An Exploration Model for Tabular Orebodies

By GEORGE S. KOCH, Jr.,* S.B., M.A., Ph.D. and RICHARD F. LINK,† B.S., M.S., Ph.D.

SYNOPSIS

A mathematical simulation model has been devised to guide the layout of surface or underground mine workings, drill stations, and drill holes that are established in order to search for tabular ore bodies. The ore bodies may be contained within veins, sedimentary beds (including reefs), or other tabular structures.

Programmed for a time-shared digital computer, the model contains both stochastic and deterministic variables. Stochastic or natural variables include the number of ore shoots and their sizes, orientations, locations and values. Deterministic or controllable variables include (i) locations and costs of workings (for underground exploration); (ii) drill station locations and costs; and (iii) drill hole azimuths, inclinations, lengths and deflections (if any).

For various assumptions about the natural variables, the model is run while the controllable variables are changed deliberately. Thus, layouts are devised to determine the percentages of the total number of assumed ore shoots found at different costs. In order to choose among the various layouts, cost-effectiveness measures are calculated.

INTRODUCTION

Given a block of ground containing ore bodies, the mining geologist or mining engineer faces the problem of devising plans for their discovery, development and mining. Usually, a generalized plan is made, either formally or informally, based on the available geological and geophysical information. Drill holes are then bored, and mine workings driven in order to expose the block of ground for visual examination. As this work proceeds, the initial plans are modified and refined.

The amount of initial planning that is worthwhile depends upon the knowledge derived from experience and theory about the geological habit of the ore bodies, in particular their pattern of occurrence. If enough is known, it may be useful to organize and extend this knowledge with a mathematical model corresponding to the physical situation. The behaviour of the model can then be studied by mathematical simulation, thus providing information that can aid in planning exposure of the block of ground.

The purpose of this report is to explain a mathematical model used to guide the discovery, development, and mining of ore bodies in a lead and silver mine in the Coeur d'Alene district, Idaho, U.S.A. The behavior of this model and its relation to the actual physical situation are discussed. In general, the model may also be applied to many other types of lens-shaped and tabular ore bodies, for instance, to stratiform ore bodies including lenses of uranium ore in sedimentary beds and gold ore shoots in reefs.

This paper is a condensation of another paper by Koch *et al* (1972). A time-shared computer program in FORTRAN IV to perform the calculations is also available (Schuenemeyer *et al*, 1971).

THE MATHEMATICAL MODEL

The mathematical model represents the geometry of ore bodies, mine workings, and drill holes in the Galena mine. The model is devised for a physical model, shown in isometric projection in Fig. 1. The ore bodies are represented as ellipses, and the drifts and drill holes as straight lines. The co-ordinates correspond to those in a block of ground in the Galena mine.

The mathematical model contains both natural and controllable variables (Table I). The natural variables include the sizes, center locations, number of targets, and the assumption that the targets are of uniform value per square unit of area. The controllable variables include level data, drift orientations, drill-station locations, drill-hole specifications, costs and the probability of detection given an intersection (a function of core size, angle of intersection, place of intersection with target and value of target).

TABLE I.

NATURAL AND CONTROLLABLE VARIABLES OF THE MATHEMATICAL MODEL

Natural variables
Target size (uniform, with major and minor axes specified by the user, or non-uniform, with major and minor axes derived from a specified probability distribution).
Target orientation (strike, dip, and pitch or plunge specified by the user. Strike, dip and pitch may be fixed or may be obtained from probability distributions with specified parameters).
Target center (north and east co-ordinates, and elevation, obtained from a uniform probability distribution).
Controllable variables
Level data (elevation of starting level, level interval, number of levels).
Drift orientation (starting co-ordinate, length, azimuth and inclination of each drift).
Drill station location (number of stations for each drill hole).
Drill hole specification (for each station, number of drill holes, and azimuth, inclination and length of each hole).
Cost (costs of drifting per foot, drilling per foot and station per station).

The calculations are grouped into the following program elements:

(i) *Establish the block of ground.* Define bounding co-ordinates for the six planes bounding the selected block of ground (Fig. 1).

(ii) *Establish the number of targets and their locations.* Select a number of targets from one to 30. Locate the center of each target by choosing three uniform random numbers to designate distances along the x, y, and z axes.

* Professor of Geology, University of Georgia, Athens, Georgia, U.S.A.

† Vice President, Artronic Information Systems, Inc., New York, New York, U.S.A.

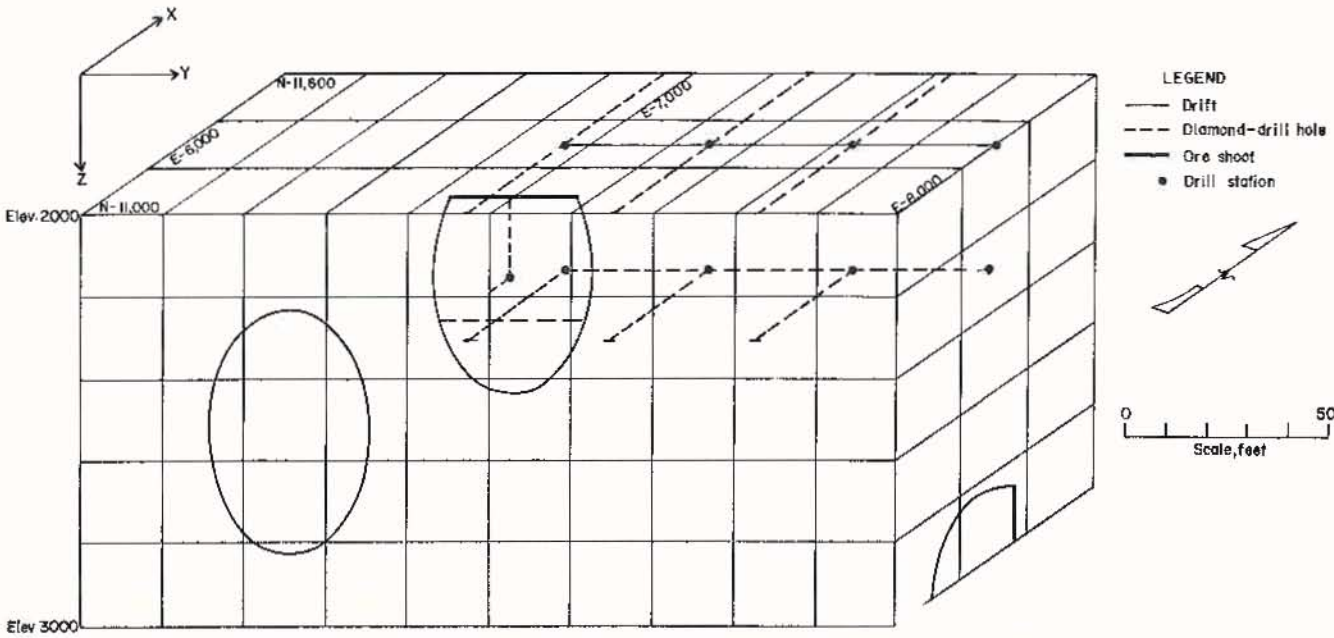

Fig. 1. Isometric projection of the physical model.

(iii) *Establish drift locations.* Select a number of drifts from one to four, define the eastern boundary of the uppermost drift, define the drift lengths, calculate the cost of drifting.

(iv) *Establish the number of drill stations per drift.* Select a number of drill stations from one to 11, determine the station co-ordinates evenly spaced within each drift, calculate the cost of cutting stations.

(v) *Establish the number and lengths of drill holes.* For each drill station, select the number of holes and their direction or directions.

(vi) *Simulate drilling of the holes.* Test whether the specified holes are entirely within the block of ground. If so, continue, otherwise stop. Calculate the total cost of all holes. For each target, calculate the number of drill hole intersections, if any.

(vii) *Calculate the total cost of development.* Sum the costs from elements (iii), (iv) and (vi) to find the total cost.

(viii) *Calculate the cost-effectiveness.* Cost-effectiveness equals the fraction of targets found divided by the total cost of development.

(ix) *Calculate the average probability of target recognition, given that at least one target is intersected.*

BEHAVIOR OF THE MODEL

In a typical problem, explained in detail by Koch *et al* (1972), there are four drifts evenly spaced 150 ft vertically apart, and extending the full length of the block of ground, 10 drill stations per drift, and holes bored north and south from each drill station extending to the edges of the block of ground. There are five targets. The problem is replicated 100 times, that is, 100 different configurations of five targets are selected by choosing random numbers, while the configuration of mine workings and boreholes remains constant.

The computer output for this problem is as follows:

(i) The unit costs of drifting, cutting drill stations and boring diamond-drill holes are specified by the user. The selected configuration of drifts, drill stations and drill holes determines the total costs of each of these items, which are summed to give the total development costs.

(ii) Table II is a frequency distribution of targets hit. In the body of the table, the first column lists the total number of targets hit from zero to the maximum number of five hit and the second column gives the frequency distribution. For example, three targets were hit five times, a total of 15 targets in all out of the 500 targets in the 100 replications. The remaining columns give a breakdown of the number of intersections of targets by the number of drill holes specified at the column heading. For the previous case of three targets hit, five of these 15 targets were hit by one drill hole, seven by two drill holes, two by three drill holes, and one by four drill holes. For this particular configuration of drill holes and targets of uniform size, the maximum number of drill holes that can intersect a target is four. The last line of this table lists the totals of the entries in each column.

TABLE II.

TYPICAL FREQUENCY DISTRIBUTION OF TARGETS HIT

Number of targets	5
Average number hit	4.57
Standard deviation	0.69
Number of replications	100

Targets hit	Frequency	1	2	3	4	5	6	7 or more
0	0							
1	0							
2	2	0	3	1				
3	5	5	7	2	1			
4	27	35	36	26	11			
5	66	124	111	74	21			
Total	100	164	157	103	33	0	0	0

(iii) A frequency distribution of cost effectiveness, which includes the class interval, frequency, relative frequency percentages and relative cumulative frequency percentage, is prepared.

(iv) A frequency distribution of the average probability of target recognition is made.

(v) The last item of output is one or more maps. Each map shows the configuration of mine workings, drill holes, and targets for any selected elevation (usually a mine level) and for any replication. Figure 2 is a typical map for another problem, selected to show two intersecting ore shoots and the intersections on a level of inclined boreholes drilled from other levels above and below it.

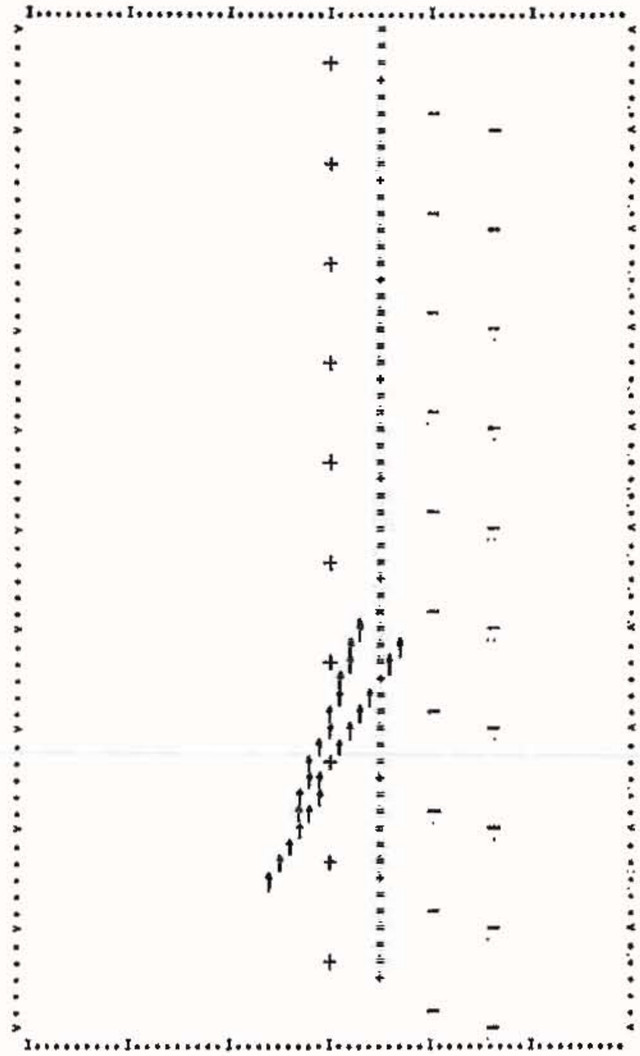

Fig. 2. Typical level map. Explanation of symbols: Ore body (↑); Drift (=); Intersection with the level of upward bore hole (+), of downward bore hole (−).

RESULTS

After several problems are run on the computer, the results can be summarized and interpreted. Lack of space permits explaining only one case.

Consider devising a layout to explore for five uniform sized targets (axes 200 ft by 600 ft) in the block of ground of Fig. 1. Table III gives the results obtained for different numbers of drifts and drill stations.

For each entry the simulation is replicated 100 times. From each drill station, two drill holes are bored due north and due south, each 300 ft long, across the entire block of ground. The means increase rather regularly from the case of one drift and one drill station (upper left-hand corner) to the case of four drifts and 11 drill stations (lower right-hand corner).

TABLE III.

MEAN NUMBERS OF TARGETS HIT FOR DIFFERENT NUMBERS OF DRIFTS AND DRILL STATIONS

Number of drifts	Number of drill stations per drift										
	1	2	3	4	5	6	7	8	9	10	11
1	0.23	0.47	0.79	1.00	1.22	1.40	1.69	1.95	2.06	2.25	2.56
2	0.33	0.68	1.01	1.38	1.68	2.07	2.19	2.56	3.02	3.32	3.56
3	0.43	0.88	1.30	1.72	2.09	2.58	2.91	3.31	3.91	4.30	4.49
4	0.45	0.97	1.46	1.89	2.33	2.80	3.21	3.66	4.38	4.77	4.95

Each mean has a corresponding standard deviation developed through simulation (Table IV). These simulation results may be interpreted through the statistical theory for the binomial distribution. Consider a certain layout of drifts, drill stations, and drill holes, together with a single target with a fixed shape, size, and orientation but with a center located at random. Let the probability of the target being intersected by at least one drill hole be some constant p, the size of which depends upon the layout of holes and the target geometry, especially the target size.

TABLE IV.

COMPARISON OF STATISTICS OBTAINED BY SIMULATION AND BY BINOMIAL THEORY FOR ONE DRIFT AND FROM ONE TO TEN DRILL STATIONS

Drifts	Stations	Mean	p	Theoretical standard deviation	Simulated standard deviation	Difference	Theoretical coefficient of variation	Simulated coefficient of variation	Difference
1	1	0.230	0.046	0.468	0.490	−0.022	2.037	2.130	−0.094
1	2	0.470	0.094	0.653	1.080	−0.427	1.388	2.298	−0.909
1	3	0.790	0.158	0.816	0.780	0.036	1.032	0.987	0.045
1	4	1.000	0.200	0.894	0.950	−0.056	0.894	0.950	−0.056
1	5	1.220	0.244	0.960	1.080	−0.120	0.787	0.885	−0.098
1	6	1.400	0.280	1.004	1.020	−0.016	0.717	0.729	−0.011
1	7	1.690	0.338	1.058	1.090	−0.032	0.626	0.645	−0.019
1	8	1.950	0.390	1.091	1.120	−0.029	0.559	0.574	−0.015
1	9	2.060	0.412	1.101	1.000	0.101	0.534	0.485	0.049
1	10	2.250	0.450	1.112	1.020	0.092	0.494	0.453	0.041

If several targets, n in number, have centers located independently and randomly, the situation is a classical binomial one with n independent trials, each with a probability p of success (with a 'success' defined as the intersection of a target by one or more drill holes). Thus, the following formulae apply:

Mean number of targets hit $= np$

Standard deviation of number of targets hit $= \sqrt{np(1-p)}$

Coefficient of variation $= \sqrt{(1-p)/np}$

In Table IV, the estimates of standard deviations obtained by simulation are compared with those calculated by the above theory. The standard error of estimate of p is

$$\sqrt{p(1-p)/k},$$

where k is the number of independent trials. The values in the table show that calculated standard deviations and coefficients of variation agree closely with those obtained by simulation.

In Fig. 3 the maximum, mean, and minimum numbers of targets hit in 100 replications are shown for one drift and from one to 11 drill stations. The vertical bars are 90 per cent confidence intervals for the means obtained in 100 replications. Figure 3 shows that for one drift the mean number of intersected targets increases linearly as the number of drill stations increases. However, the minimum number of targets intersected is zero, regardless of how many drill stations are cut, whereas the maximum number of targets found ranges from two for one station to five for five or more stations. Thus, under the conditions of this simulation, there is no guarantee of finding even one target through driving one drift even if 11 drill stations are cut and two drill holes are driven from each station.

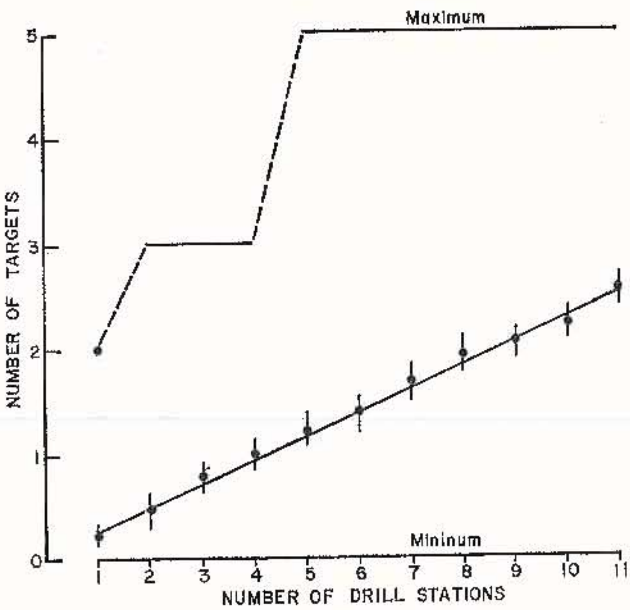

Fig. 3. Maximum, mean, and minimum number of targets hit, for one drift and from one to 11 drill stations. Five uniform size elliptical targets (axes 200 ft by 600 ft), orthogonally distributed. Vertical bars are 90 per cent confidence intervals for the means.

APPLICATION OF THE MODEL

The mathematical model was applied to the problem of exploring at depth for ore bodies in the Galena mine, Wallace, Idaho, U.S.A. The Galena mine produces silver, lead ore from steeply dipping fissure veins in pre-Cambrian quartzites, phyllites, argillites, and slates. Kesten (1961) reviews the mine geology.

In order to introduce the physical situation into the mathematical model, data from the developed mine must be summarized numerically. We hand-fitted ellipses to longitudinal sections of the ore shoots developed on 33 known veins. The areas of these ellipses are distributed approximately lognormally, as are the sizes (as well as the grades) of many ore bodies and ore deposits (Koch, *et al*, 1970-71). However, the ellipse axial ratios are distributed approximately normally.

Next, we calculated the density of ore bodies in the developed mine, and from this value estimated the density of ore bodies in the block of ground to be explored at depth, assuming that the observed number of 33 ore bodies is a random sample from a Poisson distribution. Target centers were assumed to be distributed uniformly.

Finally, we summarized the orientations of the ore bodies, using computer programs that implement methods of Fisher and Watson (Koch, *et al*, 1970-71). The poles of 30 of the vein surfaces lie on a great-circle girdle and those of the other three vein surfaces define a cluster. The plunges of the ore bodies define two clusters. Probability distributions corresponding to these empirical girdle and cluster distributions were obtained by trial and error.

Thus, we were able to introduce distributions of the natural variables (Table I) into the model.

We then assigned values to the controllable variables (Table II) in order to devise layouts for such problems as finding (i) not less than one ore body on the average, (ii) a specified percentage of the ore bodies, (iii) all ore bodies larger than a certain size, etc.

The numerical results are detailed in the full paper by Koch, *et al*, 1972. We conclude that the mathematical model corresponds satisfactorily to the physical situation and is so flexible that nearly any specified problem (for example, see the three above) can be solved readily.

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