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A BASIC PERSPECTIVE ON THE ROLES OF CLASSICAL STATISTICS, DATA SEARCH ROUTINES, CONDITIONAL BIASES AND INFORMATION AND SMOOTHING EFFECTS IN ORE BLOCK VALUATIONS

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SYNOPSIS

The major role of classical statistics in the origin and development of geostatistics is stressed. Particular attention is drawn to the fact that both elementary and simple kriging techniques are names given to the classical bivariate and multi-dimensional correlation and regression procedures respectively. It is shown that the validation of any ore valuation technique via classical statistical correlations between block estimates and follow-up values inside the block remain essential, that the data accessed must be adequate for all blocks and/or the mean grade should be introduced via simple kriging. The effects of any remaining conditional biases on grade-tonnage and profit estimates are discussed as well as the relationships between the extent of data accessed and the technique, information and smoothing effects.

THE ROLE OF CLASSICAL STATISTICS

The role of classical statistics in the origin and development of geostatistics has not received the recognition it deserves. In fact, there was a period, hopefully now past, when geostatistics was presented as a development distinct and separate from classical statistics. This led to the introduction of a new geostatistical 'language' with new terms and concepts, justified in part but many of which have their parallels or even equivalents in classical statistics. This, no doubt, contributed to the fact that relatively few classical statisticians were involved over the years in the development of geostatistics and even some antagonism and scepticism surfaced in certain circles. It is, therefore, appropriate to view the relationship between classical and geostatistics in proper perspective.

The origin and early developments of geostatistics were undoubtedly founded on classical statistics and will be covered briefly under the main headings of:

frequency distribution modelling,
modelling of spatial inter-correlations, and
multidimensional correlation and regression analyses and estimates.

From a practical point of view the massive data bases available on the South African gold mines provided the indispensible foundation for these classical statistical analyses.

FREQUENCY DISTRIBUTION MODELLING

Following the unsuccessful attempts by Watermeyr (1919), Sichel (1947) proposed the *lognormal (2 parameter) distribution* for gold grades on the Witwatersrand mines. At about the same time De Wijs (1951/3) was analysing grade variations via the differences between pairs of values, a concept which later developed into the variogram. The lognormal model formed the basis of all the local developments in the 1950's, until the *3 parameter lognormal* was proposed (Krige, 1960), a model which was far more flexible and is still the most widely used model for grade distributions. It also largely eliminated the biases in the mean grade. More recently Sichel et al (1992) developed the new *Compound Lognormal and the Log-generalised Inverse Gaussian distributions* to cater for cases which cannot be covered by the 3-parameter lognormal model.

The use of the appropriate distribution model leads to

i) the elimination of biases in and improvements of the quality of estimates of the mean,
ii) a logical way of dealing with the proportional effect,
iii) the definition of proper confidence limits for individual block estimates, and
iv) improvements in the quality of correlation and spatial structure analyses.

Where data is very limited the Bayesian principle of accepting a distribution model as known from experience to apply to comparable ore bodies, is particularly applicable.

SPATIAL STRUCTURES

An early geostatistical concept was that of the nature and size of *data support*. This was first analysed by Ross (1950), via a study of the lognormal distributions and variances for individual 'point' samples, averages of samples in development stretches of varying lengths and of oreblock values. This was further developed by Krige (1951/1952) in correlating ore block estimates with internal follow-up block averages of stope values obtained as a block was mined out. This provided the first direct evidence of *spatial correlation and structure*. This also led to the concept of the *variance-size of area relationship* (Krige, 1952), which was subsequently shown to be directly linked to the De Wijsian variogram; also to a basis for estimating the *block variance* and the estimation of *grade-tonnage curves* directly from borehole results.

In classical statistics the equivalents of the variogram are the *correllogram and the covariogram.* The former was used by Krige (1962), and others at the time when Matheron suggested the variogram. The correllogram differs only in so far as it introduces the discipline of the specification of the relevant population of values and hence the population mean and variance.

CORRELATION AND REGRESSION

The present kriging techniques originated from the application of classical correlation and regression techniques as applied to gold ore block valuations on the Witwatersrand (Krige, 1951) Orthodox block valuations were based on the averages of all available samples around the peripheries or part peripheries of ore blocks. These were correlated, using the lognormal model, with the averages of much larger numbers of samples obtained from stope faces advancing through the blocks as these were mined out. The correlation explained clearly the reason for the feature observed from the early days of the Witwatersrand, namely the under and over valuation of blocks valued as low and high grade respectively. These *conditional biases* were shown to be an inevitable regression effect wherever estimates are subject to error and do not incorporate a wide spread of information. The regressed estimates then recommended were, in principle, a weighted average of the peripheral values and the mean grade of the relevant mine section (i.e. the mean of all values outside the ore block). This provided the required wide spread of data, and in fact was *the first elementary kriging technique.*

This first kriging estimator was clearly based on classical statistical regression and soon developed (Krige, et al 1963) to *a multidimensional correlation and regression estimator* based on regularised data blocks around the oreblock to be estimated and on the levels of correlation and covariance observed between such data and ore blocks in the mined out parts of the mine section concerned. In this way the 'outside' data, catered for previously by the population mean, was now allowed to carry individual weights. Thus the data blocks close to the ore block, and more highly correlated therewith, carried more weight than data blocks further away and with a lower correlation level. These early estimators, then called weighted moving averages, were, therefore. effectively *simple kriging estimators* as the mean grade of the mine section still featured in the technique. *Simple kriging is, therefore, a typical classical multidimensional correlation technique*. Also as the data blocks used were not all of the same size, the technique was in fact *simple co-kriging.*

The implications of this and comparisons with the geostatistical ordinary kriging, which does not use the mean grade, calls for a more detailed examination.

CONDITIONAL BIASES, SEARCH ROUTINES, SMOOTHING AND INFORMATION EFFECTS

The elimination of conditional biases was the first achievement of geostatistics and still remains the most important goal for any kriging exercise. It has been shown (Krige et al, 1992) that, if the quality of an estimator is measured by its error variance, *the major contribution to the overall improvement achieved by kriging techniques (compared to orthodox techniques) can be attributed to the elimination of conditional biases*. In a different analysis (Krige et al,1989) the improvement was measured by the effect on the estimated total profits, and the extent of the contribution from the elimination of conditional biases was also confirmed as major.

It is evident, therefore, that the most sophisticated kriging techniques which do not ensure the elimination of conditional biases, will be sub-optimum and could, in fact, be worse than the original regression or elementary kriging techniques.

The main reason for any remaining conditional biases in any kriging technique, which does not incorporate the mean grade, e.g ordinary kriging, is *a limited search routine*. A search routine, even if allowing for the access of up to say 40 point values, but with a limited search radius, will still result in many ore blocks not close to data to be valued on small numbers of values. This will inevitably result in conditional biases. Furthermore, due to practical problems associated with the inversion of very large matrices, the number of data points accessed cannot be increased indefinitely. This problem can be solved by:

(i) increasing the search radius to well beyond the range of the variogram,

(ii) by regularising the point data into data blocks, and/or

(iii) by introducing the mean grade of the mine section via simple kriging or an equivalent procedure.

The main argument advanced against the use of simple kriging is that the population mean is not known accurately and/or is not stable throughout the mine section concerned. This argument is invalid because the estimate of the population mean is always far more efficient than any estimate of the associated variance or variogram, whatever the density or spread of data may be. Also, if necessary, the section can be subdivided into more homogeneous sub-areas provided the local means can be estimated with reasonable accuracy.

The aspect of the search radius was covered in a recent paper (Krige,1994) and the relevant arguments are reproduced in *Annexure 1*. The same paper also discussed in detail the serious effects of conditional biases and the relationship with the smoothing effect and biases in tonnage-grade and profit estimates. Technique and information effects were also analysed. Typical parameters from a gold mine were used to simulate elementary, ordinary and simple kriging as well as orthodox procedures for block valuations. These block valuations were then compared with the actual grades.

These comparisons, summarised in *Table 1,* show for each of the 3 estimates and for perfect valuation the variances of the block estimates, the correlation coefficients between the estimates and follow-up block values, the slopes of the regressions of real on estimated values and the error variances of the block estimates. *The results for 'ordinary' and 'simple' kriging should be accepted as representative of all kriging techniques where the data accessed for block valuations is either inadequate,i.e. too restricted, or fully adequate, respectively.*

Estimates	*Variance of Ests.*	*Corr.Coef.*	*Regr.Slope*	*Error Var.*
Orthodox	0.41	0.7	0.49	0.31
Regressed-El.Krig	0.10	0.7	1.00	0.10
Ordinary Kriging	0.21	0.76	0.75	0.095
Simple Kriging	0.13	0.80	1.00	0.07
Perfect	0.20	1.00	1.00	0.00

TABLE 1 : Showing comparative results of alternative block valuation techniques

ERROR VARIANCES

These show a clear improvement from the orthodox estimates (0.31) to regressed and ordinary kriged estimates at 0.10 and 0.095 respectively and to 0.07 for simple kriging. In this example the total improvement is 0.24 (0.31 - 0.07), and *the elimination of conditional biases* via regression (i.e. elementary kriging) *constitutes 0.21 or some 87% of the total.*

CONDITIONAL BIASES

The slopes of the regressions of follow-up values on estimates provide a measure of the conditional biases of the orthodox estimates (slope 0.49) and of the ordinary kriging estimates (slope 0.75). The effects of the biases on the tonnage-grade curves are shown in Figs. 1 and 2.

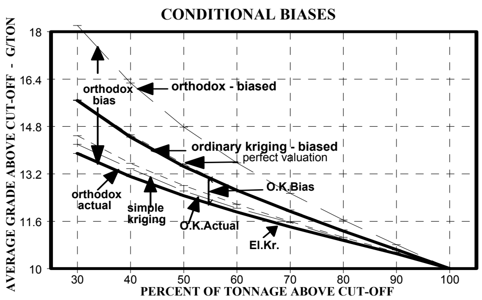

FIGURE 1 : EFFECTS ON TONNAGE/GRADE CURVES

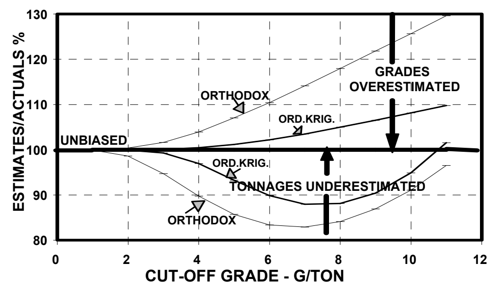

FIGURE 2 : CONDITIONAL BIAS EFFECTS ON TONS & GRADES

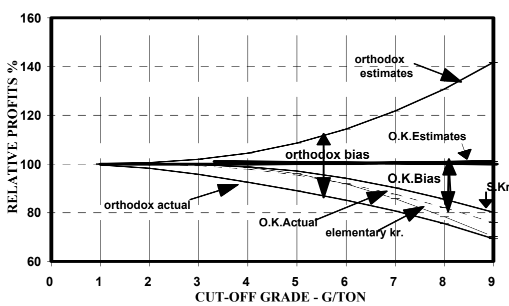

FIGURE 3 :COND.BIAS EFFECTS ON REL.PROFITS

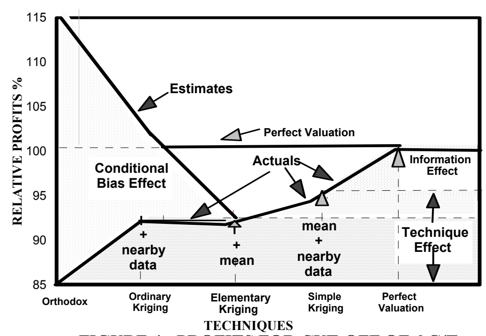

FIGURE 4 : PROFITS FOR CUT-OFF OF 6 G/T

It is clear that where conditional biases are present, the tonnages and grades above cut-off will tend to be under- and over-estimated respectively and that the tonnages and grades which will actually materialise from these estimates, will be worse than those on simple kriging. Similarly, relative profits as measured by:

(% ore mined)*(average grade above cutoff - cutoff)

as shown in *Figs. 3 and 4*, show the serious overestimation of profits where conditional biases are not eliminated.

The need for eliminating conditional biases in all ore reserve estimation procedures is, therefore, evident. Apart from an advanced warning via the slope estimated from the kriging output for each block i.e. block variance, kriging variance and Le Grange multiplier), the only practical way of testing for conditional biases is via follow-up correlations as discussed above. Such follow-up correlations will show not only

(i) the presence of any remaining conditional biases, but will also
(ii) provide a practical measure of the average error variances of block valuations compared to the theoretical kriging variances, as well as
(iii) the nature of these error distributions for establishing realistic confidence limits.

Experience on the Witwatersrand goldfields have shown that because of the highly skewed distributions of the point values, the best way of ensuring conditional unbiase is via:

(i) regularising the point data into data blocks so that an adequate spread of data is accessed for block valuations and
(ii) using simple kriging.

TECHNIQUE AND INFORMATION EFFECTS

Figure 4 also shows the extent of the *technique effect* on relative profits. Orthodox estimates using only peripheral block values show a biased estimate 15% higher than the maximum possible level with perfect valuation; also actually realised profits are at only 85% of this maximum. The introduction of nearby data via ordinary kriging shows profits at an estimated level of 100% (i.e. equal to perfect valuation) whereas the level which will actually be realised will be some 92%. Elementary kriging which does not use nearby data but introduces the mean grade of the mine section shows the estimated profit and the realised level both also at 92%. However, the use of nearby data as well as the mean via simple kriging improves the estimated and actual profit level to 94%. *The technique used is, therefore, important.*

The gap of 6% between simple kriging and perfect valuation can be reduced by further improvements in technique but these are likely to be minor as the main contributor to this gap is *the information effect*, i.e. the limited extent of the data available. This effect can only be reduced by more intensive and if possible, better placed sample data.

The smoothing effect results from the fact that the variance of the distribution of any set of block estimates, provided the estimates are conditionally unbiased, is always lower than the corresponding variance of the distribution of real block values, i.e. of perfect estimates. This is shown in Table 1 and Figure 5 where the logvariances of the regressed and simple kriged estimates (i.e. the only conditionally unbiased estimates) are at the low levels of 0.10 and 0.13 respectively compared to that for real block values (i.e. perfect estimates) of 0.20. The variances of the orthodox and ordinary kriged estimates at 0.41 and 0.21 show negative or no smoothing. They are misleading and lead to false and biased estimates as discussed above. In several recent publications this smoothing effect is attributed to kriging (incorrectly) and not, correctly, to the information effect. The fact is that any estimation procedure is never perfect because information is always limited and *if the procedure is to be conditionally unbiased, some smoothing effect is inevitable and essential.*

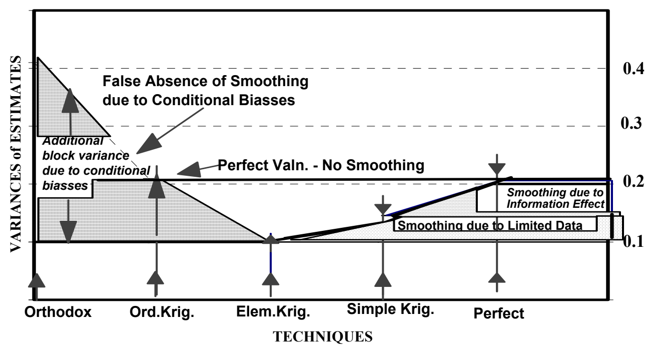

FIGURE 5 : ESTIMATE VARIANCES AND SMOOTHING

Theoretically, this is clear from the definition of the slope of the line of regression of actual (Y) on estimated block values (X), i.e.:

$$\text{slope} = r(\sigma y/\sigma x)$$

If the slope is to be unity for unbiasedness and r is less than unity because estimates are never perfect, ($\sigma y/\sigma x$) must be larger than unity, i.e. the standard deviation (or variance) of the real block values must be larger than that of the estimated values. The gap between these two variances, i.e. the smoothing effect can, therefore, only be reduced by increasing the correlation between estimates and real values, i.e. by improving the efficiency of the valuation technique or by providing more data. No mathematical manoeuvring can achieve this objective.

Simulation has been shown to produce a set of block values for a mine section with a variance set at that of the actual block values (i.e. with no smoothing effect), but such a simulation is only one of an infinite number of equally possible simulations and, therefore,

(i) a single simulation, although not subject to smoothing will show individual block values subject to the undesirable effect of conditional biases,

(ii) repeated simulations, when averaged will produce block valuations equivalent to that of the kriging technique used in the simulation procedure i.e. it cannot eliminate the smoothing effect for a specific ore reserve assessment, and

(iii) it can only provide a global estimate of the grade-tonnage curve. This can also be obtained directly from the mean grade, the block variance, the information effect and the appropriate frequency distribution. Also the ore blocks which will actually be above a specified cut off cannot be identified.

CONCLUSIONS

Classical statistics should continue to play its vital role.

It is essential in all ore reserve assessments to eliminate conditional biases and to confirm this by follow-up studies wherever possible.

The data search routine for block valuations must be adequate and, if necessary, using regularised data blocks, and the use of simple kriging should be considered.

The smoothing effect in kriging is inevitable and essential and can be reduced only by increasing the level of available data and/or by improved techniques.

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ANNEXURE 1

Example demonstrating the need to access enough data including, where necessary, data beyond the range of the semivariogram.

ORE BLOCK (Y)   AND DATA BLOCKS: (X_1/X_3) X_3 BEYOND RANGE

$Y \quad X_1$	
	X_3
X_2	

Y = Ore block to be valued $\sigma_y^2 = 0.8$

X_1, X_2 = Normal size data blocks $\sigma_{x1}^2 = \sigma^2_{x2} = 1.0 = \text{sill}$

X_3 = Large data block beyond range $\sigma^2_{x3} = 0.2$

Covariances: $\sigma_{1.2} = \sigma_{1.y} = 0{,}5$
$\sigma_{2.y} = 0{,}4$
$\sigma_{3.y} = \sigma_{3.1} = \sigma_{3.2} = 0$

Weights required for data blocks X_1/X_3 = a_1/a_3
Weight for population mean of data blocks = a_m
Le Grange mutiplier = μ

1. ORTHODOX VALUATION (X_1 & X_2 only)
   $a_1 = a_2 = 0.5$
   Error variance = $V_o = 0.8 + 2(0.5)^2 0.5 + (0.5)^2 1 + (0.5)^2 1 - 2(.0.5)0.5 - 2(0.5)0.4$
   $= 0.70$

2. ELEMENTARY KRIGING (REGRESSION, X_1,X_2 and mean)
   $1.0a_1 + 0.5a_2 = 0.5$
   $0.5a_1 + 1.0a_2 = 0.4$

   $a_1 = 0.4 \quad a_2 = 0.2 \quad a_m = 0.4$
   Error variance = $V_{ek} = 0.8 - 0.4(0.5) - 0.2(0.4)$
   $= 0.52$

3. ORDINARY KRIGING (X_1 and X_2 only)
   $1.0a_1 + 0.5a_2 - \mu = 0.5$
   $0.5a_1 + 1.0a_2 - \mu = 0.4$
   $a_1 + a_2 = 1.0$

   $a_1 = 0.6 \quad a_2 = 0.4 \quad \mu = 0.3$

Error variance $= V_{ok} = 0.8 - 0.6(0.5) - 0.4(0.4) + 0.3$
$= 0.64$

4. ORDINARY KRIGING (X_1,X_2 and X_3)

$$
\begin{aligned}
1.0a_1 + 0.5a_2 + 0.0a_3 - \mu &= 0.5 \\
0.5a_1 + 1.0a_2 + 0.0a_3 - \mu &= 0.4 \\
0.0a_1 + 0.0a_2 + 0.2a_3 - \mu &= 0.0 \\
a1 + a2 + a3 &= 1
\end{aligned}
$$

$a_1 = 0.4421$ $a_2 = 0.2421$ $a_3 = 0.3158$ $\mu = 0.0632$

Error variance $= V_{ok3} = 0.8-0.4421(0.5)-0.2421(0.4)-0+0.0632$
$= 0.545$

As X_3 increases in size its variance within the total population will decrease, until it reaches zero when it covers the whole population. The effect is as follows:

		ORDINARY		*KRIGING*		*SIMPLE KRIG.*
	X_1+X_2 only	+X_3 small	X_3 large	X_3 larger	X_3 maxm	X_1+ X_2 +mean
σ^2_{x3}	0.0	0.2	0.1	0.05	0.0	0.0
a_1	0.6	0.4421	0.4235	0.4125	0.4	0.4
a_2	0.4	0.2421	0.2236	0.2125	0.2	0.2
a_3	0.0	0.3158	0.3529	0.3750	0.4	0.4
μ	0.3	0.0632	0.0353	0.0188	0.0	0.0
ErrorVar	0.64	0.545	0.534	0.5275	0.52	0.52

Note that in the last case the results coincide with those for simple kriging, No.2 above, because the value for X_3 is then equivalent to the population mean.