, the SMU size is 10 m x 5 m x 5 m with an assumed RC grade control drilling grid of 25 m x 25 m. During the early production period, certain areas were drilled on a 12.5 m x 12.5 m grid.

Panel estimation methods

Both ordinary kriging (OK) and simple kriging (SK) techniques are applied in developing the panel (50 m x 50 m x 3 m) resource estimation grade models. However, for post-processing of the recoverable resources, which are used for Whittle optimization, the SK panel grades are used. This is as a result of the relatively efficient SK estimates as reflected by higher kriging efficiencies observed for the SK panel estimates, especially in areas with limited data, as demonstrated in this paper (see section on analyses of kriging efficiency and slope of regression). In this regard, the SK panel estimates are conditionally unbiased. The simple kriging process uses a local or ‘global’ (i.e. regression slopes (Assibey-Bonsu, 2014).

The kriging process can even be negative if KV>BV. Such a situation is unacceptable and the block valuations will be worthless; yet the authors have encountered several such cases in the literature where the data accessed per block was inadequate and ordinary kriging efficiencies were negative (Krige, 1996). It should be noted that unlike ordinary kriging, the minimum kriging efficiency under simple kriging is zero.

Regression slope and critical control limits to avoid negative kriging efficiency estimates

Regression slope (slope) of actual block values on the estimates can be written as:

Regression slope = \( \frac{(BV - KV + \text{LM})}{(BV - KV + 2\text{LM})} \) \[3\]

where LM is the Lagrange multiplier and BV and KV are as defined above.

In order to avoid negative efficiency of block estimates, the following critical control limit test has been proposed for regression slopes (Assibey-Bonsu, 2014).

Where only a global estimate of all blocks is practical, all blocks will be valued at the global or sub-domain mean, i.e.

\[ KV = \frac{1}{BV} \]

For perfect valuations: \( KV = 0 \).

The dispersion variance \( (DV) \) of the estimates is \( BV \), and then:

\[ KE = \frac{(BV-KV)}{BV} \]

Usually blocks are valued imperfectly. With no conditional biases:

\[ DV = BV - KV, \text{ and } KE = \frac{(BV-KV)/BV}{DV/BV}. \]

However, with conditional biases present this relationship does not hold and then:

\[ DV > (BV - KV) \]

The kriging efficiency can even be negative if \( KV>BV \).
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$$KV = BV$$

and

kriging efficiency = 0

Substituting $KV=BV$ in Equation [3]:

Regression slope = $\frac{ILM_1}{2ILM} = 0.5$

Thus, a regression slope less than 0.5 will always lead to a negative block kriging efficiency estimate. This highlights the danger of accepting block estimates that have a slope of regression less than 0.5.

The critical regression slope limit of 0.5 should only be used to identify blocks that will result with negative kriging efficiencies. Ideal slopes of regression should be greater than 0.95, as proposed by Krige (1996).

**Typical ordinary kriging efficiencies and slope of regression as observed at Tarkwa Mine**

In providing kriged estimates on the mine, and particularly where limited data is available, significant conditional biases have been observed with OK, as demonstrated by the large negative kriging efficiencies and poor slopes of regression associated with a substantial number of the OK estimates as demonstrated in Figure 4. These significant conditional biases as observed with the OK estimates, particularly in areas with limited data, have adverse consequences on ore and waste selection for mine planning, as well as financial planning. As a result, relatively efficient local mean SK panel estimates, as reflected by higher kriging efficiencies, are used for resource panel conditioning, which forms the basis of the recoverable resource estimates at the mine.

**Post-processing**

The parent kriged blocks/panel ($50 \times 50 \times 3 \text{ m}$) are subjected to post-processing as per the indirect recoverable resource methodology discussed in this paper (see also Assibey-Bonsu and Krige, 1999). The methodology incorporates the information effect and change of support correction for the relevant SMUs. A final production grade-control grid of $25 \times 25$ has been assumed in deriving the recoverable resources, which is based on the expected final production grade control drilling on the mine.

Table I shows the variances of different support sizes of respective geozones for the A Reef. The figure further provides the corresponding average error variances for the $50 \times 50 \times 3 \text{ m}$ panels and the $5 \times 10 \times 3 \text{ m}$ SMUs. As expected, the table demonstrates the reduction in variances from sample to SMU, and from SMU to the panels.

<table>
<thead>
<tr>
<th>Geozone</th>
<th>Sample variance</th>
<th>Block variance $5 \times 10 \times 3 \text{ m (SMU)}$</th>
<th>Block variance $50 \times 50 \times 3 \text{ m (panel)}$</th>
<th>Average error variance $50 \times 50 \times 3 \text{ m}$</th>
<th>Average error variance $5 \times 10 \times 3 \text{ m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.052</td>
<td>0.335</td>
<td>0.083</td>
<td>0.051</td>
<td>0.142</td>
</tr>
<tr>
<td>2</td>
<td>0.745</td>
<td>0.247</td>
<td>0.059</td>
<td>0.035</td>
<td>0.080</td>
</tr>
<tr>
<td>3</td>
<td>1.722</td>
<td>0.577</td>
<td>0.166</td>
<td>0.091</td>
<td>0.236</td>
</tr>
<tr>
<td>4</td>
<td>3.205</td>
<td>0.812</td>
<td>0.232</td>
<td>0.129</td>
<td>0.440</td>
</tr>
</tbody>
</table>

The post-process output provides recoverable tonnages, grades, and metal content estimates above respective cut-offs per block/panel. The probability recoverable post-processed tons, grades, and metal estimates for specific cut-offs derived for the respective panels are used for Whittle mine planning optimization. Thus, for the mine planning purposes, the base model provides recoverable grade-tonnage estimates based on $5 \times 10 \times 3 \text{ m}$ SMUs derived, taking into account the production equipment on the mine.

**Resource classification**

The Mineral Resource classification is a function of the confidence of the whole process from drilling, sampling, geological understanding, and geostatistical relationships. The following aspects or parameters are considered for Mineral Resource classification:

- Sampling – quality assurance and quality control (QA/QC)
- Geological confidence
- Kriging efficiency and slope of regression
- Deviation based on the lower 90% confidence limit estimates

**Production reconciliation results**

Table II illustrates typical results for comparisons of resource and grade control models within a common volume for one of the relevant geozones.

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the areas of the mine (the Pepe open pit area). The table shows good in situ reconciliation of the resource model, which is based on the SK indirect post-processing technique, when compared to the grade control models. Where differences are observed in the reef groupings they are due to geological re-zoning of the A Reef as A, A1, A2, and A3 for the resource model, which were subsequently combined as A Reef during mining at the Pepe area (i.e. for the grade control model, A Reef is equivalent to combined A, A1, A2, and A3 reefs for the resource model).

Table III further shows percentage errors of the reconciliations (as per Table II, for the resource model A, A1, A2, and A3 are combined as A, i.e., on the same basis as the grade control model). The table shows individual reef percentage errors of -0.7%/3.9% and -5%/3.3% for tons and grade respectively. On a ‘global’ combined basis, percentage errors of -0.1%/-0.95% are observed for tons and grade respectively. These percentage errors indicate that the SK-based indirect recoverable resource models provide reliable grade, tonnage, and metal estimates inputs for mine planning and financial forecasts.

Figures 5 and 6 also show typical reconciliations of grade control models with production as observed on the mine. The figures also demonstrate good reconciliation between grade control models and that observed during production. As the resource models and the grade control models reconcile well (Tables II and III), this shows that a good reconciliation exists between the proportional post-processed recoverable resources estimates and production at the mine. The study further shows that in areas where adequate grade control data exists, good OK efficiencies/regression slopes are observed. However, this is not the case for OK estimates for long-term areas of the mine, where drilling data is on a relatively large grid and significant conditional biases have been observed on OK panel estimates.

Conclusions

The study shows that appropriate application of an indirect post-processing technique provides efficient recoverable resource estimates for mine planning and financial forecast. The study further shows that it is critical that the conditioning panel estimates used for the post-processing are conditionally unbiased, if the corresponding recoverable resources estimates are to provide the lowest level of uncertainty for mine planning and financial forecasts. The study shows that kriging efficiency and slope of regression provide useful tools to measure the extent of conditional biases.

Acknowledgements

The authors are grateful to Gold Fields for permission to publish this paper based on a case study of the Group’s Tarkwa Gold Mine.

Table II

Comparison of Indirect recoverable resource and grade control models (Pepe Area)

<table>
<thead>
<tr>
<th>Reef</th>
<th>Resource model Tons</th>
<th>Grade (g/t)</th>
<th>Grade control model Tons</th>
<th>Grade (g/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9 321 811</td>
<td>1.38</td>
<td>10 385 781</td>
<td>1.39</td>
</tr>
<tr>
<td>A1</td>
<td>672 512</td>
<td>1.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>190 334</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>274 553</td>
<td>1.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1 569 111</td>
<td>0.91</td>
<td>1 586 705</td>
<td>0.94</td>
</tr>
<tr>
<td>E</td>
<td>1 081 803</td>
<td>0.79</td>
<td>1 123 823</td>
<td>0.90</td>
</tr>
<tr>
<td>F2</td>
<td>838 558</td>
<td>0.40</td>
<td>856 168</td>
<td>0.38</td>
</tr>
<tr>
<td>G</td>
<td>758 547</td>
<td>0.90</td>
<td>774 728</td>
<td>0.88</td>
</tr>
<tr>
<td>Total</td>
<td>14 717 229</td>
<td>1.22</td>
<td>14 707 205</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table III

Percentage errors, reconciliation of indirect recoverable resource (RM) and grade control models - Pepe Area (for RM, A Reef is: A, A1, A2, and A3 combined from Table II)

<table>
<thead>
<tr>
<th>Reef</th>
<th>Tons (%)</th>
<th>Grade (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.70%</td>
<td>-1.02%</td>
</tr>
<tr>
<td>C</td>
<td>-0.15%</td>
<td>3.30%</td>
</tr>
<tr>
<td>E</td>
<td>3.88%</td>
<td>1.27%</td>
</tr>
<tr>
<td>F2</td>
<td>2.10%</td>
<td>-5.00%</td>
</tr>
<tr>
<td>G</td>
<td>0.80%</td>
<td>-2.22%</td>
</tr>
<tr>
<td>Total</td>
<td>-0.07%</td>
<td>-0.95%</td>
</tr>
</tbody>
</table>

Figure 5—Typical reconciliation of grade control model vs production (tons)

Figure 6—Typical reconciliation of grade control model vs production (grade)
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References


