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Synopsis

Waste dumps are widely used to discharge stripped mine wastes in open pit mining methods. As mining progresses, the height of the waste dumps will increase gradually, which may induce sliding failure once the height exceeds a critical value. Therefore, estimation of the maximum dumping height is crucial to the life-cycle use of a waste dump. However, published information about the maximum height of mine waste dumps are extremely rare. In this study, the maximum dumping height is estimated based on the strength-reduction technique by numerical simulations. The influences of dump geometry and properties on the maximum height are investigated. The results show that the maximum dump height decreases as the dump slope inclination angle α or the unit weight of dump materials γ increases, but decreases when the shear parameters of dump materials (cohesion c and internal friction angle ϕ) increase. The maximum height starts to increase with the ground inclination β when shear failure occurs at the interface between the dump and the base. Furthermore the effects of the dump width B, Young's modulus E, and Poisson's ratio μ of dump materials on the maximum height are almost insignificant. Good agreement was found between numerical results available in the literature and those obtained in this study.

Keywords

waste dump, maximum dumping height, numerical models, stability.

Introduction

In surface mining methods, large quantities of wastes are generated when removing the soil and rock overlying on the ore. This results in great challenges for mine waste management. Some of these mine wastes are used as backfilled materials in the excavated areas or underground mine stopes, but most are deposited on the surface as waste dumps (Bao *et al.*, 2019; Chai, 2020; Rahul, Rai and Shrivasta, 2015; Wang and Chen, 2017; Zou *et al.*, 2018).

Since the waste dump consists of a mixture of loose soil and various rock types, the dump slope is prone to instability. Slope failure at a waste dump can cause serious environmental problems and economic loss, and may even threaten the lives of nearby people (Bao *et al.*, 2019; Blight and Fourie, 2005; Lazăr *et al.*, 2012; Shakesby and Whitlow, 1991; Wang *et al.*, 2020; Zhan *et al.*, 2021). Therefore, it is crucial to ensure the stability of a waste dump slope throughout its life-cycle.

Over the years, extensive work has been conducted on existing waste dumps regarding failure mechanisms (Chen, 2018; Wang and Chen, 2017; Zhan *et al.*, 2021), stability assessment (Adamczyk *et al.*, 2013; Behera *et al.*, 2016; Cho and Song, 2014; Gao *et al.*, 2017; Kainthola *et al.*, 2011; Wang, Zhang, and Lin., 2019), and stabilization of waste dumps (Chaulya *et al.*, 1999; Gilbertson and Williamson, 1974; Rai, Khandelwal, and Jaiswal, 2012; Rajak *et al.*, 2021; Ranjan *et al.*, 2016). The failure of a waste dump during construction has been reported (Blight and Fourie, 2005; Chen, 2018; Kasmer, Ulusay and Gokceoglu., 2006; Poulsen *et al.*, 2014; Steiakakis, Kavouridis, and Monopolis, 2009). Steiakakis, Kavouridis, and Monopolis, (2009), found that the failure of a waste dump at South Field Mine in northern Greece occurred at the third phase of deposition and caused about 2.5 Mm² of waste materials to flow out of the dump boundary. Thus, research on sound design of waste dumps is valuable, but little detailed work on this aspect is available in the published literature to the author's knowledge although some efforts have been made to investigate the factors influencing dump stability based on the factor of safety obtained from numerical simulations (Behera *et al.*, 2016; Rai, Khandelwal, and Jaiswal, 2012; Upadhyay, Sharma, and Singh 1990; Yang and Ang, 2009).

Generally speaking, the stability analysis of a waste dump is similar to that of an ordinary slope (Griffiths and Marquez, 2007; Liu, Nian, and Wan, 2010). However, mine wastes are continuously produced by the mining process, thus the height of waste dump increases gradually. Besides, to make the best of the land resources as well as for economic reasons, the footprint of a mine waste dump should be minimized (Verma, Deb, and Mukhopadyay, 2017). Thus, it is of great importance to estimate the maximum height of waste dumps in the design procedure before the dumping operation starts.

In this study, a numerical method is presented to obtain the maximum dumping height of waste dumps under dry conditions. The strength reduction method is applied for the stability analysis of the dump slope using FLAC_{3D}. Then, the influence of dump geometry and properties on the maximum dumping height is investigated. The numerical results are compared results in the literature for validation.

Numerical model

FLAC3D numerical software (Fast Lagrangian Analysis of Continua in 3 Dimensions) is based on the finite difference method, which is applied here for the estimation of maximum heights of waste dumps (Itasca., 2013). The validation of FLAC^{3D} has been conducted by the author based on cylindrical hole problems and detailed descriptions are available in Chai (2020). The strength reduction technique, based on the limit equilibrium method, is used for the dump slope stability analyses with FLAC^{3D}. In this case, a strength reduction factor F_a is introduced to obtain the reduced cohesion (c, kPa) and friction angle φ, \circ) according to Equations [1] and [2].

$$c_r = \frac{c}{F_r} \tag{1}$$

$$\varphi_r = \arctan \frac{\tan \varphi}{F_r}$$
 [2]

where c (kPa) and φ (°) are the cohesion and internal friction angle of a slope, respectively. A series of numerical calculations is conducted using different values of c_r and φ_r by adjusting F_r with a bracketing solution approach. The last stable calculation with a reduction factor F_{rs} and the last unstable calculation with a reduction factor F_{ru} can finally be obtained. Then, the average value of F_{rs} and F_{ru} is calculated as the factor of safety of the studied slope (F) in the numerical analyses.

A typical physical model of a mine waste dump is shown in Figure 1a, which is taken as the reference case. The height of the dump H (m) is considered as the vertical distance from the toe of the dump slope to the crest. The width of the dump *B* is

15 m, which is the horizontal distance from the crest of the dump slope to the back of the dump. A unit thickness in the direction perpendicular to the diagram is used for the length L (m). The inclination angle of the waste dump α (°) and the ground inclination β (°) are selected as 30° and 0°, respectively. The dump and base are both regarded as elastoplastic materials obeying the Mohr-Coulomb criterion. The dump is characterized by dry unit weight $\gamma = 18 \text{ kN/m}^3$, Young's modulus E = 20 MPa, Poisson's ratio μ = 0.3, internal friction angle φ = 20°, and cohesion c = 20 kPa. The dilation angle ψ (°) and the tensile strength T_{\circ} (kPa) of the waste dump are both selected as zero. The base is relatively hard, with unit weight $y_b = 26 \text{ kN/m}^3$, Young's modulus $E_b = 200 \text{ MPa}$, Poisson's ratio μ_b = 0.2, internal friction angle φ_b = 40°, cohesion c_b = 300 kPa, tensile strength T_{ob} =100 kPa, and dilation angle ψ_b =0°.

Figure 1b shows the numerical model of a waste dump built with FLAC^{3D} for the reference case. To simulate a plane-strain condition in FLAC_{3D}, a unit thickness in the y-direction is used in the three-dimensional (3D) model and the front and back boundaries are fixed in the y-direction. Displacements along the left and right lateral boundaries are prohibited in the x-direction. The top boundary of the model is free to move in all directions, while the bottom boundary is fixed in all directions. The optimal mesh size is 0.5 m after a series of mesh sensitivity analyses.

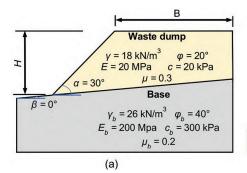
To obtain the maximum height (H_m) of the waste dump, a string of numerical simulations is conducted with different dump heights H. The height when the factor of safety F is equal to 1.0 can be regarded as the maximum dumping height H_m . Table I shows a detailed sequence of numerical simulations with different dump geometries and properties. Case zero is the reference case used to show the procedure for obtaining the maximum dumping height H_{m} . Cases 1-8 are conducted to investigate how these parameters influence the maximum height H_{m} .

Numerical results

Reference case (case zero)

Figure 2 shows the contours of maximum shear strain increment for the reference case in the last stable and unstable condition with dump height H = 45 m, 40 m, and 35 m. For the last stable cases shown in Figures 2(a), 2(c), and 2(e) there are some elements where the maximum shear strain increment is relatively large, but a continuous sliding surface from the toe to the crest of the dump slope cannot be observed; whereas shown in Figures. 2(b), 2(d), and 2(f) a continuous sliding surface occurred for all three heights, suggesting failure of the dump slope.

According to the numerical results, the factors of safety for the waste dump when H = 45 m, 40 m, and 35 m are 0.971, 1.013, and 1.052, respectively. To obtain the maximum height H_{m} of the waste



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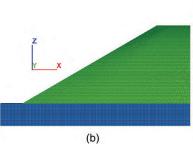


Figure 1—(a) A physical model and (b) a numerical model constructed with FLAC^{3D} of a waste dump

Table I
Numerical simulations for dump stability analyses (with γ_b = 26 kN/m³, E_b = 200 MPa, μ_b = 0.2, φ_b = 40°, and c_b = 300 kPa)

Case	Figure	α (°)	β (°)	<i>B</i> (m)	γ (kN/m³)	c (kPa)	φ (°)	E (MPa)	μ
0	1-3	30	0	15	18	20	20	20	0.3
1	4	Var.	0	15	18	20	20	20	0.3
2	5, 6	30	Var.	15	18	20	20	20	0.3
3	7	30	0	Var.	18	20	20	20	0.3
4	8	30	0	15	Var.	20	20	20	0.3
5	9	30	0	15	18	Var.	20	20	0.3
6	10	30	0	15	18	20	Var.	20	0.3
7	11	30	0	15	18	20	20	Var.	0.3
8	12	30	0	15	18	20	20	20	Var.

Var. = Variable

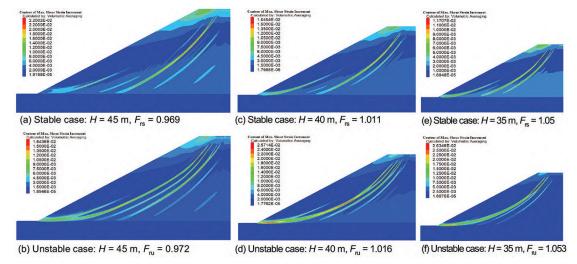


Figure 2—Contours of maximum shear strain increment for the reference case with different dump heights

dump, additional numerical simulations using different heights H are conducted and the factor of safety F is calculated. Figure 3 presents the variation of F with dump height in the range of 10 m to 50 m for the reference case obtained from FLAC^{3D}. With an increase in the dump height H, the factor of safety F is found to decrease gradually, corresponding well with the existing numerical results (Behera et al., 2016; Kainthola et al., 2011; Yang and Ang, 2009). Therefore, the maximum height of the waste dump for the reference case is about 40 m.

Effect of model geometry

In order to store more mine waste in a dump within a limited land, the dump slope has to be steeper. Ascertaining the effect of dump slope inclination angle α on the maximum height of a waste dump is of great benefit for safe design and construction. Figure 4 shows the variation in the maximum height H_m as the inclination angle of the dump slope increases from 25° to 50° (case 1 in Table I) obtained from numerical modelling in FLAC^{3D}. The maximum dumping height H_m is 85 m when the inclination angle α of the dump slope is 25°, decreasing to 14 m as α increases to 50°. An equation based on the curve-fitting technique is also presented in Figure 4, with a correlation coefficient R^2 of 0.9946.

The base on which a waste dump is constructed is sometimes inclined. Figure 5 presents the variation in the maximum height H_m as the ground slope β varies from zero to 20° (case 2 in Table

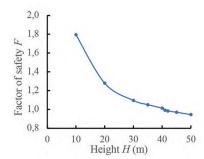


Figure 3—Variation of the factor of safety F with the dump height H for the reference case (case zero)

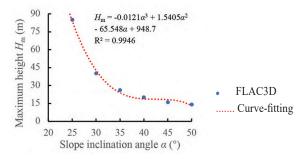


Figure 4—Variation in maximum dumping height H_m with slope inclination angle α (case 1 in Table I)

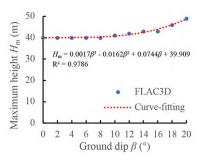


Figure 5—Variation in maximum dumping height H_m with the ground inclination β (case 2 in Table I)

I) and a curve-fitting formula with R^2 = 0.9786 (correlation coefficient). The maximum height of the dump remains constant at 40 m when the ground slope β is less than 8°. However, as the slope β increases from 8° to 20° , the maximum height H_m increases from 40 m to 49 m. The result indicates a change in the failure mechanism, which can also be confirmed by the contours of the maximum shear strain increment of case 2 shown in Figure 6. As shown in Figure 6a, when $\beta = 6^{\circ}$, the potential sliding surface is tangential to the base and failure occurs inside the dump slope. As β increases to 12° (Figure 6b), the failure surface intersects the base, indicating shear failure of the interface between the dump and the base.

The maximum height H_m of waste dumps for different values of dump width B varying from 1 to 40 m (Case 3 in Table I) is presented in Figure 7. The maximum height is seen to be almost

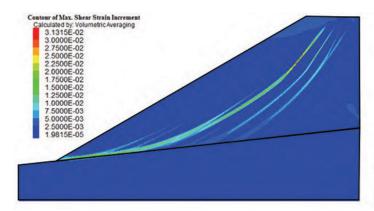
insensitive to the dump width. This is important because the dump width is desired to be as large as possible to accommodate more waste materials in practice.

Effect of dump material properties

Figure 8 shows the influence of the unit weight γ of dump materials on the maximum height H_m of waste dumps (case 4 in Table I) obtained from numerical simulations in FLAC_{3D}. The maximum dumping heigh H_m is 45 m when the unit weight γ of dump materials is 16 kN/m³, and reduces to 28 m when γ increases to 26 kN/m3. An analytical equation is also given in Figure 8 to estimate the maximum height on the basis of γ .

Figure 9 shows the variation of maximum height H_m of waste dumps as a function of the dump cohesion c (case 5 in Table I). The maximum dumping height is 20 m when cohesion c is 10 kPa, increasing to 78 m when c = 40 kPa. The results suggest that the maximum height tends to increase linearly as the cohesion c of dump materials increases from 10 to 40 kPa. This is a result of the higher shear strength induced by the increase in dump cohesion c. A linear equation, shown in Figure 9, is proposed based on the curve-fitting method for the estimation of the maximum height $H_{...}$.

The variation of maximum dumping height H_m as a function of the internal friction angle φ of dump materials ranging from 5° to 25° is exhibited in Figure 10 (case 6 in Table I). The maximum height H_m is estimated at 11 m when the friction angle is 5° , and increases rapidly to 91 m when $\varphi = 25^{\circ}$. This also results from the higher shear strength due to the increase in the internal friction



(a) Case 2 with H = 40 m and $\beta = 6^{\circ}$

Figure 6—Contours of maximum shear strain increment for case 2

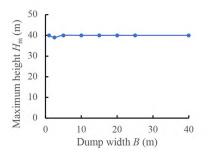
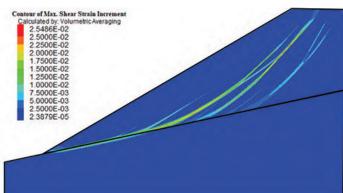


Figure 7—Variation in maximum dumping height H_m with the width of a waste dump B (Case 3 in Table I)



(b) Case 2 with H = 42 m and $\beta = 12^{\circ}$

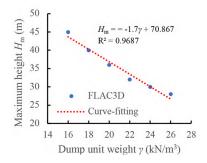


Figure 8—Variation of maximum dumping height H_m of a waste dump with the unit weight γ of dump materials (case 4 in Table I)

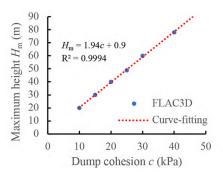


Figure 9—Variation in maximum dumping height H_m of a waste dump with cohesion c of dump materials (case 5 in Table I)

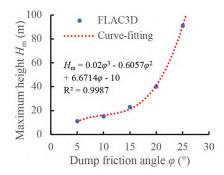


Figure 10—Variation in maximum dumping height H_m of a waste dump with the internal friction angle φ of dump materials (case 6 in Table I)

angle φ of waste dumps. For better estimation of the maximum height, an equation with R^2 = 0.9987 (correlation coefficient) is also given in Figure 10.

Figure 11 presents the variation in maximum dumping height H_m as a function of Young's modulus E of dump materials (case 7 in Table 1). It is observed that the maximum height H_m is insensitive to variation in Young's modulus between 20 MPa and 20 Gpa. In addition, Young's modulus is seen to be related more to the deformation of waste dumps rather than to the stability. For example, the total displacement of the waste dump decreases from 0.52 m to 0.36 m as Young's modulus increases from 20 MPa to 20 Gpa according to the FLAC^{3D} simulations.

Figure 12 shows the effect of Poisson's ratio μ of dump materials on the maximum dumping height H_m (case 8 in Table I). The maximum height H_m remains almost constant when Poisson's ratio varies from 0.2 to 0.4. Similar to the effect of Young's modulus, Poisson's ratio can influence the displacement of waste dumps, especially the horizontal displacement. For instance, the horizontal displacement is 0.33 m for case 8 with μ = 0.2 and H = 40 m, increasing to 0.58 m when μ = 0.4 and H = 40 m.

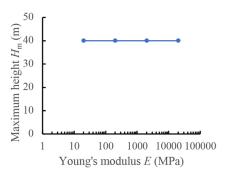


Figure 11—Variation in maximum dumping height H_m of a waste dump with Young's modulus E of dump materials (case 7 in Table I)

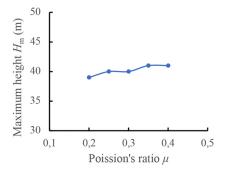


Figure 12—Variation in maximum dumping height H_m of a waste dump with Poisson's ratio of dump materials μ (case 8 in Table I)

Discussion

Validation of numerical simulations

The numerical results suggest that FLAC_{3D} is a useful tool for estimating the maximum height of waste dumps. FLAC_{3D} has been validated by investigating cylindrical hole problems (Chai, 2020). To further verify the reliability of the numerical results in this study, two more cases, shown in Table II, are considered to compare with published results in the literature (Griffiths and Marquez, 2007; Liu, Nian, and Wan, 2010; Rai, Khandelwal, and Jaiswal, 2012).

Rai, Khandelwal, and Jaiswal, (2012) applied FLAC to analyse the stability of a two-dimensional (2D) waste dump for verification of numerical results (case A in Table II). The dump geometry and properties are shown in Table II. The base is made up of the same materials as the dump. Using the parameters of case A presented in Table II and a similar numerical model shown in Figure 1b, the factor of safety *F* is calculated at 1.25. This agrees well with the numerical result of Rai, Khandelwal, and Jaiswal, (2012), in which the factor of safety *F* was estimated at 1.25.

Table II Parameters for the validation case											
Case	α (°)	β (°)	В (m)	<i>Н</i> (m)	γ (kN/m³)	c (kPa)	φ (°)	E (Mpa)	ν	F	
A	45	0	15	10	20	9.4	33	720	0.32	1.25	
В	26.57	0	15	10	20	40	0	100	0.3	1.175	

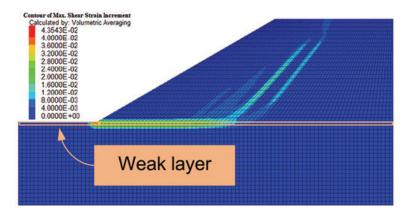


Figure 13—Contours of maximum shear strain increment for a waste dump with a weak layer between the waste dump and the base

Case B (in Table II) is taken from Griffiths and Marquez (2007) and modified by Liu, Nian, and Wan, (2010) who give more detailed parameters in their article. The same material is also used for the dump and base. The factor of safety for case B simulated using the 2D plane-strain model by ABAQUS is 1.175, with detailed parameters shown in Table II. By applying the parameters of case B to the model constructed in FLAC^{3D}, the factor of safety is estimated at 1.16. The difference between the two values is about 1.3%, suggesting the validation of the numerical result in this paper.

In general, good agreements are observed between the numerical results obtained from FLAC^{3D} and those available in the literature, which largely validates the applicability and reliability of the numerical results in the study.

Effect of base properties

In this study, a relatively stiff base is considered for the analyses of dump stability. The influences of base properties on the maximum dumping height are also investigated. It is observed that the maximum height is nearly insensitive to the base properties. For instance, as the internal friction angle of the base increases from 10° to 40°, the maximum height remains unchanged at 40 m. However, the waste dump may also be constructed on soft ground or a weak layer (Wang and Chen, 2017). Figure 14 shows the contours of maximum shear strain increment for the last unstable case of a waste dump with a 0.5-m-thick weak layer, which is characterized by $\gamma_W = 21 \text{ kN/m}^3$ (unit weight), $c_W = 0$ (cohesion), φ_w = 10° (internal friction angle), E_w = 200 MPa (Young's modulus), and $\mu_{\rm W}$ = 0.2 (Poisson's ratio). Other parameters of the waste dump and the base are the same as those used in the reference case (case zero in Table I). As shown in Figure 14, the waste fails by sliding along the weak layer. The calculated maximum dumping height is 13 m, which is much lower than the maximum height of 40 m for the reference case without the weak layer. Therefore, more detailed investigations of the influence of weak layers on the maximum dumping height are still required.

The factor of safety

In this study, the maximum dumping height was taken as the height when the factor of safety F = 1.0. However, in practice, a higher safety factor than unity (*e.g.*, 1.2) is usually applied for the consideration of dump stability (Adamczyk *et al.*, 2013; Cho and Song, 2014). The corresponding dumping height can be estimated using the same method as for the reference case. According to Figure 3, the dumping height of the waste dump is about 24 m

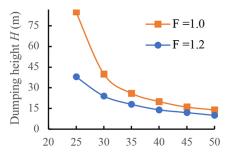


Figure 14—Variation in dumping height H of a waste dump with slope inclination angle α for different values of safety factor F

for the reference case when F = 1.2. Figure 14 shows the variation in dumping height as a function of slope inclination angle α for factor of safety 1.0 and 1.2. The same trend is observed for the influence of slope inclination angle α on the dumping height for both values of F.

Limitations

In this study, a totally dry condition is assumed for the waste dump material. This is fairly reasonable, as water can drain quickly from waste dumps consisting of numerous different sizes of rocks. However, in high rainfall areas, the influence of water on the maximum height of waste dumps cannot be neglected. More efforts are needed to fully understand it.

It will be of great value if a single analytical equation can be proposed to estimate the maximum height of waste dumps by incorporating all the factors that have an impact. It seems unrealistic due to the large number of variables. However, the development of artificial neural networks (ANN) may make it possible. Additional work is required regarding this aspect.

Other factors can also be investigated in the future, including heterogenous dump materials, a more complicated dump geometry, the influence of nearby mining activities, *etc*.

Conclusions

A numerical procedure has been presented for estimating the maximum height of waste dumps under totally dry conditions using plane-strain models constructed in $FLAC^{3D}$. Extensive numerical simulations were conducted to investigate the influence of dump geometry and properties on the maximum dumping height.

The results indicate that the maximum dumping height decreases as the slope inclination angle α or the unit weight γ of dump materials increases, whereas a higher cohesion c or internal friction angle φ of the dump materials increases the maximum height. The maximum dumping height is almost insensitive to the ground slope β for small values (e.g., β < 8° for case 2). However, as the ground slope β continues to increase (e.g., β > 8° for case 2), the maximum height tends to increase due to the occurrence of shear failure at the interface between the waste dump and the base. The dump width is seen to have a negligible effect on the maximum dumping height. The effects of Young's modulus E and Poisson's ratio μ of dump materials on the maximum dumping height are also insignificant.

The above findings are of great benefit for the preliminary design of waste dumps as regards the estimation of the maximum dumping height.

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