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The influence of conditional bias in optimum ultimate pit planning

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Kriging estimates of geological resources are sometimes conditionally biased because the kriging plan contains too few samples. This may be done purposely in order to match the variance of the estimates to the true block dispersion variance.

This work aims at evaluating the influence of conditional bias over the ultimate optimum pit design in relation to its shape, size and NPV (Net Present Value of profits). It has been restricted to two real case studies: a 'carrot shaped' gold porphyry in northern Greece and a manto-type exotic copper deposit located in northern Chile.

The methodology applied to each deposit consisted of generating a 'real block model' and four resource estimates by the use of conditional simulation and the application of four different kriging plans to the drillhole databases respectively. The estimation models vary in smoothness and conditional bias. Ultimate optimum pits were determined for the 'real' and the four different kriging models. Pit comparisons lead to the following conclusions:

- In the worst case, conditional bias overestimated the project's NPV by 32% and 5% for the gold and copper deposits respectively.
- The overestimation of high grades is more relevant than the underestimation of low grades. This resulted in a tonnage over-extraction of 148% and 1% for the gold and copper deposit respectively. This difference was attributable to the vastly different geometry of the deposits.
- The smoothing effect of kriging, without or very little conditional bias, produced open pits that were different to the 'true' (ideal) ones and furthermore, underestimated the project's NPV by 10% and 6% for the gold and the copper deposits respectively.
- Other factors that influence the optimum open pit design and to some extent control the effect of conditional bias are: the cut-off grade, the orebody geometry and distribution of grades within it and the amount of overburden.

Keywords: conditional bias, ultimate pit design

Introduction and conditional bias

Kriging has become the main tool for the estimation of geological resources worldwide. In some applications however, kriging estimates suffer from conditional bias mainly due to the use of too few samples or composites in the kriging plan. This is sometimes done purposely in order to match the variance of the estimated block values to the true block dispersion variance.

The objective of this work is to evaluate the influence of conditional bias over the ultimate optimum pit design in relation to its shape, size and NPV (Net Present Value of profits). The study was limited to two real world applications: a 'carrot shaped' gold porphyry in northern Greece, which is still in an exploration stage, and a manto-type exotic copper deposit located in northern Chile, which has been in production since 2001.

In simple words, conditional bias implies that for local estimation, the means of the true block values differ from those of the estimated block values for different grade categories. Typically in open pits, the estimated block values are the kriging estimates and the 'true block values' correspond to the average of the blast holes found within the blocks.

Conditional bias can, and often will occur, even if there is no overall bias in the estimation. The most common effect is that low-estimated grades are underestimated whereas high-estimated grades are overestimated, therefore the regression line of the true block values (Y) on the estimated block values (X) differs from the first bisector and has a slope lower than 1.0. An example of a scatter plot with conditional bias is shown in Figure 1, in which the linear regression line (thin line) cuts across the first bisector (thick line). The estimates have no global bias since the underestimated and overestimated areas shown in gray are similar in size, therefore the means of the estimated and true block values are very similar.

Possible effects of conditional bias in open pit design are illustrated in Figure 2.

The top pair of graphs corresponds to the true and estimated grade distributions of the gold porphyry, which has a superficial and a deep high-grade zones. The overestimation of these, especially the deep one can cause severe tonnage over-extraction for a given set of economic parameters.

The bottom pair of graphs corresponds to the true and estimated grade distributions of the copper manto-type deposit. The over-extraction effect in this case is not at all obvious and depends on the cut-off grade being considered.

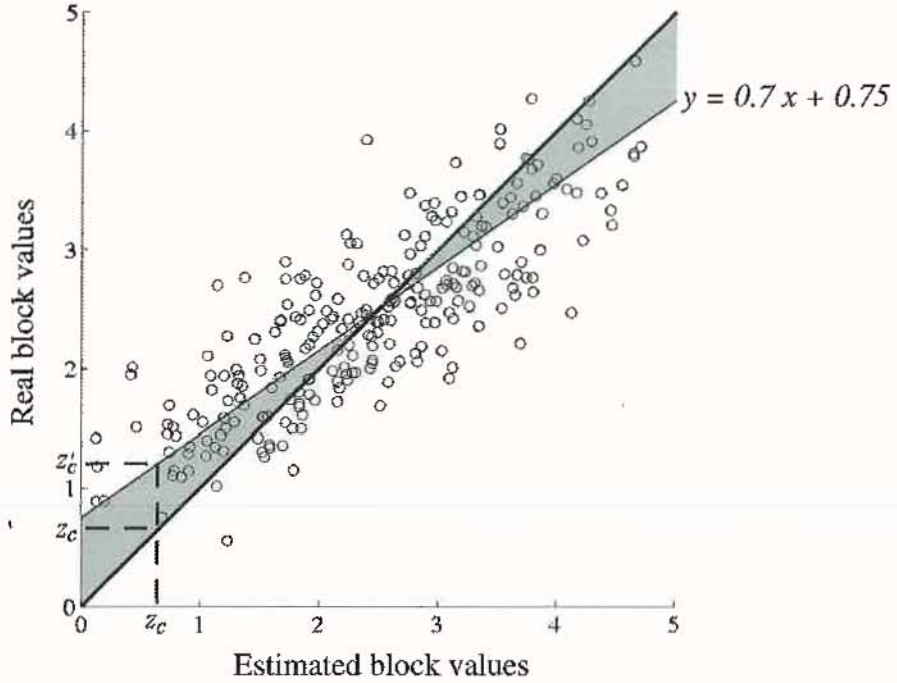


Figure 1. Illustration of conditional bias

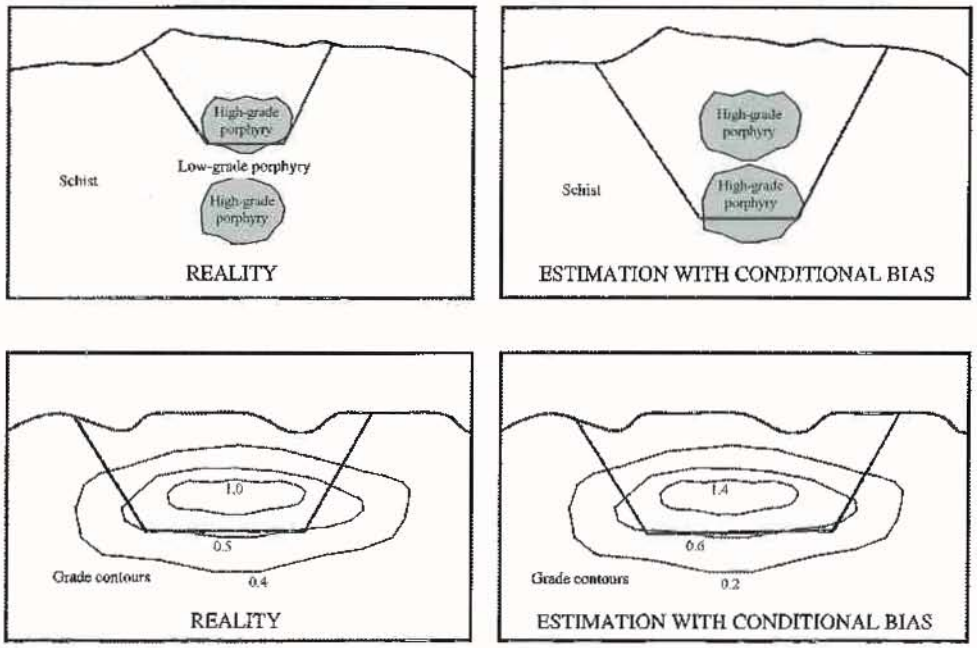


Figure 2. Effects of conditional bias on pit design

It is apparent, however, that for both deposits overestimation in the high-estimated grade area will cause overestimation of the grade of the material extracted as ore, which leads to the overestimation of the NPV.

Available data and methodology

A real gold porphyry drillhole database and a geological block model were made available for this work. The prospect is located in the Chalkidiki province in northern Greece and has been drilled using an irregular 50 m by 50 m grid. The deposit consists of a 'carrot-shaped' mineralized porphyry surrounded by schist that contains lower grade disseminated mineralization. Figure 3 depicts a typical plan and section showing the main rock types. This deposit contains a small high-grade zone near surface and a large one between 400 m and 600 m of depth.

A drillhole database and a geological model of a manto-type exotic copper deposit were also made available for this study. It is interesting to note that this deposit is very different from the gold porphyry in shape and grade distribution. This deposit is located in Chile's Second Region. This orebody was drilled using an irregular 70 m by 70 m grid of vertical drillholes. Production began in

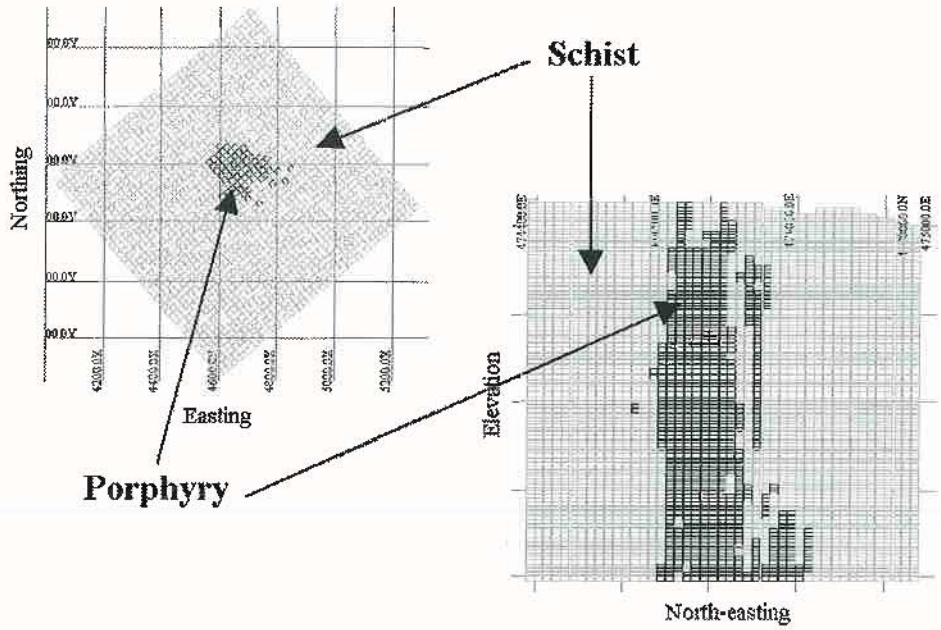
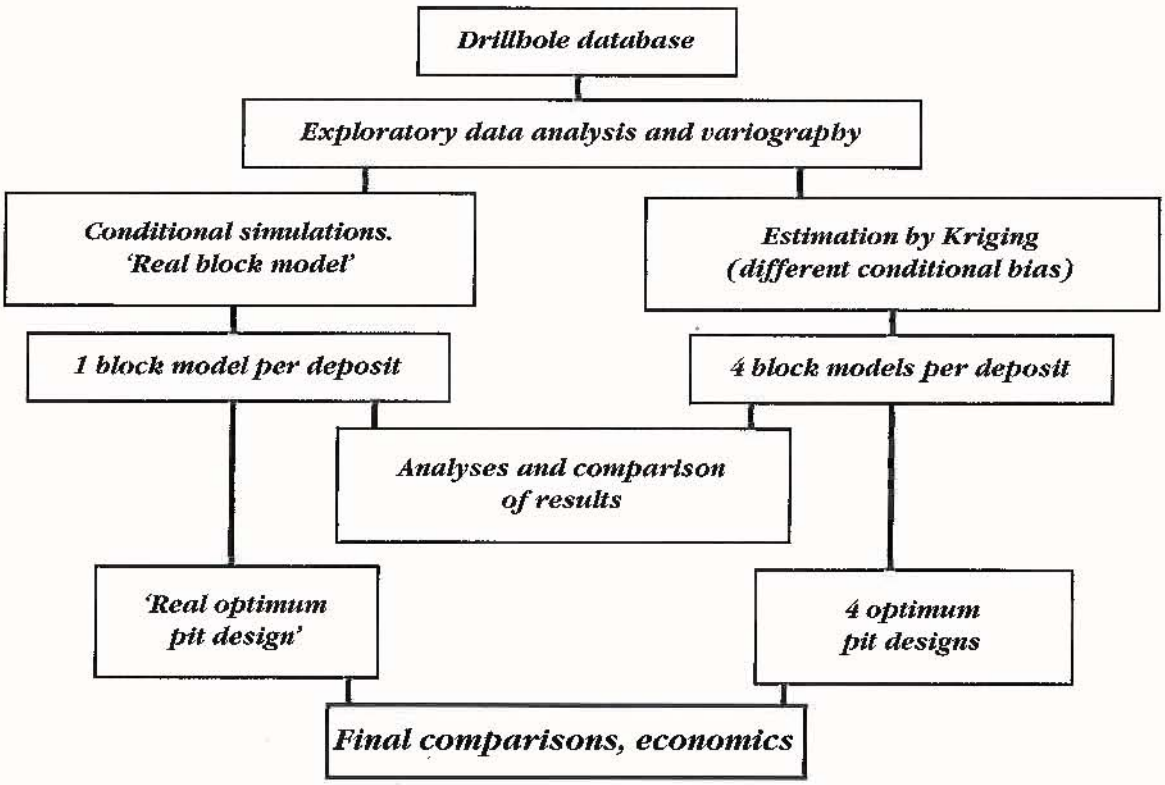


Figure 3. Plan and section showing main rock types—gold porphyry



2001. Figure 4 depicts a typical plan and section showing the main estimation domains, which roughly correspond to low, high and very high grades zones.

In order to quantify the differences between optimum ultimate pit designs based on estimated block models with varying degrees of conditional bias and grade smoothness it was necessary to carry out the comparisons against the ultimate pit design obtained from the 'real block model grades'. As these are unknown for the two deposits, they were obtained by Gaussian point-support simulation using the turning bands method (Journel and Huijbregts, 1978). The results were then averaged into block grades. Eleven simulations were done and ranked according to the total metal content. The median was accepted for comparison purposes.

On the other hand, four estimated block models were obtained by kriging for each deposit. These block models vary in the degree of conditional bias and smoothness. The simulations as well as the kriging models were based on the drillhole databases.

The methodology is outlined in the diagram above.

As shown in the above diagram, analyses were carried out in the following stages:

- Database revision and validation.

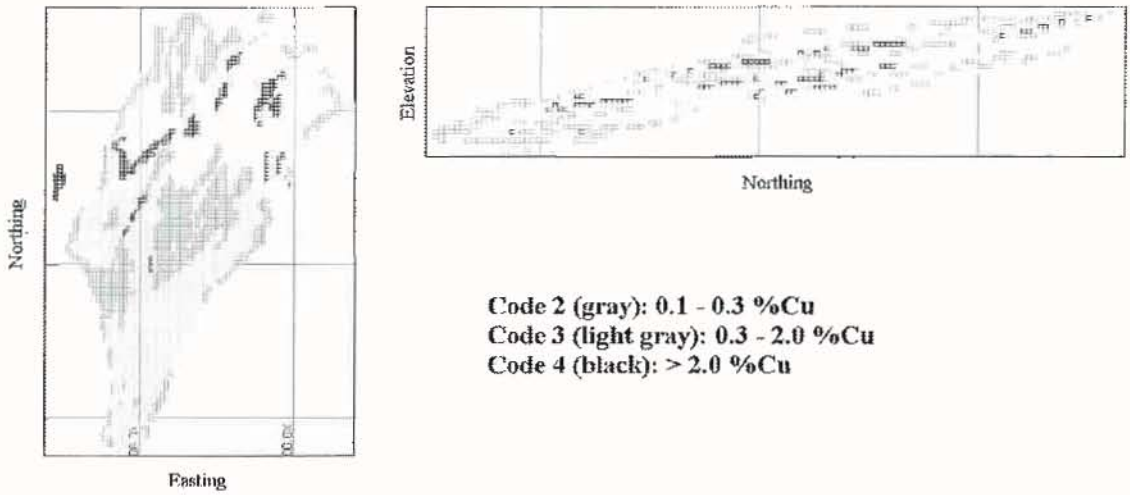


Figure 4. Plan and section showing estimation domains—exotic copper deposit

- Exploratory data analysis and variography for the element of interest in each deposit.
- Block model estimation by ordinary kriging. The following runs were made for each deposit:
  - *Kriging 1*: Estimation with minimum or no conditional bias, achieved using 8 octants in the search and between 2 and 3 composites per octant.
  - *Kriging 2*: Estimation with minimum or no conditional bias achieved using 8 octants in the search and between 2 and 6 composites per octant. This model is over-smoothed. No attempts were made to correct for such a smoothing effect, although post-processing techniques have recently been proposed for this purpose (Assibey-Bonsu and Krige, 1999).
  - *Kriging 3*: Estimation with a strong conditional bias achieved using 2 angular sectors in the search and only 2 composites per sector.
  - *Kriging 4*: Estimation with intermediate conditional bias achieved using 4 quadrants in the search and only 2 composites per quadrant.
- Conditional Gaussian simulation using the turning bands method. Eleven point-support simulations were carried out for each deposit. These were ranked according to the total metal content. The median simulation was accepted and averaged into blocks of the same size as those used for kriging. This model was considered as the 'real block model'.
- Statistical validation of the simulation and kriged models.
- Analysis and quantification of the conditional bias by means of scatter plots between the 'real' and estimated block grades.
- Calculations of optimum ultimate pit designs for the 'real' and estimated block models using the Lerchs-Grossmann algorithm. These pits were not made operational by smoothing or designing ramps so that results would not be distorted.
- Final comparisons included the following aspects:
  - Pit volumes indicating total ore and waste removed.
  - Grade-tonnage curves for final pits and their phases as well as comparison against the 'real' case.
  - Estimation of the Net Present Value of Profits (NPV) generated by the four estimates and comparisons against the 'real case'. It is worth mentioning that, for each kriged block model, the NPV was calculated using the estimated grades; for this reason, it does not reflect what will actually be achieved in the exploitation stage, but what is *expected* to be achieved according to the block model.
  - Analyses of other factors that act in conjunction with the conditional bias and have a bearing in the final results, such as cut-off grade, geometry and depth of the orebody.

Detailed description of composites and block models

For the *gold porphyry*, a total of 14,252 five-m composites were available. These corresponded to a total of 121 diamond drillholes bored in an approximate 50 m by 50 m grid. Composite data included gold grade as well as a rock type code (porphyry or schist). The total volume drilled was of approximately 600 m by 800 m on surface and 1000 m in depth. Total tonnage amounted to approximately 1,300 million tonnes.

The block model consisted of 45 rows, 45 columns and 92 levels. Blocks used were 20 m by 20 m by 10 m high. The model was rotated 49.33 degrees to the north-east in order to coincide with the interpreted sections. The 186,300 total blocks were coded according to the rock type (1 = porphyry, 2 = schist).

The exotic copper deposit consists of a manto striking N50E and dipping 11 to 15 degrees to the NW. Exploration was carried out by means of an approximate 70 m by 70 m grid of diamond and reverse circulation drillholes. A total of 7,037 ten-m composites were available for analyses. The total volume covered by drilling was of the order of 3,200 m by 1,700 m by 395 m in depth and the total mineralized tonnage amounted to some 4,900 million tonnes.

The block model consisted of 140 rows, 160 columns and 40 levels. Blocks used were 12.5 m by 12.5 m by 10 m high. The 986,000 blocks were coded according to the estimation domain. These were 1 = waste, 2 = (0.1% to 0.3%) total copper, 3= (0.3% to 2.0%) total copper and 4 = (greater than 2.0%) total copper. These domains were hand-interpreted in sections considering the grade distribution as

well as the geology. Three-dimensional solids were then constructed by means of tie lines. The estimation was finally carried out within these domains.

Geostatistical evaluation

Gold porphyry

Results of the exploratory data analysis are shown in Table I.

Experimental and fitted variograms for each rock type are shown in Figure 5 and summarized in the following equations, in which the distances in parenthesis refer to the ranges in the main anisotropy directions:

Code 1: $\gamma_{Cod\,1}(h) = 2.0 + 2.8\ sph$ (30 m, 30 m, 30 m) + 2.3 *sph* (70 m, 70 m, 240 m) + 5.5 cibic (∞, ∞, 270 m)

Code 2: $\gamma_{Cod\,2}(h) = 0.2 + 0.35\ sph$ (130 m, 80 m, 390 m) + 0.35 *sph* (260 m, 240 m, 480 m) + 0.12 *sph* (∞, 150 m, 390 m) + 0.25 *sph* (∞, 300 m, ∞).

Quantification of conditional bias by means of fitted linear regression models to 'true block values' as a function of estimated (kriged) block values for all rock types combined is shown in Table II. As an example, Figure 6 shows the regression line and the first bisector for the four kriging runs done on the porphyry rock-type. As expected, the first two kriging runs have acceptable conditional bias, the third one shows a severe effect and the fourth one is worse than the first two but better than the third one.

Exotic copper deposit

Statistical analysis results are shown in Table III.

Empirical and fitted variograms for each estimation domain, excluding waste, are shown in Figure 7 and summarized in the following equations, in which the distances in parenthesis refer to the ranges in the main anisotropy directions:

Code 2: $\gamma_{Cod\,2}(h) = 0.005 + 0.025\ sph$ (250 m, 250 m, 40m) + 0.03 *sph* (100 m, 100 m, 40 m)

Code 3: $\gamma_{Cod\,3}(h) = 0.16 + 0.106\ sph$ (140 m, 140 m, 55 m) + 0.075 *sph* (90 m, 90 m, 35 m)

Code 4: $\gamma_{Cod\,4}(h) = 0.4 + 0.54\ sph$ (60 m, 60 m, 20 m).

Conditional bias results for domains 2 to 4 combined are shown in Table IV. As an example, Figure 8 shows the results for domain 3, which is the main ore producer. Results are similar to those obtained for the gold porphyry.

Open pit optimization

Gold porphyry

The following economic parameters were used for the open pit optimization.

Base gold price:	$300/ oz
Gold recovery:	95% (agitation leaching)
Processing cost:	$12/tonne of ore
Sales cost:	$12/oz
Mining cost:	$0.65/tonne of material
Variable transportation cost:	$0.008/10 m
Discount rate:	10%
Pit angle:	50 degrees for all sectors

Pit optimizations were carried out using the Lerchs-Grossmann algorithm, which allows the generation of phases or nested pits for increasing metal prices and mining costs. The gold prices used ranged between $120/0z and

Table I
Clustered and declustered statistics—gold porphyry

Statistic	Code 1: Porphyry		Code 2: Schist		
	Clustered	Declustered	Clustered	Declustered	Totals
No. of Composites	3,083	3,083	11,169	11,169	14,252
Minimum [g/t]	0.036	0.036	0.020	0.020	0.020
Maximum [g/t]	51.352	51.352	27.884	27.884	51.352
Mean [g/t]	2.303	1.496	0.729	0.701	1.069
Coeff. of variation	1.106	1.219	1.282	1.314	1.481
Median [g/t]	1.640	0.957	0.436	0.412	0.552

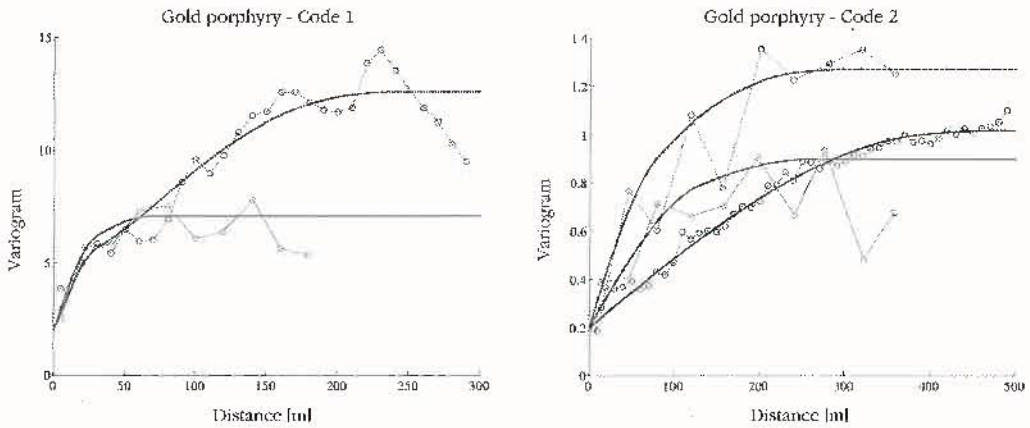


Figure 5. Empirical and fitted variograms for the gold porphyry along the main directions of anisotropy

Table II
Conditional bias results for the gold porphyry (porphyry + schist)

Kriging Run	Regression Line	ρ^2	Estimated Mean	Real Mean	No. of Blocks
Kriging 1	Y = 0.94X + 0.08	0.836	0.612	0.653	86,485
Kriging 2	Y = 0.94X + 0.07	0.832	0.610	0.653	86,485
Kriging 3	Y = 0.56X + 0.26	0.727	0.620	0.653	86,485
Kriging 4	Y = 0.73X + 0.17	0.794	0.615	0.653	86,485

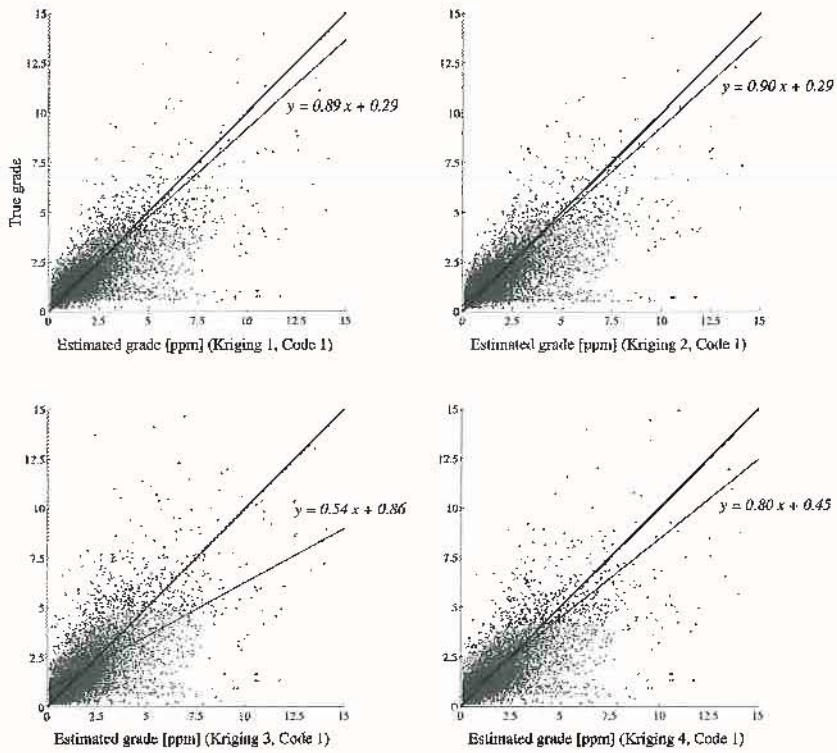


Figure 6. Conditional bias for kriging runs—gold porphyry

Table III
Statistical analysis results—exotic copper deposit

Statistic	Code 1 (Waste)	Code 2	Code 3	Code 4	Total (excluding waste)
Number	3562	1062	2078	265	3405
Minimum [%]	0.000	0.008	0.031	0.192	0.008
Maximum [%]	4.968	2.608	5.040	9.652	9.652
Mean [%]	0.074	0.324	0.930	2.169	0.837
Coeff. of variation	2.391	0.794	0.647	0.448	0.878
Median [%]	0.033	0.264	0.755	2.051	0.604

$390/oz in steps of $45/Oz. Economic evaluations were based on a gold price of $300/oz. A comparison of pit optimization results between the 'real block model' and the four estimations is shown in Table V.

Manto-type exotic copper deposit

The economic parameters used for the pit optimizations were as follows.

Copper price:	$ 0.95/Lb
Copper recovery %:	15.38 * ln (cotal Cu) + 73.966
Processing cost:	$2.65/tonne of ore (Crushing + Heap leaching)
SX-EW cost:	$0.0844/lb
Mining cost:	$0.65/tonne of material
Variable transportation cost:	$0.008/10 m
Discount rate:	10%
Pit Angles:	50 for north-west and 55 for south-east

Copper prices ranging from 0.38 $/lb to 0.95 $/lb in steps of 0.14 $/lb were used to generate pit phases for this case

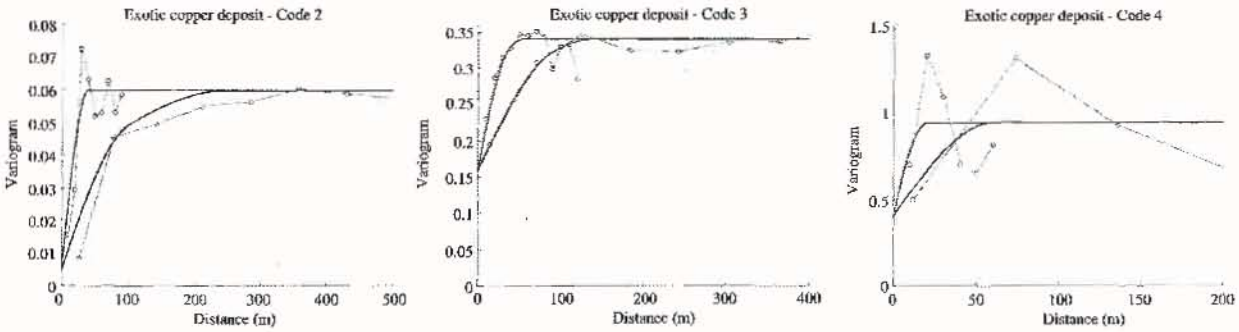


Figure 7. Empirical and fitted variograms for the exotic copper deposit along the main directions of anisotropy

Table IV
Conditional bias results—exotic copper deposit (domains 2 to 4)

Kriging Run	Linear Regression	ρ^2	Estimated Mean	Real Mean	No. of Blocks
Kriging 1	Y = 1.00X + 0.01	0.842	0.750	0.762	97,095
Kriging 2	Y = 1.02X + 0.00	0.841	0.746	0.762	97,095
Kriging 3	Y = 0.68X + 0.25	0.746	0.753	0.762	97,095
Kriging 4	Y = 0.88X + 0.11	0.814	0.747	0.762	97,095

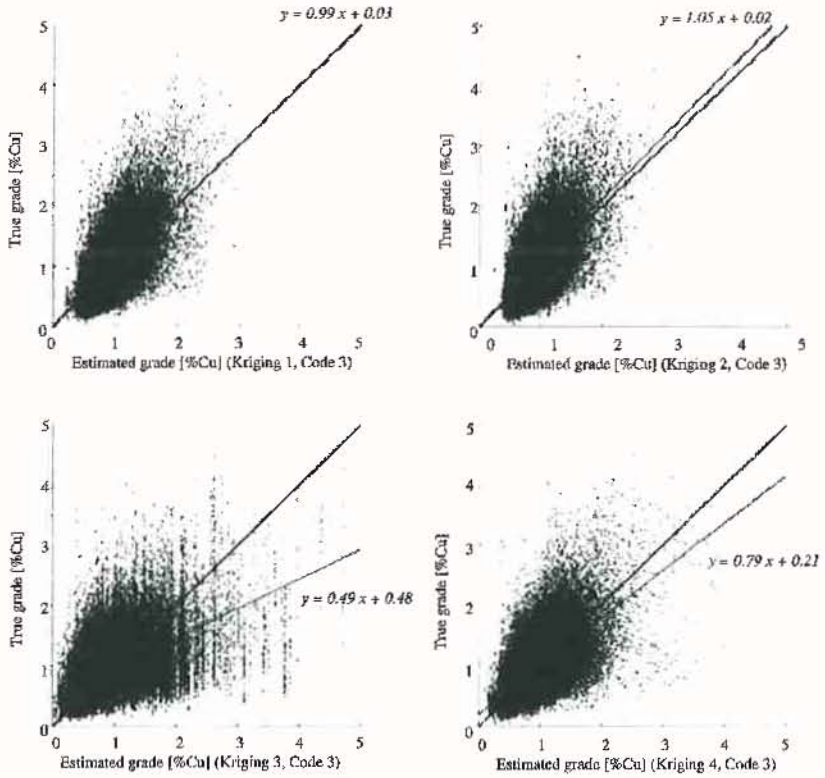


Figure 8. Conditional bias for kriging runs—exotic copper deposit

Table V
Pit optimization comparative results—gold porphyry

	Price [US$/oz]	Total tonnage [kt]	Ore tonnage [kt]	Ore/waste ratio	Head grade [g/t]	Recoverable fine gold [koz]	NPV [kUS$]	Cutoff g/t
Reality	300	16795	11237	0.49	2.737	942	98692	1.431
Kriging 1	300	18494	13841	0.34	2.581	1093	105923	1.431
Kriging 2	300	18441	13376	0.38	2.457	1004	88986	1.431
Kriging 3	300	41679	17679	1.36	2.706	1467	130359	1.431
Kriging 4	300	19278	13722	0.40	2.644	1106	109915	1.431

Table VI
Pit optimization comparative results—exotic copper deposit

	Price [US$/lb]	Total tonnage [kt]	Ore tonnage [kt]	Ore/waste ratio	Head grade [g/t]	Recoverable fine copper [klb]	NPV [kUS$]	Cutoff [%]
Reality	0.95	512186	231003	1.22	0.952	3678362	702019	0.28
Kriging 1	0.95	506406	238070	1.13	0.905	3544421	677475	0.28
Kriging 2	0.95	505834	238869	1.12	0.898	3516536	661883	0.28
Kriging 3	0.95	515046	228606	1.25	0.954	3662319	739898	0.28
Kriging 4	0.95	508803	234742	1.17	0.917	3560034	697394	0.28

study. Economic evaluations were based on 0.95 $/lb. Comparative results are shown in Table VI.

Conclusions

This study leads to the following conclusions.

- Conditional bias causes underestimation in low-estimated grade areas and overestimation in high-estimated grade areas, thus alters the distribution of estimated grades relative to the real grades. In the two cases considered the overestimation of high-estimated grades, in the worst case, was responsible for waste over-extraction that amounted to 24,884 kt (148.2%) and 2,860 kt (0.56%) for the gold and copper deposits respectively. This effect is much more pronounced in the gold porphyry due to its geometry.
- Conditional bias produces an overestimation of the project's NPV. In the case of the gold porphyry, the 'real' figure was of US$ 98.692 million, the model with largest conditional bias overestimated it by US$ 31.667 million (32.1%) while the model with moderate conditional bias caused an overestimation of 11.4%. For the exotic copper deposit, the 'real' NPV was US$ 702.019 million; the model with largest conditional bias overestimated it by US$ 37.879 million (5.4%).
- The smoothing effect of kriging, with very little or no conditional bias, underestimated the project's NPV. This underestimation was of the order of 9.8% and 5.7% for the gold porphyry and the manto-type copper deposits respectively.
- Other factors that influence the optimum open pit design and to some extent control the effect of conditional bias are: the cut-off grade, the orebody geometry and distribution of grades within it and the amount of overburden.
- Conditionally unbiased kriging models produce open pits, which are different to the 'true' ones because of the smoothing effect of this estimation technique.

Acknowledegments

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