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A Dispersion Model of Autogenous Particle Separations, with Specific Application to the Batch Jigging of Particles

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A novel mathematical model is presented which describes the behaviour of beds of particles, of uniform size and shape but different densities, during autogenous separations.

The approach used in the formulation of the dispersion model is a phenomenological one based on a statistical view, which leads to a diffusion-like representation of the autogenous-separation process. Although the model currently considers only density-related phenomena, it accounts quantitatively not only for the density-segregation aspects of the process but for the interactions between the particles.

Quantitative predictions of the dispersion model are compared with the results obtained in tests using a specially constructed experimental batch jig, and it is shown that the model provides an adequate quantitative description of the process for particles of uniform size and shape.

It is pointed out that, for the description of more complicated autogenous separations, a more rigorous formulation of the particle drag and density-segregation terms will have to be incorporated into the dispersion model. Nonetheless it is concluded that the model as presented in this paper provides not only a novel insight into the mechanisms of autogenous separations but a means by which to analyse similar systems.

Introduction

The recent revival of interest in gravity separation as an area of mineral processing is particularly manifest in the attention being paid to a class of separation devices that operate on autogenous principles. The material fed into these separators becomes stratified in a bed of similarly stratifying particulate material, usually under the influence of gravity.

Autogenous separators are particularly attractive to mineral processors because of their relatively low capital and operating costs. Common examples of autogenous separators in use today are jigs and Reichert cones, although many devices of this class are used in diverse mineral processing applications.

Mathematical analysis is a well-established technique for the optimization, design, and control of mineral-processing equipment. The purpose of such quantitative analysis is the derivation of mathematical relationships that allow the degree of separation to be predicted following changes in either operating conditions or equipment design. Of particular interest are those mathematical models capable of providing a dynamic representation of particle separations.

Investigators have traditionally tried to quantify the phenomenon of autogenous separations by considering the motion of each particle separately and expressing its response to varying hydrodynamic conditions. In this single-particle hydrodynamic force-balance approach, an attempt is made to relate the separation of particles with different physical characteristics to differences in the hydrodynamic forces acting on each particle. Because it is difficult to consider the behaviour of all the particles in a large ensemble simultaneously, the approach is idealized, and single particles are considered in isolation. As a result, the quantitative description that evolves from this approach is a gross simplification of what actually happens, and bears little direct resemblance to the physical system in question.

In reality, such particulate separations represent an enormously complicated system of interacting particles.[1,2] There are many variations of the basic approach described above, but the relation of the behaviour of single isolated particles to the behaviour of such particles in a dense bed of material is a difficulty that is common to all of them.[3,4] It is therefore hardly surprising that no rigorous quantitative analysis of particulate separations has emerged from these techniques.

Another theory of particle separations, often referred to as the 'potential theory', has been proposed by a number of investigators, among whom are Mayer,[5] Kirchberg *et al*,[6] and Schafer.[7] The proponents of the potential theory avoid the difficulties associated with the hydrodynamic force-balance approach by considering the properties of the bed of particles as a whole, rather than the behaviour of individual particles.

This approach involves analysis of the stratification of a bed of particles containing material of different densities in terms of the tendency for the gravitational potential energy of the system to be reduced by the adoption of distributions of material that lower the overall centre of gravity of the bed of particles. According to the theory, the change in the potential energy represented by the changing centre of gravity is minimized only after complete stratification has been achieved. The potential theory of particle stratification poses certain problems in that it gives no fundamental description of the dynamics involved in the approach of the system to stable configuration, and cannot account for the stratification of material with a spread of particle sizes. Moreover, the prediction that the material will be perfectly stratified once the potential energy of the bed has been minimized is inaccurate, since it does not describe the natural dispersion of the material in such beds.[8]

The dispersion model of particle separations

A fundamental feature of autogenous particle separations is that the separation of the different generic particles is never perfect.[9-11] This can be regarded as being due to remixing of the material within the autogenous bed.[10] The dispersive mixing of homogeneous material in uniformly fluidized beds has been successfully modelled using classical diffusion theory.[11-13] This approach, which has its origin in the early work on Brownian motion and is based on conditions of long particle displacement times and large sample populations, can be regarded as the Langevin stochastic differential equation.[13]

In the present work, it is proposed that an approach analogous to diffusion mixing models in fluidized beds can be applied to separations involving particulate material with a spread of densities. In a homogeneous fluidized bed, the motion of a particle can be described by a set of Langevin equations.[9] If $y(t)$ is the position of such a particle and $v(t)$ its velocity then, in the absence of external forces, the particle force balance can be expressed in the following form:[14]

$$m \frac{dv(t)}{dt} + \beta v(t) = A(t). \qquad [1]$$

In the formulation of the above force-balance equation, it has been assumed that the friction force acting on a particle of mass m is proportional to the velocity $v(t)$. The term $A(t)$ represents the 'collision force'.[14] This term is microscopically complicated since it describes the collision forces to which the particle is subjected as it moves through a bed of similar particulate material. However, on a macroscopic scale, it can be regarded as a process of independent stationary increments. The function $A(t)$ is considered to be a Gaussian white noise stochastic process.[15]

Hence in Equation [1] the influence of the surrounding fluid medium on the motion of the particle is assumed to be split into two parts: the dynamic friction to which the particle is subject and a noise component. The linear nature of the dynamic friction force is an idealization that is 'borrowed' from the theory of microscopic Brownian particles, and has no further fundamental significance within the context of particle motion in autogenous beds.

In an autogenous separating device, the influence of an external force field on particle motion will be significant. It is consequently necessary to include an external force term, $K(t)$ in Equation [1]. The force-balance equation describing the motion of a particle within an ensemble of similar particles situated in some external force field will therefore take the general form:

$$m\frac{dv(t)}{dt} + \beta v(t) + mk(t) = A(t). \qquad [2]$$

Equation [2] is simply the classical equation of mathematical physics in which Stokes' law is assumed to apply, and that includes some noise term that accounts for the interactions between the particles.

Since interparticulate acceleration is assumed to be small, the inertial term is taken to be insignificant, and is dropped from Equation [2]. This equation can consequently be reformulated in terms of particle displacement as follows:

$$\frac{dy(t)}{dt} = - \frac{mk(t)}{\beta} + \frac{A(t)}{\beta} \quad . \qquad [3]$$

In this model, the displacement of the particle, rather than its velocity, undergoes a random walk.[9] Hence, Equation [3] should, by the nature of the stochastic formulation chosen, be rewritten in a more rigorous manner as an Itô stochastic differential equation:

$$dy(t) = - \frac{mk(t)}{\beta} \, dt + dw(t) . \qquad [4]$$

The function $w(t)$ is known as the Wiener process, which has independent increments and well-defined properties.[13] The process $y(t)$ described by the Itô Equation [4] is Gaussian, and has the Markov property.[14,16]

The evolution of the conditional density $p(y,t;y_0,t_0)$ where $t>t_0$, of this Markov process, which is described by Equation [4], satisfies the Kolmogorov forward equation, which takes the following form:[14,16]

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial y}\left[\frac{mk(t)}{\beta} p\right] - \frac{1}{2}\frac{\partial^2}{\partial y^2}\left[2Dp\right] = 0, \qquad [5]$$

where D is the variance parameter of the Wiener process $w(t)$,[13,14] and $p=p(y,t;y_0,t_0)$.

If $p(y,t;y_0,t_0)$ is interpreted as the probability of a given generic particle being found in the interval y to $y + dy$ at time t then, when it is applied to a large number of similar particles, it can be interpreted as the concentration of those particles in the region y to $y + dy$ at time t. With this interpretation of the conditional density $p(y,t;y_0,t_0)$, Equation [5] becomes the generalized 'dispersion' equation describing one dimensional segregation of the material in an autogenous bed situated in an external force field.

The dispersion model of autogenous separation processes as expressed in Equation [5] is an entirely generalized description of particle behaviour. In principle, the derivation of this model relies on the same approach as that adopted in the traditional hydrodynamic force balance description. However, in the case of the dispersion model, the interactions between the particles are included as a noise term and the force balance as a whole is interpreted as a stochastic differential equation. The dispersion model has been formulated to include only one space co-ordinate, which is parallel to some external force field. In principle, the equation can be reformulated to include a second or third spatial dimension as well.

The dispersion model of batch jigging

As a test of the basic validity of the dispersion model, it was formulated in a more specific sense to describe the process of particle jigging. Despite the apparent simplicity of jigging from a superficial point of view, a detailed examination of the particle mechanisms involved shows that it presents one of the most complex separation phenomena to quantify. Not only must the interactions between the particles be described, but the continuously varying fluid pulsations and their influence on the bed as a whole must be accounted for. The model presented here as a dispersion model essentially involves a phenomenological approach, and assumes that the process can be represented as a continuum.

Clearly the dispersion representation of jigging can not include all the details of the autogenous separation process at this stage. In this initial attempt to quantify the segregation of particulate material in jig beds, therefore, only density-related phenomena are considered.

Studies of batch jigging were selected to validate the dispersion model of autogenous particle separations, since they provide a particularly suitable, yet simple, means by which the dispersion model can be tested and the parameters of the model measured. Batch jigs are ideal for this purpose, since the initial conditions of the bed can be preselected to represent any configuration of material distribution, and the various process variables - such as bed depth and pulsation characteristics - can be kept constant and suitably adjusted. Batch jigs also have various operational advantages over continuous jigs, because they are not complicated by factors such as material feeding and withdrawal mechanisms and cross-flow velocity gradients. Consequently, the physical conditions selected in batch jigging studies can be easily quantified by the choice of suitable model parameters.

In the formulation of Equation [4] to describe particle behaviour in a batch jig bed, the drag term β was assumed to represent the friction to which a particle will be subjected as it penetrates the jig bed. A more rigorous mathematical formulation of this term has not been proposed in the literature as yet, and it is assumed here to be constant for a given bed of material. The external force term $k(t)$, which in this case describes the action of gravity on the particulate material, is expected to be attenuated in some way by the buoyancy effect due to the local density of the surrounding the particle of interest. In an initial attempt to derive a specific quantitative description of particle jigging, it was therefore assumed that the gravitational force term in Equation [4] takes on the linear form:

$$\frac{m_i k(t)}{\beta} = \frac{Vg}{\beta}(\rho_i - \bar{\rho}) \ , \qquad [6]$$

where the bed density, $\bar{\rho}$, is assumed to be a linear combination of all the densities of the species:

$$\bar{\rho} = \sum_{i=1}^{n} c_i \rho_i \qquad [7]$$

The substitution of the gravitational force term as formulated in Equation [6] into Equation [4] allows the Itô stochastic equation for particle jigging to be expressed as follows:

$$dy_i(t) = -K_\mu(\rho_i - \bar{\rho})dt + dw(t) \ , \qquad [8]$$

where K_μ represents the 'drift coefficient' and is assumed to be a constant for a given system of particles of uniform size. From expression [6] it is evident that the drift coefficient can be expressed in the following form:

$$K_\mu = \frac{Vg}{\beta} \ .$$

The drift coefficient is proportional to the penetration velocity of the particle, and characterizes the mobility of a given type of particle.

The variance parameter, D, of the Wiener process, $w(t)$, is assumed to be a constant and takes on the value $2\ K_D$, where K_D is called the 'diffusion coefficient', and is a measure of the dispersive mixing of particles in the jig bed.

The Kolmogorov equation associated with the Markov process, y_i, as described by Equation [8], takes the form:

$$\frac{\partial c_i}{\partial t} = - \frac{\partial [K\mu(\rho_i - \bar{\rho})c_i]}{\partial y} + K_D \frac{\partial^2 c_i}{\partial y^2} \quad , [9]$$

where the conditional probability density is re-interpreted as the concentration c_i.

Experimental validation of the model

In an initial attempt to establish the validity of the dispersion model, it was fitted to experimental data on batch jigging that had been collected for the purposes of the present investigation. The model parameters K_μ and K_D were adjusted to fit the data, and the degree of the correlation between the predictions of the model and the experimental

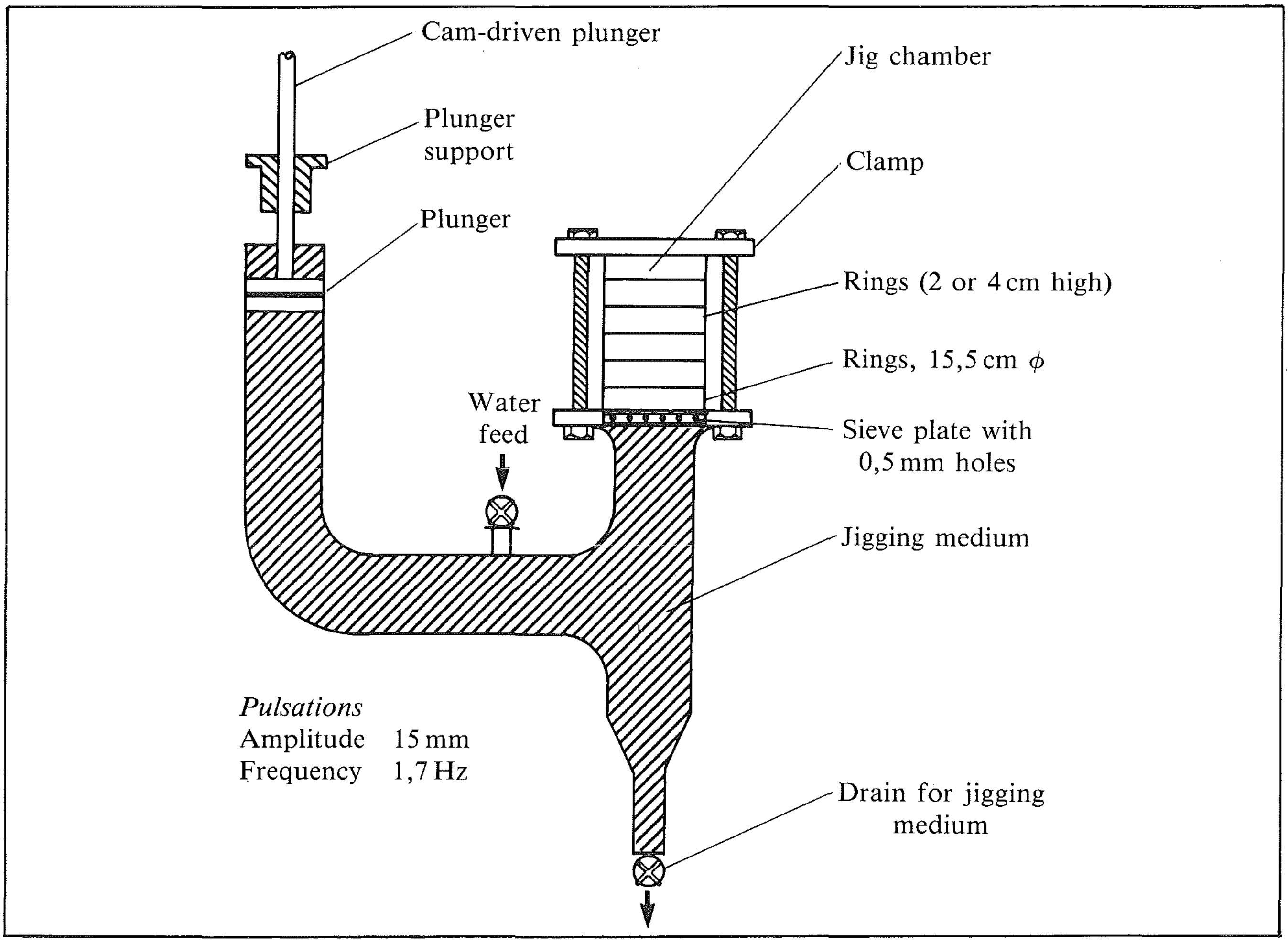
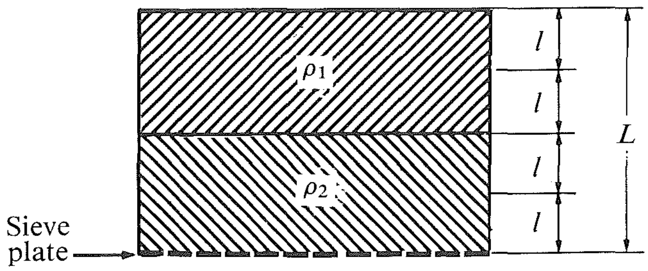


FIGURE 1. Diagram of batch jig showing all essential features

data was studied.

For these experiments, a batch jig was constructed as in the form of a 'U'-tube (Figure 1). A cam-driven plunger to produce pulsations in the jigging medium (water in this case) was fitted onto the end of one 'arm' of the U-tube, and the jigging chamber, consisting of a number of stacked rings of stainless steel, was located at the end of the other arm of the U-tube. This arrangement facilitated accurate sectioning of the bed into layers as well as dismantling of the bed during sampling. Hence a minimum amount of material was displaced when samples were taken for analysis to show the vertical distribution of particles of different densities.

The test particles consisted of 3,5 mm polyvinylchloride (PVC) cubes of uniform size but three different densities: 1200, 1300, and 1500 kg m^{-3}. In the batch jigging tests, a pre-selected configuration of material of non-equilibrium distribution (i.e. with denser material placed above less-dense material) was carefully loaded into the stacked rings above the sieve plate (see Figure 2). Water was then introduced into the bed, and the material was subjected to the desired number of pulsations. When the pulsations had stopped, the water was drained from the bed, and a specially constructed sampling tray was fitted over each of the stacked rings in turn, the top of the tray being aligned with the base of the ring concerned. Thus each ring could be moved horizontally onto the tray, and the bed material within that segment could be removed from the jig and subjected to a float-and-sink analysis in a determination of the particle-density distribution. To generate a complete time-history of the stratification process for a given configuration of particles, a series of experiments was conducted under the same initial conditions, each experiment being terminated after a different pre-selected number of pulsations.



l Height of bed section (2 cm)
L Height of jig bed (8 cm)
$\rho_1 > \rho_2$

FIGURE 2. Initial distribution of material for binary batch jigging tests

The first stage in the analysis of the bed material from the batch jigging experiments consisted of a study of the stratification characteristics of particles of uniform size but two distinct densities. In these tests, which were termed 'binary' batch jigging tests, equal volumes of the two selected density fractions were charged to the jigging chamber. Care was taken to ensure that the packing of the bed would be consistent by levelling of the new material to the exact height of a bed segment. The initial profile selected for the material in these tests was stepped, the lower- density particles being placed in the lower half of the bed above the sieve plate.

The dispersion Equation [9] together with no-flux boundary conditions applied at the surface of the jig bed and sieve plate, was integrated numerically by use of the Crank-Nicolson implicit method.[17] The numerical integration scheme was incorporated in a software simulation program (which was written in FORTRAN) in which the various model parameters and initial conditions could be selected prior to integration.[9] The results predicted by the model for the three binary systems tested are presented in Figure 3.

These binary density systems of uniform particles represent the simplest possible jigging system involving some form of particle segregation that can be described by the dispersion model. In an effort to present a simplified analysis of the stratification that occurs in such a binary test, it is convenient to characterise the distribution of material in terms of 'misplaced material'. Misplaced material is defined as the sum of all the high-density material in the top half of the bed and all the low-density material in the bottom half of the bed expressed as a percentage of the material originally charged to the jig (assuming that equal volumes of the two density fractions were originally charged to the jig bed as illustrated in Figure 2). The material that is initially charged to the jig bed therefore represents 100% misplaced material. The curves in Figure 3 represent the predictions of the model for the binary batch tests, which were obtained by the selection of suitable values for the parameters K_μ and K_D. These curves of misplaced material versus time display the typical characteristic of autogenous separations, namely, a redistribution

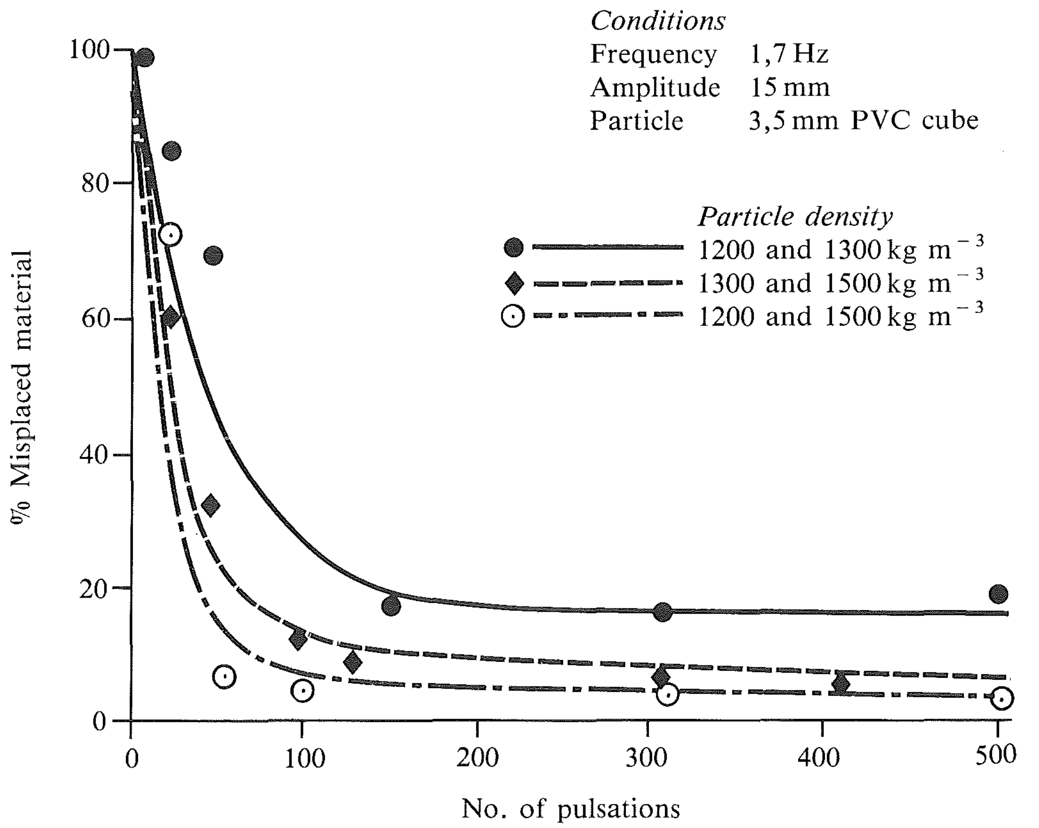


FIGURE 3. Dispersion-model prediction of the results of three binary batch jigging tests (K_μ = 10; K_D = 1)

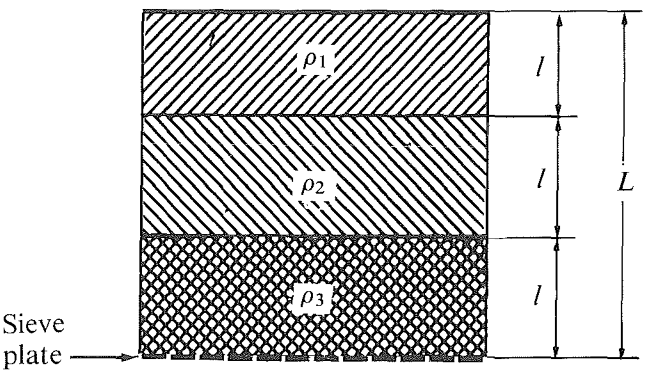
of bed material, which finally evolves into some steady-state distribution of the material. As the same numerical values of the parameters K_μ and K_D were used in the descriptions of all three tests, this feature appears to confirm the independence of these parameters on density in a system of mono-sized particles.

A steady-state solution for the dispersion model describing a binary system, which is presented in the Addendum, shows that the steady-state concentration profile for a given species of particles is, in fact, a function of the ratio K_μ/K_D. By the selection of a suitable numerical value for this ratio, the predictions can be fitted to the steady-state data, while the actual magnitude of either of the parameters determines the dynamic characteristics of the non-steady-state profile.

The multidensity behaviour of a batch jig was also investigated for a system using three densities (ternary system). The initial configuration of the bed in these tests is shown in Figure 4, from which it can be seen that the bed was charged with equal volumes of the three density fractions corresponding to the three bed layers. The procedure in these tests was the same as that used in the binary tests, and the parameters determined in the fitting of the binary data were used in the simulation of these ternary systems. In the simulation of a multidensity batch system of n densities, it is necessary to solve only n-1 coupled dispersion equations. Since it can be shown that c_n calculated from the relation

$$c_n = 1 - \sum_{i=1}^{n-1} c_i \qquad [10]$$

satisfies the dispersion Equation [9]. The equations are coupled because the bed-density term $\bar{\rho}$ will be a function of the concentrations of n-1 species. Predicted curves for a series of ternary density tests on PVC cubes, which are presented in Figures 5 and 6, again display the typical evolution of a non-steady-state distribution of material into some steady-state concentration profile. The data for these tests are presented in terms of the volume fraction of a given species per layer of bed.



l Height of bed section (4 cm)
L Height of jig bed (12 cm)
$\rho_1 = 1500$ kg m^{-3}, $\rho_2 = 1200$ kg m^{-3}, $\rho_3 = 1300$ kg m^{-3}

FIGURE 4. Initial distribution of material for ternary batch jigging tests

Conclusions

A dispersion model of autogenous separations, which was explicitly formulated to describe batch jigging systems of particles of uniform size and shape, dynamically characterizes the fundamental feature of such separations. This fundamental characteristic or phenomenological feature can

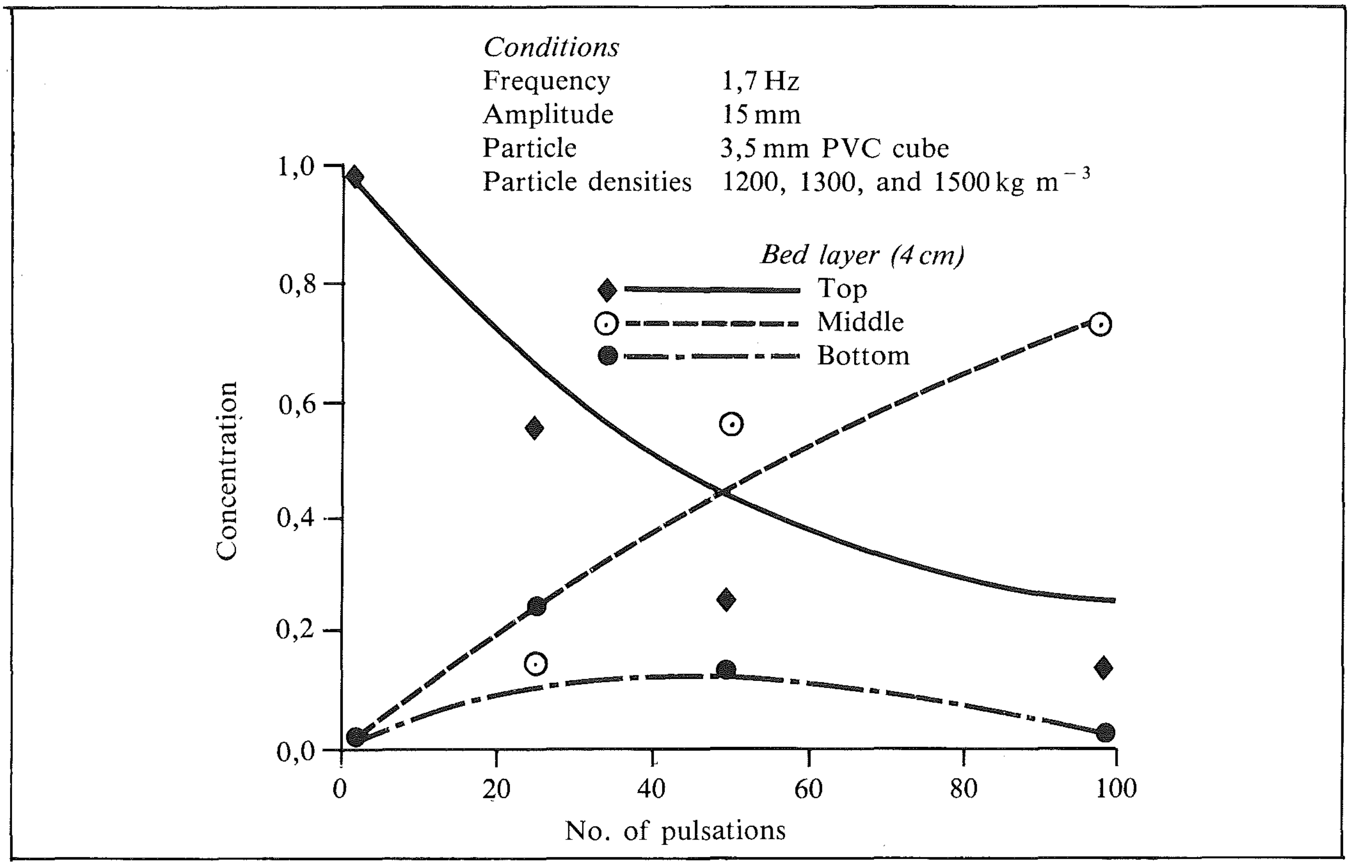


FIGURE 5. Dispersion-model prediction of the distribution of the 1500 kg m^{-3} density fraction in a ternary system (K_μ = 10; K_D = 1)

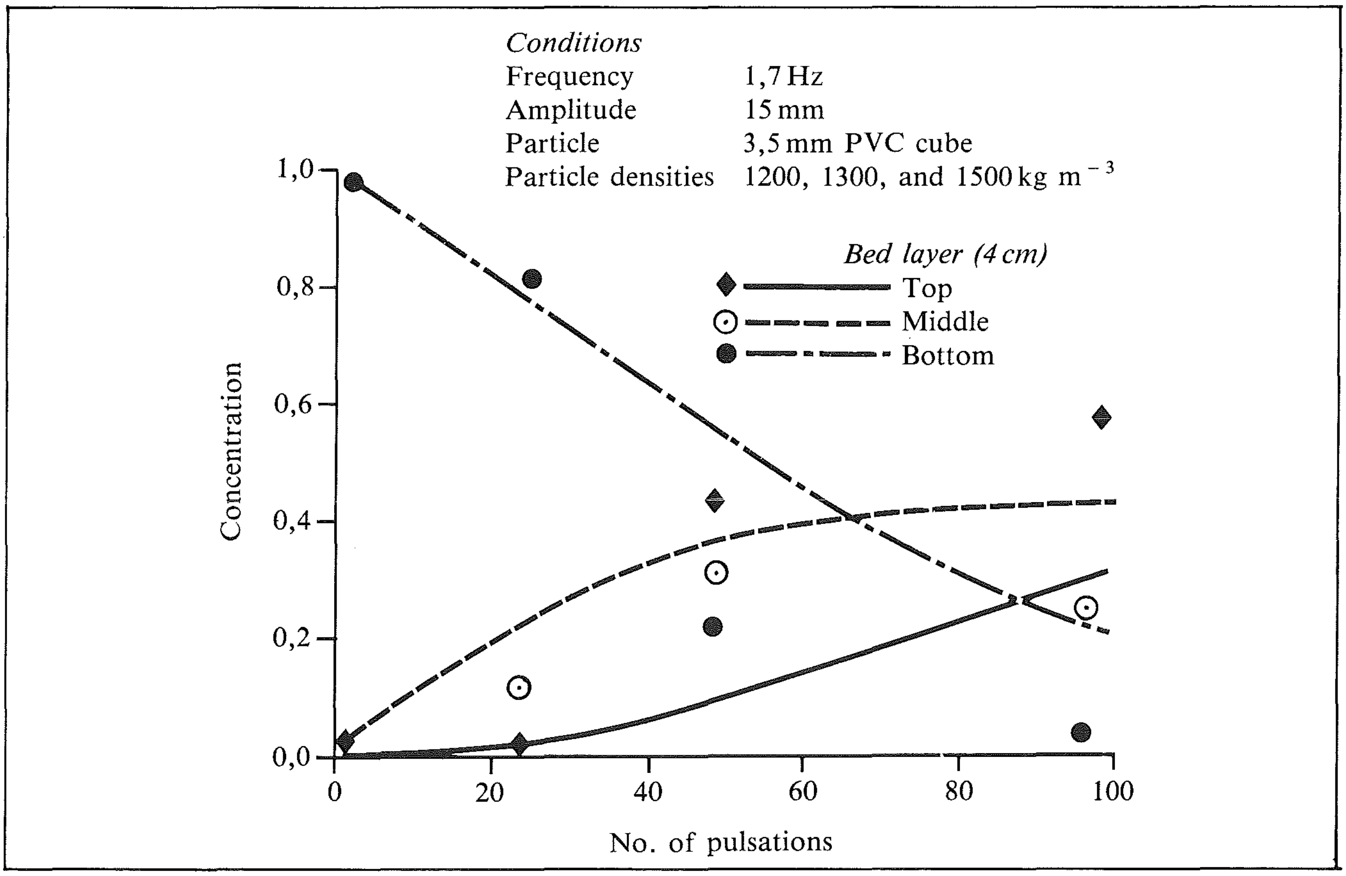


FIGURE 6. Dispersion-model prediction of the distribution of the 1 200 kg m^{-3} density fraction in a ternary system (K_μ = 10; K_D = 1)

be described as follows. If a mixture of particles of different densities is pulsed continuously, the distribution of the material in the plane parallel to the external force field will undergo some rearrangement until a steady-state re-distribution is achieved. Neither the traditional single-particle hydrodynamic force balance description nor the potential theory describe this fundamental characteristic of autogenous density separations, but the dispersion model quantitatively accounts for the density-segregation aspects of the process as well as the interactions between the particles.

The dispersion model of autogenous separations, as presented in this paper, deals exclusively with the behaviour, in batch jigs, of beds containing a mixture of particles of a number of different densities but uniform size and shape. The model contains further idealizations in that the terms for the particle drag and gravitational segregation as, incorporated in Equation [8], are assumed to be linear. Moreover, the model parameters K_μ and K_D are assumed to be independent of the density distribution of the material and constant for a given feedstock.

However, although the dispersion model provides an adequate quantitative description of the batch jigging process for particles of uniform size and shape, it is expected that, for the description of more complicated autogenous separations, a more rigorous formulation of the particle drag and density segregation terms will have to be incorporated into the dispersion model. Nevertheless, the dispersion model provides not only a novel insight into the mechanisms of autogenous separations, but a potential means by which further system can be analysed.

Nomenclature

Symbols

$A(t)$	Gaussian white noise process
β	Particle drag term (kg s^{-1})
c	Volumetric concentration
g	Gravitational acceleration term (kg s^{-2})
$k(t)$	Force term (kg s^{-2})
L	Bed depth (m)
m	Mass (kg)
n	Species
p	Conditional density function ($p(y,t;y_0,t_0)$)
t	Time (s)
$v(t)$	Velocity (m s^{-1})
$w(t)$	Wiener process: $w(t)=\frac{A(t)}{\beta}\,dt$
y	Space coordinate (m)
V	Volume of particle (m^3)
D	Variance parameter of the Wiener process $w(t)$
K_μ	Drift coefficient (m^3 s^{-1})
K_D	Diffusion coefficient
ρ	Density (kg m^{-3})

Subscripts

i	Species index
0	Initial state

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Addendum

Steady-state Solution to the Dispersion Equation for a Binary Density Batch Jigging System

At steady state the term

$$\frac{\partial c}{\partial t}$$

in Equation [9] becomes zero, and consequently the equation can be expressed as the following ordinary differential equation:

$$\frac{dc}{dy} = -\frac{K_\mu}{K_D} c_i(\rho_i - \bar{\rho}). \qquad [11]$$

In a binary density system with species i = 1 and i = 2, the bed-density term can be expressed as

$$\bar{\rho} = c_1\rho_1 + (1 - c_1)\rho_2 .$$

This expression can be rearranged to correspond to the density segregation term in Equation [11]

$$\rho_1 - \bar{\rho} = (1-c_1)(\rho_1 - \rho_2) . \qquad [12]$$

Substitution of Equation [12] into Equation [11] and solving for c_i gives

$$c_i = \frac{1}{1 + H\, e^{\alpha y}} , \qquad [13]$$

where

$$\alpha = \frac{K_\mu}{K_D}(\rho_1 - \rho_2)$$

and

$$H = \frac{e^{\alpha L(1-c_{ai})} - 1}{e^{\alpha L} - e^{\alpha L(1-c_{ai})}} .$$

c_{ai} is taken to be the average initial concentration of species i in a bed layer.