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Something for nothing?—An unconventional approach to extracting three-dimensional grade information from two-dimensional estimation of thin, tabular reef deposits

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This paper presents an initial investigation into the viability of co-kriging the grades of multiple mining cuts in a two dimensional, thin reef scenario. Deposits with gradational mineralized zones, in the perpendicular sense, are not as well served by the use of reef accumulation and thickness as deposits characterized by a sharp distinction between ore and waste rock. South African Bushveld Complex platinum deposits and Witwatersrand gold deposits are examples of this distinction. The calculation of an optimal mining cut is of paramount importance in the mining of Bushveld platinum. Two-dimensional kriging of a constant stoping width mining cut is an effective method for resource calculations once a cut is decided, but this begs the question of how to decide on the cut. Comparison of multiple cuts is used as a way of achieving this decision on many of Anglo Platinum's operations. Although the kriging of each cut is valid in isolation, occasional negative values occur when incremental grades between overlapping cuts are calculated. Negative incremental grades cannot reflect reality. They are thought to be related to the inability of the method to include the strong continuity in the perpendicular direction demonstrated by the relevant variograms. Co-kriging of related variables has been investigated as a potential means to reimpose this inherent order relationship. Various co-kriging options have been investigated. The method has been tested on a real data set from an established platinum operation. One of three types of anomaly has been investigated so far. The results show that co-kriging the mining cut grade as the primary variable, with just one secondary variable, the reef thickness, eliminates most occurrences. Severe anomalies remaining are arguably indicative of population mixing and might be more effectively treated by rezoning the data.

Aim—To investigate the viability of co-kriging the grades of multiple mining cuts as a method for resolving anomalies in the estimation of such cuts on Bushveld platinum deposits, with applicability to other thin reef type deposits with a gradational not sharp cutoff between ore and waste in the direction perpendicular to the reef.

Keywords: co-kriging, cross variograms, ordinary kriging, platinum, two-dimensional, grade profile

Introduction

The situation

Bushveld complex platinum deposits are thin, shallow dipping reef deposits. (The term 'thin' is used in the sense that the reef is a metre or less in thickness, whereas the deposit extends over tens or hundred of square km; in other words distance in one dimension << distances in other two dimensions.) It is not enough to calculate the usual reef thickness and accumulation (Journel and Huijbegts[1]) as per Witwatersrand gold deposits (Storrar[2]) because there is not a sharp cutoff from ore bearing rock to waste. Resource estimation needs dilution not by zero but from a variable lower grade envelope. Where the reef is thinner than the minimum stoping width, one method is to krige the constant support mining cut. Strictly speaking, a cut is a mixture of populations—reef, hangingwall and footwall typically have different distributions, although there is evidence that the mineralized zone, at least on some operations, is not rigidly associated with the stratigraphic units. On average, though, the proportions of different populations are the same. A stronger justification for this empirical solution is the relatively symmetrical rather than skew distribution and associated clear variogram of the mining cut as the variable, as shown in Figures 1 and 2. Note: The term PGE refers to the combined grade of the platinum group elements and the naming convention is explained in the Appendix. Ordinary kriging of cuts is capable of producing block models with regression slopes and kriging efficiencies approaching those achieved on South African gold operations.

This approach works well provided that the mining cut is not to be varied. In much of the South African platinum mining industry, however, the attainment of the 'best cut' is of paramount importance. Estimating multiple cuts off the same data, each with its own variogram, and then choosing that which gives the best grade or quantity of metal for a given region, is a pragmatic way to achieve this. Ore accounting requires a breakdown into hangingwall, reef and footwall material. This, and even a coarse representation of the grade profile through the reef, can be generated from suitably overlapping cuts, which is useful in an industry accustomed to working in terms of the grade profile seen on

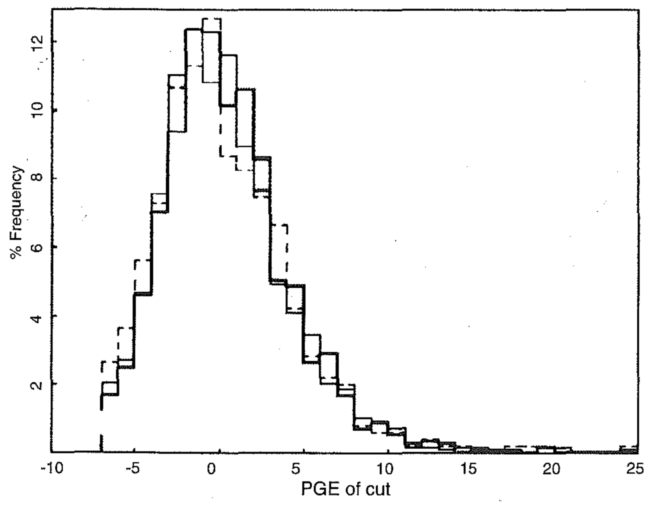

Figure 1. Grade distributions, relative to an arbitrary constant, for three different mining cuts with the same 80cm stoping width but containing 0, 10 or 20cm of hangingwall material. Solid line: PGE 0080. Bold line: PGE 1080. Dashed line: PGE 2080. Units are g/t

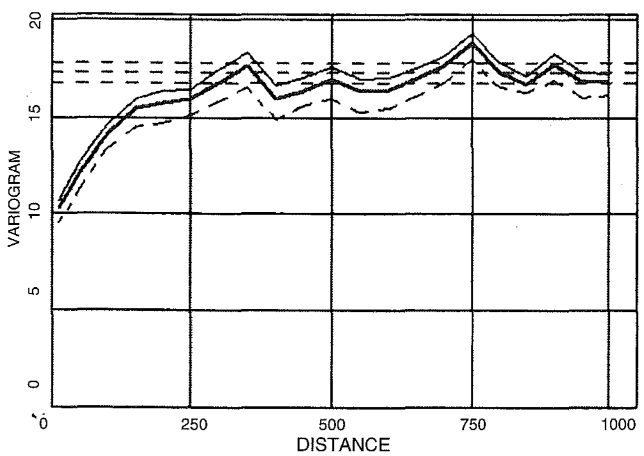

Figure 2. Variograms for the variables of Figure 1. The dashed horizontal lines indicate the variances. Note the different variances of the different cuts, associated with different populations over the same support. Units: Variances are in (g/t)², distances in m

underground samples or borehole intersections. (Note: Semi-three-dimensional estimation—not in true Cartesian coordinates but in layers referred to a geological contact and projected on to the horizontal or a representative reef plane—is another approach to the same objective, but its drawback is the high kriging variances of the stacked thin blocks.)

The problem

Even with 'clean' data and stringent control of the search neighbourhood, anomalous blocks with one or more negative grade increments are occasionally encountered in kriged multi-cut models. Figure 3 shows the frequency of occurrence in a real case.

The anomalous blocks illustrated in Figure 3 come from a model run with standard production methodology on data from an operating mine in the Western Bushveld. The histograms and variograms of Figure 1 belong to this data. The reef is normal thin Merenksy. A strict search of 20-50 samples within 350 m is used for the same five variables: reef thickness, reef accumulation, and three fixed stoping width mining cuts whose positions overlap systematically. From these, the grades of the portion of footwall immediately below the reef, the hangingwall component above the reef, and a further pure footwall component can be calculated. See the Appendix for details. Of the 621 blocks estimated 43 were anomalous.

Negative kriging weights occurred in 20 per cent of the estimations (1833 estimates out of 10605). The model was rerun with an option to exclude negative kriging weights and again tested for anomalies (in the sense of negative incremental grades between overlapping mining cuts). Half the anomalous blocks occurred at the same positions as before. Negative kriging weights due to the shielding typical of closely spaced data cannot therefore be the only—or even the main—cause of the anomalies.

Examining the quality of the estimates, just under half of the standard production run's anomalous blocks (18 out of 43) were from estimates with regression slopes of 0.82–0.95 and kriging efficiencies of 0.41–0.73: good quality individual estimates nevertheless resulting in anomalous grade increments between the cuts.

Possible cause

Two-dimensional estimation of multiple cuts has a problem analogous to that of indicator kriging (Dowd[3]): an order relationship that cannot be ensured mathematically. In the environment being studied the accumulation has to be a monotonically increasing function of cut width on individual underground sample sections or borehole reef intersections. Overlapping cuts share information, thus negative grade increments are not possible on individual

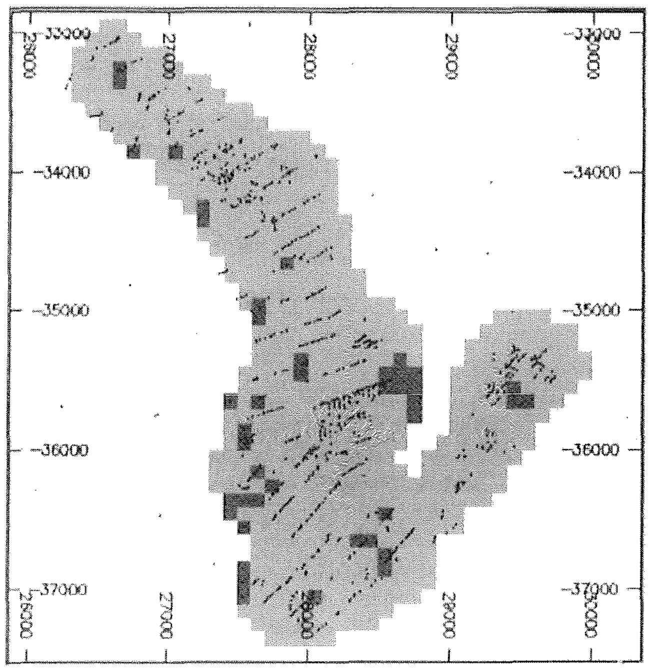

Figure 3. Production method block model (light grey) showing where anomalous blocks occur (dark grey—one or more negative grade increments between overlapping mining cuts estimated independently by ordinary kriging). The dots represent data locations. The coordinates are Cartesian: X (m) increases eastwards, Y (m) increases northwards

cuts. Kriging in two dimensions with different variogram models per cut means that even if the same underground sample sections or borehole intersections are selected by the same search, the kriging weights are not necessarily the same for the different cuts. It is feasible that negative increments can occur, although not likely. Because there is strong continuity through the reef, as shown by the variogram in Figure 4, the cut data are usually sufficiently well behaved to preserve the order relationship. Platinum is however notorious for its high variability, nugget effects of 50 to 60 per cent of the total sill being typical. Outliers can and do generate anomalies.

A possible solution

Co-kriging of the multi cuts is proposed.

The applicability of co-kriging

Co-kriging is often used when the variable of interest, the 'primary' variable, is sparsely sampled but other, 'secondary', variable(s) are more plentifully available and correlated with the primary variable. Journel and Huijbregts[4] point out that only in the case of undersampled primary variables is there any benefit to be had if the data are collocated (measured at the same points) and a linear co-regionalization exists (the cross variograms are replicas of each other except for scaling factors).

Myers[5] showed that block co-kriging is just as valid as point co-kriging, hence the kriging variances of co-kriged blocks may be compared with the results of single variable kriging methods such as ordinary kriging. The example in Isaaks and Srivastava[6] demonstrates the kind of improvement to be gained.

The co-kriging of multiple cuts at the same locations, deliberately restricted to data containing all the cuts, is a case of collocated co-kriging that is fully sampled. However, because the cuts mix different populations, their cross variograms should not be regarded as representing a linear co-regionalization even though they may look similar, see Figure 5. Co-kriging can therefore be expected to improve the estimation compared with ordinary kriging of the individual variables. The question is by how much.

Method

The software

Datamine is used to krige multi-cut models at Anglo Platinum. The identification of anomalous blocks was done in Datamine. The direct and cross variograms were generated and their models fitted in Datamine. Co-kriging is not implemented in Datamine, so the program cokb3d of the public domain GSLIB software (Deutch and Journel[7]) was used with Datamine as the 'graphics engine'. As a precaution, it was first established that ordinary kriging produces the same results from Datamine and the GSLIB program kt3d to the limits of single precision output when run on identical data with identical search definitions, variogram models and grid definitions (GSLIB) or block model prototypes (Datamine).

Co-kriging choices

The GSLIB programs and subroutines can be edited, so options that are not present in the default program can potentially be implemented. In this early stage of investigation it was decided to work with cokb3d in its unaltered Stanford University 1998 form, which has fairly tight restrictions on the numbers of variables and samples, and restrict the ordinary kriging options of Datamine and kt3d to match.

GSLIB provides three types of co-kriging—simple co-kriging, not further considered here, and two ordinary kriging options: the traditional method and the standardized method (Deutch and Journel[7]) depending on how the non-bias condition is handled. The traditional method is, effectively, two non-bias conditions: the kriging weights of the primary variable sum to one and the weights of the secondary variable(s) sum to zero. The standardized method imposes a single non-bias condition, together with a rescaling of the secondary data so that their mean(s) equal the mean of the primary data. It has been noted that the traditional method reduces the effect of the secondary variable(s) very rapidly, so tending towards the ordinary kriging case, and predisposes the occurrence of negative kriging weights for reasons other than shielding (Isaaks and Srivastava[6]). These characteristics seemed inherently unsuited to the present project but the traditional method was also tested.

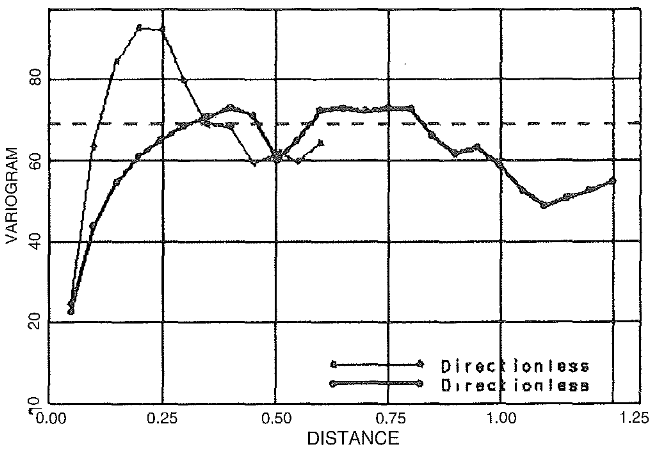

Figure 4. Variogram of PGE grade in the direction perpendicular to the reef. Thick line—entire mineralized zone. Thin line —reef stratigraphic unit only. Units: as for Figure 2

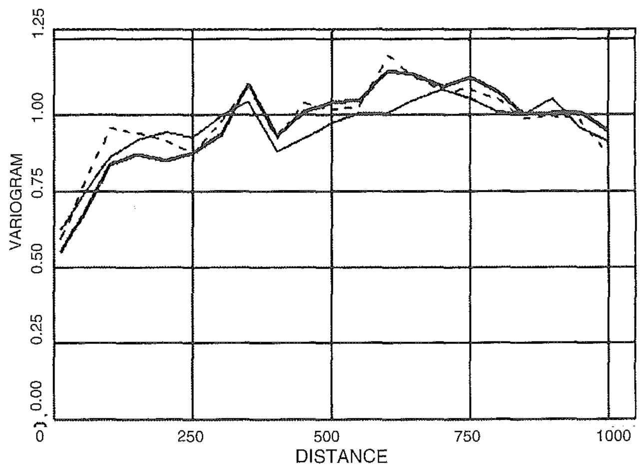

Figure 5. Cross variograms for the variables required to calculate the below-reef-to-footwall increment: PRP0000 (p), ACC0000 (a) and PGE0070 (c). The vertical scale is normalized by the relevant cross variances. Solid line—(c,p); bold line—(a,p); dashed line—(a,c)

Default restrictions on the number of secondary variables (two or less) and samples (up to twenty primary, up to twenty secondary) were left unchanged and the search parameters of the base case were adjusted to conform to these limitations. Thus co-kriging with identical sample locations to the ordinary kriging case could be achieved by the following:

- primary variable plus 1 secondary variable—20 primary data, 20 secondary data
- primary variable plus 2 secondary variables—10 primary data, 20 secondary data, equivalent to 10 of each.

The base case

The benchmark or 'base case' has to be more restricted than the method applied in the production model to ensure that identical data are used for the ordinary kriging and co-kriging, as explained above. Compared with the strict search (20–50 samples within the variogram range) of the production model, restriction to 10 samples or 20 samples predictably reduced the regression slope and kriging efficiency of all the block estimates. The number of anomalous blocks increased from 43 to 52 out of 621 but the locations of the previous anomalous blocks remained consistent. For example, 3 out of 8 below-reef anomalies from good estimates for the production method model increased to 5 out of 9 in the base case with regression slopes now in the range 0.50–0.8 instead of 0.8–0.9 previously.

To keep the number of variables manageable for this feasibility stage of the investigation, it was decided to work only on the incremental grade anomaly of the reef-to-foot wall transition.

Variography

The variograms and cross variograms were calculated from the full data set of 1481 locations for the cuts to get the maximum information, even though the kriging at this stage was done on data reduced to 1087 locations by the requirement that the sampling completely cover all the cuts being estimated. No top cutting was imposed. Table I lists the variogram models. Isotropic modelling was justified by consideration of the variogram contours. Anisotropy was not significant for the cuts. Even with thickness and accumulation, the inner structure was reasonably isotropic and directional anisotropy was only evident beyond about 150 m.

Co-kriged cases

The variables to co-krige are reef thickness, reef accumulation, and the so-called 'clean cut'–the mining cut from the top reef contact to bottom of the usual mining cut (see the Appendix and Figure 6 for an explanation of this terminology). For a mining cut of 80cm stope width which includes 10cm of hangingwall, the clean cut is the 70cm cut with no hangingwall. This would seldom be mined in practice but it is needed to calculate the grade increment from the bottom of the reef to the bottom of the actual mining cut.

Results

One secondary variable

By co-kriging only two of the three related variables, it is possible to ensure that the same samples are used for the primary and secondary variables. Which pairs should be chosen to krige the three variables? Reef thickness and accumulation are usually correlated, so the cut incorporating footwall material is not expected to change the result. Accumulation and thickness are thus co-kriged with each other, swopping primary and secondary status. The 'clean cut' grade is a function of both so it can be co-kriged with either. In practice the more effective secondary variable turned out to be thickness.

Two secondary variables

There are now choices regarding numbers of points. The variables are collocated. Only 10 samples of the primary variables and 20 samples of the two secondary variables will thus ensure that the sample locations are identical for all three variables. Conversely 20 primary and 20 secondary samples per block will result in only the 10 closest locations supplying the secondary information whilst the full 20 locations supply the primary data as for the ordinary kriging case.

Kriged values

The effects of the above choices on removing the anomalies are compared in Table II. Co-kriging the clean cut grade with only one secondary variable, the reef thickness, gives the most satisfactory results.

The traditional option (primary weights sum to 1,

Table I
Variogram models

Variable	Nugget	Structure 1		Structure 2		Structure 3	
		Sill	Range (m)	Sill	Range (m)	Sill	Range (m)
PRP0000	0.012487	0.011253	146	0.010312	388	-	-
ACC0000	4.263525	1.763791	36	1.190559	194	2.419355	457
PGE0070	7.964869	5.413602	32	3.179416	158	5.054771	339
PGE0080	8.463393	0.998294	34	0.998294	160	3.898525	337
PGE1080	7.483081	3.547454	46	2.933471	187	3.103148	337

Variable		Nugget	Structure 1		Structure 2	
Primary	Secondary		Sill	Range (m)	Sill	Range (m)
PGE0070	PRP0000	0.17199	0.104333	114	0.041125	338
PGE0070	ACC0000	7.934472	2.96198	162	2.52651	336
PRP0000	ACC0000	0.138831	0.079376	133	0.072817	351

Table II

Co-kriging schemes for comparison with the ordinary kriging cases. The kriged variables required to calculate the grade increment are shown as functions of the primary and secondary data variable(s). The success rating is indicated by the number of anomalies resolved out of the total number of ordinary kriging anomalies. a=accumulation, p=perpendicular thickness, c=clean cut

Co-kriged model Variables (as function of)	**c.f.** Points	**Ordinary kriged model** Variables	Points	**Success**
a(a,p), p(p,a), c(c,p)	(20,20)	a(a), p(p), c(c)	(20)	8/9
a(a,p), p(p,a), c(c,a)	(20,20)	a(a), p(p), c(c)	(20)	7/9
a(a,p), p(p,a), c(c,p,a)	(20,20)	a(a), p(p), c(c)	(20)	8/9
a (a,p,c), p(p,a,c), c(c,p,a)	(20,10,10)	a(a), p(p), c(c)	(20)	unstable
a (a,p,c), p(p,a,c), c(c,p,a)	(20,10,10)	a(a), p(p), c(c)	(10)	unstable
a (a,p,c), p(p,a,c), c(c,p,a)	(10,10,10)	a(a), p(p), c(c)	(10)	unstable

secondary weights to 0) proved so unsatisfactory that it was abandoned after the one secondary variable stage of testing. It is not included in Table II.

The 'standard' method of co-kriging thus looks promising on collocated data with all the variables fully sampled. However, it is when undersampled data are reintroduced that the method becomes most effective. In a test co-kriging on all the data, regardless of missing values in some variables, a 100 per cent removal of the ordinary kriging anomalies resulted. This awaits full verification and analysis.

Discussion

A reason for the anomalies

It is worth examining the reasons for the negative grade increments. Of the two blocks in Table II that were not resolved by co-kriging, one was from a poor data configuration (low regression slope, negative kriging efficiency) and so not worth further examination. The other, though, was a 'good' estimate. Datamine has facilities allowing the data used to estimate a block to be displayed along with the kriging weights. These revealed that one extremely high outlier in reef accumulation and, more importantly, thickness, was the cause. Examination of the reef thickness revealed similar but less severe outliers in the respective search neighbourhoods of all the other reef-to-footwall anomalous blocks in the base case.

Why co-kriging is effective

Co-kriging reduces the effect of the outliers. For all but one anomaly, despite outlier reef thickness greater than the stoping width, the order relationship for the coarse block grade profile was preserved because the weight of the outlier was reduced compared with the weight calculated by ordinary kriging.

Conclusions

Co-kriging of multiple cuts has proved to have an approximately 90 per cent success rate in solving anomalous grade increments between reef and footwall on one typical production data set. It has also proven the validity of the multi-cut method of determining the best cut by demonstrating that ordinary kriging mostly gives reasonable results. It is only in the presence of severe outliers that the order relationship is likely to be violated.

The co-kriging study is now being applied to the hangingwall anomalies and footwall anomalies further away from the reef.

It is not envisaged that cokriging will necessarily become routine. If results from other sites and the above types of anomalies should confirm this initial study, and if the benefits are deemed sufficient, the use of automated variogram fitting tools would need to be investigated to make the workload of fitting the cross variograms manageable. At present he role of cokriging is likely to be more of a fall back method in places where stationarity is not assured or it is suspected that more than one population may be present but the sparsity of data precludes separating them.

Acknowledgements

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Appendix

Figure 6 illustrates the calculation of grade increments from multi cuts. The coarse grade profile calculated from the overlapping cuts is shown schematically next to the stratigraphic information. The reference is the TRC (top reef contact). The following conventions apply when naming the cuts:

- Fixed stoping width cuts: number of cm on the far side of the contact and total stoping width. Thus in the data set used, where the reference is TRC, PGE1080 is the grade of a fixed cut with 10 cm of hangingwall and, by inference, 70 cm of reef plus footwall material. Similarly PGE0070 is the reef and footwall part only of the 1080 cut, also known as the 'clean cut'. PGE0080 is the grade of a hypothetical cut including no hangingwall, only reef and footwall, over the same overall stoping width as the 1080 cut. The 2080 cut (not illustrated) would extend further into hangingwall and take correspondingly less footwall.
- Variable thickness quantities are defined by all of the reef stratigraphic unit plus specified cm of hangingwall and footwall. Thus the true perpendicular thickness and accumulation of the reef are referred to as PRP0000 and ACC0000 in the tables.

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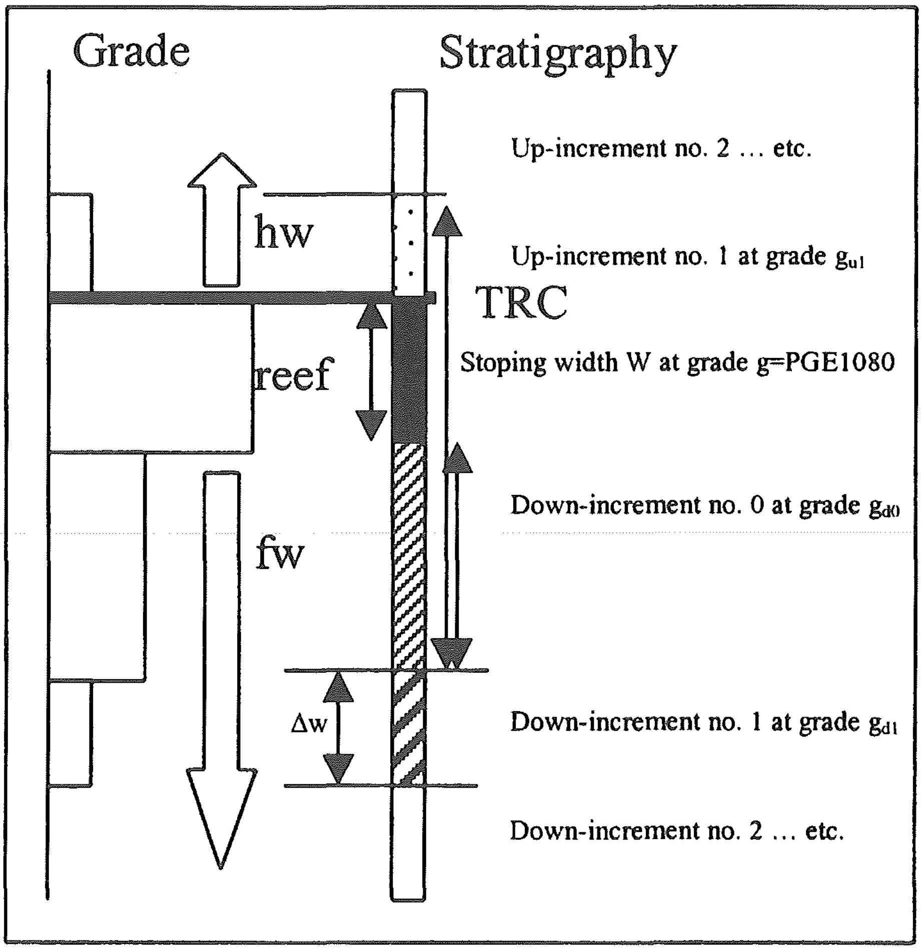

Figure 6. The relationship between the different fixed cuts and the amounts of reef, hangingwall and footwall material in a given cut. The 0070 cut consists of the black and thin diagonally hatched regions, the 10 cm wider 1080 cut adds the dotted portion of hangingwall, and the 0080 cut is the same width as the 1080 but covers the black, thin hatched, and thick hatched regions of the schematic stratigraphic column

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