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A Quantitative Model for Gravity Separation Unit Operations that rely on Stratification

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A class of gravity separation unit operations in ore-dressing relies for its operation on the production of a stratified bed. Heavier material gravitates downwards, and separation is achieved by splitting the ore horizontally, with the recovery of a dense stream and a lighter stream. The important ore-dressing unit operations that fall in this class are the jig, the Reichert cone and the pinched sluice. A new quantitative model based on Mayer's potential theory is developed here. This model is developed completely for the simple case of binary mono-size particulate materials, but the extension to units operating with feed material distributed over particle size and density is discussed in principle. The model can be used to predict the performance of these units and is very suitable for simulation. The model is tested successfully against experimental data obtained from a batch jig and a Reichert cone.

Introduction

The jig, cone and sluice are very important ore-dressing unit operations. These units have one very important characteristic in common: the particulate solid material in the feed forms a bed in the unit, the bed stratifies under the influence of gravity and separation of dense and lighter particles is affected by splitting the stratified bed in the horizontal plane using a suitable mechanical arrangement. The performance of these units is determined by the efficiency of stratification which in turn depends on the characteristics of the feed material. Performance depends on the nature of the feed.

The development of a quantitative model for the prediction of the performance of these units has not been successfully achieved although numerous researchers have attempted to do so. Sivamohan and Forssberg[1] have recently reviewed the literature and discuss the most important attempts at quantitative modelling.

The approach that is used here is based on the potential energy theory of Mayer[2] who showed that a bed of particles will stratify in such a way as to minimise the potential energy of the bed. In practice this ideal stratification never occurs because various dispersive forces are always present which tend to disperse the particles by random motions and thus destroy the ideal stratification profile. The actual stratification achieved represents a balance between the stratification forces and the dispersive forces.

This paper deals only with the steady-state equilibrium stratification and does not consider the dynamics associated with the attainment of an equilibrium stratified bed. These dynamics, which will be important if the residence time in the unit is

not sufficient to achieve equilibrium or whenever the dynamics are relevant for control purposes, will be the subject of a much more comprehensive study. The emphasis in this paper is on the effect of particle density differences and the effect of particle size distribution is not considered in detail. The model in its present form must be calibrated against experimental data before it can be used in practical simulation studies. Further work is under way to include the effect of particle size.

The stratification potential

Consider a densely-packed bed of mono-sized spherical particles of uniform density. If a single particle of the same size but different density is introduced into the bed it will be rigidly held and will not move. However, if the bed is expanded to such an extent that some random relative motion is possible among the particles, it will be observed that the odd particle will tend to move downwards or upwards depending on whether it is more or less dense than the surrounding particles. This relative motion will always act so as to decrease the potential energy of the bed. This is illustrated in Figures 1a and 1b.

The potential energy of the entire assembly of particles changes because of the change in position of the odd particle. This change in potential energy can be calculated by considering the exchange of position of two particles.

$$\Delta E = g\upsilon_p\rho' h_2 + g\upsilon_p\rho h_1 - g\upsilon_p\rho' h_1 - g\upsilon_p\rho h_2 \quad (1)$$

Here E is the potential energy of the assembly, υ_p the volume of a single particle, ρ' the density of the odd particle and ρ the density of the surrounding particles. Puting $\Delta h = h_2 - h_1$, Equation (1) becomes

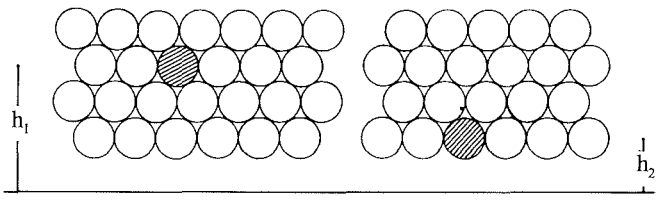

FIGURE 1a (*left*). Dense-packed bed with a single high density particle.

FIGURE 1b (*right*). Stratification after bed expansion to allow relative motion among the particles. The single high density particle has moved downwards to reduce the potential energy of the bed.

$$\Delta E = g\upsilon_p(\rho'-\rho)\Delta h \quad (2)$$

$$\frac{\Delta E}{\Delta h} = g\upsilon_p(\rho'-\rho) \quad (3)$$

If $\rho' \geqslant \rho$ then $\frac{\Delta E}{\Delta h} \geqslant 0$ and the system potential decreases as the position of the heavier particle is lowered.

Thus it is only necessary to allow the bed to expand sufficiently to permit relative motion among the particles, and stratification will occur. The particles will move relative to each other so as to minimise the total potential energy of the assembly. The movement of the particles is facilitated by the expansion of the bed caused by the jigging motion in a jig or by the particle-particle collisions in a bed of solids flowing down the floor of a sluice or a cone.

E can be regarded as the stratification potential of a particle of density ρ' in an assembly of uniformly sized particles of having a local average density ρ. The force that causes the stratification is equal to the negative gradient of the stratification potential. Thus the stratification force experienced by a particle of density ρ' in an environment of particles having an average density ρ is given by

$$\text{Stratification force} = -\frac{dE}{dh} = -g\upsilon_p(\rho'-\rho) \quad (4)$$

The stratification force is opposed by

the resistance experienced by the particles due to contact and collisions with other particles and by the viscous drag of the carrier fluid. This resistive force is a strong function of the particle size and it also depends on the mechanism that causes the bed expansion. It will be specific to each type of unit.

Equilibrium stratification

The force experienced by a particle because of the local gradient of the stratification potential of the particle will cause it to move relative to its neighbours so as to decrease the overall potential energy. The stratification does not happen instantaneously, and the stratification becomes more and more complete the longer the particle assembly remains in the unit. The dynamics of the attainment of the fully stratified bed are not considered in this paper, and it is assumed that sufficient residence time is provided to ensure that an equilibrium stratification pattern is established prior to splitting the bed.

The ideal stratification pattern that minimises the potential energy will never be achieved because the random motion of the individual particles will act to decrease any gradients in the concentration profile of any particle type. For example, the ideal stratification pattern for a mixture of only two types of mono-sized particles will be a perfect separation into two layers, with all the denser particles lying below the lighter particles. In practice the random motion of the particles will act to flatten the very steep concentration gradient at the boundary between the two layers. This process will have the characteristics of a diffusive motion and can be described by a Fickian Equation of the form

$$\text{Diffusive flux} = -D \frac{dC}{dh} \qquad (5)$$

In Equation (5) D is a diffusion coefficient which will depend on the particle size, shape and bed expansion mechanism. C is the volumetric concentration of particles of a particular type.

The stratification force produces motion of the particles which can be characterised by a specific mobility u for each particle type. u is defined as the penetration velocity achieved by a particle in the absence of any dispersive forces under a unit stratification potential gradient.

The flux of particles due to the stratification potential is given by

$$\text{Stratification flux} = - Cu \frac{dE}{dh} \qquad (6)$$

The bed attains its equilibrium stratification pattern when the stratification flux for each particle type is exactly balanced by the corresponding diffusive flux - the stratification fluxes attempting to sharpen the concentration gradients and the diffusive flux tending to flatten them. In the simple case of two particle types with the same size and different density the equilibrium stratification pattern will consist of a layer of dense particles below and lighter particles above, with a diffuse region between in which the concentration changes rapidly but not suddenly from one type to the other.

It is important to understand the difference between the ideal and equilibrium stratification patterns when assessing the performance of a gravity separation unit operation.

The equilibrium balance of fluxes gives

$$D \frac{dC}{dh} = -Cu \frac{dE}{dh} \qquad (7)$$

This equation can be integrated immediately to give

$$C = C^o \exp(-\frac{u}{D} E) \quad (8)$$

where C^o is the concentration of particles at a horizontal plane where E = 0.

Equation (8) shows that the concentration of a particular particle type must be an exponential function of the stratification potential for that type. In order to establish the equilibrium concentration profile it is necessary to obtain the stratification potential profile by integration of Equation (4).

It is convenient to transform the independent variable in Equation (4) from the position h above the bottom of the bed to volume of solid contained in the layer of the bed below h. Since only particles of the same size are being considered the packing density ρ_B which represents the volume of solid per unit volume of bed is constant. The volume fraction of solid in the layer of the bed below height h is given by

$$V = \frac{hA\rho_B}{h_B A\rho_B} = \frac{h}{h_B} \quad (9)$$

where A is the floor area and h_B the height of the bed. Equation (4) becomes

$$\frac{dE}{dV} = gh_B \upsilon_p(\rho' - \rho) \quad (10)$$

which may be integrated from E = 0 at V = 0 to give

$$E = gh_B \upsilon_p(\rho' V - \int \rho dV) \quad (11)$$

Substitution in Equation (8)

$$C = C^o \exp(-\alpha V\rho' + \alpha \int \rho dV) \quad (12)$$

where $\alpha = \frac{ugh_B \upsilon_p}{D}$ is called the specific stratification constant.

The constant C^o is established by requiring that

$$\int_o^1 CdV = C^f \quad (13)$$

where C^f is the volumetric concentration of the component in the feed.

The binary mixture of mono-size particles

The simplest example of gravity [illegible] by stratification occurs with the binary mixture of mono-size particles all having the same shape. The analysis of this highly idealised simple case is not trivial and provides valuable insight into the computational procedures required to use the proposed model effectively. The analysis is presented below.

Consider a gravity separation unit to be fed by a simple binary mixture having volume fraction C_1^f of component 1 and volume fraction C_2^f of component 2. The densities of the components are ρ_1 and ρ_2 respectively. The potential energy model can be used to calculate the separation performance of the unit as follows.

Equation (12) can be written for each component

$$C_1 = C_1^o \exp(-\alpha_1 V\rho_1 + \alpha_1 \int \rho dV) \quad (14)$$

$$C_2 = C_2^o \exp(-\alpha_2 V\rho_2 + \alpha_2 \int \rho dV) \quad (15)$$

at any horizontal plane in the stratified bed. In a bed of mono-sized particles d_1 $\alpha_2 = \alpha$ and the constant β_{12} given by

$$\beta_{12} = \alpha(\rho_1 - \rho_2) \quad (16)$$

is called the binary stratification index for the mixture in the unit. C_1 and C_2 are volumetric concentrations so

$$C_1 + C_2 = 1 \quad (17)$$

$$\frac{C_1}{1 - C_1} = \frac{C_1^o}{C_2^o} \exp(-\beta_{12} V) \quad (18)$$

Equation (18) can be rearranged to

$$C_1 = \frac{C_1^o/C_2^o \exp(-\beta_{12} V)}{1 + C_1^o/C_2^o \exp(-\beta_{12} V)} \quad (19)$$

Equation (13) gives

$$C_1^f = \int_o^1 C_1 dV = \int_o^1 \frac{C_1^o/C_2^o \exp(-\beta_{12}V)}{1 + C_1^o/C_2^o \exp(-\beta_{12}V)} dV \quad (20)$$

$$= -\frac{1}{\beta_{12}} \ln\left(\frac{1 + C_1^o/C_2^o \exp(-\beta_{12})}{1 + C_1^o/C_2^o}\right) \quad (21)$$

Equation (21) can be solved explicitly for C_1^o/C_2^o to give

$$\frac{C_1^o}{C_2^o} = \frac{1 - \exp(-\beta_{12}C_1^f)}{\exp(-\beta_{21}C_2^f) - 1} \exp(\beta_{12}) \quad (22)$$

The concentration profile in terms of the volume fraction of the bed from the base is given by Equations (19) and (22) together.

The separation is achieved by splitting the bed at some horizontal position into a light upper layer and a heavy lower layer as shown in Figure 2. The volumetric yield of solids to underflow that is achieved in the equipment determines the effective splitting level.

The recovery of the separated components as a function of the volumetric yield of total solids, V_s, is given by

$$\text{Recovery of component 1} = \frac{\int_o^{V_s} C_1 dV}{\int_o^1 C_1 dV}$$

$$= \frac{-1}{\beta_{12}C_1^f} \ln\left(\frac{1 + C_1^o/C_2^o \exp(-\beta_{12}V_s)}{1 + C_1^o/C_1^o}\right) \quad (23)$$

The recovery of component 2 is obtained by symmetry.

Recovery of component 2 =

$$\frac{-1}{\beta_{21}C_2^f} \ln\left(\frac{1 + C_2^o/C_1^o \exp(-\beta_{21}V_s)}{1 + C_2^o/C_1^o}\right) \quad (24)$$

Equations (23) and (24) are plotted in Figure 3 as a function of the parameter β for a feed containing 20% of heavy mineral by volume. These curves may be regarded as the performance curves for gravity separators of this type. The larger the numerical value of the binary stratification index the more efficient the separation device.

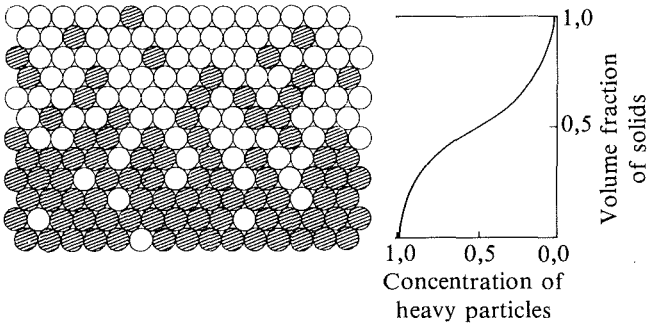

FIGURE 2. Stratified bed showing the dispersion caused by random motion. The graph shows the concentration profile calculated from Equations (19) and (20). The bed is split in the horizontal plane to produce heavy and light products.

Comparison with experimental data

The model developed above has been tested against three sets of experimental data. The first set was obtained by D.A. Vetter[3] in an experimental batch jig. The material tested consisted of binary mixtures of PVC 3,5 mm cubes having relative densities of 1,2 1,3 and 1,5. The jig used allowed the determination of the recovery of each particle type to be determined in each of four horizontal layers in the bed after the equilibrium stratification pattern had been established. The experimental data are shown in Figure 4 together with the best fitting model simulation. Because the particles were all of the same size a single value of α was required to fit all the data. The effectiveness of the model in correlating the data over a fairly wide density differential range is evident. A value for α = 0,09 was used to fit the data which represents very efficient stratification in the experimental jig used. Somewhat smaller values of this parameter can be expected in industrial jigs.

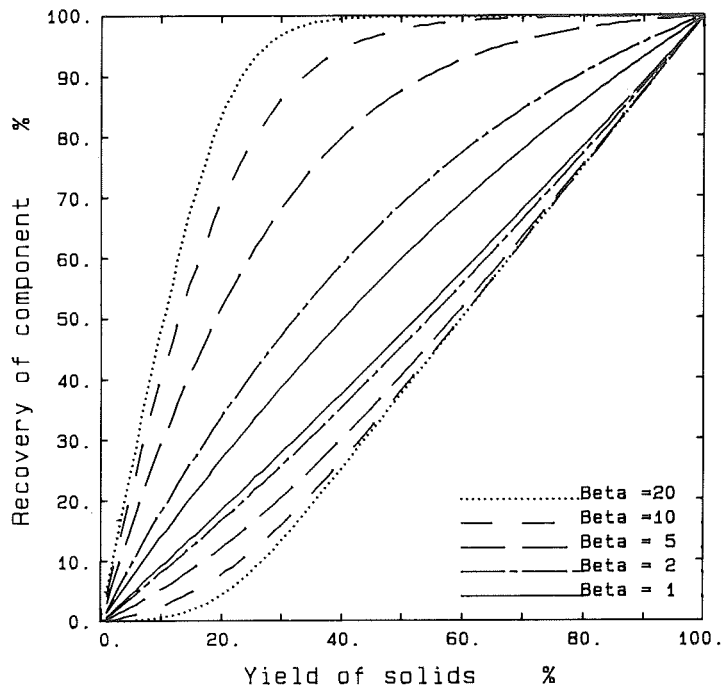

FIGURE 3. Performance curves for stratifying gravity separators operating on a mono-sized feed containing 20% heavy mineral by volume. The absolute value of β for each curve is shown. Heavier particles have a positive value of β and lighter particles have negative values of β. Curves with positive β are above the diagonal and vice versa.

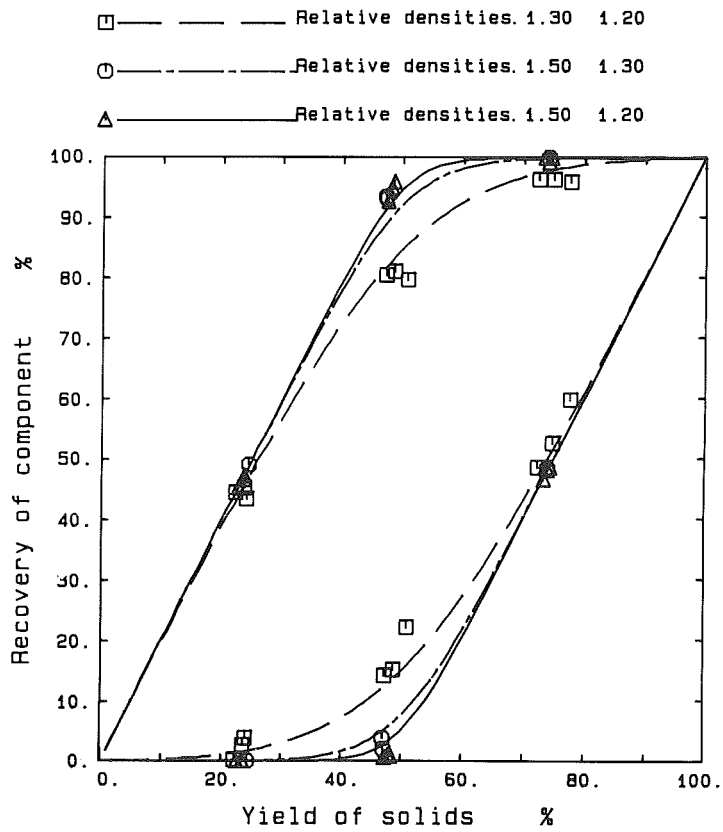

FIGURE 4. Comparison between the model and experimental data obtained by Vetter[3] in a batch jig with mono-sized PVC cubes of varying density. The lines plotted were calculated from Equations (23) and (24). A single value of the parameter α = 0,09 was used to describe all the data.

The second set of data was obtained by Vetter[3] using coal screened in a narrow size fraction + 425μm - 850μm. Binary mixtures were made synthetically by mixing two narrow density fractions obtained by dense liquid separation. This model is tested against this data in Figure 5. In this case it was necessary to allow the value of α to vary for each data set. The non-uniform nature of the particle shape yielded values of the specific stratification constant significantly lower than that found for the uniform PVC cubes. Values of α found are given in Table 1.

The third set of data was obtained by Prof E. Forssberg[4] at the University of Lulea using a Reichert cone to beneficiate two samples of iron ore. The model developed in this paper is not strictly applicable to these data because the feed material was distributed with respect to size and density so does not constitute a mono-size binary mixture. The model is tested against this data in Figures 6 and 7 and it is clear that the simple model does provide an adequate description of the average behaviour of a two-component system in a Reichert cone. The cone has a smaller specific stratification constant α than the batch jig and consequently will give less sharp separations requiring many more stages to achieve a given separation

Table 1. Values of the specific stratification constant for separation of coal fractions in a batch jig.

Specific gravities of binary fractions	α
2,85 and 1,35	0,012
1,55 and 1,35	0,035
1,45 and 1,35	0,048

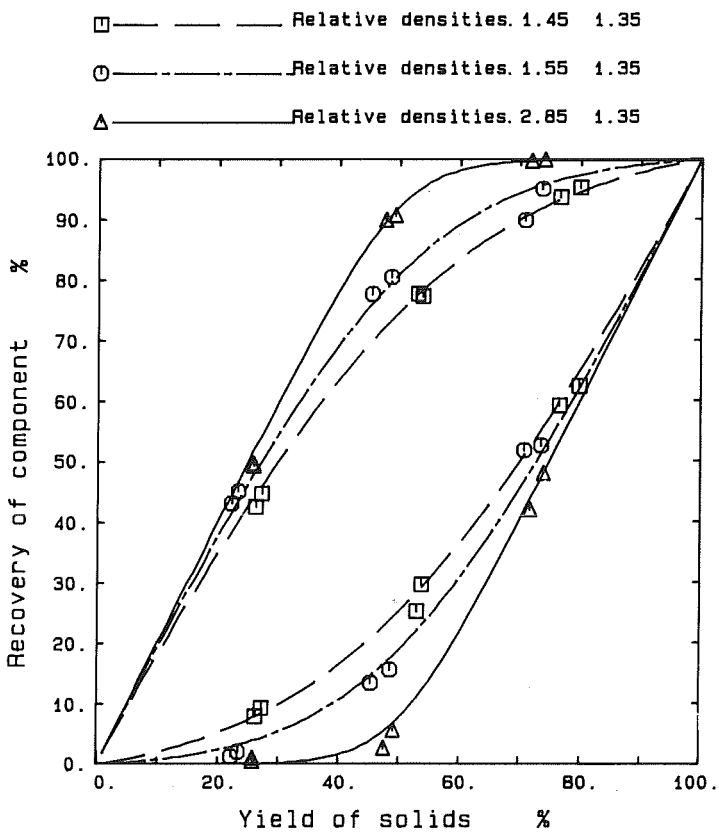

FIGURE 5. Comparison between theoretical model and experimental data of Vetter[3] obtained in a batch jig with closely sized fine coal particles. Synthetic binary mixtures were made by mixing two narrow density fractions.

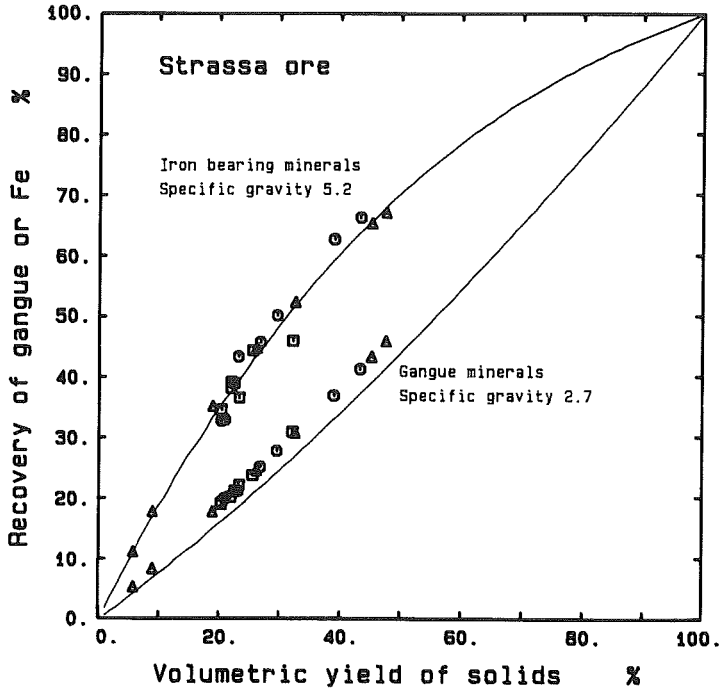

FIGURE 6. Comparison between theoretical model and data obtained by Forssberg[4] when beneficiating iron ore containing 22% haematite by volume on the Reichert cone. □ Constant feed rate, constant slot width, varying % solids in feed. ○ Constant feed % solids, constant slot width, varying feed rate. △ Constant feed % solids, constant feedrate, varying slot width. The lines were calculated from Equations (23) and (24) with $\alpha = 0{,}008$.

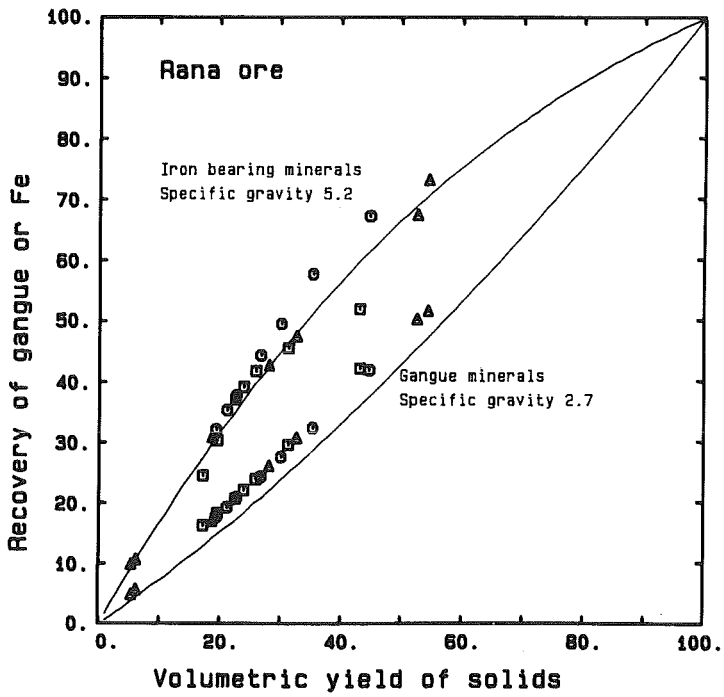

FIGURE 7. Comparison between theoretical model and data obtained by Forssberg[4] when beneficiating iron ore containing 30% haematite by volume on the Riechert Cone. □ Constant feed rate, constant slot width, varying % solids in feed. ○ Constant feed % solids, constant slot width, varying feed rate. △ Constant feed % solids, constant feedrate, varying slot width. The lines were calculated from Equations (23) and (24) with $\alpha = 0{,}009$.

efficiency.

The model provides a useful general correlation for all stratifying gravity separation units. The following dimensionless groups are defined for binary separations

$$\Phi_{ij} = R_i \; \alpha(\rho_i - \rho_j) \; C_i^f$$

where R_i is the recovery of species i to underflow

$$\theta_{ij} = \frac{1 - \exp(-\alpha(\rho_i - \rho_j)C_i^f)}{1 - \exp(-\alpha(\rho_i - \rho_j)C_i^f)}$$

$$\Psi_{ij} = \frac{1 + \theta_{ij}\exp\left(-\alpha(\rho_i - p_j)(V_s - C_i^f)\right)}{1 + \theta_{ij}\exp\left(\alpha(\rho_i - \rho_j)C_i^f\right)}$$

Equation (23) and (24) become

$$\Phi_{ij} = -\ln \Psi_{ij} \qquad (25)$$

and a plot of Φ_{ij} against Ψ_{ij} reduces all the experimental data to a single straight line as shown in Figure 8. Equation (25) or Figure 8 can be used to predict the performance of a stratifying gravity separator provided only that an estimate is available for the specific stratification constant $\alpha$for the particular unit and feed material.

The excellent correlation of all the experimental data on a single graph confirms the utility and effectiveness of the simple model developed here.

Multicomponent mixtures and the effect of particle size distribution

The simple model presented above must be developed further to provide an accurate density variation is positive the particle tends to sink and vice versa. Similar dislocations can be caused by a mismatch in shape between an individual particle and its packed neighbourhood, as illustrated in Figure 11. These local density variations are very difficult to analyse and not much work has been done in random packings of particles of irregular shape. Much research needs to be done in this area.

Particle size and shape also have a significant effect on some of the other par-

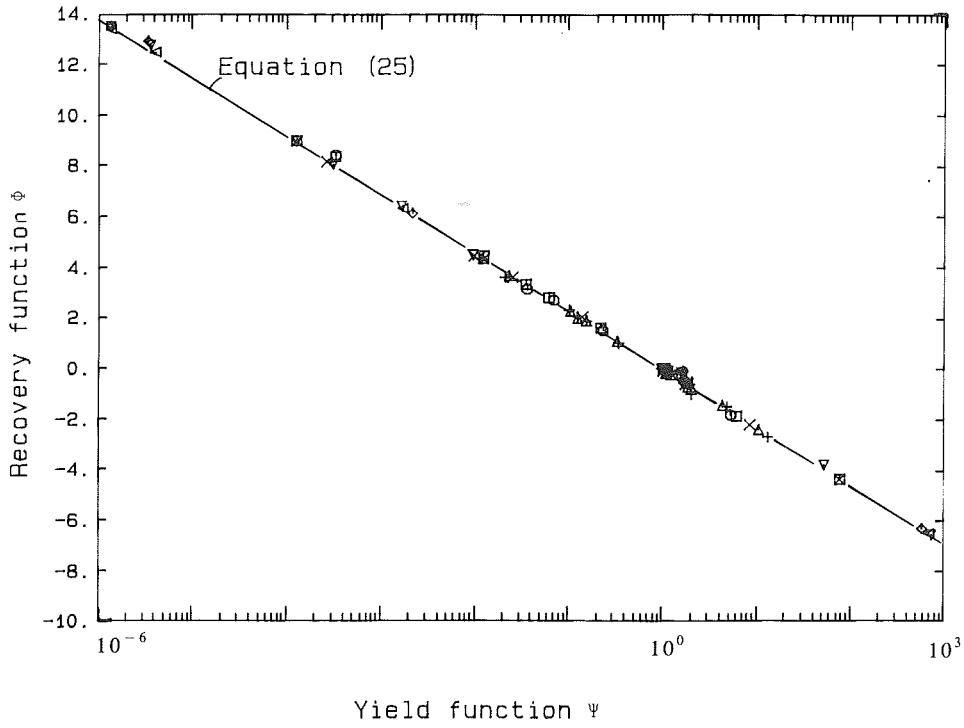

FIGURE 8. Generalised performance curve according to Equation (25) for gravity separators processing binary feed materials. Vetter's[3] data for PVC cubes and coal in a batch jig are shown.

ameters of the model. The penetration velocity u, the packing density ρ_B and the effective diffusivity D are all strong functions of size and shape. These influence the specific stratification constant α , and it will be necessary to measure α experimentally and correlate the measured values with the particle characteristics. The availability of a viable model for these important industrial units makes such correlations worthwhile.

Discussion

The model developed in this paper provides an important addition to the repertoire of available models for ore-dressing operations. The author has promoted the development of such models for use in the simulator MODSIM for ore-dressing plants. Models for stratifying gravity separation unit operations have been conspicuously absent until now, and the present model has been incorporated into MODSIM[5] which is now widely used throughout the world.

A similar model was developed in a particularly good MSc thesis by D. A. Vetter [3] who described the particle motion by a stochastic differential equation. The description of the industrially important case when the feed material contains particles having a range of specific gravities and is not graded to a fairly narrow size range.

The presence of particles of several different specific gravities is best handled theoretically by classifying the feed into a finite number of discrete gravity intervals each characterised by a representative specific gravity. The model must then be extended to cater for several density classes. The extension is not difficult, but the simple analytical solution given in Equation (19) is no longer available. Numerical solutions are not difficult to achieve using Equation (12) for the concentration profile and noting that the average density at any height is given by

$$\rho = \Sigma C_i \rho_i \tag{26}$$

Ongoing work in the author's laboratory is aimed at the development of efficient numerical algorithms for this case.

The effects of variable particle size and shape are significantly more difficult to handle. A number of parameters in the model are comparatively strong functions of the particle size and the particle size distribution. Most importantly, the stratification potential is a function of the particle size and shape. This is illustrated by the sketches in Figures 9, 10, and 11 adapted from Mayer's classic paper. The dislocation created by the insertion of a particle smaller or larger than average particle size in the neighbourhood gives rise to a local density variation around the particle. If this

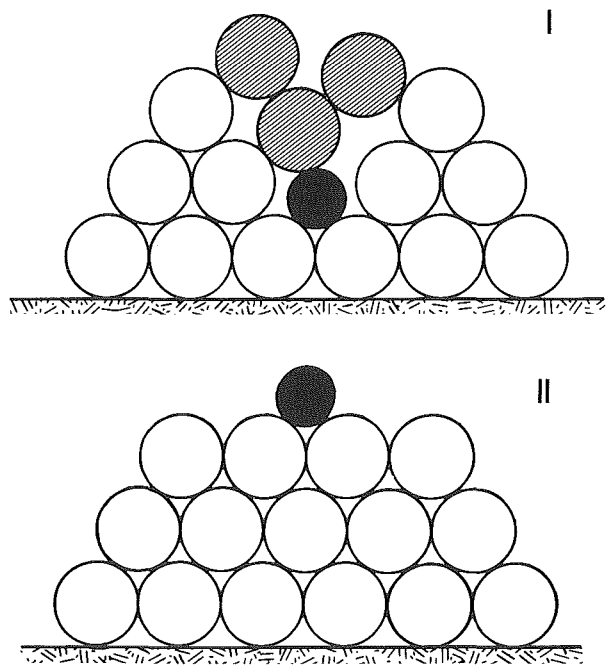

FIGURE 9. Dislocation caused by a smaller grain in a bed of larger grains. After bed expansion the smaller grain moves upwards to reduce the total potential energy of the bed. After Mayer[2].

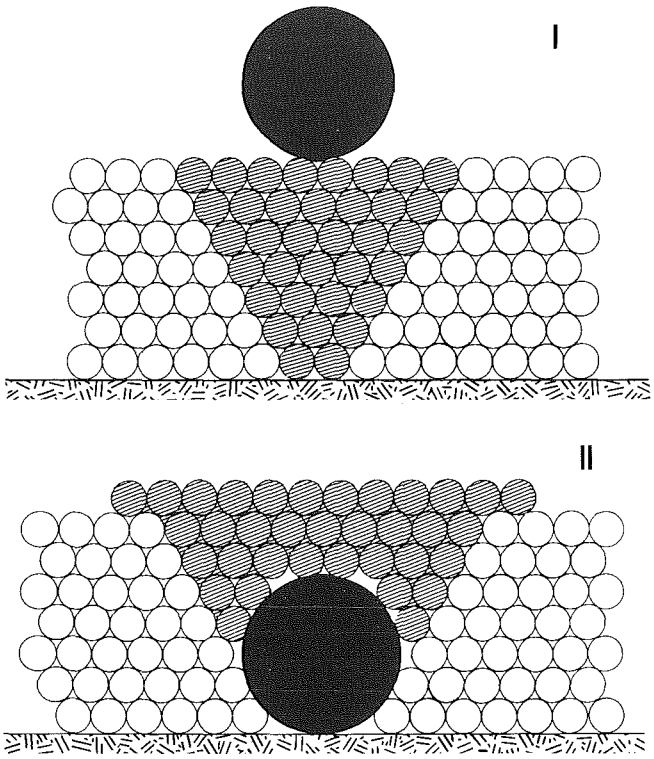

FIGURE 10. Reduction in potential energy of the bed after bed expansion due to downward movement of larger particle. After Mayer[2].

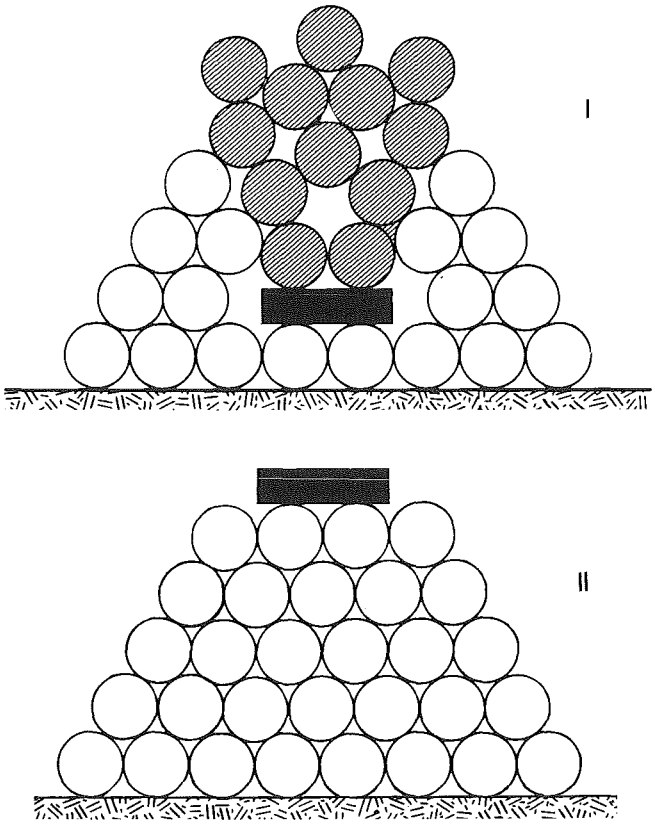

FIGURE 11. Dislocation caused by mismatch of shape and consequent upward movement of odd-shaped particle after bed expansion. After Mayer[2].

stratification and dispersive forces were included and Vetter demonstrated that statistically stable stationary solution of the stochastic differential equation existed. Vetter's approach has the added advantage of handling the practically important case where residence time in the unit is insufficient for the establishment of the equilibrium stratification profile. We plan to develop and extend Vetter's ideas in a future research programme.

References

1. SIVAMOHAN, R. and FORSSBERG E. A review of the literature on intermediate and fine particle gravity concentration. International Journal of Mineral Processing. 15, 1985. pp. 155-181.
2. MAYER F.W. Fundamentals of a potential theory of the jigging process. In: N Arbiter (Editor). Proc. 7th Int. Mineral Processing Congress N.Y. Gordon and Breach, New York. Vol.1, 1964. pp. 75-86.
3. VETTER D.A. Mathematical model of a fine coal batch jig MSc Thesis, University of Natal, Durban, 1987.
4. FORSSBERG E. Private Communication. 1980.
5. KING R.P. MODSIM : A modular approach to the design, balancing and simulation of ore-dressing plant flowsheet. Univ. of the Witwatersrand, Dept. of Metallurgy and Materials Engineering. Report Gen/2/86, 1986.