# Statistical Valuation of Diamondiferous Deposits

By H. S. SICHEL,* D.Sc.

SYNOPSIS

The valuation of diamondiferous deposits depends on (i) the density distribution, (ii) the size distribution of stones and (iii) the selling price structure prevailing at a particular time.

In the density distribution the stochastic variable is defined as the number of individual stones found per sample unit irrespective of the stone size. The density distribution is discrete and allows for barren sample units. As diamonds often occur in clusters, a Poisson mixture distribution is derived which is entirely new, as existing discrete distributions cannot represent the stone number frequencies found in nature.

For the sizes of diamonds, which are measured by weight, a new compound logarithmic normal distribution is developed.

Combination of (i), (ii) and (iii) leads to a meaningful appraisal of diamondiferous ground expressed in monetary value per production unit of mining.

Finally, confidence limits are calculated which allow for inferences from sampling to recovery.

## INTRODUCTION

In the marine deposits of South West Africa diamonds occur in clusters formed by natural traps such as gullies, potholes and storm beaches. Diamonds, referred to in this paper also as *stones*, constitute discrete units of varying size (weights) measured in carats per stone. To obtain a meaningful valuation of diamondiferous mining deposits one must take into account at least three different aspects:

(i) the *stone density* per unit volume or per unit area in the horizontal plane, that is, the number of stones irrespective of size, colour, shape and quality in a given mining unit,

(ii) the size by weight distribution of individual stones, and

(iii) the revenue structure expressed either as monetary value per stone or money per carat. The latter concepts are taken as functions of stone size and it is assumed that individual diamonds are of *average* shape, colour and quality.

By suitable combination of (i), (ii) and (iii), it is possible to arrive at the monetary revenue from a unit mining volume or area, which is expressed as, for example, rands or dollars per m$^3$ or per m$^2$. If one knows how much it costs to mine and treat one cubic metre of ground, then it is easy to see whether a particular section of the mine is payable or unpayable depending on whether the expected revenue rate exceeds or does not exceed this 'pay limit'.

## THE STONE DENSITY DISTRIBUTION

A sample unit may be a section of a trench, a pit or a borehole expressed in terms of volume or cross-sectional area in the horizontal plane. The number of stones discovered in a sample of gravel may be $r = 0, 1, 2, 3 \ldots$ For the sample units in vogue the majority of occurrences are zero which means that a large number of barren units exist *within* a diamondiferous deposit. On the other hand, there is the extremely rare occurrence of several hundred stones being present in one sample unit where this unit is situated over a natural diamond trap such as a deep pothole.

Altogether the typical shape of the discrete frequency distribution of the number of diamonds per sample section is that of the reversed J-shape type with a very long upper tail. In exceptional cases the distribution has a maximum at $r = 1, 2$ or $3$ stones per section with a long upper tail similar to that in the general case. Conventional discrete distribution models such as the negative binomial do not fit the observed field data at all, due primarily to the large number of barren sections and the unusual long tail of the observed frequency distributions. As the stones have been deposited in clusters and as the within-cluster variations seem to be fairly consistent, one thinks immediately of a mixture of Poisson distributions with widely differing *average* stone densities per cluster. Such a compound Poisson probability law should have a mixture distribution capable of much skewer forms than the Pearson Type III which leads to the Negative Binomial distribution and which had proved totally inadequate in representing our field observations.

A compound Poisson distribution is defined as

$$\phi(r) = \int_0^\infty p(r|\lambda)\, f(\lambda)\, d\lambda, \qquad \ldots\ldots\ldots\ldots (1)$$

where

$$p(r|\lambda) = \frac{e^{-\lambda}\lambda^r}{r!} \qquad \ldots\ldots\ldots\ldots (2)$$

and $f(\lambda)$ is the mixing distribution.

A very flexible mixing distribution is

$$f(\lambda) = \frac{1}{2}\left(\frac{2\sqrt{1-\theta}}{\alpha\theta}\right)^{\gamma} \frac{1}{K_\gamma(\alpha\sqrt{1-\theta})}\lambda^{\gamma-1}\exp\left\{-\left(\frac{1}{\theta}-1\right)\lambda - \frac{\alpha^2\theta}{4\lambda}\right\} \qquad \ldots\ldots\ldots (3)$$

where $-\infty < \gamma < \infty$, $0 < \theta < 1$ and $\alpha > 0$ are the three parameters. Function $f(\lambda)$ is intermediate to Pearson's Type III and Type V distributions and in the limit it can assume either form. $K_\gamma(\,.\,)$ is the modified Bessel function of the second kind of order $\gamma$.

Substitution of equations (2) and (3) into (1) gives the new family of discrete frequency distributions

$$\phi(r) = \frac{(\sqrt{1-\theta})^{\gamma}}{K_\gamma(\alpha\sqrt{1-\theta})}\frac{(\alpha\theta/2)^r}{r!}K_{r+\gamma}(\alpha), \qquad \ldots\ldots (4)$$

where $r = 0, 1, 2 \ldots \infty$.

If we write $\beta = \alpha\theta/2$, we obtain an alternative form of (4),

$$\phi(r) = \frac{(\sqrt{1-2\beta/\alpha})^{\gamma}}{K_\gamma(\alpha\sqrt{1-2\beta/\alpha})}\frac{\beta^r}{r!}K_{r+\gamma}(\alpha) \qquad \ldots\ldots (5)$$

*Acting Director, Graduate School of Business Administration, University of the Witwatersrand, Johannesburg.

This family of distributions was first published by Sichel (1971) and encompasses most of the better known discrete distributions such as the Poisson, negative binomial, geometric, Fisher's logarithmic, Yule, Good, Waring and Riemann distributions. If in equations (4) or (5) $\gamma$ is made negative, an entirely new set of discrete distributions is generated. For the stone density, $\gamma = -\frac{1}{2}$ is a good empirical choice and equations (4) and (5) become, respectively,

$$\phi(r) = \sqrt{\frac{2\alpha}{\pi}} \exp(\alpha\sqrt{1-\theta}) \frac{(\alpha\theta/2)^r}{r!} K_{r-1/2}(\alpha) \quad . \ . \ . \ . \ (6)$$

$$= \sqrt{\frac{2\alpha}{\pi}} \exp(\alpha\sqrt{1-2\beta/\alpha}) \frac{\beta^r}{r!} K_{r-1/2}(\alpha) \quad . \ . \ . \ . \ .(7)$$

The above distribution has two parameters which have to be estimated from the observed data, that is, $\alpha > 0$ and $0 < \theta < 1$ or $\alpha > 0$ and $\beta > 0$.

Parameter $\theta$ characterizes the *tail* of the distribution. The nearer it is to unity, the longer the tail. For $\theta = 0$ the distribution becomes a Poisson and hence it has a short tail. Parameter $\beta$ characterizes the frequencies at the *start* of the distribution. If $\beta < 1$ we deal with a reverse J-shaped curve and if $\beta > 1$ we have a unimodal distribution. There are two ways to estimate parameters $\alpha$ and $\beta$. In the first method we equate the observed proportional frequency $\hat{\phi}$ (0) for the barren units ($r = 0$) and the observed *average* number of stones per sample unit $\bar{r}$ (including the barren units) to the zero proportion and the mean in the population. This gives the estimates

$$\hat{\alpha} = [-\ln\hat{\phi}(0)] \left[1 - \frac{\ln\hat{\phi}(0)}{2[\bar{r} + \ln\hat{\phi}(0)]}\right] \quad . \ . \ . \ . \ . \ .(8)$$

and

$$\hat{\beta} = \frac{\bar{r}}{\hat{\alpha}}\left(\sqrt{\bar{r}^2 + \hat{\alpha}^2} - \bar{r}\right) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ .(9)$$

from which

$$\hat{\theta} = \frac{2\hat{\beta}}{\hat{\alpha}}. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ .(10)$$

This method is fairly efficient provided a large proportion of the observed frequencies is in the zero class.

The *maximum likelihood* method yields

$$\frac{1}{n}\sum_{i=1}^{n} \frac{K_{r-3/2}(\hat{\alpha})}{K_{r-1/2}(\hat{\alpha})} - \frac{1}{\hat{\alpha}}(\sqrt{\bar{r}^2 + \hat{\alpha}^2} - \bar{r} + 1) = 0, \quad .(11)$$

which is an equation containing parameter estimate $\hat{\alpha}$ only. With the help of tables of the function

$$\Lambda(r|\hat{\alpha}) = \frac{K_{r-3/2}(\hat{\alpha})}{K_{r-1/2}(\hat{\alpha})}$$

prepared by the author, it is relatively simple to find $\hat{\alpha}$ from equation (11). Equations (9) and (10) will then give estimates $\hat{\beta}$ and $\hat{\theta}$. The theoretical (expected) frequencies are worked out numerically from the recurrence formula

$$\phi(r) = \frac{\beta}{\alpha}\left(\frac{2r-3}{r}\right)\phi(r-1) + \frac{\beta^2}{r(r-1)}\phi(r-2). \quad . \ .(12)$$

All one has to know for the start of computations in (12) is

$$\phi(0) = \exp[-\alpha(1 - \sqrt{1-\theta})] \quad . \ . \ . \ . \ . \ .(13)$$

and $\phi(1) = \beta\phi(0)$. $\quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ .(14)$

The conventional $\chi^2$-test will indicate whether the observed frequencies are in agreement with the theoretical stone density distribution in equation (6) or (7).

## ESTIMATION OF THE AVERAGE STONE DENSITY PER SAMPLE UNIT

The population mean, that is, the average number of stones per sample unit, is given by

$$D = E(r) = \frac{\beta}{\sqrt{1-\theta}} = \frac{\alpha\theta}{2\sqrt{1-\theta}}. \quad . \ . \ . \ . \ . \ . \ .(15)$$

The maximum likelihood estimate for this population mean is the simple arithmetic mean in the (statistical) sample of $n$ (physical) sample units. Thus

$$\hat{D} = \frac{1}{n}\sum_{i=1}^{n} r_i = \bar{r}. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ .(16)$$

The exact sampling distribution of the arithmetic mean $\bar{r}$ was shown to be again equation (7) with parameters $\alpha$ and $\beta$ being replaced by $n\alpha$ and $n\beta$ and the variable $r$ being replaced by $r/n$ (Sichel, 1971).

The Pearsonian shape coefficients of the sampling distribution of the mean are

$$B_1(\bar{r}_n) = \frac{1}{nD}\frac{[2(2-\theta) + \theta^2]^2}{2(1-\theta)(2-\theta)^3} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ .(17)$$

and

$$B_2(\bar{r}_n) = 3 + \frac{1}{nD}\frac{8 + 4\theta + 4\theta^2 - \theta^3}{2(1-\theta)(2-\theta)^2}. \quad . \ . \ . \ . \ .(18)$$

Numerical evaluation of formulae (17) and (18) for values of $\theta$ and $D$ as they apply to marine deposits along the South West African coast indicates that the distribution of the average stone density estimate $\hat{D}$ is highly skewed for a (statistical) sample size of 100 units and does not sufficiently approach 'normality' even for 1 000 sample units. From equations (17) and (18) and the percentage points given by Johnson, *et al* (1963), the author has drawn up a table for 95 per cent central confidence limits for the population stone density $D$ making the assumption that $\theta = 2\beta/\alpha$ is 0,97. The latter is a very typical constant for the marine deposits discussed here.

## THE STONE SIZE DISTRIBUTION

According to one geological theory diamonds were transported from inland down the Orange River and then by sea currents up (north) and down (south) along the coast. The stones were deposited on the beaches by marine action and the combination of marine currents and waves and of geological diamond traps on land was responsible for the continuous sorting of the stone sizes by weight. One would expect then that the diamond traps would catch the larger stones preferentially and that the average stone weights would decrease as distance from the river mouth increased. Intensive sampling has actually proved this hypothesis to be correct with some notable exceptions due to 'reworking, wash-outs and secondary deposits in other river beds'.

For relatively small mining areas, and particularly for a single-trench unit, the size distributions appear to follow the two-parameter lognormal law. This is best seen from a straight line plot of the observed cumulative frequency distributions of sizes on log-probability paper.

In areas near the river mouth the diamonds have relatively large mean sizes coupled with high variability, whereas in mining blocks far removed from the Orange River the diamonds have low mean sizes and low variability. Geologically speaking, diamonds are well sorted according to size in these areas. A fair number of small stones are found in all areas irrespective of whether the average size is small or large.

An interesting feature is the departure from log-normality of diamond size as the mining area, from which an observed stone weight distribution originates, is increased. In particular, the size distribution of diamonds obtained from recovery plants, which serve large mining areas, is distinctly hyper-lognormal in as much as the cumulative frequency distribution of the recovered product curves strongly, either convex downwards or even S-shaped, on a log-probability plot.

In general, it is very important to estimate and forecast the upper tail of the size spectrum as accurately as possible. Because of the rarity of large diamonds, the price structure is non-linear with respect to size and a small number percentage of the larger stones contribute the major proportion of total revenue.

A general mathematical model for the size distribution of diamonds found in larger mining areas or obtained in the recovery plants, must take cognisance of the geological mode of deposition as discussed previously. From field observations we do know that in localized areas on the same beach horizon stone sizes are well represented by the two-parameter lognormal distribution, that is,

$$f(z) = \frac{1}{\sqrt{2\pi}\,\sigma z} \exp[-(\ln z - \xi)^2/2\sigma^2], \quad \ldots\ldots (19)$$

where $z > 0$ is the diamond weight in carats/stone and $-\infty < \xi < +\infty$ and $\sigma > 0$ are the two parameters.

It is important to note that the different beaches in the marine deposits are of widely varying geological ages and hence, for equation (19) to hold, one has to stay within one particular beach.

If observations are obtained from *different* localities within the *same* beach or from *different* overlapping beaches within the *same* locality one must expect parameters $\xi$ and $\sigma^2$ to be different, resulting in a *mixture* of lognormal distributions which, in most cases, will no longer follow the lognormal pattern of equation (19). From the remarks with respect to the sorting action of the sea and the trapping by the bed rock formation on land, it is not unreasonable to assume that, *within* a particular beach deposit, parameters $\xi$ and $\sigma^2$ are both functions of the distance $d$ from the mouth of the Orange River. Furthermore, in view of sorting, $\xi$ and $\sigma^2$ should be related inversely to $d$ and should be related directly to each other. As a first assumption we may formalize the latter proposition as

$$\xi = a + b\sigma^2, \quad \ldots\ldots (20)$$

where $a$ and $b$ are two constants.

If we make the transformation $x = \ln z$ in (19) and substitute (20) for parameter $\xi$, we obtain

$$\psi(x|\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp[-(x - a - b\sigma^2)^2/2\sigma^2], \quad \ldots (21)$$

where $-\infty < x < +\infty$ and $a$, $b$ and $\sigma^2$ are parameters.

If we assume the mixing distribution of the logarithmic variance $\sigma^2$ to be distributed according to a Pearson Type III function

$$\delta(\sigma^2) = \frac{\rho^\tau}{\Gamma(\tau)} (\sigma^2)^{\tau-1} \exp[-\rho\sigma^2], \quad \ldots\ldots (22)$$

where $\rho > 0$ and $\tau > 0$ are two parameters and $\sigma^2 \geqslant 0$, then the compound normal distribution of the natural logarithms of individual diamond sizes may be obtained from

$$\Omega(x) = \int_0^\infty \psi(x|\sigma^2)\, \delta(\sigma^2)\, d\sigma^2. \quad \ldots\ldots (23)$$

Substitution of (21) and (22) into (23) and integration gives

$$\Omega(x) = \sqrt{\frac{2(2\rho + b^2)}{\pi}} \frac{1}{\Gamma(\tau)} \left(\frac{\rho}{2\rho + b^2}\right)^\tau \times \exp[b(x-a)] k_{\tau-1/2} (\sqrt{2\rho + b^2}\,|x-a|) \quad \ldots (24)$$

where $x = \ln z$ $(-\infty < x < +\infty)$ and $\rho$, $\tau$, $a$ and $b$ are parameters. $k_{\tau-1/2}(\cdot)$ is the auxiliary modified Bessel function of the second kind of order $\tau - 1/2$.

Introduce new parameters

$$c = \frac{b^2}{2\rho + b^2}, \quad s = \sqrt{2\rho + b^2}, \quad \text{and } \nu = \tau - 1/2.$$

Consequently

$$1 - c = \frac{2\rho}{2\rho + b^2} \quad \text{and } \sqrt{c}\, s = b.$$

Equation (24) becomes

$$\Omega(x) = \frac{s(1-c)^{\nu+1/2}}{2^\nu \sqrt{\pi}\, \Gamma(\nu + 1/2)} \exp[\sqrt{c}\, s(x-a)]\, k_\nu(s|x-a|), \quad .(25)$$

where $-\infty < x < +\infty$.

This compound normal distribution has four parameters, that is,

$a$, for location, where $-\infty < a < +\infty$,
$s$, for spread, where $s > 0$,
$c$, for skewness, where $0 \leqslant c < 1$,
and $\nu$, for kurtosis, where $\nu > -1/2$.

If $\sqrt{c}$ is positive, the distribution of $x$ is positively skew and if $\sqrt{c}$ is negative the distribution is negatively skew. For $\sqrt{c} = 0$ we have a symmetrical distribution which is leptokurtic for parameter $\nu$ being small. As $\nu \to \infty$, we approach normality. If we write $v = s(x - a)$ then equation (25) becomes

$$\Omega(x) = s(1-c)^{\nu+1/2} \exp[\sqrt{c}\, v]\, T_\nu(|v|), \quad \ldots (26)$$

$$\text{where } T_\nu(|v|) = \frac{1}{2^\nu \sqrt{\pi}\, \Gamma(\nu+1/2)} k_\nu(|v|), \quad \ldots\ldots (27)$$

$$v = s(x-a) \text{ and } |v| = |s(x-a)| = s|x-a|.$$

As the $T_\nu(|v|)$ function in equation (27) was tabulated for ordinates by Elderton (1929) and for the probability integral by Pearson, *et al* (1932), it is relatively simple to calculate ordinates and probabilities numerically for $\Omega(x)$ in equation (26).

By making the transformation $z = \exp(x)$, where $z$ is the weight of a diamond in carats/stone, we obtain from (25)

$$p(z) = \frac{s(1-c)^{\nu+1/2}}{2^\nu \sqrt{\pi}\, \Gamma(\nu+1/2)} \frac{1}{z} \exp[\sqrt{c}\, s(\ln z - a)]\, k_\nu(s|\ln z - a|) \quad \ldots\ldots (28)$$

for $z \geqslant 0$.

Equation (28) may be called the compound lognormal distribution with parameters $a$, $s$, $c$ and $\nu$. It arises from a mixture of two-parameter lognormal distributions with different logarithmic variances $\sigma^2$ and a linear relationship between the means $\xi$ and variances $\sigma^2$ of the logarithms of the observations. As such it has applications far beyond the framework of the present investigation.

While the size distribution of the stones (in carats/stone) is very skew (equation (28) ), the natural logarithms of diamond weights, as given in equation (25), are only moderately and positively skew. Hence moment estimates for the parameters of the latter distribution should be fairly efficient.

The first four central moments of the compound normal distribution in equation (25) are as follows:

$$\mu_1'(x) = a + \frac{(2\nu + 1)\sqrt{c}}{s\,(1 - c)}, \qquad \ldots\ldots\ldots\ldots (29)$$

$$\mu_2\,(x) = \frac{(2\nu + 1)\,(1 + c)}{s^2(1 - c)^2}, \qquad \ldots\ldots\ldots\ldots (30)$$

$$\mu_3\,(x) = \frac{2(2\nu + 1)\,(3 + c)\sqrt{c}}{s^3(1 - c)^3}, \qquad \ldots\ldots\ldots (31)$$

$$\mu_4\,(x) = 3(2\nu + 1)\,\frac{(2\nu + 3)\,(1 + c^2) + 2(2\nu + 7)c}{s^4(1 - c)^4}. \quad .. (32)$$

The Pearsonian shape coefficients are

$$B_1(x) = \frac{\mu_3^2(x)}{\mu_2^3(x)} = \frac{4c(3 + c)^2}{(2\nu + 1)(1 + c)^3} \qquad \ldots\ldots (33)$$

and

$$B_2(x) = \frac{\mu_4(x)}{\mu_2^2(x)} = 3 + \frac{6(c^2 + 6c + 1)}{(2\nu + 1)(1 + c)^2}. \qquad \ldots (34)$$

The compound normal distribution of equation (25) is applicable if, and only if,

$$B_2(x) \geqslant \tfrac{1}{2}\,[3B_1\,(x) + 6]. \qquad \ldots\ldots\ldots\ldots (35)$$

## ESTIMATION OF PARAMETERS OF THE COMPOUND LOGNORMAL DISTRIBUTION

From the natural logarithms of the observed diamond weights $\hat{B}_1(x)$ and $\hat{B}_2(x)$ are calculated, from which

$$\hat{q} = \frac{3\hat{B}_1}{2\hat{B}_2 - 3\hat{B}_1 - 6}. \qquad \ldots\ldots\ldots\ldots\ldots (36)$$

Parameter $c$ is estimated by iteration from

$$\hat{q} = \hat{c}\left(\frac{3 + \hat{c}}{1 - \hat{c}}\right)^2. \qquad \ldots\ldots\ldots\ldots\ldots (37)$$

Next, one calculates

$$\hat{\nu} = \frac{2\hat{c}(3 + \hat{c})^2}{\hat{B}_1(1 + \hat{c})^3} - \frac{1}{2}, \qquad \ldots\ldots\ldots\ldots\ldots (38)$$

and

$$\hat{s} = \sqrt{\frac{(2\hat{\nu} + 1)\,(1 + \hat{c})}{\hat{\mu}_2(1 - \hat{c})^2}}, \qquad \ldots\ldots\ldots\ldots (39)$$

where $\hat{\mu}_2$ is the sample variance of the natural logarithms of the observed stone sizes.

Finally, we have

$$\hat{a} = \hat{\mu}_1' - \frac{(2\hat{\nu} + 1)\sqrt{\hat{c}}}{\hat{s}(1 - \hat{c})}, \qquad \ldots\ldots\ldots\ldots (40)$$

where $\hat{\mu}_1'$ is the sample mean of the natural logarithms of the observed stone sizes.

If we wish to estimate the average stone size (in carats/stone) from the parameter estimates, we use

$$\hat{\mu}_1'(z) = \left[\frac{1 - \hat{c}}{1 - (1/\hat{s} + \sqrt{\hat{c}})^2}\right]^{\hat{\nu}+1/2} \exp\,(\hat{a}). \qquad \ldots (41)$$

From estimation exercises for large mining areas in South West Africa, it would appear that $\nu = 4$ could be used as a parameter known *a priori*. In such a case equation (38) can be solved for $\hat{c}$ by putting $\hat{\nu} = 4$. The shape coefficient $\hat{B}_2$ need not be determined at all. In the remaining calculations one proceeds as before.

## THE GRADE OF DEPOSITS

Mining engineers express the 'grade' of a diamond deposit in terms of total carats per production unit such as *carats per hundred load* or *carats/m²* or *carats/m³*. For the marine deposits of South West Africa the last two grade definitions are in vogue. Both serve a useful purpose.

Grade expressed as carats/m² appears to be more fundamental and natural as it is related to the mode of geological deposition which must have been more bi- than tri-dimensional when the action of the sea over a beach is considered. The third dimension, depths of gullies and potholes in the bedrock formations, is independent and coincidental.

However, a mining plant treats a volume of gravel and is unaware of the area from which this gravel was gathered. Furthermore, the cost of mining to a depth of 3 m is higher than that of mining to a depth of 1 m, even if the planar grades are identical.

Mathematically, the true unknown planar grade is defined as

$$g_A = \frac{D\theta_o}{A} \qquad \text{(carats/m}^2\text{)}$$

where $D$ is the average density, that is, the *average* number of stones per sample unit, such as trench sections, pits or boreholes. Parameter $\theta_o$ is the *average* diamond size in carats/stone and $A$ is the cross-sectional *area* of a sample unit. In a particular prospecting campaign, $A$ is usually constant. Correspondingly, the volumetric grade is defined as

$$g_V = \frac{D\theta_o}{\bar{V}} = \frac{D\theta_o}{A\bar{d}} \qquad \text{(carats/m}^3\text{)}$$

where $\bar{V}$ is the *average* volume of sample units and $\bar{d}$ is the *average* depth of trenching, pitting or coring.

It is obvious from these definitions that an identical mining grade may be obtained in totally different circumstances. For example, $D\theta_o$, the number of carats in an average sample unit, may be made up by numerous small stones or by a single large stone. Yet the financial reward will be larger for the latter if shape, colour and quality are the same.

Even if $D$ and $\theta_o$ are the same for two cases, the monetary reward will be higher if the spread of individual stone weights around the average size $\theta_o$ is larger.

From the foregoing it should be clear that the mining 'grades', as defined for diamondiferous ground, are not above criticism.

## MONETARY REVENUE PER MINING UNIT

For economic decision-making and for forecasting purposes a better concept than the grades discussed in the previous section is the revenue per mining unit such as rands or dollars per m² or per m³.

For a diamond of average shape, colour and quality, its price is dependent on its size. The price structure changes from time to time and is subject to market forces such as supply and demand. Generally speaking, the revenue for a stone increases non-linearly with stone weight although shape, colour and quality may change the size-revenue relationship drastically.

If, for average conditions, a diamond of weight $z$ carats per stone fetches $\omega(z)$ *rand per carat*, then the price per stone is

$$\eta(z) = z\omega(z) \quad . \quad \text{(rand/stone)} \qquad \ldots\ldots\ldots\ldots (42)$$

The average revenue per carat-stone size relationship may be expressed as a polynomial, that is,

$$\omega(z) = \sum_{j=0}^{m} a_j z^j \quad \text{(rand/carat)} \qquad \ldots\ldots\ldots\ldots (43)$$

hence

$$\eta(z) = \sum_{j=0}^{m} a_j z^{j+1} \quad \text{(rand/stone)} \qquad \ldots\ldots\ldots (44)$$

where $a_j$ are the constants which are determined from the price structure and $m$ is the highest order of the polynomial.

If $z_i$ is the weight of the $i$-th stone in a group of $n$ diamonds, then the average rand per stone in this group is

$$\bar{R}_s = \frac{1}{n} \sum_{i=1}^{n} z_i\, \omega(z_i) = \frac{1}{n} \sum_{i=1}^{n} \eta(z_i) \qquad \ldots\ldots\ldots (45)$$

and the average rand per carat in this group of $n$ stones

$$\bar{R}_c = \frac{\bar{R}_S}{\theta_o} . \qquad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

Formulae (42) to (46) are independent of the size distribution of $z$ and hence are applicable for any distribution shape, whether lognormal or not.

If $f(z)$ is the stone size distribution with an average weight of $\theta_o$ carats per stone, then, from equation (46), the average revenue per carat is given by

$$\bar{R}_c = \frac{1}{\theta_o} \int_0^{\infty} z f(z) \omega(z)\, dz \quad , \quad \text{(rand/carat)} \qquad \ldots (47)$$

If we deal with small homogeneous mining areas the stone sizes are distributed according to the two-parameter lognormal distribution as defined in equation (19). In that case the average weight is

$$\theta_o = \exp(\xi + \tfrac{1}{2}\sigma^2) \qquad \text{(carats/stone)}.$$

Now, if we substitute equations (19) and (43) into (47) we obtain, after integration, the average revenue per carat as

$$\bar{R}_c = \sum_{j=0}^{m} a_j \theta_o^{\,j} e^{j(1+j)\sigma^2/2} \qquad \text{(rand/carat)}. \qquad \ldots (48)$$

Formula (48) shows very clearly that in a group of stones of various sizes, the average revenue per carat is dependent on the mean weight $\theta_o$ *and* the (logarithmic) variance $\sigma^2$ of the size distribution. The larger the variability, as measured by $\sigma^2$, the larger the monetary return per carat *even if the average weight $\theta_o$ remains constant.*

Finally, from the previous definitions of grades, one obtains the average revenue per $m^2$ and per $m^3$ as

$$\bar{R}_A = \bar{R}_c g_A = \frac{D}{A} \sum_{j=0}^{m} a_j \theta_o^{\,j+1} e^{j(1+j)\sigma^2/2} \ \text{(rand/m}^2\text{)} \quad (49)$$

and

$$\bar{R}_V = \bar{R}_c g_V = \frac{D}{A\bar{d}} \sum_{j=0}^{m} a_j \theta_o^{\,j+1} e^{j(1+j)\sigma^2/2} \quad \text{(rand/m}^3\text{)} \qquad \ldots (50)$$

As $A$, $\bar{d}$ and the $a_j$'s are given constants, the primary variables determining revenue are $\theta_o$, $\sigma^2$ and $D$. It is, therefore, of importance that prospecting campaigns will be undertaken in such a way that

(i) the average stone size $\theta_o$,

(ii) the logarithmic variance of the stone sizes $\sigma^2$, and

(iii) the stone density per sample unit $D$

may be estimated separately.

## SAMPLE ESTIMATES AND CONFIDENCE INTERVALS

### *Average stone size*

The unknown true average weight of diamonds $\theta_o$ (in carats/stone) in a limited and homogeneous mining area may be estimated by the arithmetic mean,

$$\bar{z} = \frac{1}{n} \sum_{i=1}^{n} z_i,$$

or by the maximum likelihood estimator $t$ for a two-parameter lognormal population (Sichel, 1966). As the number of stones in a sample usually exceeds $n = 20$ and as the underlying logarithmic variance is rarely larger than unity, one may replace $t$ by the simpler $t'$-estimator, that is,

$$t' = \exp(\bar{x}_e + \tfrac{1}{2} V_e), \qquad \ldots\ldots\ldots\ldots (51)$$

where

$$\bar{x}_e = \frac{1}{n} \sum_{i=1}^{n} x_i,$$

$$V_e = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{n-1}{n} \hat{\sigma}_e^2$$

and

$$x_i = \ln z_i.$$

If 90 per cent central confidence limits are required, one may use Sichel's tables (1966). If one would like any other confidence level, one may use

$$\left.\begin{matrix} UL_{1-p} \\ LL_p \end{matrix}\right\} = t' \exp\left[\frac{V_e}{2(n-1)}\left(1 + \frac{V_e}{2}\right) \pm k_p \sqrt{\frac{V_e}{n-1}\left(1 + \frac{V_e}{2}\right)}\,\right] \qquad \ldots (52)$$

where $k_p$ is the standardized normal deviate cutting off a proportion $p$ in the tail of the distribution.

### *Average grade*

For the density distribution given as equations (6) or (7) the mean stone density per sample unit was estimated by

$$\hat{D} = \frac{1}{N} \sum_{i=1}^{N} r_i = \bar{r}.$$

Here $r_i$ is the number of stones found in the $i$-th sample unit and $N$ is the total number of sample units which are investigated. We may write

$$S = \sum_{i=1}^{N} r_i \quad \text{and} \quad \hat{D} = \frac{S}{N}. \qquad \ldots\ldots\ldots (53)$$

For the marine deposits in South West Africa one may assume that $\theta = 2\beta/\alpha \approx 0{,}97$, as found empirically from observations. Tables for 95 per cent central confidence limits for the density per sample unit $D$ were constructed and a small section is reproduced in Table I.

TABLE I

SMALL SECTION FROM TABLE OF 95 PER CENT CENTRAL CONFIDENCE LIMITS FOR STONE DENSITY, $D$, IF $\theta = 2\beta/\alpha = 0{,}97$.

| $S$ | $F_U$ | $F_L$ |
|---|---|---|
| 320 | 1,483 | 0,585 |
| 330 | 1,475 | 0,591 |
| 340 | 1,467 | 0,597 |
| 350 | 1,460 | 0,602 |
| 360 | 1,453 | 0,608 |
| 370 | 1,446 | 0,613 |

One enters Table I with the *total* number of diamonds $S$ which were recovered in the $N$ sample units, to read off multiplying factors $F_U$ and $F_L$. The lower and upper 95 per cent confidence limits for the planar grade estimate are

$$\hat{g}_{A,L} = \frac{F_L S \hat{\theta}_o}{A\,N} \quad \text{and} \quad \hat{g}_{A,U} = \frac{F_U S \hat{\theta}_o}{A\,N} \qquad \ldots\ldots (54)$$

and for the volumetric grade estimate we have

$$\hat{g}_{V,L} = \frac{F_L S \hat{\theta}_o}{A N \bar{d}} \quad \text{and} \quad \hat{g}_{V,U} = \frac{F_U S \hat{\theta}_o}{A N \bar{d}}. \qquad \ldots\ldots (55)$$

It may be noticed that formulae (54) and (55) do not take into account the variability introduced by the average size $\hat{\theta}_o$-estimate. The variability of the average density $\hat{D}$-estimate is large in comparison with that of $\hat{\theta}_o$ and hence not much harm is done by considering $\hat{\theta}_o$ as a parameter in the above equations.

The variance of the grade estimator, $\hat{g}_A = \hat{D}\hat{\theta}_o/A$, may be approximated closely by

$$\operatorname{var}(\hat{g}_A) = \frac{\theta_o^2}{A^2 N}\left[\frac{D(2-\theta)}{2(1-\theta)} + D\sigma^2\left(1 + \frac{\sigma^2}{2}\right) - \frac{\sigma^4}{2}(1-\phi(0))\right] \qquad \ldots\ldots (56)$$

Here, $\theta_o$ is the average stone size, $A$ is the cross-sectional area of sample unit, $N$ is the number of sample units, $\theta$ is the parameter in the stone density distribution, $D$ is the average stone density $= \beta/\sqrt{1-\theta}$, $\sigma^2$ is the parameter in the lognormal stone size distribution, and $\phi(0)$ is the proportion of barren sample units in the stone density distribution.

The second and third terms in the square bracket of equation (56) are small in comparison with the first. This is why the variability in estimator $\hat{\theta}_o$, can be neglected, as stated previously when determining confidence limits for the grade.

*Applications*

Table II shows the observed stone density distribution as found in the prospecting trenches in Block X on the $D$-beach. Altogether $N = 118$ trench units gave rise to $S = 345$ stones. The theoretical density distribution of equation (6) was fitted to the data using the maximum likelihood method for estimating the parameters which are shown at the bottom of the table. The fit is satisfactory as judged by the $\chi^2$-test.

The corresponding stone size distribution of equation (19) was fitted to the observed diamond weights for the same Block X on the $D$-beach. The observations are given in Table III. It will be noticed that the number of individual stones in Table III is $n = 74$ and *not* $S = 345$. In the past, only a fraction of the stones discovered in the trenches were weighed individually whereas the remainder were weighed in groups and their numbers counted. As this procedure results in a loss of a large quantity of information, the present practice is to weigh each diamond individually. Parameter estimates are shown at the bottom of Table III and the $\chi^2$-test indicates good agreement between observations and the two-parameter lognormal model.

TABLE II

STONE DENSITY DISTRIBUTION OF BLOCK X (D BEACH)

| No. of stones $r$ | Observed no. of sections $f_o$ | | Expected no. of sections $f_E$ | | $f_o - f_E$ | $(f_o - f_E)^2/f_E$ |
|---|---|---|---|---|---|---|
| 0 | 41 | | 36,0 | | +5,0 | 0,694 |
| 1 | 19 | | 26,8 | | −7,8 | 2,270 |
| 2 | 16 | | 16,2 | | −0,2 | 0,002 |
| 3 | 8 | | 10,1 | | −2,1 | 0,437 |
| 4 | 6 | | 6,6 | | −0,6 | 0,054 |
| 5 | 5 | 11 | 4,6 | 8,0 | +3,0 | 1,125 |
| 6 | 6 | | 3,4 | | | |
| 7 | 1 | | 2,5 | | | |
| 8 | 3 | 6 | 2,0 | 6,0 | 0,0 | 0,000 |
| 9 | 2 | | 1,5 | | | |
| 10 | 5 | | | | | |
| 11 | 3 | | | | | |
| 13 | 1 | 11 | | 8,3 | +2,7 | 0,878 |
| 19 | 1 | | | | | |
| 21 | 1 | | | | | |
| Total | 118 | | 118,0 | | 0,0 | 5,460 |

$P(\chi^2 \geqslant 5{,}460 \mid 5\ DF) = 0{,}362\,46$

$\hat{\alpha} = 1{,}592\,81$ $\qquad \hat{\theta} = 0{,}935\,12$

$\hat{\beta} = 0{,}744\,73$ $\qquad \bar{r} = 2{,}923\,73$

TABLE III

STONE SIZE DISTRIBUTION OF BLOCK X (D BEACH)

| Class interval (carats/stone) | Observed no. of stones $f_o$ | | Expected no. of stones $f_E$ | | $f_o - f_E$ | $(f_o - f_E)^2/f_E$ |
|---|---|---|---|---|---|---|
| 0,0— 0,1 | 0,0 | | 0,1 | | | |
| 0,1— 0,2 | 2,0 | 9,0 | 2,1 | 7,4 | +1,6 | 0,346 |
| 0,2— 0,3 | 7,0 | | 5,2 | | | |
| 0,3— 0,4 | 8,0 | | 7,1 | | +0,9 | 0,114 |
| 0,4— 0,5 | 5,0 | | 7,6 | | −2,6 | 0,889 |
| 0,5— 0,6 | 5,5 | | 7,3 | | −1,8 | 0,444 |
| 0,6— 0,7 | 6,5 | | 6,6 | | −0,1 | 0,002 |
| 0,7— 0,8 | 6,0 | | 5,8 | | +0,2 | 0,007 |
| 0,8— 0,9 | 6,0 | | 4,9 | | +1,1 | 0,247 |
| 0,9— 1,0 | 7,0 | | 4,2 | | +2,8 | 1,867 |
| 1,0— 1,1 | 5,0 | 6,5 | 3,5 | 6,5 | 0,0 | 0,000 |
| 1,1— 1,2 | 1,5 | | 3,0 | | | |
| 1,2— 1,3 | 1,0 | | 2,5 | | | |
| 1,3— 1,4 | 1,5 | 4,0 | 2,1 | 6,4 | −2,4 | 0,900 |
| 1,4— 1,5 | 1,5 | | 1,8 | | | |
| 1,5— 2,0 | 5,0 | | 5,4 | | −0,4 | 0,030 |
| 2,0— 3,0 | 4,5 | | 3,5 | | | |
| 3,0— 5,0 | 1,0 | | 1,1 | | | |
| 5,0—10,0 | | 5,5 | 0,2 | 4,8 | +0,7 | 0,102 |
| 10,0— | | | 0,0 | | | |
| Total | 74,0 | | 74,0 | | 0.0 | 4,948 |

$\bar{x}_e = -0{,}336\,61$ $\qquad P(\chi^2 \geqslant 4{,}948 \mid 9\ DF) = 0{,}838\,75$

$\hat{\sigma}_e = 0{,}679\,84$

$t' = 0{,}897$ carats/stone

To find the 95 per cent central confidence limits for the average stone size $\theta_o$ of Block X we use equation (52).

We have $t' = 0{,}897$, $n = 74$, $V_e = \frac{73}{74} \times (0{,}679\,8)^2$ and $k_{0,025} = 1{,}96$ (from standard normal tables). Hence $L.L. = 0{,}758$ and $U.L. = 1{,}068$ carats/stone. The probability is 19 in 20 that the unknown true *average* diamond weight $\theta_o$ in Block X, on the $D$-beach, will lie somewhere between 0,758 and 1,068 carats/stone. The best *point* estimate for $\theta_o$ is 0,897 carats/stone. The planar grade estimate for Block X and associated 95 per cent central confidence limits are obtained from $\hat{g}_A = \hat{D}\hat{\theta}_o/A$ and from equation (54), respectively. The factors $F_U$ and $F_L$ in (54) are taken from Table I.

For $\hat{D} = \bar{r} = 2{,}923\,73$ stones per trench section, $\hat{\theta}_o = t' = 0{,}897$ carats/stone, and $A = 5$ m², $\hat{g}_A = 0{,}524$ carats/m² is obtained. Entering Table I with $S = 345$ stones, interpolation yields $F_U = 1{,}463\,5$ and $F_L = 0{,}599\,5$. Thus, the limits are:

$$L.L. = (0{,}599\,5)\,(0{,}524) = 0{,}314 \text{ carats/m}^2$$

$$U.L. = (1{,}463\,5)\,(0{,}524) = 0{,}767 \text{ carats/m}^2.$$

Hence the probability is 19 in 20 that the unknown true grade $g_A$ in Block X, on the $D$-beach, will lie somewhere between 0,314 and 0,767 carats/m². The best point estimate is 0,524 carats/m².

From Table II it will be seen that parameter estimate $\hat{\theta} = 0{,}935$ is lower than the $\theta = 0{,}97$ which was assumed *a priori* in the calculation of the factors in Table I. The discrepancy may be due to sampling. If it is not and if the unknown true density parameter $\theta$ in Block X is actually lower than 0,97, then the use of Table I will err on the conservative side.

It was stated previously that formula (56) brought in the variations introduced by estimating the average stone size $\theta_o$, whereas formulae (54) and (55) neglected these variations as unimportant. To prove this statement numerically, we calculated upper and lower confidence limits from (56). If $\hat{\theta}_o = 0{,}897$, $A = 5$, $N = 118$, $\theta = 0{,}97$, $\hat{D} = 2{,}923\,7$, $\hat{\sigma}^2 = 0{,}462\,2$ and $\hat{\phi}(0) = 0{,}305\,1$, then equation (56) yields $\text{var}(\hat{g}_A) = 0{,}014\,123$ and $S.E.(\hat{g}_A) = 0{,}118\,84$. Hence, $L.L. = \hat{g}_A - 1{,}96\; S.E.(\hat{g}_A) = 0{,}291$ carats/m², $U.L. = \hat{g}_A + 1{,}96\; S.E.(\hat{g}_A) = 0{,}757$ carats/m². These confidence limits compare well with those derived previously. The last confidence interval is slightly wider as it brings in the variations of $\hat{\theta}_o$.

The cumulative percentage frequency distribution of stone sizes as given in Table III for Block X ($D$-beach), is plotted on logarithmic probability paper in Oosterveld (1972). The almost perfect straight line plot corroborates the remarks made previously that diamond sizes in the marine deposit of South West Africa are well represented by a two-parameter lognormal distribution provided the stones originate from a small compact mining block, on one and the same beach horizon.

A further interesting example is shown as curve ($A$) in Fig. 1 and in Table IV. The 204 sizes listed are all of stones originating from a *single* 5-m² trench section. The plot ($A$) is linear on log-probability paper and the $\chi^2$-test in Table IV is satisfactory, indicating that the two-parameter lognormal distribution holds right down to a small, and very exceptional, diamond concentration.

In contradistinction, graph ($B$) in Fig. 1 represents the cumulative frequency distribution of 1 022 stone sizes given in Table V. The data represent sizes of diamonds found in prospecting trenches covering a fairly large mining area. The graph in Fig. 1 is convex and curves strongly towards larger stone sizes indicating a compound log-normal distribution. In consequence thereof, equation (28) was fitted to the data. Parameter estimates are given at the bottom of Table V and the $\chi^2$-test indicates a very good agreement between observations and theory.

TABLE IV

STONE SIZE DISTRIBUTION IN SINGLE TRENCH SECTION

| Class interval (carats/stone) | Observed no. of stones $f_o$ | Expected no. of stones $f_E$ | $f_o - f_E$ | $(f_o - f_E)^2/f_E$ |
|---|---|---|---|---|
| 0,00—0,10 | 11.5 | 9,3 | +2,2 | 0,520 |
| 0,10—0,20 | 34,5 | 39,9 | −5,4 | 0,731 |
| 0,20—0,30 | 44,0 | 42,3 | +1,7 | 0,068 |
| 0,30—0,40 | 35.5 | 32,8 | +2,7 | 0,222 |
| 0,40—0,50 | 27,0 | 23,3 | +3,7 | 0,588 |
| 0,50—0,60 | 13,0 | 16,1 | −3,1 | 0,597 |
| 0,60—0,70 | 9,5 | 11,2 | −1,7 | 0,258 |
| 0,70—0,80 | 5,5 | 7,8 | −2,3 | 0,678 |
| 0,80—0.90 | 6,0 } 10,5 | 5,5 } 9,4 | +1,1 | 0,129 |
| 0,90—1,00 | 4,5 | 3,9 | | |
| 1,00—1,10 | 3,0 | 2,9 | | |
| 1,10—1,20 | 3,5 } 10,0 | 2,1 } 6,6 | +3,4 | 1,752 |
| 1,20—1,30 | 3,5 | 1,6 | | |
| 1,30— | 3,0 | 5,3 | −2,3 | 0,998 |
| Total | 204,0 | 204,0 | 0,0 | 6,541 |

$\hat{\bar{x}}_e = -1{,}113\,13$ $\qquad P(\chi^2 \geqslant 6{,}541 \mid 8\ DF) = 0{,}586\,9$

$\hat{\sigma}_e = 0{,}707\,01$

$\hat{\bar{z}} = 0{,}421\,81$ carats/stone.

TABLE V

STONE SIZE DISTRIBUTION OF LARGE MINING AREA

| Class interval (carats/stone) | Observed no. of stones $f_o$ | Expected no. of stones $f_E$ | $f_o - f_E$ | $(f_o - f_E)^2/f_E$ |
|---|---|---|---|---|
| 0,00— 0,15 | 9,0 | 8,89 | +0,11 | 0,001 |
| 0,15— 0,25 | 39,0 | 33,11 | +5,89 | 1,048 |
| 0,25— 0,75 | 358,0 | 371,39 | −13,39 | 0,483 |
| 0,75— 1,25 | 257,5 | 255,50 | +2,00 | 0,016 |
| 1,25— 1,75 | 137,0 | 127,24 | +9,76 | 0,749 |
| 1,75— 2,25 | 69,5 | 70,01 | −0,51 | 0,004 |
| 2,25— 2,75 | 40,5 | 43,44 | −2,94 | 0,199 |
| 2,75— 3,25 | 28,0 | 26,57 | +1,43 | 0,077 |
| 3,25— 3,75 | 20,5 | 19,42 | +1,08 | 0,060 |
| 3,75— 4,25 | 16,5 | 12,67 | +3,83 | 1,158 |
| 4,25— 4,75 | 7,5 | 10,73 | −3,23 | 0,972 |
| 4,75— 5,25 | 7,0 | 7,36 | −0,36 | 0,018 |
| 5,25— 5,75 | 2,5 | 5,52 | −3,02 | 1,652 |
| 5,75— 6,25 | 4,5 | 4,19 | +0,31 | 0,023 |
| 6,25— 6,75 | 2,0 } 4,0 | 3,58 } 6,24 | −2,24 | 0,804 |
| 6,75— 7,25 | 2,0 | 2,66 | | |
| 7,25— 7,75 | 3,0 } 5,0 | 2,66 } 4,70 | +0,30 | 0,019 |
| 7,75— 8,25 | 2,0 | 2,04 | | |
| 8,25— 8,75 | 1,0 | 1,64 | | |
| 8,75— 9,25 | 1,5 } 3,0 | 1,33 } 5,22 | −2,22 | 0,944 |
| 9,25— 9,75 | 0,5 | 1,23 | | |
| 9,75—10,25 | 0,0 | 1,02 | | |
| 10,25—14,75 | 7,0 | 5,31 | +1,69 | 0,538 |
| 14,75—21,75 | 3,0 | 2,45 | | |
| 21,75—32,25 | 1,0 } 6,0 | 1,12 } 4,49 | +1,51 | 0,508 |
| 32,25—48,25 | 2,0 | 0,51 | | |
| 48,25—71,25 | 0,0 | 0,41 | | |
| Total | 1 022,0 | 1 022,00 | 0,00 | 9,273 |

$\hat{a} = -0{,}661\,00$ $\qquad P(\chi^2 \geqslant 9{,}273 \mid 14\ DF) = 0{,}813\,1$

$\hat{s} = 3{,}854\,49$

$\hat{c} = 0{,}063\,45$ $\qquad \hat{\bar{z}} = 1{,}529$

$\hat{v} = 4$

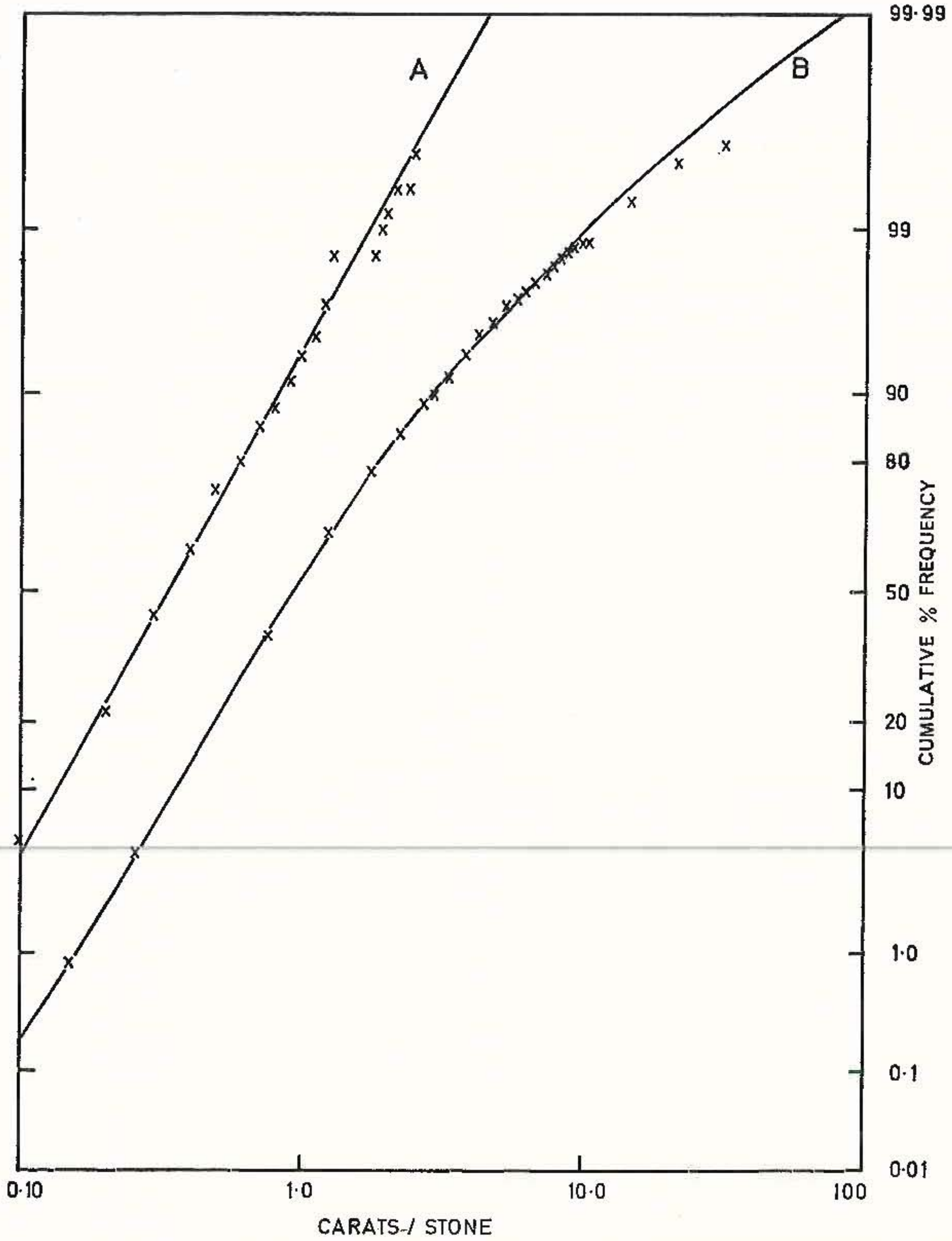


FIGURE 1 : CUMULATIVE PERCENTAGE FREQUENCY DISTRIBUTIONS OF OBSERVED STONE SIZES (in carats/stone), PLOTTED ON LOGARITHMIC PROBABILITY PAPER, FOR (A) 204 DIAMONDS RECOVERED IN PROSPECTING A SMALL MINING AREA AND FOR (B) 1022 DIAMONDS RECOVERED IN PROSPECTING A LARGE MINING AREA. THE SMOOTH CURVE IN (A) AND (B) REFER TO THE THEORETICAL DISTRIBUTIONS FITTED TO EQUATIONS (19) AND (28) RESPECTIVELY.

The estimation of parameters for the size distributions of diamonds is of particular practical help if one wishes to forecast revenue and sizes of stones to be expected in future mining operations. In a small sample, and virtually all samples of diamondiferous ground are small due to the minute weight ratio of valuable material to gangue, the probability of including the larger stones in a sample is low. Hence, any statistical estimate with respect to stones of a larger size than the largest found in a finite sample, is of some importance.

Again, the revenue estimate per mining unit has a much smaller standard error if it is based on the parameter estimates than if it is deduced directly from the actual stone sizes found in the sample.

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