Optimizing Medium-Term Operational Plans for a Group of Copper Mines

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SYNOPSIS

Since 1964, Nchanga Consolidated Copper Mines, Ltd., has been developing and using production scheduling methods for optimizing its metallurgical operations in the short term (up to one year). Medium-range metallurgical planning has been possible only by successive evaluations of a single-year L.P. A modified version of Rosenbrock's direct search technique has now been developed to allow multi-year optimization of both mining and metallurgy for medium-range planning. The modifications to the search technique have produced a marked increase in computational efficiency for this heavily constrained problem in many variables and the system is a usable planning tool, given a reasonably powerful computer. The plans are 'optimum' in the sense that they yield the maximum present value of the operations.

Each of the resulting five-yearly mine plans forms the basis for further detailed metallurgical evaluation by the established production scheduling system.

INTRODUCTION

The group of mines and treatment plants operated by Nchanga Consolidated Copper Mines, Limited, includes three major copper mining and processing installations. They are all located in the Copperbelt region of Zambia; at Kitwe (operated by the Rokana Division), at Chingola (Chingola Division) and at Chililabombwe (Konkola Division). The principal activities of each of these three installations are shown in Fig. 1, which also indicates the transport links. The circles in Fig. 1 show where there is a choice available to the operating management. There are three areas in which such choices can be made:

1. the mining rate of each orebody,
2. the point or points along each flotation bank at which the routing of the flotation product changes, and
3. the routing of concentrates to stockpiles and the reclamation from stockpiles.

The copper ores mined in Zambia are either predominantly acid-soluble (commonly called 'oxide' ores) or else predominantly acid-insoluble ('sulphide' ores). In general, the sulphide concentrates and the high-grade oxide concentrates are treated by the smelter. The medium- and low-grade oxide concentrates, and, after roasting, the low-grade sulphide concentrates, are treated by the leach plant. The tailings from the Chingola concentrator together with reclaimed tailings from dump will be treated by the low-grade leach plant, whose first stage of operation is scheduled for 1974.

Also shown in Fig. 1 are the major constraints to be met within the overall operation. There are capacity constraints on the various plants, handling limits such as coal feed in the smelter, as well as availabilities of other materials such as acid.

PREVIOUS WORK ON OPTIMIZING COMPANY PRODUCTION PLANS

Since the company's mining and treatment operations are established at three separate locations, decisions can be made at a local level which are not necessarily the best in the context of overall company objectives.

Because of the large number of possible combinations of variables which have to be examined in the search for the 'best' production schedule for all the mines and treatment plants in the company, it is practicable to attempt such scheduling only by computer. Work started in 1964 on the design of suitable computer programs. Since 1965 production schedules have been produced for consideration by the management at the various mines and treatment plants [see Williams, *et al* (1968) and Williams, *et al* (1969)]. The scope of the scheduling system has been improved steadily throughout this period in the light of operational experience. The latest version of the established production scheduling system, called G.P.S.$\overline{\text{III}}$, is presented at this Conference (Williams, *et al*, 1972).

Although the benefits of these production scheduling systems have never been in doubt, it was felt that there were two important areas of interest to the company which were not adequately handled by these systems.

It was recognized that flotation is a non-linear process and consequently the L.P. scheduling systems had to assume a fixed mining plan and calculate the concentrate arisings by reference to tables. If this mining/metallurgy interface could be modelled, then overall optimization of mining and metallurgy could be achieved.

The second requirement was for a new medium-term (up to five years) planning system. Although the established L.P. systems had been used in the medium term, planning was possible only by successive evaluations of a single-year L.P. The new system should be capable of optimizing five separate years simultaneously, thereby avoiding any danger of sub-optimization.

The new system, known as G.P.S.$\overline{\text{V}}$., was therefore aimed at a medium-term corporate planning model, covering five years' operation and allowing the incorporation of stockpiles between the five one-year periods. Detailed metallurgy was not considered, as such detail is generally valid only over a small working range, but the new system was designed to reproduce the significant process characteristics. Since some of the processes to be modelled were non-linear, it was decided to use a direct optimization technique.

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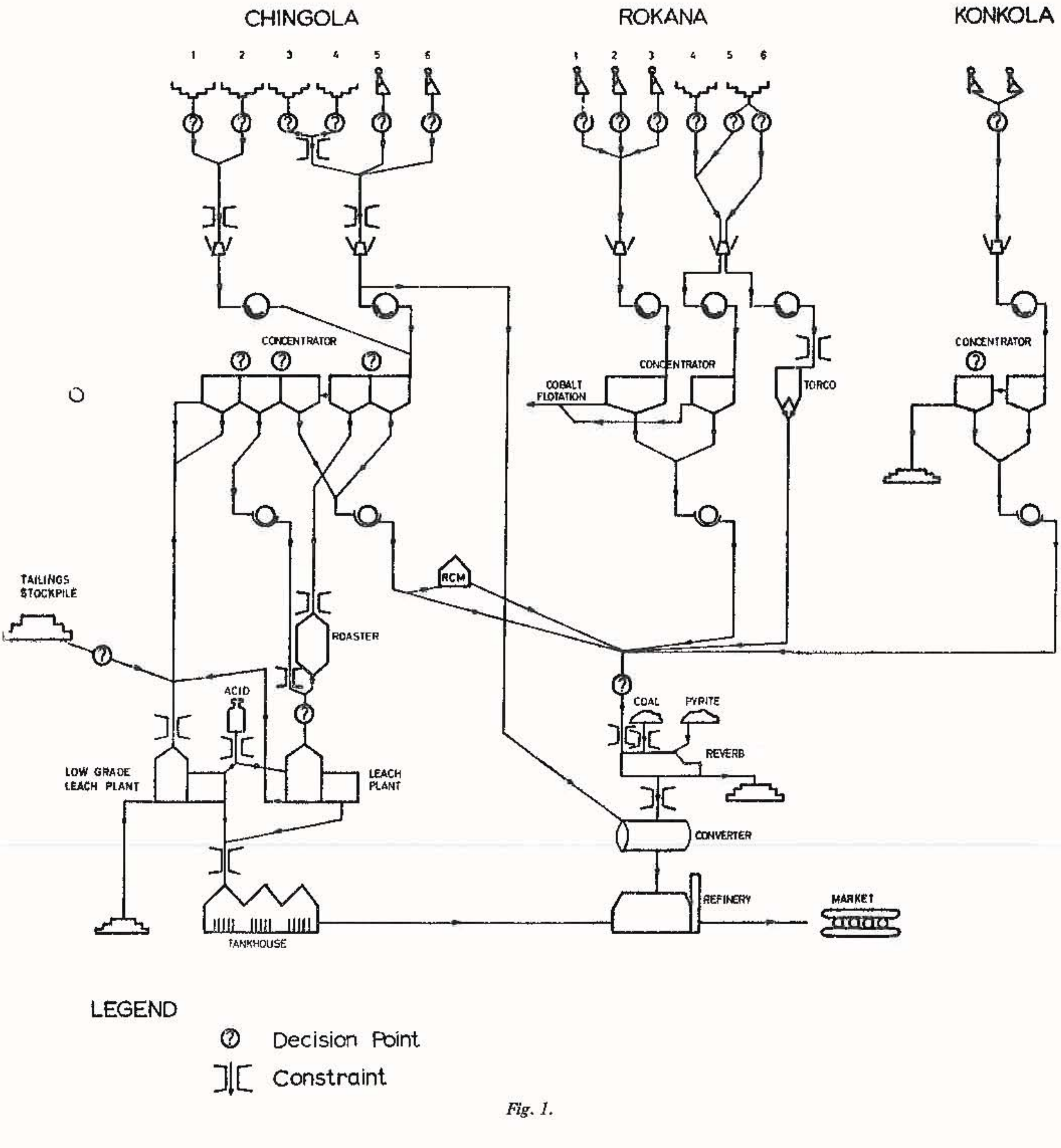

Fig. 1.

THE MEDIUM-TERM PLANNING SYSTEM

The new system involves the use of a direct search procedure to find the maximum or minimum value of a function of independent variables. The specific procedure is that due to Rosenbrock (1960), although some modifications have been made to the method of handling constraints, as described below. Rosenbrock's method, although slow compared with some others such as that due to Powell (1964), was chosen because of its robustness over a wide variety of functions. Since little was known of the behavioural characteristics of the proposed model of the mining and metallurgical activities of the company, robustness of the optimization procedure was considered to be a vital attribute in the initial stages of the project. A further advantage of Rosenbrock's method is that it does not require component functions to be continuously differentiable, so that the system modeller has freedom to incorporate conditional statements and component functions which, if necessary, contain break-points. Comprehensive flexibility is a great asset in the early stages of development of a large system, when model changes need to be implemented fairly frequently.

The optimizing system known as G.P.S.$\overline{\text{V}}$. comprises:

(i) the direct search or hill-climbing program,

(ii) the model of the company's activities, and

(iii) an output program which describes and summarizes in readable form the point in the problem space which is deemed 'optimum'.

The system is controlled by the direct search program. This program chooses a value for each of the independent variables (i.e. a possible management strategy for each of the five years under consideration) and submits this set of values to the company model. The model evaluates, for each year, the copper production, revenue accruing and the variable costs of development, mining and metallurgical operations. The 'margin' (revenue less variable cost) for each year is then obtained and discounted. The sum of these discounted margins is then the present value of the strategy, since the fixed costs are constant. The search routine then selects a new value for each of the variables and again invokes the company model, continuing the search until no further improvement can be obtained. The output program then translates this latest set of variable values into a set of mining and metallurgical plans which are output in a suitable form.

The direct search procedure is discussed in more detail in a later section of this paper.

THE COMPANY MODEL

Mining

Figure 1 illustrates the principal activities of the company which are dealt with by the optimizing system. There are four open-pit orebodies and two underground orebodies planned to be operating in the period 1973-77 at Chingola Division, two underground orebodies amalgamated into one for planning purposes at Konkola Division, and three underground orebodies and two open pit orebodies at Rokana Division. One of the Rokana open pit orebodies will provide two types of ore, one of which must be treated by flotation, the other by the TORCO plant prior to smelting. In the present version of the system, the above orebodies are the only variable sources of primary ore. Two additional variable sources of primary material are the previously stockpiled tailings at Chingola which can be fed to the low-grade leach plant at that Division, and pyrite from Nampundwe Mine which is principally a source of sulphur but which also contains a small amount of copper. The planning system takes cognisance of ore from two other small mines and of an exchange of concentrates with Roan Consolidated Mines, Ltd., but the respective tonnages concerned are assumed to be fixed. These factors do not significantly affect the solution to the problem of optimization between one set of conditions and another, but their incorporation in the system leads to 'acceptable' production summaries in the printed output from the system without management needing to allow for these fixed components.

It is assumed that if a quantity of ore is mined in a particular year, then the development work involved will be carried out in the previous year (development being considered either as stoping for underground ore sources, or stripping for open pit sources).

Concentrating

The model of concentrating is based on a simple first-order exponential model for mineral flotation:

$$Q = HM(1 - e^{-Kt})$$

where Q = cumulative percentage of material floated in time t.

H = Head grade (sulphide copper in sulphide bank, oxide copper in oxide bank).

M = Mineral factor, i.e. reciprocal of percentage grade of copper in mineral.

K = Flotation parameter.

In practice some minerals float more readily than others and so each ore source is divided into two classes of mineral, one with a high flotation performance and the other with a low performance. Two terms are then introduced into the flotation model to describe the behaviour of each.

A third term is introduced to incorporate the gangue flotation effect, and other refinements model the effect of oxide flotation in the sulphide bank and sulphide flotation in the oxide bank. As can be seen from Fig. 1, at the Chingola concentrator the optimization routine chooses the cut-off points between the various concentrates. At Konkola the flotation is simpler and only the high-grade oxide/low-grade oxide cut-off point is variable, and at Rokana the flotation does not require a sophisticated model. The concentration and subsequent recovery of cobalt at Rokana Division are not included in the G.P.S.$\underline{\text{V}}$. system.

Concentrate treatment at Chingola

The low-grade sulphide concentrate from the Chingola concentrator is roasted prior to being fed into the leach and electro-winning plants. The second oxide fraction is fed to leach directly. Acid and lime requirements are calculated (jointly) for the leach plant and low grade leach plant, but other process details are not represented in the system. The low grade leach plant processes the final oxide fraction, current tails, leach residue and tails from dump.

Concentrate treatment at Rokana

Coal (including the coal equivalent of oxygen enrichment) lime-rock and pyrite are calculated as dependent variables and are limited as necessary.

The electro-refining plant is modelled by a fixed recovery and a plant capacity.

Although most of the company's acid requirement is produced at the Rokana Division Acid Plant from sulphur dioxide generated by the smelter, the present version of the planning system does not attempt to deal with the implied interaction.

THE OPTIMIZATION TECHNIQUE

The Direct Search procedure

The procedure is that due to Rosenbrock (1960). The mathematics are fairly complex but the essential characteristic of the procedure is that the search always proceeds, as nearly as possible, along the direction of an improving ridge. There is, of course, no means for guaranteeing that the final result of the search is the overall, or global, maximum rather than a local peak but, from the characteristics of the problem and by using a number of different starting points, it is usually possible to determine whether a 'freak' result has occurred, particularly if the problem is highly constrained.

Treatment of constraints

In most optimization problems some or all of the variables are required, or constrained, to be within a given range. Rosenblock deals with this problem by penalizing entry into a very small 'boundary region' just inside the constraint. Although this method works, it makes the computational efficiency very low for a heavily constrained problem. The procedure was therefore modified as below.

(i) *Independent variables*

The 'independent' variables are those which are not calculated from other variables. Let us consider the variable x_i which is constrained to lie between a lower limit L_i and an upper limit U_i, i.e.

$$L_i \leqslant x_i \leqslant U_i$$

The variable x_i is constrained and, for computational efficiency, we would much prefer it to be unconstrained. We therefore transform the variable x_i to be a function of a new, unconstrained variable y_i. The variable y_i is independent and the variable x_i has become dependent. The transform most commonly used is:

$$x_i = L_i + (U_i - L_i)\sin^2 y_i$$

It is evident that y_i can assume any value and x_i will still lie between L_i and U_i.

(ii) *Dependent variables*

It is in the constraining of dependent variables that the greatest difficulties arise. Insisting that constraints be met causes computational inefficiency with all known methods and, for this size of problem, efficiency is important. After experimenting with various techniques the method selected was that of using penalty functions for constraint violation, that is, to modify the objective function by subtracting a function which is the weighted sum of squares of the amounts by which the constraints are violated. Obviously, the greater the violation the greater the penalty, and if no violation occurs the objective function is unmodified. This method of dealing with constraints is reasonably efficient and, in practice, provides quite valuable information about the behaviour of the function in the vicinity of the optimum in terms of the relative importance of the constraints on which the solution lies. The main criticism levelled at this method is that a solution can be (and usually is) obtained which lies outside the constraints imposed on the problem. In an industrial environment, however, it is rarely possible to quote an exact constraint on a variable, so that a small violation does not, very often, make the solution impracticable. Moreover, any particular violation can be made as small as desired by suitably adjusting the penalty function. An advantage of the method is that the magnitude of the violation yields valuable information about the value of relaxing the constraints. If a constraint were on plant handling capacity, for example, and were violated, the penalty incurred by this violation would be equal to the value of extending the handling capacity by that amount. Ideally, of course, the penalty functions should be the actual cost of relaxing the constraints and the optimization then would also optimize the plant requirements. At the time of writing, these actual costs are not yet available and the penalty functions in current use are arbitrary ones.

VALIDATION

Shorter-range planning models are usually easily verified by comparing the predicted output from the model with the output actually achieved. In the case of G.P.S.$\overline{\text{V}}$. the medium-term inputs to existing plant are sometimes quite different from current practice and it is then not possible to compare predicted output with production statistics. In addition, G.P.S.$\overline{\text{V}}$. contains models of plant which are not yet operational.

In this case, therefore, the only real validation possible is the extent to which operating management consider that results from the system are realistic. It is possible to verify those parts of the system which will hardly change from the current method of operation, particularly as only the major process details are required, but the rest of the system can be judged only by the credibility of the answers it provides.

It must be emphasized that the inclusion of, say, a linear model for a process does not imply that the process is necessarily thought to be linear, but only indicates that the output from the process appears 'reasonable' for the particular range of input used. The simple model of flotation which is used similarly does not imply that flotation is a simple first-order process, but it does give acceptable answers for a range of inputs.

As was stated earlier, one of the reasons for initiating the development of G.P.S.$\overline{\text{V}}$. was the need to model the mining/treatment interface, thereby enabling a production scheduling system to be devised, which could incorporate variable mining rates, Splaine (1969). Because of the difficulties in producing a mathematical model of flotation which can predict plant behaviour to any reasonable accuracy, a number of different approaches have been tried. As no single method has shown itself to be superior to the others, the mathematical model chosen has been the one which is most convenient to use in the system under consideration. When a production scheduling system uses a given mining plan, the head grade of the flotation feed can be calculated immediately, and these data used to choose flotation model parameters which give results for this grade which are known to be reasonable. In the case of a feed which is outside present experience, parameters are chosen to produce results which management expect. In the case of G.P.S.$\overline{\text{V}}$. head grades are not known at the start and consequently a more generalized flotation model is required.

Although results from the G.P.S.$\overline{\text{V}}$. flotation model have been discussed with management, it is difficult to assess the range of inputs for which they are acceptable. Development of the model is continuing in an attempt to improve the results, but should a model be devised which is generally more acceptable, it will be used in place of the present model.

Cost information for the system is supplied by Financial Management after discussions about the general situation suggested by the initial runs. Cost data are then given with a restriction on the range of conditions for which the data are valid; if the system moves out of this range, then new costs are input, calculated on the activity level suggested by the system.

RESULTS

Having developed the system so that management accepted the results, three runs were made:

(i) Using the company's present mining plan. This run was carried out in order to provide a basis for comparison with other runs.

(ii) Allowing the system to change the mining rate of every ore source by up to about 20 per cent from the company plan.

(iii) Allowing the mining flexibility in (ii) and allowing 10 per cent extra smelter capacity.

The purpose of carrying out these runs was partly to check that the changes predicted by the system were sensible and also to indicate to management the type of question which the system is capable of answering.

Basic results from the three runs are shown in Table I where certain important variables have been listed together with an indication of whether the variables are:

F — given a prescribed fixed value

U — at the upper limit

— — in between limits

L — at the lower limit.

As mentioned before, the mining rates for all ore sources are fixed in the first time period.

TABLE I

		COMPANY PLAN					FLEXIBLE MINING					EXTRA SMELTER				
		YEAR					YEAR					YEAR				
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
CHINGOLA MINING	1	F	F	F	F	F	F	—	—	—	—	F	—	U	U	—
	2	F	F	F	F	F	F	L	L	L	L	F	L	—	U	—
	3	F	F	F	F	F	F	L	L	L	L	F	L	L	L	L
	4	F	F	F	F	F	F	L	L	L	L	F	L	L	L	L
	5	F	F	F	F	F	F	U	—	U	U	F	U	U	U	U
	6	F	F	F	F	F	F	L	L	L	L	F	—	—	U	U
KONKOLA MINING		F	F	F	F	F	F	U	—	U	—	F	U	U	U	U
ROKANA MINING	1	F	F	F	F	F	F	—	—	—	—	F	U	U	U	U
	2	F	F	F	F	F	F	—	—	—	—	F	U	U	U	U
	3	F	F	F	F	F	F	—	—	—	—	F	U	—	U	U
	4	F	F	F	F	F	F	L	L	L	L	F	L	L	L	L
	5	F	F	F	F	F	F	L	L	L	L	F	L	L	L	L
	6	F	F	F	F	F	F	L	L	L	L	F	L	L	—	—
SMELTER	Feed	—	—	—	—	—	—	—	—	—	—	—	U	U	U	U
	Matte	—	U	U	—	—	—	U	U	U	U	—	U	U	U	U
	Coal	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
LEACH PLANT	Feed	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	Roaster	U	U	—	U	U	U	U	U	U	U	U	U	U	U	U
LOW GRADE LEACH PLANT	Feed	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
	Acid	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
MARGIN		3.051					3.171					3.369				
COPPER		1 295					1 309					1 401				
ORE		54 390					54 383					59 641				

A comparison of runs 1 and 2 shows which ore sources are the most profitable to operate; at Chingola the system would choose to cut back on sources 2, 3, 4 and 6 and mine source 5 generally at its upper limit. Source 1 is not mined at the upper limit due to capacity constraints on the treatment plants. Similarly at Rokana mining of sources 4, 5 and 6 is reduced, but sources 1, 2 and 3 are not required to be at their maxima.

Increasing smelter capacity increases production from the Rokana underground sources and mines extra ore to fill the enlarged smelter from Konkola and Chingola sources 1, 2 and 6. Although the results of this run are relatively obvious, they confirm that the model is behaving in the right manner, and also suggest which ore sources should produce the additional ore.

In each case all three treatment plants are full. (The only exception is the smelter in the first run where the matte handling constraint was not met in every year, but the matte produced was actually very close to the upper limit). The sulphide flotation at Chingola chooses to fill the roaster leaving as much capacity as possible in the smelter for other concentrates. As much dump tails as required to fill the low-grade leach plant are taken.

By allowing mining flexibility the total margin (that is, revenue minus total variable cost) is increased by approximately four per cent for virtually identical quantities of ore mined and copper produced. Increasing smelter capacity by 10 per cent together with the flexible mining gives an increase in margin of 10 per cent and increases the copper produced by approximately eight per cent.

CONCLUSIONS

The system described represents an attempt to produce a medium-term corporate planning system. Although in the earlier stages of development scepticism was expressed at the idea of building a company model, as G.P.S.$\overline{\text{V}}$. was refined, often in the light of such criticism, the system output became more realistic and gradually more interest was expressed in the system.

In its present state of development G.P.S.$\overline{\text{V}}$. is thought to have overcome the initial hurdles of acceptance, but it still has some way to go in this area. However, for some time now, the development has concentrated on removing idiosyncrasies and on making the system usable; future work on the system will concentrate on updating the plant models and assumptions included in the system, thereby improving the quality of the results.

The system is currently entering its final stage of being taken over by the Mining and Metallurgical Services Department which is responsible for evaluating company strategy. It will be used to investigate medium-term planning and provide solutions which will then be evaluated in more detail by the G.P.S. $\overline{\text{III}}$ system already mentioned.

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