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A Robust Approach to the Design of Milling Circuit Control Systems in the Presence of Modelling Errors

A. MONTINI, M.H. MOYS and I. MACLEOD

Department of Electrical Engineering, University of the Witwatersrand, Johannesburg

The problems encountered in the design of control systems for milling circuits when the model of the system and the model parameters are uncertain are discussed. Considering these uncertainty factors has led to the development of a technique that gives a quantitative assessment of real system stability, reveals the trade-offs between performance and stability and helps the designer in deciding what level of model complexity to use in the controller design. Furthermore, it allows for on- or off-line optimization of the milling circuit performance as it provides the base for the design of a controller which is stable over the plant operating range.

Introduction

The last two decades have seen many developments in the control theory of linear and nonlinear systems. Frequency and state space approaches[1-3] have been used to design and optimize control systems in the metallurgical field.[4-5] Many theoretical problems have drawn the theoreticians' attention, while others, sometimes very important, have been completely disregarded. These factors have created a growing scepticism as to whether many of the theoretical approaches are relevant to the control engineer.

One of the easily identifiable weaknesses of recently developed control theories is that they do not consider or do not place the right design emphasis on the uncertainty of the system model. Control engineers generally assume a simplified plant model in the design of a control system, but do not usually check if a controller designed using an approximate model of the real plant will stabilize and attain to prescribed performances when used on the real plant.

A grinding mill circuit is an example of a complex multivariable, nonlinear and time-varying system, and it requires a sophisticated measurement and control strategy. One line of attack on this problem is to improve the measurement side of it. The measurement of conditions inside the mill is currently being explored.[6-8] These measurements provide direct monitoring of conditions inside a mill and may detect changes in milling conditions within 25-45 seconds, while measurements made on the discharge from the mill respond with time constants of minutes[9,10] to change in feed conditions. Much tighter control of mill behaviour is therefore possible. A second line is to design better control systems through a deeper understanding of the underlying theories of modelling and control.[4,11-15] Although many of these theories are elegant approaches, unfortunately there is not yet much sign[5] of their practical use in the metallurgical field. Perhaps the main reason is that many of the advanced mathematical tech-

niques have not included a design step that evaluates the robustness to modelling errors and model structure uncertainty. This step can provide better results in terms of stability and achievable performance of the system, and may even simplify the overall design process.

It is the scope of this paper to :

1. Highlight the control problems which may occur when the plant model is not accurate.
2. Introduce a technique to evaluate the robustness of a feedback controller to model uncertainties.
3. Give stability conditions and a design procedure easily integrable with standard design techniques.
4. Discuss the problems encountered in the design of a multivariable robust controller for a grinding pilot ball mill.
5. Discuss the role of robust control theory concepts in metallurgical and specifically in milling control applications.

Modelling error effects on linear feedback systems

Problem statement

Consider the problem of designing a feedback controller for a system whose transfer function $G_p(s)$ is known to have modelling errors as follows:

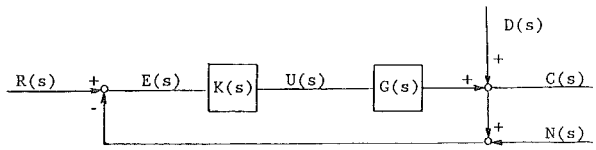
i) the real $G_p(s)$ is regarded as too complex for the design exercise, therefore an approximate system model is used;

ii) the structure and parameters of $G_p(s)$ are unknown or only partially known;

iii) a linear approximation of an input-output bounded nonlinear $G_p(s)$ has to be used for feedback controller design.

Suppose that a *NOMINAL MODEL G(s)* has been chosen to represent the system and a controller K(s) has been designed to produce the required stability and performance characteristics of this nominal model.

Thus:



$$C = G\,K\,(I+G\,K)^{-1}\,(R-N) + (I+G\,K)^{-1}\,D \qquad [1]$$

Where : N is the measurement noise
R is the command input
D is the disturbance
U is the manipulated input
C is the controlled output.

The error signal is :

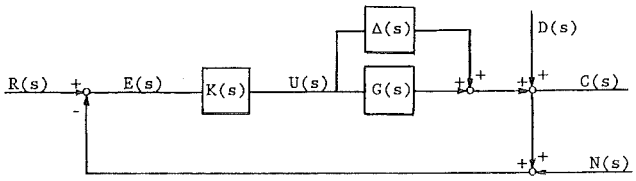
$$E = R-C$$
$$= (I+G\,K)^{-1}\,(R-D) + G\,K\,(I+G\,K)^{-1}\,N \qquad [2]$$

Can this controller K(s) stabilize the real system $G_p(s)$ and produce the required performance ?

It seems obvious to assume[16] that if G(s) is the nominal model of the plant, the real unknown plant model can be enveloped by the relation :

$$G_p(s) = G(s) + \Delta(s) \qquad [3]$$

where $\Delta(s)$ is called the additive perturbation of the nominal model. $\Delta(s)$ gives an assessment of all possible perturbations due to the real unknown plant around the nominal approximation. Therefore a proper closed loop block diagram representation of the real perturbed system is the following:



and

$$\Delta H_{ol} = \Delta(s)\,K(s) \qquad [4]$$
$$\Delta H_{cl} = (I + G_p\,K)^{-1}\,\Delta H_{ol} \qquad [5]$$

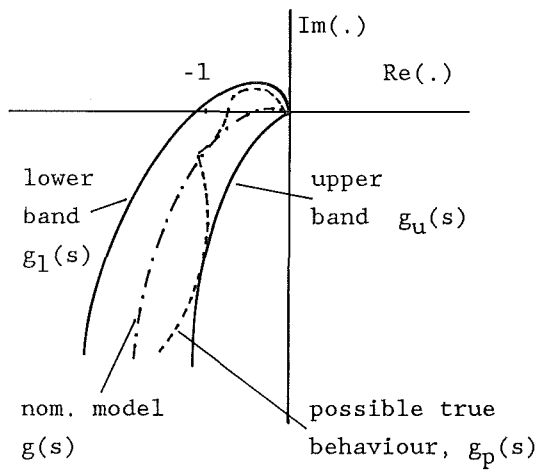
Where : ΔH_{cl} and ΔH_{ol} are the changes in closed loop and nominal open loop systems due to $\Delta(s)$.

Equations [1-5] summarize benefits and conflicts of a design. Loop sensitivity may be decreased by increasing the gain of K, thereby reducing the gain of the sensiti-

vity operator $(I+GK)^{-1}$, this also reduces changes in the closed loop transfer function due to the perturbation Δ. An increase in gain is accompanied by increasing noise measurement effects; furthermore the open loop gain GK can only be increased up to a certain point because of (a) physical limits on the manipulable inputs, and (b) stability requirements if the system is, as it may quite often be, non-minimum phase. It can be anticipated that considering this sensitivity problem will affect the obtainable performance of the closed loop system but this is unavoidable if there is considerable system model uncertainty.

Stability conditions for perturbed systems

The stability concept of a perturbed system can be illustrated by referring to the following sketch of a single input-single output open loop perturbed system:



Provided the open loop system is stable then the Nyquist stability criterion requires that the (-1,0) point must not be encircled by the real system locus, i.e. by the nominal model g(s) + the perturbation bound $\delta(s)$ locus :

$$| g_p(s) + 1 | \geq 0 \qquad [6]$$

i.e.

$$| g(s) + \delta(s) + 1 | \geq 0 \qquad [7]$$

The extension to the multivariable systems is similar and can be written as :

$$\det | G(s) + \Delta(s) + I | \neq 0 \qquad [8]$$

It can be shown[17] that using the similarity transformation of the matrix G(s) :

$$X(s)^{-1} G(s) X(s) = \mathrm{diag} \{ q_i(s) \} \qquad [9]$$

the stability requirement [8] in the n-dimensional problem can be written as:

$$| q_i(s) | + \sum_{j=1}^{n} | r_{ij}(s) | > 1.; \; i=1,n \qquad [10]$$

where: s belongs to the Nyquist Contour D,

$r_{ij}(s)$ is the ij-th element of $X^{-1} \Delta(s) X$,

$q_i(s)$ is the i-th diagonal element of $(X^{-1} G(s) X)$,

X is a similarity scaling complex matrix.

Let $F(s)=\mathrm{diag}\{f_i(s)\}_{1\leq i\leq n}$ be a final shaping matrix of compensators $f_i(s)$, and $L(s)=\mathrm{diag}\{l_i(s)\}_{1\leq i\leq n}$ be a positive matrix representing the required closed system performance specification,[17] then an upper bound to the relative stability of the perturbed system $G_p(s)$ is obtained if:

$$|f_i(s)||q_i(s)|+|f_i(s)| \sum_{j=1}^{n} |r_{ij}(s)| > l_i(s) \quad i=1,n; \qquad [11]$$

It can be proven[17] that by manipulating the parameters of the compensators $f_i(s)$ to satisfy [11] a minimization of the conservativeness required to ensure a stable controller of $G_p(s)$ is achieved.

The modelling problem

To specify a control strategy it is necessary to have a model of the system to be controlled. Despite its imperfect structure, we aim to find the set of model parameter values that provides the best fit of the model to the data. The quality and/or complexity of the model depends on what that model has to be used for. The important point to note is that a band of model uncertainty may be drawn to envelope the nominal model. There are many approaches on how to define the uncertainty band of a dynamic model.[18-21] In this paper a very simple approach is used.[17]

Let $G_p(s)=\{g_{pij}(s)\}$ be an uncertain n x n plant transfer function matrix and let $y_{pij}(t)$ denote the set of the i-th system responses to the j-th manipulable input set:

$$x_j(t) = \{ x_j(t),\ p_j(t) \le x_j(t) \le q_j(t) \} \quad j=1,n; \quad [12]$$

where $p_j(t)$ and $q_j(t)$ are some saturation functions for the jth actuator.

Let a(t) and b(t) be two matrix of the extrema of the measured plant behaviour whose elements a_{ij} and b_{ij} are defined as follows:

$$\left|\int_0^T a_{ij}(t)\ x_j(T-t)\ dt\right| \le \min_t |Y_{pij}(t)| \quad [13]$$

$$\left|\int_0^T b_{ij}(t)\ x_j(T-t)\ dt\right| \ge \max_t |Y_{pij}(t)| \quad [14]$$

where T is a time which is large enough to ensure the correct identification process of the plant dynamics.

Defined the Laplace matrix functions :

$$G_u(s) = L\{ b(t) \} \quad [15]$$

$$G_l(s) = L\{ a(t) \} \quad [16]$$

where L{.} is the Laplace operator, $G_u(s)$ and $G_l(s)$ will be called upper and lower n x n bound transfer functions corresponding to the upper and lower n x n bound time domain matrix functions a(t) and b(t).

If a controller has been designed to stabilize even the worst case (from the stability condition point of view) of perturbed system, then the controlled system output c(t) to setpoint demand $c_{set}(t)$, provided that the manipulable input satisfy to limits in [12], is:

$$\int_0^T [c_u(t)-c(t)]^2\ dt \le \int_0^T [c_u(t)-c_l(t)]^2\ dt \quad [17]$$

where $c_u(t)$ and $c_l(t)$ are the closed loop system responses calculated by using a plant transfer function matrix represented by $G_u(s)$ and $G_l(s)$ respectively.

Design procedure

Computer Aided Graphics Design is becoming a powerful tool for solving iterative design problems. In this section a design procedure based on the theory developed above is described. The procedure is based on the assumption that the perturbation model of the system is represented by a band of transfer functions as described in the previous section. An example of how this band of functions have been derived and used for controller design is reported in the following section. In this procedure the design will be carried out considering the two, lower $G_l(s)$ and upper $G_u(s)$, transfer function models simultaneously.

The basic idea of this method is that a simple nominal model can be chosen between the upper and lower model, and this nominal model can be used to design a compensator to stabilize limit band conditions of the perturbed system G_p. This procedure allows one to see how a certain choice for a nominal model is going to affect the limit band of performance and the achievable closed loop characteristics. The optimization in the choice of the average nominal model is then a result of the relation between refinements of this model structure and improvements in performances due to this structure. The main tool is a theory that can quantify stability bounds as developed in the previous sections, made accessible through the use of computer graphics facilities.

Procedure

Given a plant represented by the perturbation $G_p(s)=G(s)+\Delta(s)$:

1) Identify the perturbation matrix $\Delta(s)$. This can be done either by input-output analysis or by other methods.[18-21]
2) Define 'upper' and 'lower' transfer function matrices (equations [15-16]) under the experimental requirement given by [12] for the manipulable input of the plant.

3) Choose a suitable 'simple' transfer function nominal model within the model bands [15-16].
4) Design a decoupling matrix X(s) to pseudo-diagonalize the nominal model.
5) Let :

$$Q_u(s) = X(s)^{-1}\, G_u(s)\, X(s) \qquad [18]$$

$$Q_l(s) = X(s)^{-1}\, G_l(s)\, X(s) \qquad [19]$$

Plot as s follows the Nyquist contour D the functions:

$$\Gamma_{ui}(s) = \sum_{j=1}^{n} q_{uij}(s)\, f_i(s) \qquad [20] \quad i=1,n$$

$$\Gamma_{li}(s) = \sum_{j=1}^{n} q_{lij}(s)\, f_i(s) \qquad [21] \quad i=1,n$$

where: q_{uij} and q_{lij} are the i-j-th elements of Q_u and Q_l respectively,
f_i is the i-th diagonal element of the diagonal matrix F(s).

6) Let $L(s)=\mathrm{diag}\{l_i(s)\}_{i=1,n}$ be a matrix representing a set of requirements for closed loop behaviour in the Laplace domain. Choose the average parameters for $f_i(s)$ in [20-21] such that :

$$|\Gamma_{ui}(s) - l_i(s)| \geq 0 \qquad [22]$$

$$|\Gamma_{li}(s) - l_i(s)| \geq 0 \qquad [23]$$

$$i=1,n \quad s \epsilon D$$

7) Simulate the time domain response of the perturbed control system to different setpoint demands and disturbance signals while checking saturation effects [12] for final control elements. If saturation conditions are evident then the f_i's gain must be decreased or a new nominal model must be chosen and the design repeated starting from Step 3.

Design example

Grinding mill circuit control is one of the most debated topics in applied Multivariable Controller Design.[4,12] A milling system model is normally nonlinear and time varying. Provided that a range of operations has been identified, a suitable nonlinear dynamic model can be chosen to approximate the real plant. Generally no attempt is made to give guidelines for the choice of the nominal model or to quantify what effect the modelling errors would have on the controller performance. This is generally assessed on the plant when the quality of the control system and therefore the quality and range of acceptability of the controller's performances are tested.

In the design application reported below, the proposed approach is used to show how these effects can be assessed before tests need to be performed on the real plant.

The system or plant used is a pilot grinding circuit shown in Figure 1. This system has been described in other papers.[6,9,10] The operating output variables used to describe the system are the %solids in the slurry inside the mill, inferred from a secondary type of measurement Θ_D (D_TH),[9,10] the flow and density of the slurry fed to the classifier or cyclone (CYFR and CYFD). Manipulated inputs are in this case the feed rate of solid (FSRT), and the flowrates of the water added to the mill (FWRT) and to the mill discarge sump (SWRT).

This plant's dynamic model has been studied by input-output analysis. The system was taken to steady-state points in the operating range and several upwards and downwards and different size step tests in the manipulable variables were performed and the system output responses recorded. One such filtered and normalized test response set of CYFR to FWRT is reported in Figure 2. Depending on the operating variable values and the size of step used, different models might have been used to represent these dynamics. No attempt to fit proper models through them has been made. Upper and lower model bounds have been traced to envelop the set of data. Only for the sake of comparison the average model has been obtained (see Figure 2). The same analysis has been carried out to inspect the whole matrix of system responses to the system inputs. Figure 3 is therefore obtained. Table 1 reports the derived average nominal model $G_{aver}(s)$ to fit the system responses. The question addressed at that point is concerned with

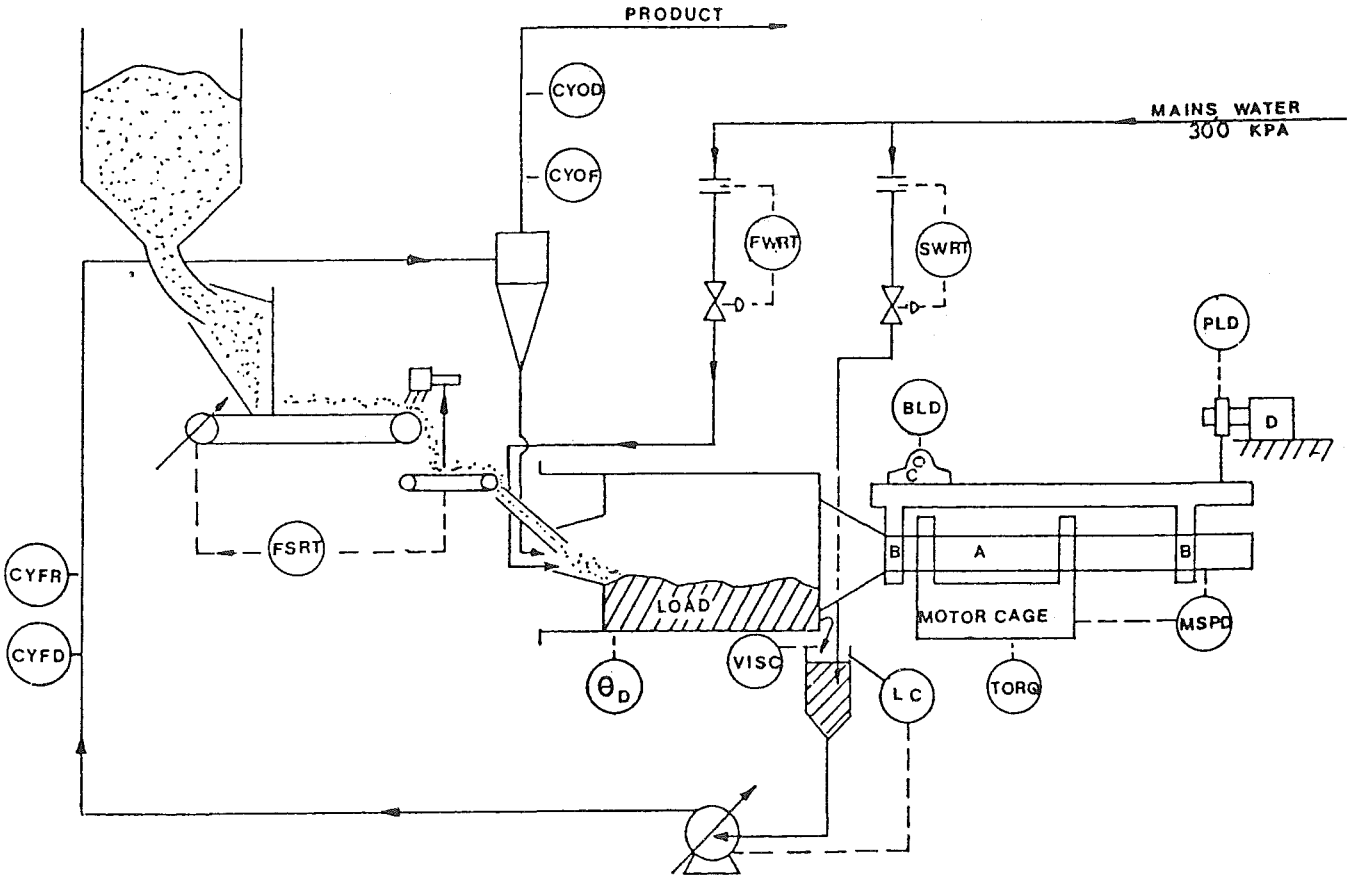


FIGURE 1. Pilot grinding mill

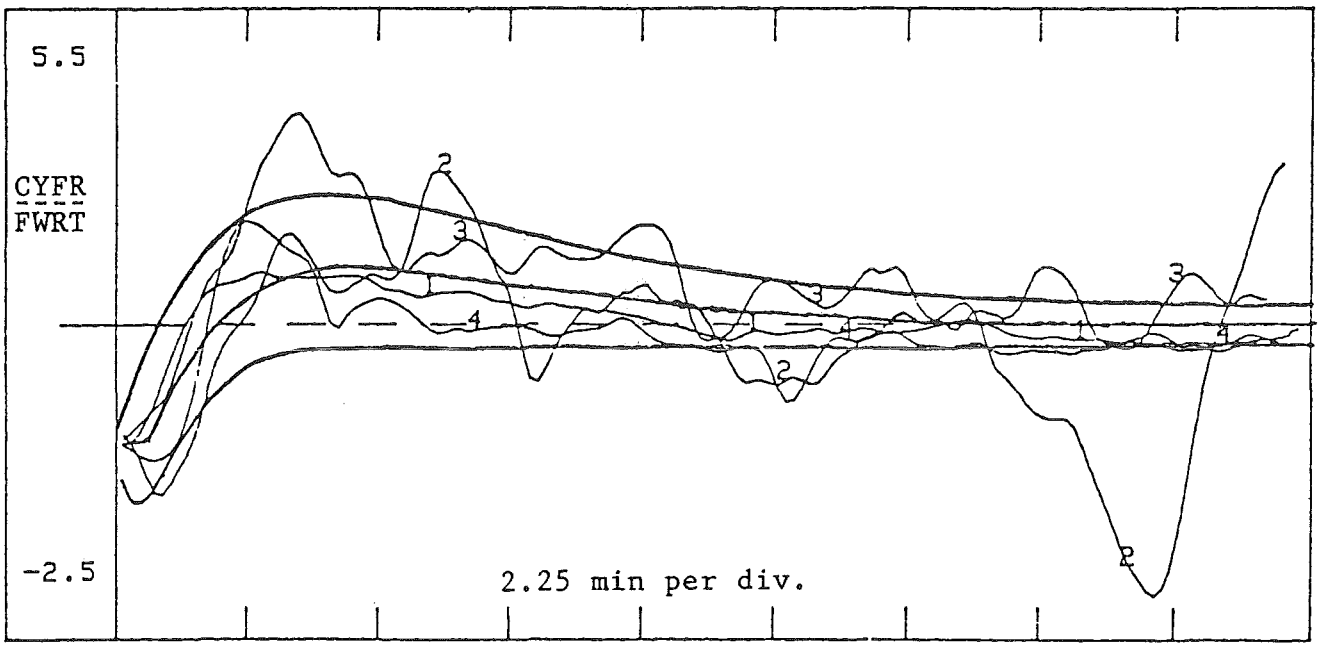


FIGURE 2. Filtered normalized step responses of CYFR to FSRT (1) CYFR response to −1,5 kg/min step change to FWRT, (2) CYFR response to −0,4 kg/min step change in FWRT, (3) CYFR response to 0,4 kg/min step change in FWRT, (4) CYFR response to 1,0 kg/min step change in FWRT

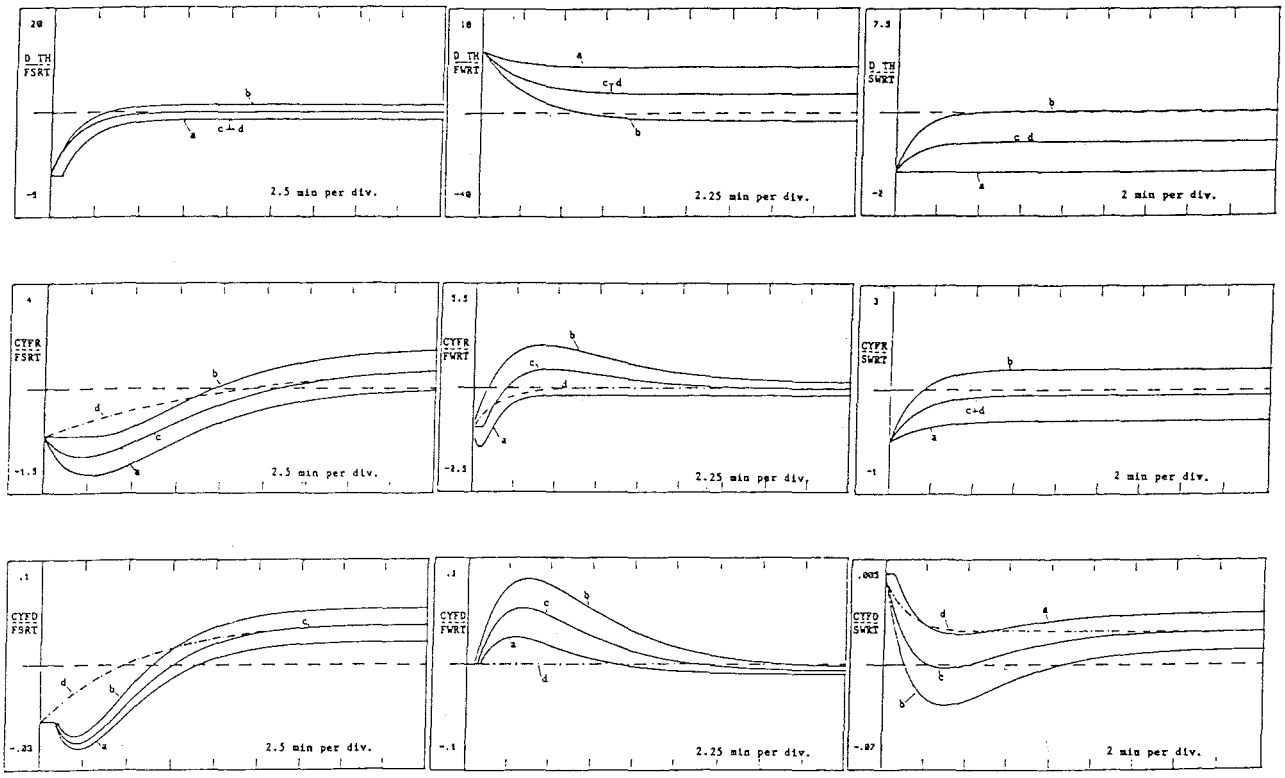


FIGURE 3. Matrix of system normalized step response bands. (a) & (b) are the bound curves, (c) the 'average' model responses and (d) the 'simple' model responses

TABLE 1 : Average model $G_{aver}(s)$.

$\frac{7.47}{1 + 80\ s}$	$\frac{-10.5}{1 + 130\ s}$	$\frac{1.4}{1 + 60\ s}$
$\frac{1.775(1-300\ s)}{(1+220\ s)(1+240\ s)}$	$\frac{1.444(1+500\ s)e^{-35\ s}}{(1+130\ s)(1+180\ s)}$	$\frac{.9}{1 + 80\ s}$
$\frac{.0597(1-145\ s)e^{-55\ s}}{(1+140\ s)(1+140\ s)}$	$\frac{-.0068(1-3300\ s)e^{-37\ s}}{(1+130\ s)(1+175\ s)}$	$\frac{-.02055(1+454\ s)}{(1+115\ s)(1+140\ s)}$

TABLE 2 : Simple model G(s) for controller design.

$\frac{7.47}{1 + 80\ s}$	$\frac{-10.5}{1 + 130\ s}$	$\frac{1.4}{1 + 60\ s}$
$\frac{1.775}{1 + 550\ s}$	$\frac{1.444}{1 + 75\ s}$	$\frac{.9}{1 + 80\ s}$
$\frac{.0597}{1 + 300\ s}$	0	$\frac{-.02055}{1 + 50\ s}$

absolute and relative stability of real plant, i.e.: how complex must the plant model used in the controller design be to produce a reliable design?

Previous design experience gained on an INA design package[11] led to the choice of a simple matrix model approximation (Table 2) in which only first order terms appear (nonminimun phase components and time delays have been dropped). Its normalized step responses are compared with the enveloped model response bands to the same inputs as shown in Figure 3 (dot lines). Considerable differences between the two model responses exist. The approximation chosen (Table 2) is so simple that decoupling this matrix is something that can be attempted by hand.[3] For the purpose of this paper this matrix has been pseudo-diagonalized by the Inverse Nyquist Array procedure[1] producing a decoupling matrix K(s).

The final inverse Nyquist plots are shown in Figure 4. The final P.I. controller diagonal matrix are then designed by simulation of the control system having the average model $G_{aver}(s)$ (Table 1) as a system model approximation with K(s) in the loop. Figures 5 to 7 show the closed loop setpoint demand tests. On the left of each figure is given the manipulable input deviations from steady state, and on the right the setpoint demand for the controlled output and the controlled output

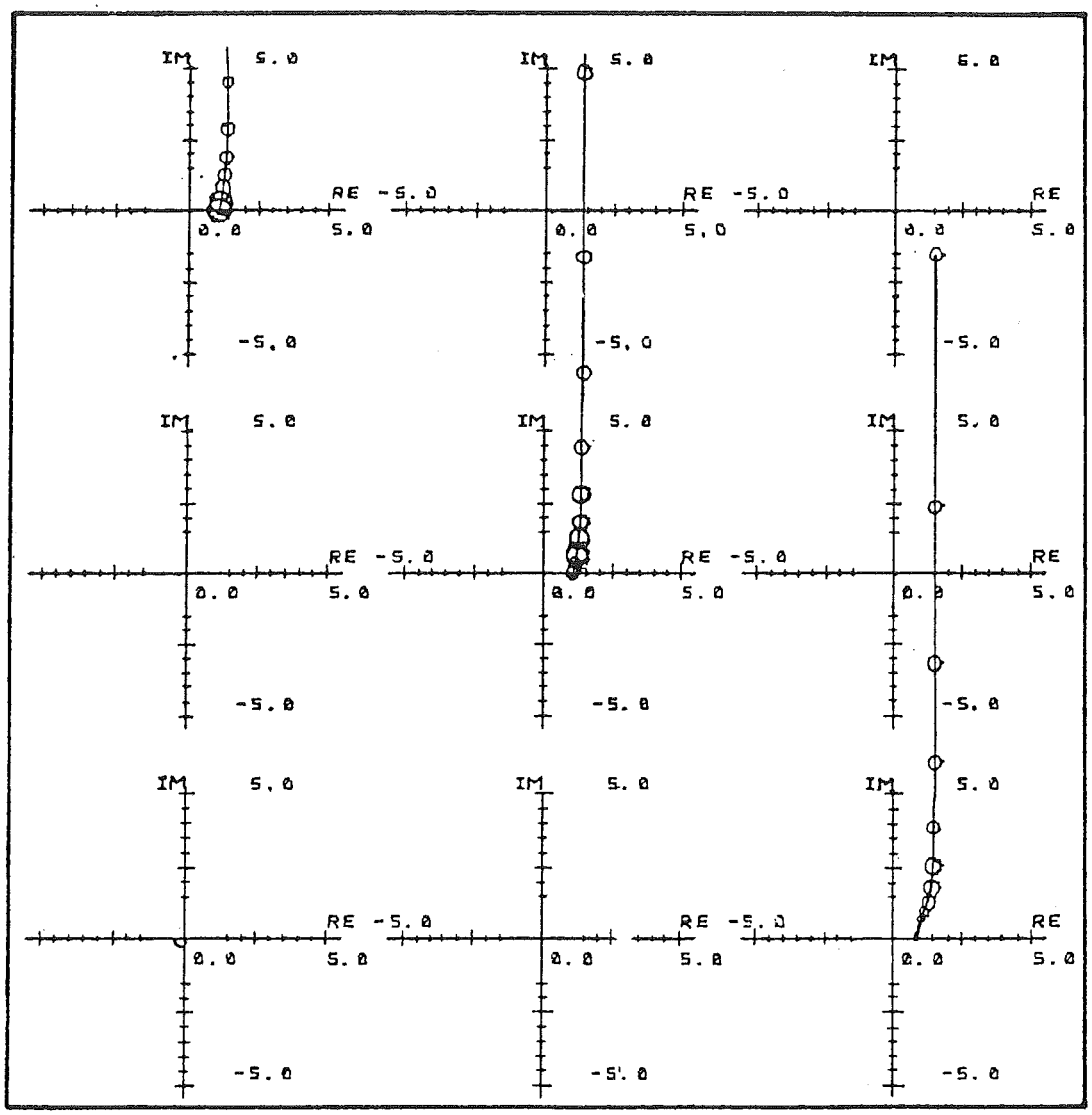


FIGURE 4. Inverse Nyquist array plots of 'simple' model

responses from steady state condition. The three P.I. controllers used are the following:

loop	prop. gain	integ. time
Θ_D / FSRT	$K_p = 5.0$	$\tau_i = 100$ sec
CYFR / FWRT	$K_p = 1.5$	$\tau_i = 75$ sec
CYFD / SWRT	$K_p = 1.0$	$\tau_i = 40$ sec

This simulation proves that if the system were represented by the 'average' system model (Table 1), the control system would be stable with little interaction and the final control elements' signals would be within operating limits. Let PID(s) be the diagonal matrix of P.I. controllers as given above. The question to answer now is if the perturbed system will be stable.

If $G_u(s)$ and $G_l(s)$ represent the upper and lower system matrix models that envelop the perturbed system (see Figure 3), limit conditions for the perturbed systems can be checked by allowing $G_u(s)$ and $G_l(s)$ to be the system model in such conditions. If this is done and we close the loop on $G_l(s)$ with decoupler K(s) and controller matrix PID(s) the system results in an unstable system as is shown by simulation plots in Figure 8 for a setpoint step demand in Θ_D. No stability problems were found for limit condition $G_u(s)$. A further proof of this is found by plotting the direct Nyquist bounds :

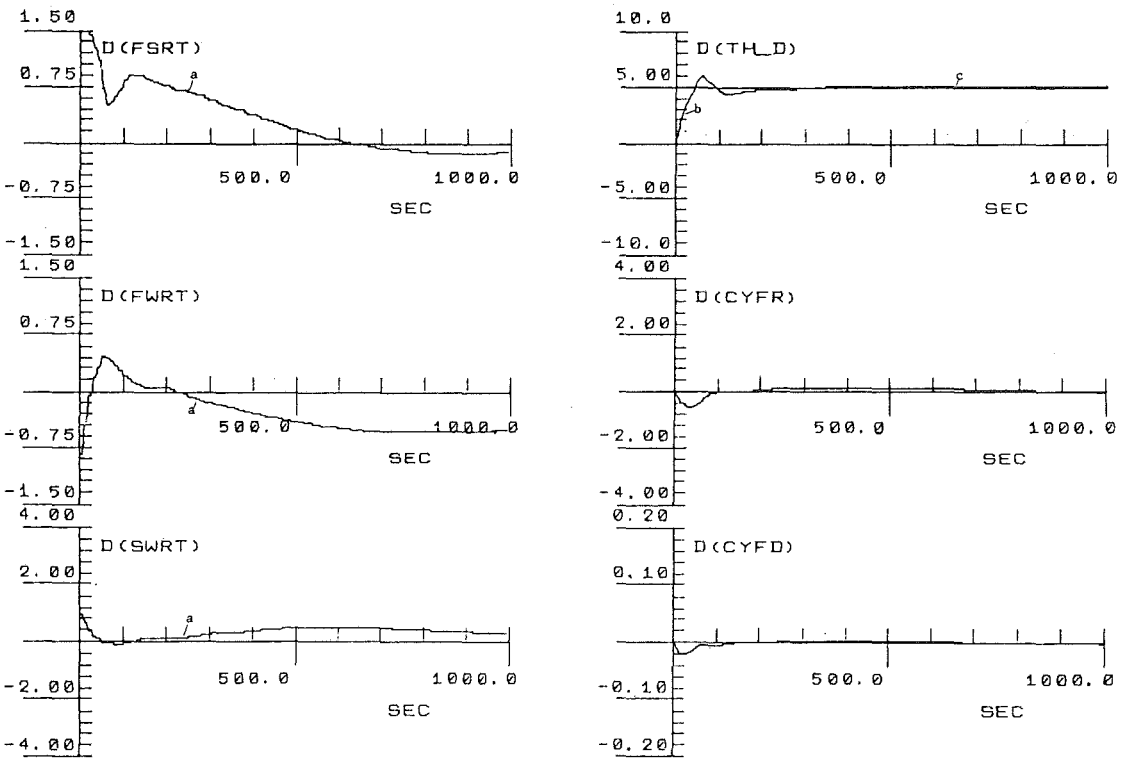


FIGURE 5. Closed loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response and setpoint.
Note: The controller is derived from the 'simple' plant model while the plant model is represented by the 'average' approximation

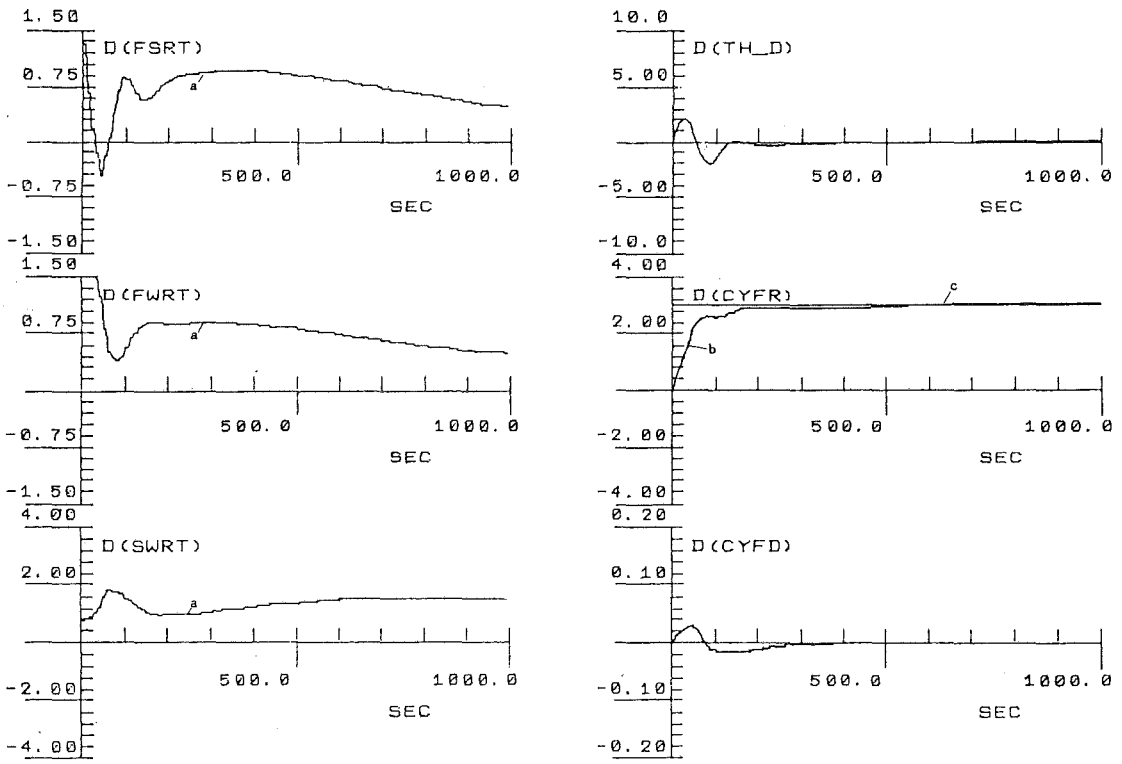


FIGURE 6. Close loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response an setpoint.
Note: The controller is derived for the 'simple' plant model while the plant model is represented by the 'average' approximation

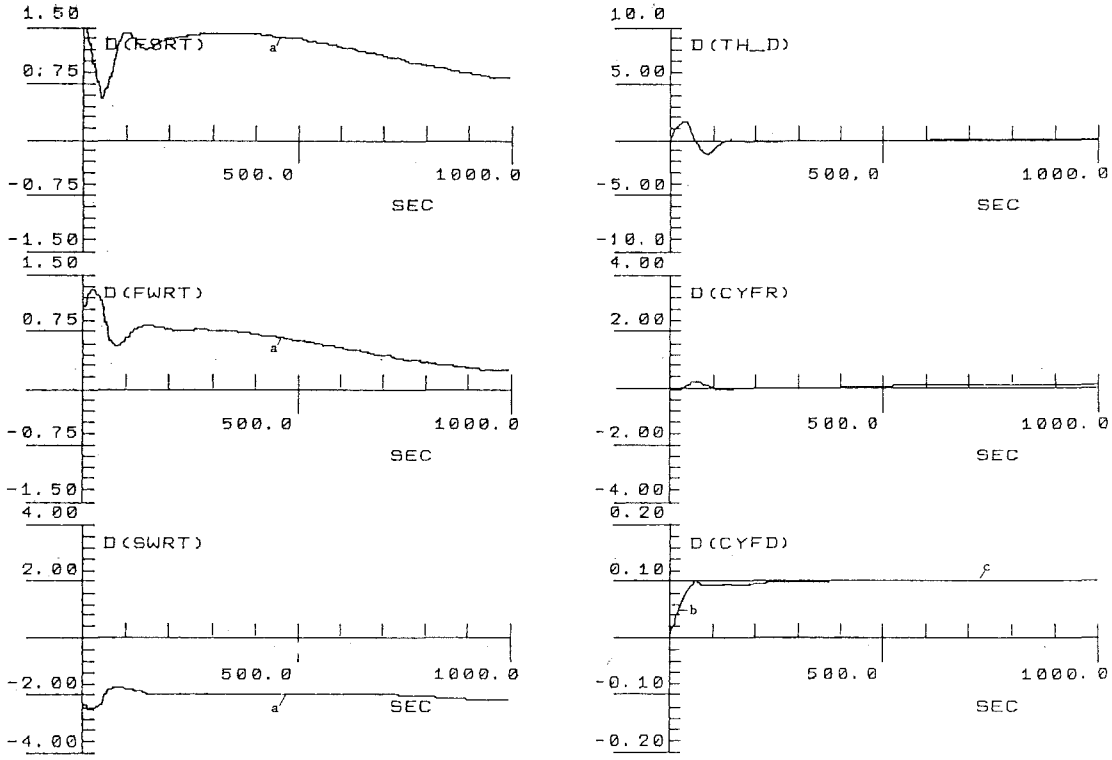


FIGURE 7. Closed loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response and setpoint.
Note: The controller is derived for the 'simple' plant model while the plant model is represented by the 'average' approximation

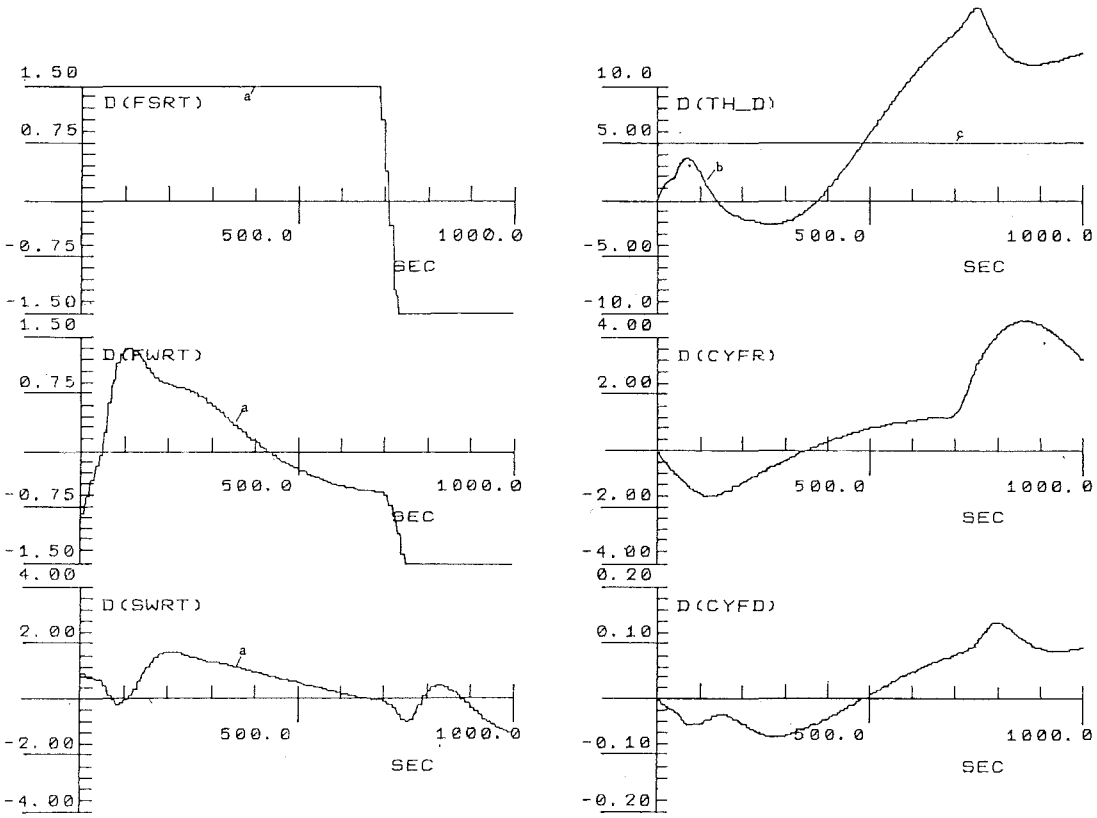


FIGURE 8. Closed loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response and setpoint.
Note: The controller is derived for the 'simple' plant model while the plant model is represented by the 'lower' approximation

$$\Gamma'_{ui}(s) = \sum_{j=1}^{n} q'_{uij}(s) \qquad [24] \quad i=1,n$$

$$\Gamma'_{li}(s) = \sum_{j=1}^{n} q'_{lij}(s) \qquad [25] \quad i=1,n$$

where: q'_{uij} is the i-jth element of $Q'_u(s)$
q'_{lij} is the i-jth element of $Q'_l(s)$

$$Q'_u(s) = G_u(s)\ K(s)\ PID(s) \qquad [26]$$

$$Q'_l(s) = G_l(s)\ K(s)\ PID(s) \qquad [27]$$

In Figure 9 is shown a matrix of direct Nyquist plots for a working range of frequencies of the $G_{aver}(s)$ K(s) PID(s) (solid line) and on each diagonal elements show the lower ($\Gamma'_{li}(s)$) and upper ($\Gamma'_{ui}(s)$) boundaries [20] and [21] respectively. In these plots the instability of the lower bound condition relative to the stability conditions [22-23] is clearly demonstrated. Figure 10 shows the array plot element (1,1) magnified. A 1.2 peak magnitude upper bound condition is also plotted as well as the bandwidth 0.707 circle condition. This figure shows that :

upper bound phase margin = 93°
lower bound phase margin = -58°
upper bound bandwidth = .053 rad/sec
lower bound bandwidth = .075 rad/sec
aver. model bandwidth = .042 rad/sec

The lower bound for the stability of the perturbed system can be modified. This is done by using a lag type of compensator:

$$f_l(s) = \frac{1 + \tau_l\ s}{1 + \alpha_l\ \tau_l\ s} \qquad [28]$$

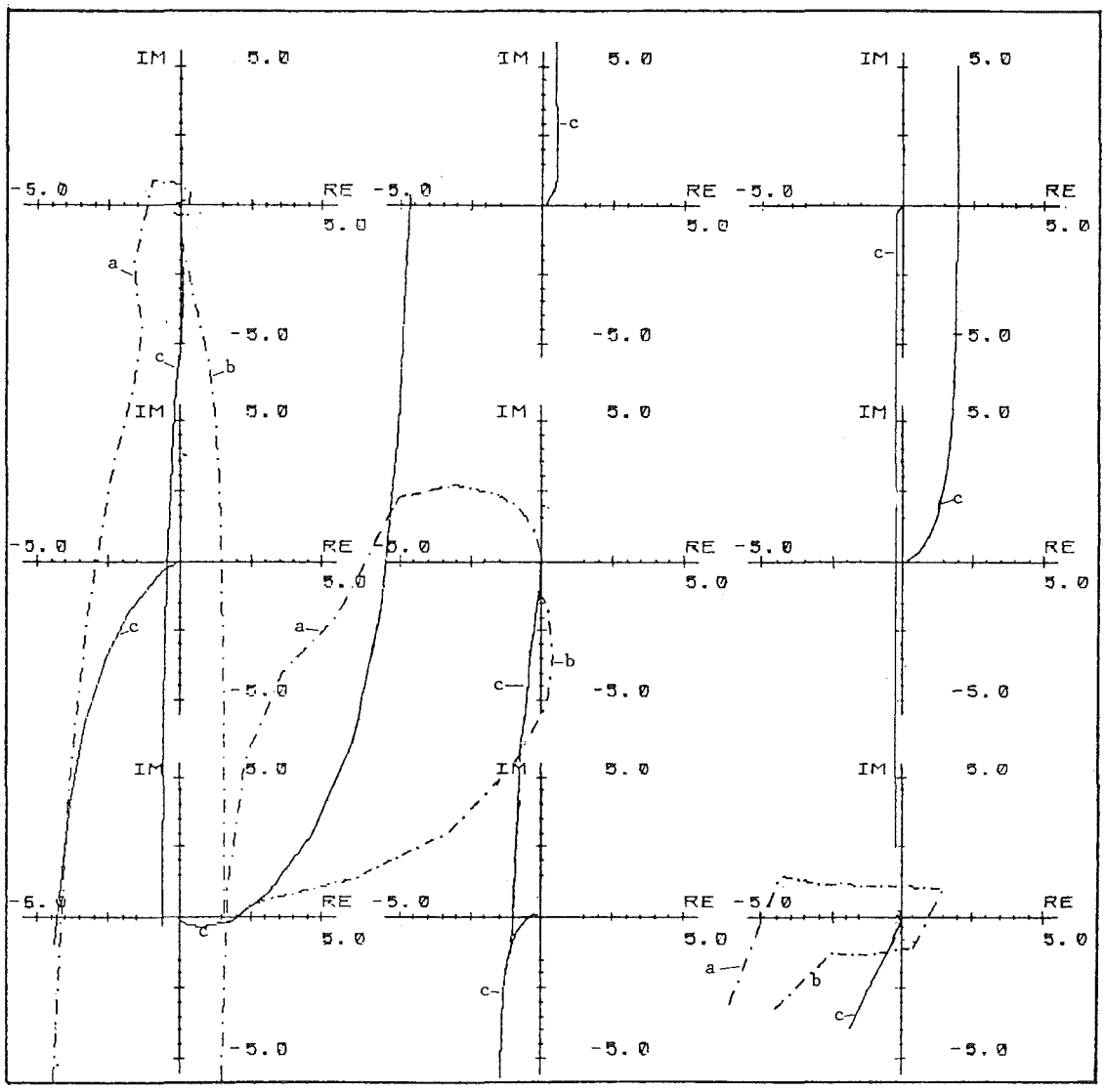


FIGURE 9. Direct Nyquist array's plots. (a) & (b) 'lower' and 'upper' bounds, (c) $G_{aver}(s)$ K(s) PID(s) element plots.

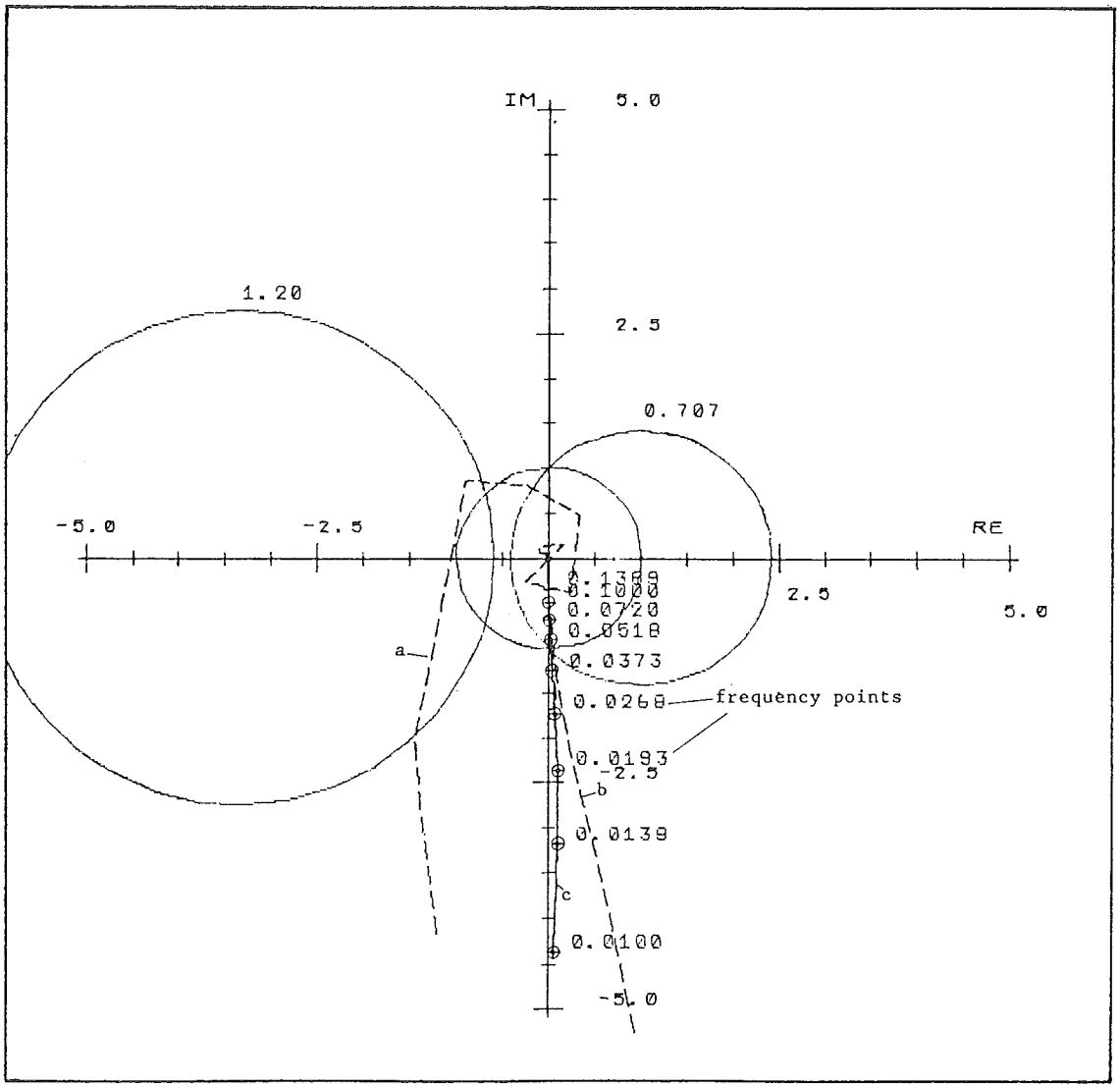


FIGURE 10. Direct Nyquist plot of array's element (1,1). (a) & (b) 'lower' and 'upper' bounds, (c) $G_{aver}(s)$ K(s) PID(s) element plot.

where $\tau_1 > 0$ and $\alpha_1 > 1$.
These parameters can be found very easily by a standard lag compensator design technique. Furthermore as the system bandwidth (and the speed of the response) decreases with lag compensation, an optimization approach can be used for their definition, i.e: find τ_1 and α_1 such that the peak magnitude is 1.2 and the bandwidth is as close as possible to what it was before this compensation were run. Figure 11 results from this analysis on loop (1,1). In Figure 12 is the final view of the perturbated bound stabilization after a similar analysis is done for loop elements (2,2) and (3,3).

The final question to be answered is if the time domain simulation satisfies to the saturation limits of the manipulable inputs. Without this proof the overall problem of perturbation matrix structuring is not ascertained. Figures 13 to 16 show the time domain simulation of the final closed loop performance of the perturbed system. The main conclusion that can be drawn is related to the settling times. These have increased quite remarkably even if they are well within working values for this type of plant.

It can be generally stated that the stabilization of a real perturbed system is obtained with a loss in the time domain

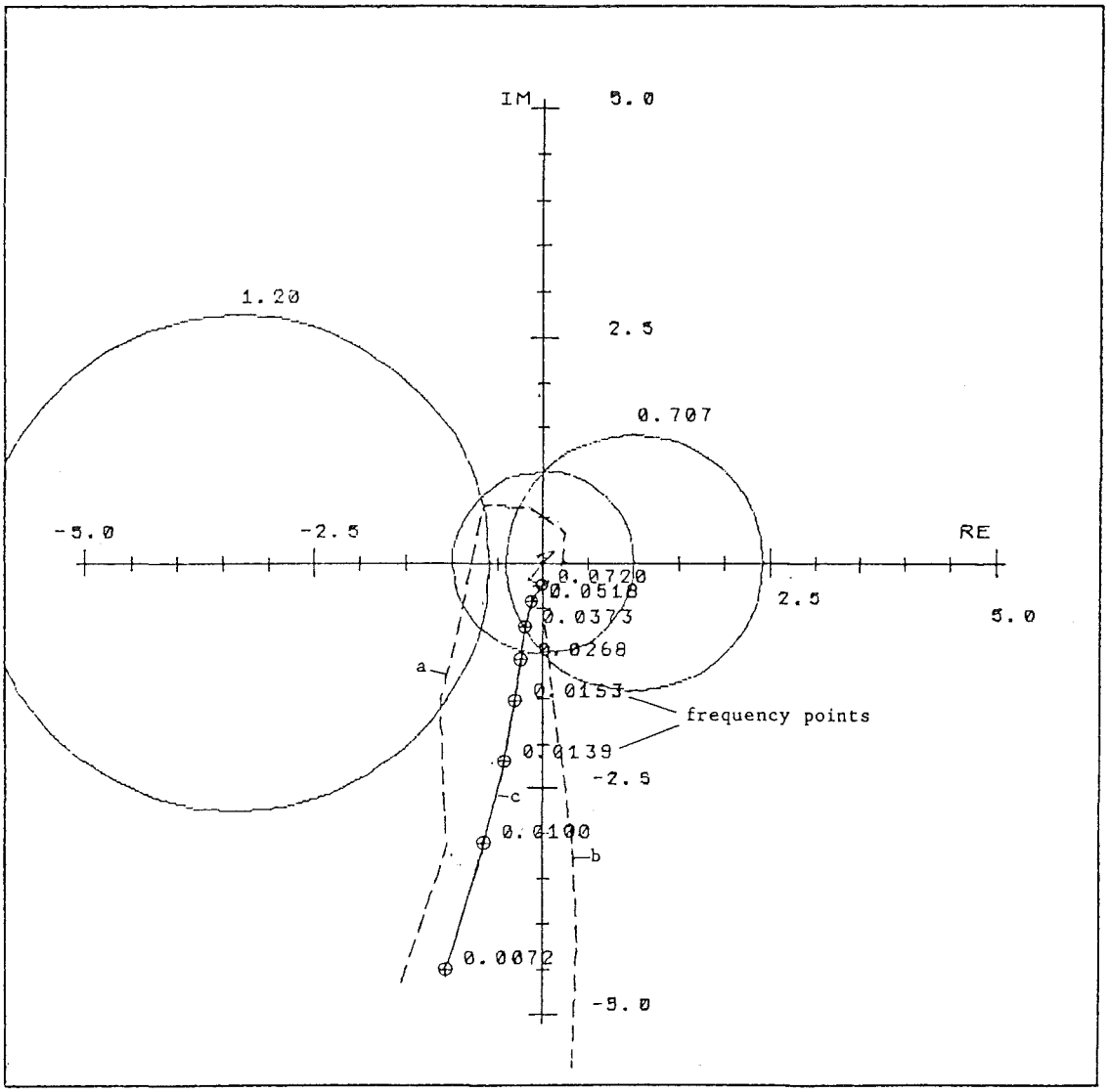


FIGURE 11. Direct Nyquist plot of array's element (1,1) after lag compensation. (a) & (b) 'lower' and 'upper' bounds, (c) $G_{aver}(s)$ K(s) PID(s) $f_1(s)$ element plot.

performance that the controlled system can achieve. The proposal technique helps the designer in correctly identifying what can be achieved by the system.

Discussion and conclusions

Some of the problems currently encountered in designing control systems for grinding circuits have been discussed for the situation where the robustness of the design to modelling errors is taken into account. This technique rationalizes the design of a control system as follows:

1. It clearly gives a quantitative assessment of control system stability under uncertainty in model structure and parameter values.
2. It reveals the design trade-offs, i.e. the price that must be paid to stabilize the perturbed system, or the relationship between bandwidth of controlled output and bandwidth of noise and disturbances.
3. It allows one to quantify what control aims can be achieved before plant tests are done.
4. It helps the designer in deciding what level of system model complexity to use for the design exercise.
5. It lends itself to Computer Aided Design

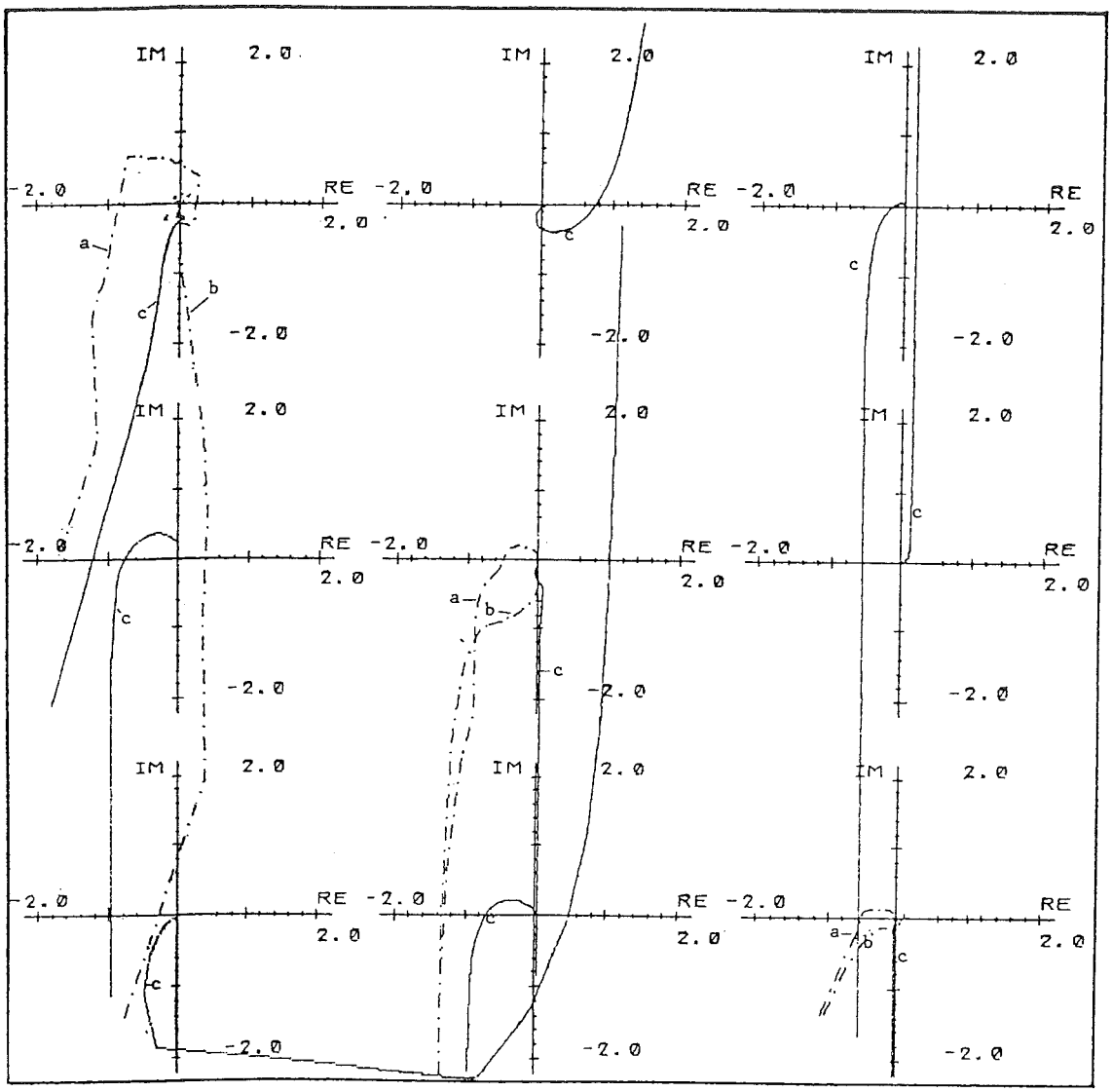


FIGURE 12. Direct Nyquist array's plots after final band stabilization. (a) & (b) 'lower' and 'upper' bounds, (c) $G_{aver}(s)$ and K(s) PID(s) F(s) element plots

by easing the control engineer's task.

The benefits for milling system control applications are the following:

6. The I.N.A. approach of Rosenbrock[1] has been estensively used to design multivariable controllers for grinding mills. This is probably due to the concept of dominance which is suitable for performance and stability analysis through the very effective graphical interpretation and because it provides some form of sensitivity assessment through the Ostrowski's circles. An important general comment on this technique is related to the I.N.A. need for diagonal dominance.

Recalling the original paper by Rosenbrock,[22] theorem 4 states and proves that if a plant model transfer function matrix G(s) contains one nonminimum phase element then it can only be made diagonal by a stable precompensator matrix K(s) but K(s) must have at least one zero in the right half complex plane. If K(s) contains nonminimun phase elements, these may be acting on other path elements of G(s) and the overall G(s)K(s) may become nonminimum phase with known effects on system performance achievability.

In processing the average model approximation (Table 1) a great deal of effort was required for its diagonalization.

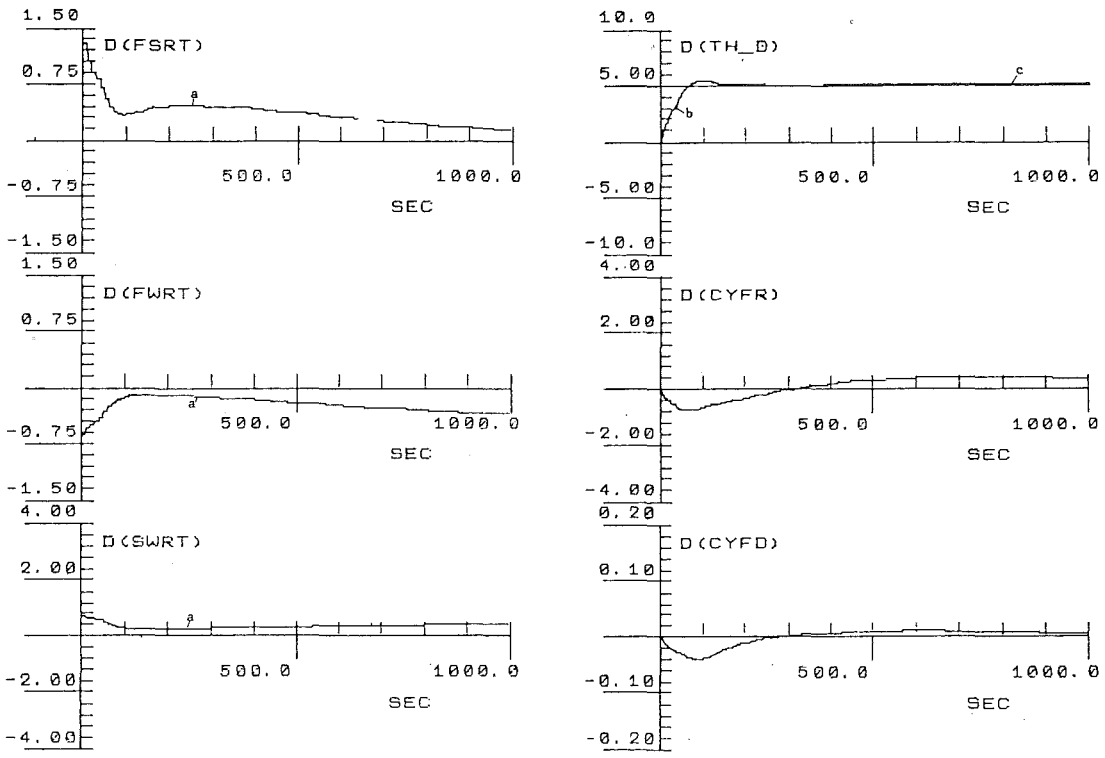


FIGURE 13. Final closed loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response and setpoint.
Note: The controller is derived for the 'simple' plant model while the plant model is represented by the 'average' model approximation.

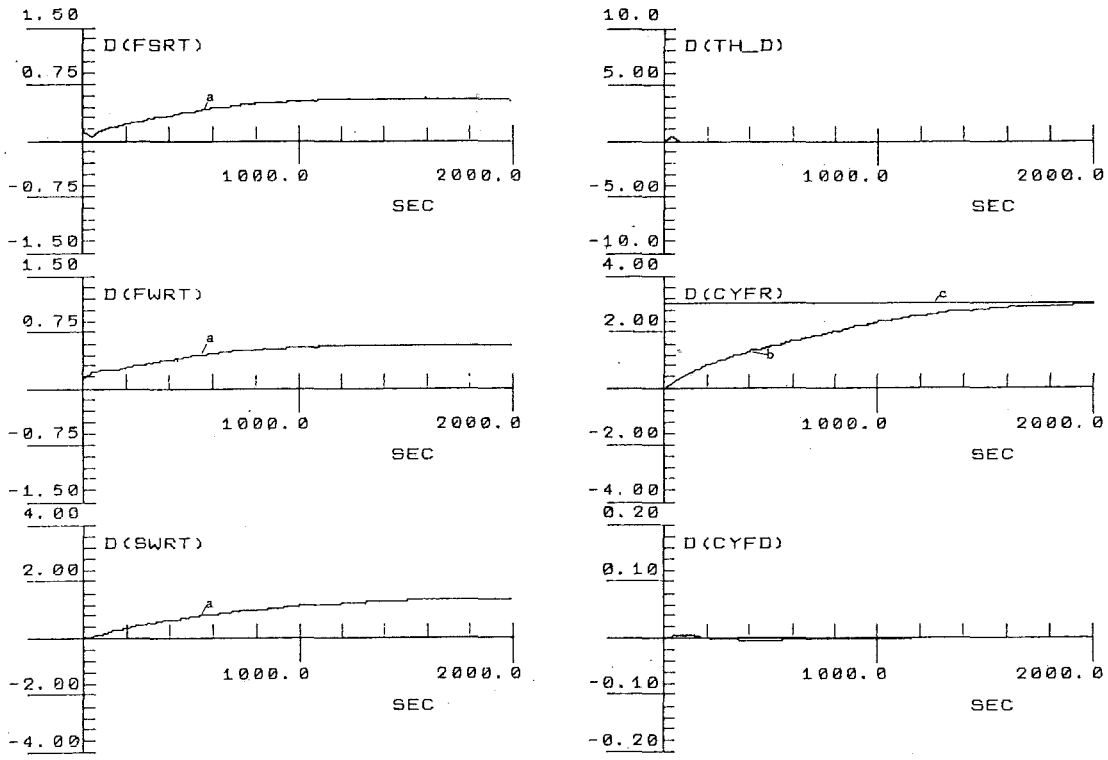


FIGURE 14. Final closed loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response and setpoint.
Note: The controller is derived for the 'simple' plant model while the plant model is represented by the 'average' model approximation

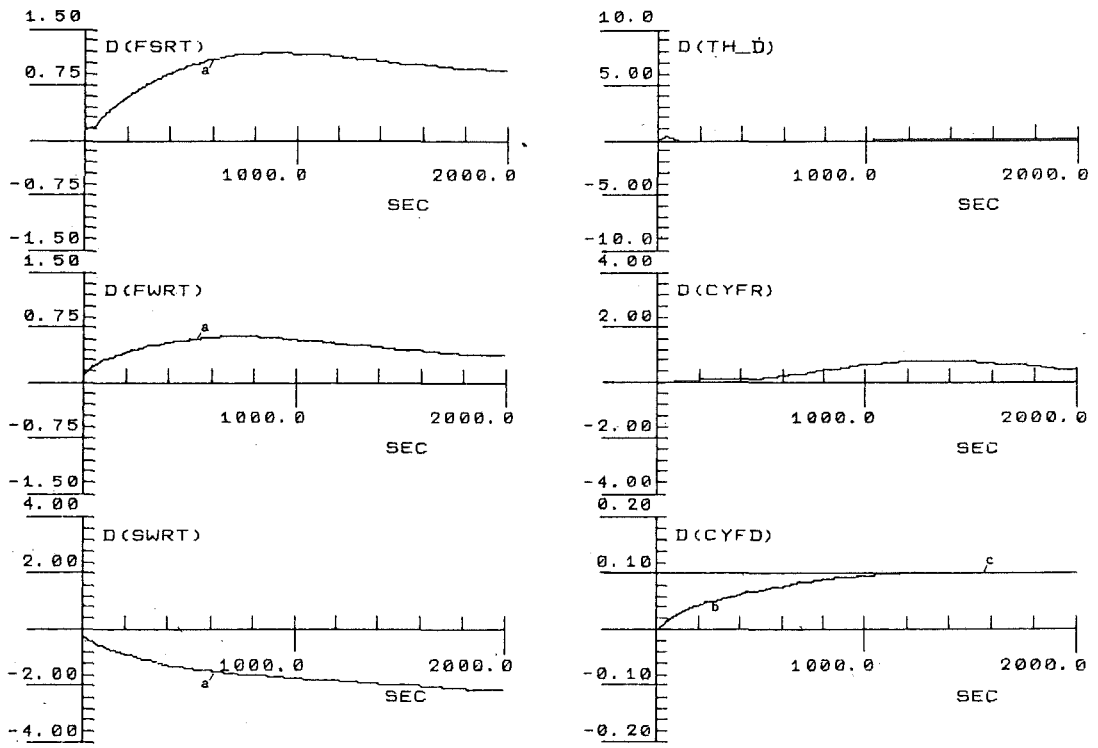


FIGURE 15. Final closed loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response and setpoint.
Note: The controller is derived for the 'simple' plant model while the plant model is represented by the 'average' model approximation

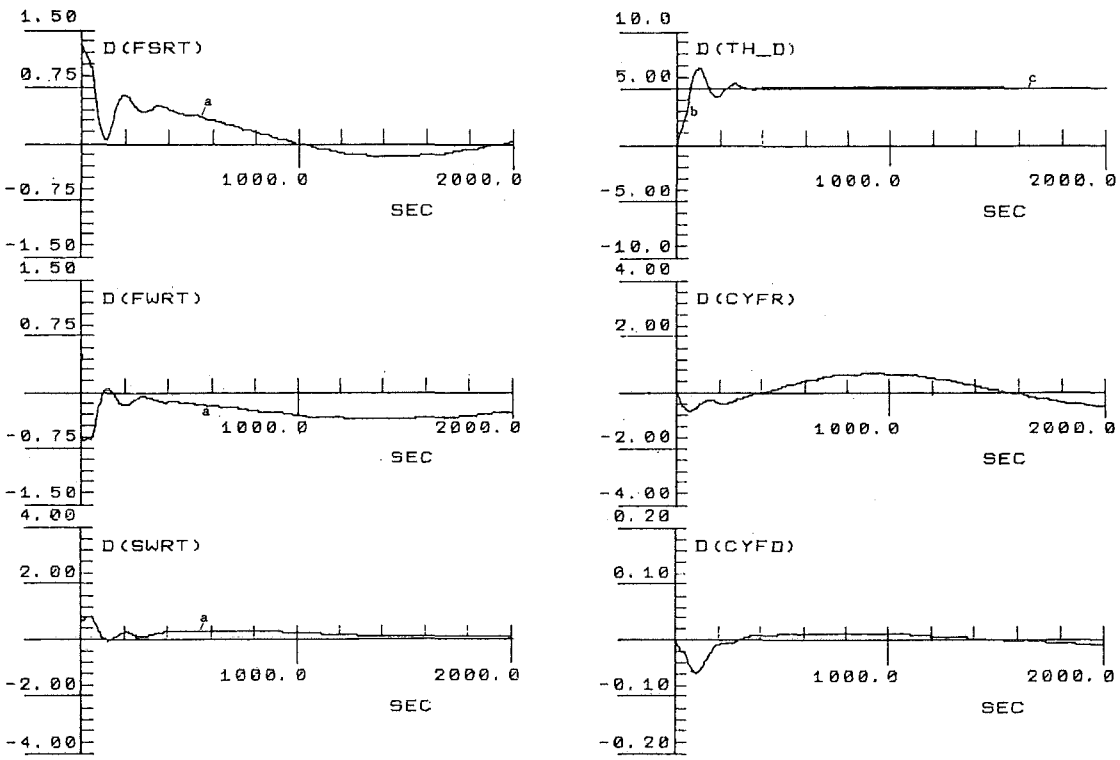


FIGURE 16. Final closed loop setpoint demand tests. (a) manipulated input perturbation, (b) & (c) controlled output response and setpoint.
Note: The controller is derived for the 'simple' plant model while the plant model is represented by the 'lower' model approximation.

The final precompensator matrix K(s) was extremely complicated as well as non-minimum phase. However by means of Robust Control Theory it was shown that it did not really need to be so complicated. The dominance concept is only a sufficient condition for stability and is sometimes too restrictive for practical applications. This highlights the importance of point 4.

7. The optimization of the milling plant can be attempted if a quite detailed state space representation of the system is known. On or off line optimization of the plant can be performed by selecting those sets of plant operating setpoints to optimize the plant efficiency and product quality. This is a constraint optimization that normally leads the plant to operate near limit conditions for cyclone or mill.[23] The plant model structure is a function of the operating point around which dynamic tests of identification were run. It has been shown by the design example that a stable controller from the I.N.A. point of view can be unstable within the operating range if the conditions for robust stability are not satisfied. Therefore one of the main benefits that the Robust Control Theory brings is that it allows for stability in the operating range and therefore allows for the optimization of the plant operation.

One thing that is made clear under the robust stability approach, is that a trade-off exists between performance and robustness. In order to ensure stability this technique might lower the system performance too much. This is still an identification problem as Δ may be too wide. This problem solution requires the use of a better model to represent the system performance. It is, however, important to note that the proposal technique tells the designer to change the plant model if prescribed performances are not achieved.

References

1. ROSENBROCK, H. H. *Computer aid design of control systems*. Academic Press, London, 1974.

2. MacFARLANE, A. G. J. *Complex variable methods for linear multivariable systems*. Taylor & Francis, London, 1980.

3. OWENS, D. H. *Feedback and multivariable systems*. Peter Peregrinus, London, 1978.

4. GOSSMAN, G. J., HULBERT, D. G., KOUDSTAAL, J., BRAAE, M. and BUNCOME, A. Application of Computer control to a gold milling circuit at the East Driefontain Gold Mine. *Proc. 12th CMMI Congress. H. W. Glen (ed)*, Johannesburg, SAIMM, 1982. pp 817-825.

5. HERBST, J. A. and RAJAMANI, K. The application of modern theory to mineral processing operations. *Proc. 12th CMMI Congress, H. W. Glen (ed)*, Johannesburg, SAIMM, 1982.

6. MONTINI, A. and Moys M. H. The dynamic behaviour of the load in a pilot ball mill as a function of axial position and design and operating variables. *Proc. 13th CMMI Congress*. Vol. 4., Singapore, May 1986. pp 17-24.

7. MOYS, M.H. The measurement of parameters describing the dynamic behaviour of the load in a grinding mill. *Mintek 50 Conference Proc.*, Johannesburg, 1985. pp 215-219.

8. VERMEULEN, L.A., OLSEN de FINE, M.J. and SCHAKOWSKI, F. Physical information from the inside of a rotary mill. *J. S. Afr. Inst. Min. Metall.* 84 (b), 1984. pp 247-253.

9. MONTINI, A. and MOYS, M.H. Developments in control of grinding circuits. *Proc. 3rd Symposium on Control Theory and Applications*. Univ. of Natal, Durban, June 1985.

10. MOYS, M. H. and MONTINI, A. The use of conductivity measurements (obtained from a probe mounted in the mill shell) in the control of grinding mills. *88th Annual General Meeting of CIM*, Montreal, May 1986. (to be published in the *CIM*

journal).
11. HULBERT, D.G. Software for the design of multivariable control systems by use of the Inverse Nyquist Array. *Report M31, Council for Mineral Technology (MINTEK)*, June 1982.
12. YLINEN, R., NIEMI, A. J. and IIVARINEN, T. A linear quadratic gaussian control algorithm for sulphide ore grinding. *Proc. 4th IFAC Symposium on Automation in Mining, Mineral and Metal Processing*. Finland, 1983. pp 163-169.
13. GOODWIN, G. and SIN, K.S. *Adaptive filtering prediction and control*. Prentice-Hall, New Yersey, 1984.
14. HINDE, A.L. and KING R. P. Difficulties in the practical implementation of mill control systems. *Pre-print 11-th CMMI Congress, paper 50*. Hong Kong, 1978.
15. GOODWIN, G.C., RAMADGE, P.J. and CAINES, P.E. Discrete Time Multivariable Adaptive Control. *IEEE Trans. Autom. Control*, Vol.AC-25, 1980.
16. DOYLE, J.C.and STEIN G. Multivariable feedback design : concept for a classical/ modern synthesis. *IEEE Trans. Automatic Control*, Vol.AC-26, No 1, February 1981.
17. MONTINI, A. On modelling effects in linear multivariable controller design. Internal report Dept. of Electrical Engineering, Univ. of the Witwatersrand. Johannesburg, Nov. 1986.
18. CLOUD, D.J. and KOUVARITAKIS, B. Statistical bounds on multivariable frequency response : an extension of the generalized Nyquist criterion. *IEE Proc.*, Vol. 133, Pt., D., No 3, May 1986.
19. JANISZOWSKI, K. Evaluation of the effects of identification errors on the dynamical properties of a process model. *Int. J. Control*, Vol. 41, No 2, 1985. pp 445-459.
20. CHEN, Y. H. On the deterministic performance of uncertain dynamic systems. *Int. J. Control* , No 5., Vol. 43, 1986. pp 1557-1579.
21. KRISHNAN, K. R., CRUICKSHANKS, A. Frequency domain design of feedback systems for specified insensitivity of time domain response to parameter variation. *Int. J. Control*,, Vol. 25, No 4, 1977. pp 609-620.
22. ROSENBROCK, H. H. Design of multivariable control systems using Inverse Nyquist Approach. *Proc. IEE*, Vol. 143, No 11, Nov. 1969. pp 1929-1936.
23. HULBERT, D.G. and BARKER, I.J. The control and optimization of the operation of milling circuits. *Proc. MINTEK 50th Conference*. Vol. 1, 1984. pp 197-204.