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An Optimizing Regulator for the Control of Autogenous Milling Circuits

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An adaptive steady-state on-line optimizing regulator is proposed as an integrated approach to the control of autogenous run-of-mine milling circuits. This optimizing regulator continuously estimates an internal model of the controlled process, and, based on this, determines mill fresh solids feed rate, mill water flow rate and sump water addition flow rate such that a circuit performance objective is optimized. The dual requirement of controlling both the mill pulp load and the load of particles of grinding media size is thus directly and simultaneously addressed.

The concept and theory underlying the operation of the adaptive optimizing regulator are discussed.

Introduction

The introduction of autogenous run-of-mine (ROM) tumbling mills has been one of the most important developments in South African milling practice during recent decades. However, the potential economic advantages of this method have been partially offset by the difficulties encountered in achieving consistent product quality and flexibility of milling circuit operation. These difficulties can very often be traced to the inadequacy of the automatic control strategies presently in use.

The problem of achieving satisfactory automatic control of autogenous and semi-autogenous ROM milling circuits has attracted interest in recent years (Flook,[1] Duckworth and Lynch,[2] Pauw and co-workers[3]). The availability of reliable digital computer-based control systems has rapidly advanced the potential for implementing comprehensive control strategies which seek an optimum economic operating point for the circuit. However, present control systems have not gone much beyond the simple objective of maintaining certain process variables at pre-specified setpoints through the use of single-loop PID controllers or multi-variable controllers. Present-day standard practice is to additionally use a separate power peak-seeking controller that uses heuristic logic to manipulate only the fresh rock feedrate.

This paper gives a preliminary overview of a research project being undertaken in the Department of Electrical Engineering at the University of the Witwatersrand, Johannesburg. The objective is to develop an advanced integrated supervisory control algorithm that continuously tracks and maintains the optimum economic operating condition of an autogenous ROM milling circuit. A requirement is that the algorithm must be implementable on a typical modern process control computer of moderate processing power.

The method proposed in the paper does not make use of detailed off-line methods or simulations of the milling circuit, but is based instead on the on-line identification of a simple dynamic process model. The gradient of a suitable objective function for the corresponding steady-state model is determined from this dynamic model, and it guides the next move of the multiple control input variables, eventually moving the plant to the optimum operating point.

The resulting adaptive optimizing regulator does not remove the need for conventional setpoint following controllers. These remain an essential part of the first-level circuit control scheme. However, the need for a separate power peak-seeking controller is removed completely. The optimizing regulator operates as a supervisor on a second level and supplies the setpoints for the conventional first-level controllers.

Results of the first phase of the research indicate that the proposed optimizing regulator appears to be very promising for application to the control of autogenous and semi-autogenous ROM milling circuits. It provides a number of advantages: it is an integrated approach to the problem and is inherently multivariable in nature; it can track a shifting optimum operating point reasonably fast; it is not very sensitive to process noise, and it provides sufficient flexibility, for example, to allow the handling of constraints and control objectives which change from time to time.

Control objectives in autogenous run-of-mine milling circuits

Duckworth and Lynch[2] point out that one of the main problems in developing control strategies for a ROM milling circuit relates to the conceptual difficulty of designing the control system to meet the control objectives. The common control objective stated by most workers, for example, Duckworth and Lynch,[2] Pauw and co-workers[3] and Lynch,[4] is to operate the circuit at the maximum throughput tonnage that will allow the desired product particle size to be maintained. Occasionally an alternative stated objective is to produce the finest possible product size for a given throughput. Various control schemes are then synthesized, but invariably these only address the real control objectives indirectly. For example, control of cyclone feed flow conditions emphasizes the optimal use of the classifier, but neglects the resulting effect on grinding efficiency in the mill, whilst power peak-seeking control emphasizes optimal grinding, but neglects the resulting effect on classifier efficiency. Furthermore, the actions of a power peak-seeking controller disturb the setpoint following controllers. Optimization of the whole circuit is not achieved.

The well-known difficulty of moving into an unstable operating condition (Duckworth and Lynch[2]) is also a symptom of the lack of co-ordination of the various individual controllers. This problem and the problem of interaction between peak-seeking logic control and the individual PID controllers have rendered existing control schemes unsatisfactory.

Recent studies into the internal mechanisms taking place in autogenous mills (Stanley,[5] Stanley,[6] Flook[7]) provide clear new indications as to how milling circuits should be controlled. Stanley[6] highlights the fact that there are two major aspects to the ROM milling circuit control problem, namely the control of the load of larger rock particles of grinding media size in the mill and the control of the mill pulp loading. There is considerable interaction and trade-off between these two aspects which makes control of ROM milling circuits particularly difficult.

Power drawn by the mill is a good indicator of grinding media load, but is also affected by changes in pulp loading. A good automatic control system must distinguish between these two possibilities in order to make appropriate corrections.

Power drawn by the mill however, is only a partial performance indicator. The transfer of power to the pulp and consequently the amount of breakage within the pulp, is determined by mill pulp loading. Optimum transfer of energy is indicated by mill rattle, discharge pulp temperature and discharge fineness.

Economically significant unmeasured disturbance variables to be dealt with by a good control scheme include changes in feed-size distribution, grinding media competence, grinding media hardness, the amount and condition of ferrous grinding media in the mill (in the case of semi-autogenous milling) and the degree of wear of the mill liner (Flook[7]).

In view of the above factors, we see the problem of autogenous ROM milling circuit control as a direct multi-input optimization problem. The three usual control inputs of interest are fresh rock feedrate, flowrate of water to the mill and flowrate of water to the sump. Depending on the values of the disturbance variables at a point in time, a unique combination of these control inputs must be determined to optimize the circuit economic performance.

In the initial phase of the research we are taking mill power as the economic criterion to be optimized. Later we will investigate the dual-criterion optimization of mill power and discharge pulp temperature.

Support for our approach is provided by the recent results reported by Pauw[3] and co-workers which show that an indirect two-input optimization approach gives promising results in milling circuit applications.

Theory of on-line optimizing regulation

A very general description of the acceptable operation of a dynamic system such as a milling circuit consists of a set of n differential equations, a set of q inequality constraints and a set of m output relationships given by:

$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{d}) \qquad [1]$$

$$\mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{d}) \leqq 0 \qquad [2]$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{u}) \qquad [3]$$

where $\mathbf{x}$ = n-vector of model state variables, $\mathbf{u}$ = ℓ-vector of manipulable control inputs, $\mathbf{y}$ = m-vector of measurable outputs and $\mathbf{d}$ = r-vector of disturbance variables.

Such a model can be used to obtain a current estimate (at time t=0) of the optimum trajectory of the real system by finding control inputs $\mathbf{u}(t)$ to minimize the integrated total of a specified objective function or performance index, Ψ, over the time period [0,T]. This optimization problem may be written as:

$$\min_{\mathbf{u}} \int_{o}^{T} \Psi(\mathbf{y},\mathbf{u},\mathbf{d}) \; dt \qquad [4]$$

such that [1], [2] and [3] are satisfied.

Note that Ψ is an economic objective function that is possibly measured directly, for example, power in the case of a milling circuit. In the more general case Ψ could be an arbitrary algebraic function of the control inputs $\mathbf{u}(t)$, plant outputs $\mathbf{y}(t)$ and disturbances $\mathbf{d}(t)$. For particular values of the inputs, outputs and disturbances, Ψ is assumed to have an extremum, a minimum or maximum point, and it is desired to keep the system operating at this extremum value while avoiding constraint violations. We call this safe tracking of the optimal operating point, optimizing regulation.

Note that Ψ is not the usual linear quadratic objective function used in optimal control theory as discussed, for example, by Herbst and Rajamani.[8]

It is well-known that [4] represents an exceptionally difficult non-linear dynamic optimization problem, and excessive amounts of computing are required for its solution. We therefore propose a simplification based on classifying process disturbances according to their frequency spectra.

Practical optimization time periods [0,T] are typically large relative to the dominant process time constants, and therefore only persistent disturbances, $\mathbf{d}_s$, with periods larger than the process settling time have an important effect on Ψ. Rapidly varying disturbances, $\mathbf{d}_f$, also called process noise, comparable to or faster than the dominant system time constants, are effectively non-existent relative to the optimization period. Also, the influence of these rapidly varying disturbances can be suppressed by using conventional single-variable or multi-variable regulatory control.

Therefore the plant can be assumed to be at quasi-steady state during the time period [0,T] for optimization purposes.

In order to implement the conventional regulatory control subsystem, a subset of the inputs $\mathbf{u}_1$ is selected to control a subset of the outputs, $\mathbf{y}_1$, in the face of disturbances, $\mathbf{d}_f$. Using the quasi-steady state assumption, optimization problem [4] can then be reformulated as a steady-state or static optimization problem, dependent on persistent disturbances $\mathbf{d}_s$:

$$\min_{\mathbf{y}_1^*, \mathbf{u}_2} \quad \Psi(\mathbf{y}_1^*, \mathbf{y}_2, \mathbf{u}_2, \mathbf{d}_s) \qquad [5]$$

such that

$$\mathbf{f}\ (\mathbf{y}_1^*, \mathbf{y}_2, \mathbf{u}_2, \mathbf{d}_s) = 0 \qquad [6]$$

$$\mathbf{g}\ (\mathbf{y}_1^*, \mathbf{y}_2, \mathbf{u}_2, \mathbf{d}_s) \leq 0 \qquad [7]$$

where

$$\mathbf{y} = [\mathbf{y}_1, \mathbf{y}_2]^T, \quad \mathbf{u} = [\mathbf{u}_1, \mathbf{u}_2]^T$$

$\mathbf{y}_1^*$ is a vector of setpoints for the conventional regulators and $\mathbf{y}_2$ is a vector of the remaining uncontrolled outputs. The resulting general control structure is shown in Figure 1.

The task of the optimizing regulator is to repeatedly solve [5] for the optimal input variables $\mathbf{u}_2$ and $\mathbf{y}_1^*$. The resulting control inputs, $\mathbf{u}_2$, are then applied directly to the plant while control inputs $\mathbf{u}_1$, have been replaced by the setpoints $\mathbf{y}_1^*$ of the associated conventional regulators. The solution is calculated every T_o seconds where the choice of T_o depends on the speed of variation of the disturbances, $\mathbf{d}_s$. In summary, the task of the optimizing regulator is to track a shifting optimum that is affected by disturbances that vary slowly compared with the dominant plant time constants. Note that this approach inherently provides manipulation of multiple control inputs.

An adaptive optimizing regulator

A wide variety of techniques have been proposed for use in digital process control computers for maintaining a process at its optimum steady-state operating point. In off-line methods (Savas,[9] Webb and co-workers,[10] Maarleveld and Rijnsdorp[11]), key process measurements are regularly supplied to a predetermined detailed steady-state process model and a static optimization procedure is then performed to find the required control inputs. These are then applied to the plant. Although a detailed nonlinear model can be used and fast static optimization algorithms are available, this method suffers from two serious disadvantages. Firstly, most economically important disturbances cannot be measured or modelled exactly. Secondly, even for processes of low complexity, off-line models are difficult to obtain and are always inaccurate owing to the impossibility of modelling all effects. Consequently, it is imperative that the optimizing regulator interacts with the operating process in some way so that all economically important

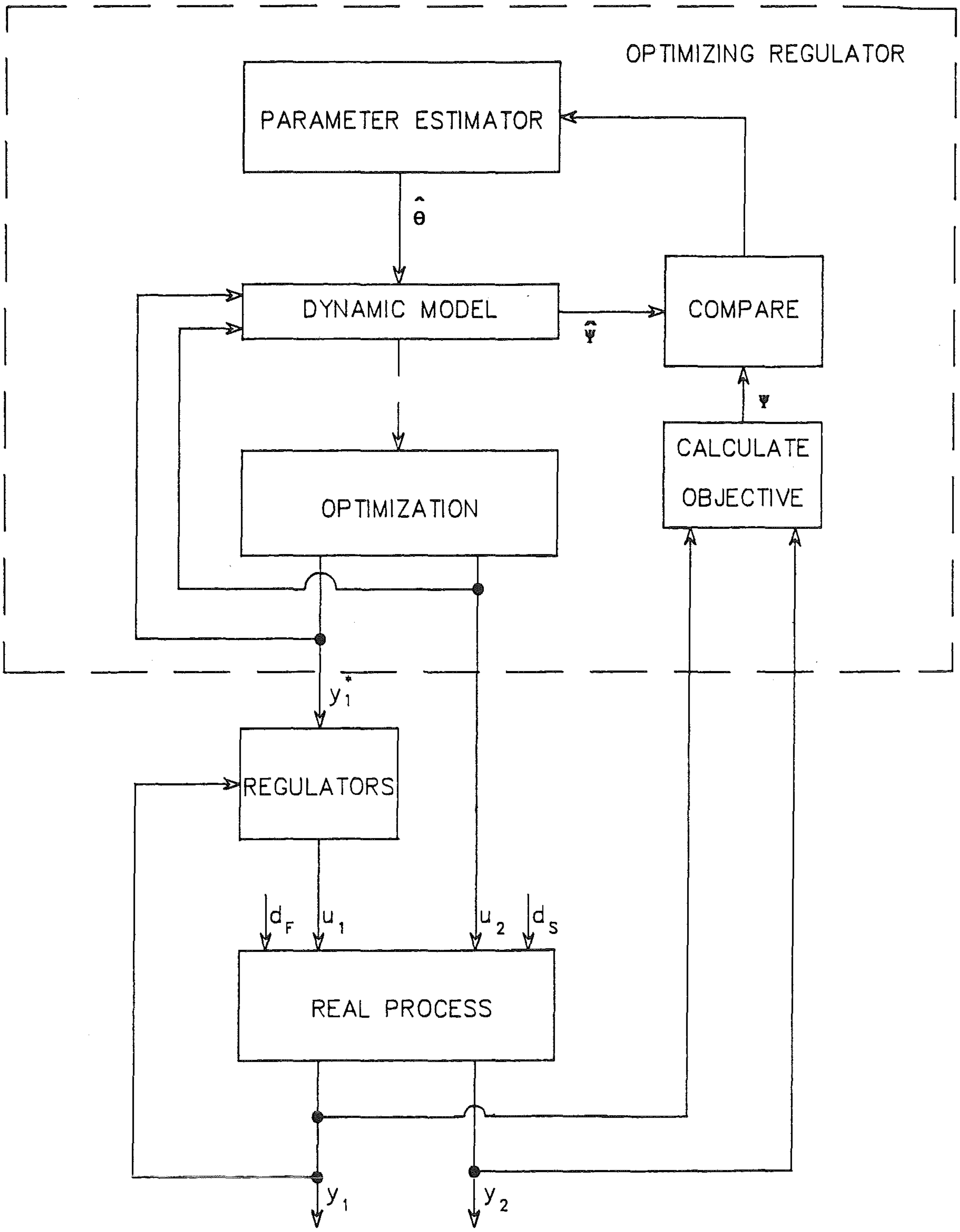

FIGURE 1. General structure of a control system incorporating an optimizing regulator

disturbances are detected as soon as they affect plant outputs, and a detailed fundamental model is not required.

Numerous on-line methods for obtaining steady-state models through direct searches on the operating plant have been proposed. Edler and co-workers[12] compare the performance of different techniques. Since steady-state information is required, measurements should be taken only after the process has settled after each change in the control inputs. This results in a very slow search procedure. On the other hand, Sawaragi and co-workers[13] have found that very complex stability problems arise if the control inputs are changed before process transients have died away. Furthermore, these methods are very sensitive to process noise (Saridis[14]).

In the present research project we have selected an approach based on a two-step procedure of regularly determining the parameters of a steady-state mathematical model and then adjusting the control inputs so that the performance index is at its extreme value. This approach is very closely linked to adaptive control and first attracted attention in the 1960s (Blackman,[15] Jacobs[16]), but lack of suitable computing hardware made practical implementation difficult. With the availability of microprocessors there has recently been renewed interest in this approach (Sternby,[17] Garcia and Morari,[18] Garcia and Morari[19]). Also, encouraging theoretical results concerning the stability and convergence properties of algorithms incorporating this adaptive two-step optimization procedure are now available (Haimes and Wismer,[20] Roberts and Williams[21]).

In order to overcome the problems of having to wait for the plant to settle to a steady-state after each adjustment to the control inputs and sensitivity to noise, the most promising approach appears to be to determine the steady-state process model parameters by recursively estimating the parameters of a simple dynamic input-output model during the transient response, as suggested by Bamberger and Isermann.[22] It is then a simple matter to extract the corresponding steady-state model. This can be used to solve optimization problems [5] and thereby determine how the control inputs should be varied in order to improve plant economic performance. The procedure is then repeated at the new operating point with a dynamic process identification followed by an optimization step, and so on until the optimal point is reached.

Outline of the algorithm and its application

Many different combinations of available on-line dynamic parameter estimation techniques (Seborg and co-workers[23]), and techniques for solving static optimization problems (Bryson and Ho[24]) can be used in implementing the approach proposed above. It is one of the aims of our research to identify the best combination for the control of autogenous ROM milling circuits. In order to illustrate the integration of these techniques into an on-line optimizing regulator, we present the internal theory in some detail.

In order to simplify explanation and highlight the basic principles, we treat here the simplest possible implementation. A number of simplifications have been made and the resulting regulator scheme is not intended to be implemented in this elementary form. In the following we select a second-order deterministic autoregressive moving average process model, a simple projection algorithm for parameter estimation and unconstrained optimization using the steepest descent technique. Also we treat the (unrealistic) case where no conventional regulatory controllers are used.

The static optimization problem [5] becomes:

$$\min_{\mathbf{u}} \quad \Psi \left(\mathbf{y}(\mathbf{u}) \ , \ \mathbf{u}\right) \qquad [8]$$

where the first component of the output vector, $y_1(u)$, is the mill power and the remaining components are other outputs. The control input vector is:

$$\mathbf{u} \quad = \quad [u_1 \ u_2 \ u_3]^T \qquad [9]$$

where u_1 = fresh rock feedrate, u_2 = mill water flowrate and u_3 = sump water flowrate. Ψ can be optimized by means of a simple gradient search algorithm,

$$\mathbf{u}_{k+1} \quad = \quad \mathbf{u}_k \ - T_o \ \nabla_{\mathbf{u}} \ \Psi\Big|_{k} \qquad [10]$$

where $\nabla_{\mathbf{u}}$ denotes the gradient with respect to $\mathbf{u}$. This algorithm is executed regularly at times $t=kT_o$, $k\varepsilon\{0,\ 1,\ 2...\}$.

Applying the differentiation chain rule to the gradient in [10] gives:

$$\nabla_{\mathbf{u}} \ \Psi \ (\mathbf{y},\mathbf{u})\Big|_k \quad = \quad \frac{\partial}{\partial \mathbf{u}} \ \Psi(\mathbf{y},\mathbf{u})\Big|_k + \frac{\partial \mathbf{y}}{\partial \mathbf{u}}\Big|_k \ \frac{\partial}{\partial y} \ \Psi(\mathbf{y},\mathbf{u})\Big|_k \qquad [11]$$

In the present example we choose to control the plant so as to achieve maximum power. The objective function is therefore:

$$\Psi(\mathbf{y},\mathbf{u}) \quad = \quad -y_1 \qquad [12]$$

Substituting this into [11] gives

$$\nabla_{\mathbf{u}} \ \Psi\Big|_k \quad = \quad -\left|\frac{\partial y_1}{\partial u_1} \quad \frac{\partial y_1}{\partial u_2} \quad \frac{\partial y_1}{\partial u_3}\right|_k^T \qquad [13]$$

In the present special case we therefore only need a model for output y_1.

We assume that a second-order ARMA model will provide an adequate representation of the behaviour of the mill power. This can be justified on the grounds that, physically, certain effects contributing to power have slow dynamics compared with others. Also, from an optimization point of view, the search directions are determined using first derivatives only and a higher order model is therefore not justified. The predicted power is thus assumed to be given by

$$\begin{aligned}\hat{y}_1(k) \quad = \quad & -a_1 y_1(k-1) \ - \ a_2 y_1(k-2) \\ & + b_{11} \ u_1(k-d) \ + \ b_{21} \ u_1(k-d-1) \\ & + b_{12} \ u_2(k-d) \ + \ b_{22} \ u_2(k-d-1) \\ & + b_{13} \ u_3(k-d) \ + \ b_{23} \ u_3(k-d-1)\end{aligned} \qquad [14]$$

Note that an estimate of the pure delay, d, is required.

This can be expressed compactly in regression form as

$$\hat{y}_1(k) \quad = \quad \boldsymbol{\phi}^T(k-1) \ \hat{\boldsymbol{\theta}}(k-1) \qquad [15]$$

where

$$\phi^T(k-1) \ = \ [-y_1(k-1), -y_1(k-2), u_1(k-d), u_2(k-d),\ u_3(k-d), u_1(k-d-1),$$

$u_2[k-d-1), u_3(k-d-1)]$

$$\theta^T(k) = [a_1, a_2, b_{11}, b_{12}, b_{13}, b_{21}, b_{22}, b_{23}] \quad [16]$$

The model parameters can be estimated by means of the following projection algorithm (Goodwin and Sin[25]):

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\Phi(k-1)[y_1(k) - \hat{y}_1(k)]}{1 + \Phi^T(k-1)\ \Phi(k-1)} \quad [17]$$

The gradient required in Equation [10] can now finally be computed from the estimated parameters by finding the steady-state solution of [14] and applying [13]. Substituting the result into [10] gives the optimizing control law directly in terms of the estimated parameters:

$$u_1(k+1) = u_1(k) + T_o \left| \frac{\hat{b}_{11} + \hat{b}_{21}}{1 + \hat{a}_1 + \hat{a}_2} \right|_k$$

$$u_2(k+1) = u_2(k) + T_o \left| \frac{\hat{b}_{12} + \hat{b}_{22}}{1 + \hat{a}_1 + \hat{a}_2} \right|_k$$

$$u_3(k+1) = u_3(k) + T_o \left| \frac{\hat{b}_{13} + \hat{b}_{23}}{1 + \hat{a}_1 + \hat{a}_2} \right|_k \quad [18]$$

Simulation results

Figure 2 shows the simulated performance of a typical fully-autogenous ROM grinding circuit under control of the optimizing regulator. In this test the optimizing regulator manipulates the fresh rock feedrate and the flowrate of water to the mill. The mill discharge sump level is controlled by a conventional proportional controller which manipulates the sump discharge pump speed. Water is added to the sump at a constant flowrate.

A simple model is used to simulate the grinding circuit. In this model, mill load is split into three size fractions, namely, particles of grinding media size, pulp and particles smaller than the required grind size. The physical mechanisms describing comminution included in the model are chipping, abrasion, attrition and impact breakage. Mill power changes dynamically with changes in solids feedrate, feed coarseness, mill water addition, sump water addition and pulp flowrate. The effects of mill overload, pulp hold-up and pulp density are included in the model. The Plitt equation is used to model the hydrocyclone. The circuit simulation was validated by qualitative comparisons with responses of real circuits.

The practical details of the results in Figure 2 were: optimization update interval T_o = 30 min., parameter estimator update interval = 3 min., optimizer started at t = 5 min., magnitude of noise added to fresh rock feedrate and flowrate of water to the mill = 1%, parameter estimator = recursive least squares with a relative dead zone in the prediction error to reduce the effects of noise and unmodelled dynamics, optimization technique = gradient algorithm. A disturbance occurred at t = 500 min. This consisted of a 10% increase in the proportion of fines in the fresh rock feed.

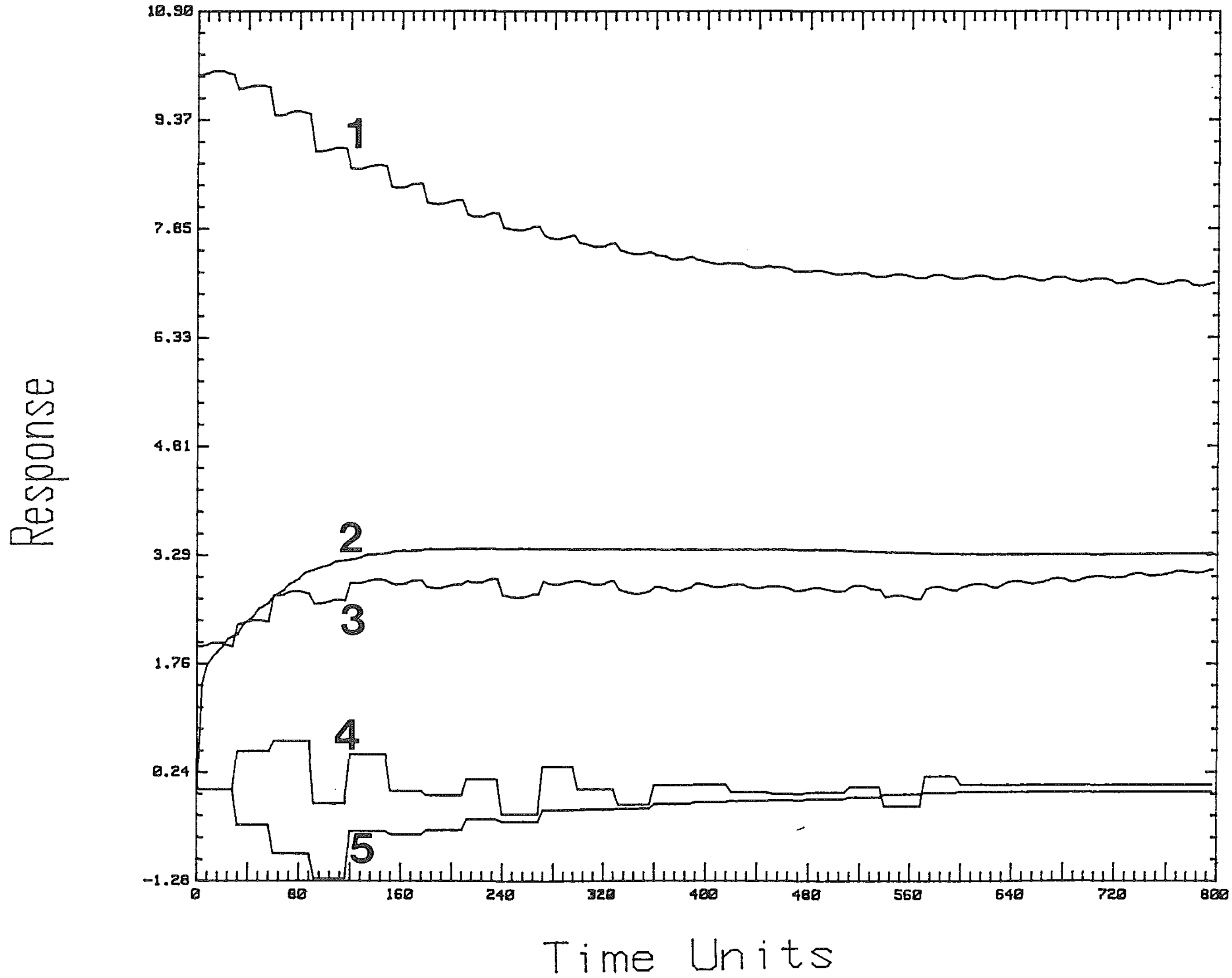

FIGURE 2. Simulated ROM circuit responses under control of optimizing regulator. Curve 1: mill water flowrate. 2: mill power. 3: fresh rock feedrate. 4: power gradient w.r.t. rock feedrate. 5: power gradient w.r.t. water flowrate

Conclusions

The concepts and theory underlying an adaptive optimizing regulator have been presented. It appears as though this kind of controller is directly applicable to the control of autogenous run-of-mine milling circuits and is suitable for implementation in a typical process control computer. It should be seen as being complementary to conventional single-variable or multivariable regulatory controllers in this application and provides a way of integrating the operation of such controllers.

A number of aspects of the proposed approach obviously require further investigation before their practical significance can be assessed.

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