(International Geostatistics Congress, Wollongong, Australia, 1996)

A PRACTICAL ANALYSIS OF THE EFFECTS OF SPATIAL STRUCTURE AND OF DATA AVAILABLE AND ACCESSED, ON CONDITIONAL BIASES IN ORDINARY KRIGING.

D.G.KRIGE
Private Consultant, South Africa

ABSTRACT : The need for routine checks on the presence and extent of any conditional biases in ore block valuations is stressed as well as the need for defining the efficiencies of such valuations. The factors affecting such biases and efficiencies and their interactions are analysed, i.e. the spatial parameters of range and nugget, the data patterns available, the data search routine which determines the total number and patterns of data used, the ore block size, the extrapolation distance (if any) and the use of point data regularised into data blocks.

1. Introduction

Ore valuation for a new mining project or an existing mine basically covers two major stages. At *the initial or first stage* the data is limited and is obtained either from a broad drill hole grid or from the initial main development grid. During *the second or final stage* more data becomes available from a closer drill hole or blast hole grid or from sampling of stope faces and auxiliary development; this is also the stage of final selection of blocks as ore (payable) or waste (unpayable). Apart from providing a basis for short and longer term mine planning and viability studies, such valuations are frequently also required to provide resource and reserve figures in the broad categories of inferred, probable, measured and proven ore to substantiate a major capital investment in a foreign country, and/or the raising of loans from, e.g. the World Bank.

The proposed internationally acceptable definitions of 'measured' resources and of 'proven' reserves do not specifically refer to defined blocks of ore. However, proven reserves are defined as that part of the measured resource on which sufficient technical and economic studies have been carried out to demonstrate that it could justify economic extraction under specified economic conditions. This implies the mining of individual ore blocks to cut off grade(s) . Therefore, the author contends that a 'proven' reserve must relate to individual blocks of ore and that the aggregate of all these blocks above the specified cut-off grade constitutes the proven reserve. Tonnage/grade estimates based on global estimates or on simulation techniques, even with an acceptable block model cannot, in my opinion, be classified as 'proven' reserves but rather as 'probable' reserves or resources.

At both stages of valuation mentioned above, individual block valuations will be subject to error due to the data limitations. The estimated error levels can provide a basis for classification of the reserves into the required categories. Therefore, the valuation technique used should ensure minimum error variances and this will be the case if the appropriate kriging and data search routines are used. These requirements are linked closely to the expected slopes of regression of the eventual follow-up values, (usually inside the blocks) on the original block estimates. Slopes of less than unity indicate the presence of conditional biases with blocks in the upper grade categories overvalued and the reverse applying to blocks valued as low grade.

Block valuations subject to conditional biases thus result in lower efficiencies and higher error variances and, if used directly for selective mining decisions, can lead to serious biases in grade, tonnage and profit estimates (Krige 1994, 1996). *Therefore, common sense dictates that individual block valuations, effected at whatever stage and for whatever purpose, should not be subject to conditional biases.* In fact, the presence and economic effects of conditional biases on the reserves and production records of South African gold mines led directly, almost half a century ago, to the birth of South African geostatistics and of kriging. Their elimination in all reserve estimates still remains a primary prerequisite.

The author's experience with many reserve and resource estimations has been disturbing. In surprisingly many operating mines follow-up validation exercises are not practised, i.e. the crucial practical test for quality and acceptability is ignored. It is also not the general practice to analyse the expected results of the kriging search routine as specified in order to test in advance for the likelihood of conditional biases in the block valuations. *With the presently available soft- and hard-ware facilities there is no excuse for such practices.*

An inherent and unavoidable effect of the kriging of individual ore blocks is the so-called 'smoothing' of the estimates. At the second or final stage, as defined above, any remaining 'smoothing' has to be accepted and can only be reduced by more and/or better data. However, first stage estimates, being based on less data, will be subject to additional 'smoothing' resulting generally in the overestimation of tonnages and underestimation of the grades above specified cut-offs. *In both stages the prerequisite of conditional unbiasedness remains essential.* Various attempts have been made to reduce or eliminate the 'smoothing' effect but this can only be achieved at the expense of introducing conditional biases in the individual block valuations. *Such a practice is completely unacceptable.*

Where the effects of smoothing is expected to be significant at the first stage, early production and financial planning can be based on global adjustments to tonnages and grades. However, individual block estimates cannot be adjusted but can either be qualified by estimates of recoverable tonnages and grades for each block or, alternatively, mine planning and financial studies can be performed on a series of acceptable simulations in order to define the overall levels of uncertainty.

This paper is aimed at highlighting the main factors responsible for conditional biases in ordinary block kriging and at defining in broad practical terms the conditions under which these biases can be avoided.

2. Main Factors Affecting Conditional Biases.

2.1. SPATIAL STRUCTURE.

The spatial structure observed and modelled reflects an inherent feature of the mineralisation and is defined by the nugget effect, the range and the type(s) of variogram models used. In these analyses single isotropic spherical and exponential models have been used for both two- and three-dimensional ore bodies. Results have shown that, generally, the spherical equivalent to an exponential range can, for purposes of this study, be accepted at about 2.5 times the exponential range.

2.2. DATA AVAILABLE.

Point data have been assumed available mainly on a 50 x 50m grid horizontally but, where specifically indicated, denser grids have also been used. In three-dimensional cases the vertical pattern has been assumed to be bore hole composites corresponding to a bench height of 12.5m. Because of practical considerations *point data regularised into data blocks* have also been studied for comparative purposes. This is relevant, for example, to the use of dense blast hole data in an open cast mine or where the data variability is so high as to result in a prohibitively high number of point data required per block to ensure conditional unbiasedness.

2.3. DATA ACCESSED.

In practice the data actually accessed in a block valuation will depend on the data pattern available relative to the block and on the search routine specified. In these studies the presence of a data point within the block has been avoided and the ore block has been assumed to be more or less centrally situated within the data grid, except where the data grid is dense and where the effect of extrapolation has been examined, i.e. where the block lies outside the data grid. The analyses effected were aimed mainly at the number of data values required for conditional unbiasedness and at the pattern of data used.

2.4 ORE BLOCK SIZE.

Except where indicated, ore blocks have been assumed to be 20 x 20m horizontally and 12.5m vertically in the three-dimensional cases. A limited number or analyses of larger block sizes have been effected.

2.5. PROGRAM USED.

A convenient 'krigtest' program was provided by the Ore Evaluation Department of Anglo-American Corporation. The input specifies the data to be used and the block and variogram details. The output consists of the kriging variance, the block variance, the dispersion variance of the estimate and the slope of regression of the actual block value on the estimated block value.

The cases run were aimed at highlighting actions which can readily be taken by the practitioner on the existing data, i.e. changing the search routine to access more data or a different pattern of data, changing the block size, calling for more data to be made available, e.g. a denser grid of bore hole values, or regularising the data into data blocks.

3 The Efficiencies of Block Valuations

It is proposed to measure efficiency as follows:

Efficiency = (BV-KV)/BV expressed as a percentage.

where BV = Block variance, i.e. the variance of actual block values.

and KV = Kriging Variance, i.e. the error variance of the block estimate.

For perfect valuations: KV=0 , the dispersion variance(DV) of the estimates = BV and the efficiency = (BV-0)/BV = 100%

Where only a global estimate of all blocks is practical, all blocks will be valued at the global mean, i.e.:

DV = 0, KV = BV and Efficiency = (BV-BV)/BV = 0%

Usually blocks are valued imperfectly. *With no conditional biases* :

DV = BV-KV and Efficiency = (BV-KV)/BV = DV/BV

However, *with conditional biases present* this relationship does not hold and then:

DV > (BV-KV) because of insufficient smoothing, and

Effic < DV/BV

= (BV-KV)/BV

The efficiency can even be negative if KV>BV. Such a situation is ridiculous and *the block valuations will be worthless; yet the author has encountered several such cases in practice where the data accessed per block was inadequate*.

A study of some 70 cases covering wide ranges of spatial and data patterns used indicated a correlation between efficiency and the regression slope (actuals on estimates) of 87.5% (see figure 1). The correlation proved to be the same between efficiency and all the main spatial and data factors combined. *Thus the slope (or the extent of conditional biases present) effectively incorporates all the major factors affecting the efficiency of block valuations.*

A study of the results for Figure 1 shows that the poor efficiencies correspond to poor spatial structures and low numbers of data accessed. As the structures strengthen and more data is used the results move up along the curve to higher levels of efficiencies.

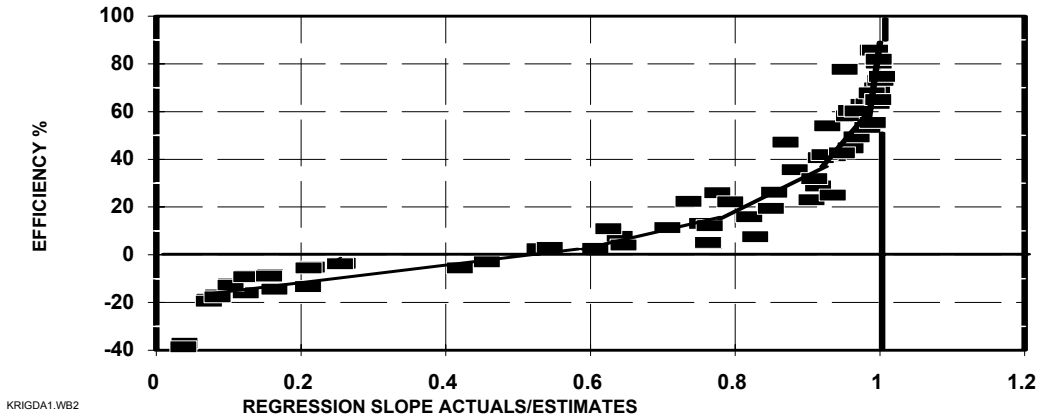
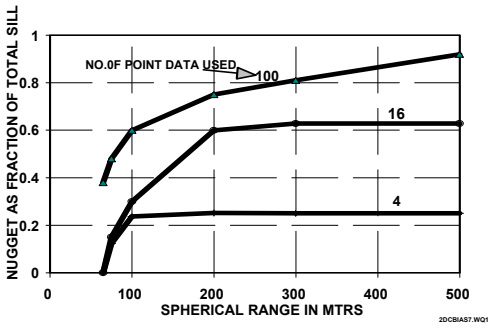


FIGURE 1:Showing correlation between efficiencies of block valuations and regression slopes

4. The Total Number of Data Required.

Figures 2 and 3 show the effective number of point data required to ensure virtual conditional unbiasedness (slopes > 95%) relative to possible nugget effects and spherical ranges for the 2D and 3D cases.



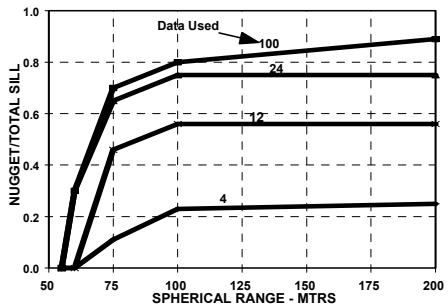


FIGURE 2:Number of data required for a 95% slope --- 2D case ---grid 50x50m,blocks 20x20m - slope<95% above curves, i.e. unacceptable.

FIGURE 3:Number of data required for a 95% slope ---3D case --- grid 50x50x12.5m,blocks 20x20x12.5m -- slope<95% above curves, i.e. unacceptable

It is clear that for ranges exceeding 100m the number of data points required are virtually the same for the 2D and 3D cases and call for the following maximum nugget effects as percentages of the total sill:

4 Data points	25%
16 Data points	60%
100 Data points	85%

For shorter ranges the maximum permissible nuggets reduce fairly rapidly to zero at a range of about 60m.

5. Number of Composites per Drill hole -- 3D Cases

Figure 4 shows the effect of data configurations used in 3D cases, particularly the effect of the number of composites per drill hole. Two spatial structures are covered, i.e. a spherical range of 100m with nugget 40% and range 75m with nugget 75 %. The higher efficiencies and better slopes in the case of the stronger spatial structure is obvious.

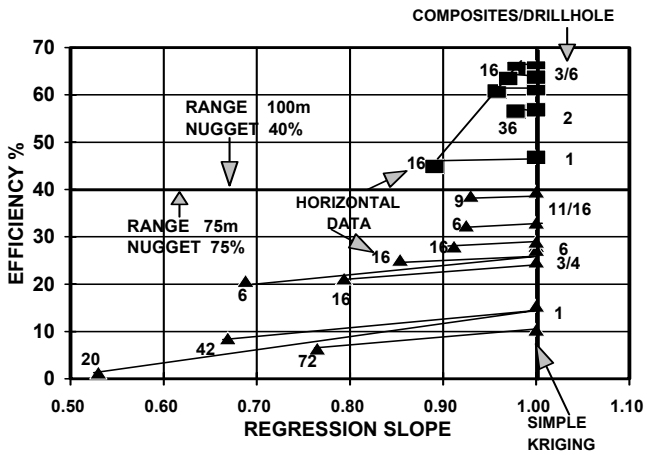


FIGURE 4: Showing effects of number of composites per drill hole on slopes and efficiencies

From figure 4 it is also evident that significant improvements in slopes and efficiencies are obtained by increasing the *number of composites per drill hole to, say, 6. Also as the data configuration used is improved, the results of ordinary kriging closely approach those of simple kriging*.

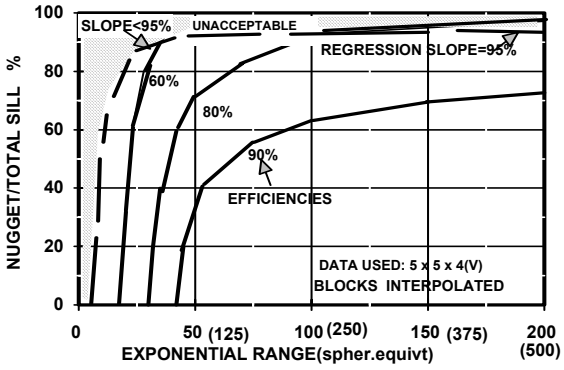
6. The Effects of Data Densities

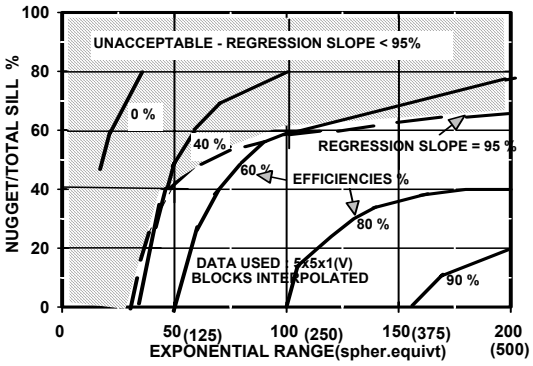
The valuator has to accept the inherent spatial structure. As shown in pars.4 and 5 above, the extent and pattern of data used can, however, be adjusted to gain the maximum advantage. Also, where the results are still unsatisfactory, additional data can be called for in the form of a closer drilling grid. The expected gains can then be weighed against the cost of the additional drilling. The results of an analysis of this factor for horizontal drilling densities of 50x50m, 25x25m and 15x15m are summarised in Figures 5 and 6 for data patterns used of 5x5x4(vert) and 5x5x1(vert) respectively and the standard block size of 20x20x12.5m.

It is evident that a denser data grid will improve the quality of valuations very significantly. Also, the close correlation between efficiencies and regression slopes is

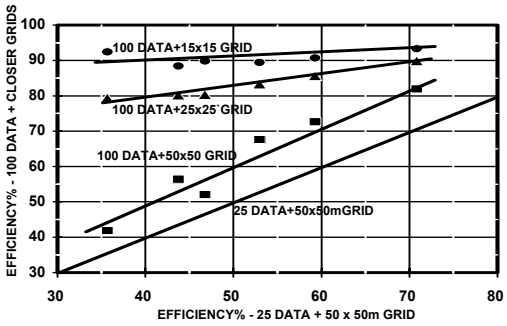
obvious. For slopes exceeding 95% the efficiencies tend to be better than 40% and generally exceed 60%.

The correlations between the efficiencies for the different data densities are shown in Figure 7 for the data patterns used for Figures 5 and 6.





FIGURES 5 and 6: Showing effects of data densities on efficiencies of block kriging for various spatial structures and data patterns used of 5x5x4 and 5x5x1 respectively.



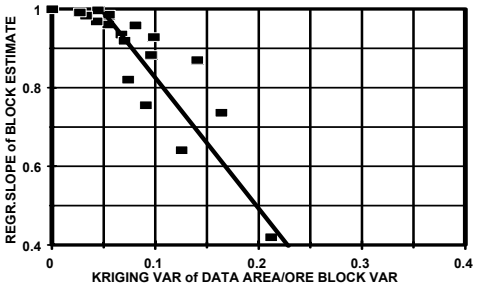


FIGURE 7 :Showing extents of improvements resulting from denser data grids-- data pattern of 5x5x4 compared to 5x5x1.

FIGURE 8: Influence of error variances of data areas accessed on slopes of regression.

7. Influence of error variances of total data areas accessed

Figure 8 shows clearly why the data patterns used and the total data area covered has such a strong effect on the regression slopes. If the area covered by the data used is itself poorly valued then the smaller ore block inside this area will obviously be valued even more poorly and will generally be subject to conditional biases. These results indicate that, unless the kriging variance for the larger data area is equivalent to some 5% of the ore block variance or less, the regression slope is likely to be lower than the required 95%. *This is a further useful criterion for the control of conditional biases.*

8. The Effects of Block Sizes

This effect has been studied for the standard data grid of 50x50x12.5m and for block sizes ranging from 10x10x12.5m to 50x50x12.5m. The nugget effect was taken at 67% of the total sill, the spherical ranges at 75 and 500m and the data pattern used in the ordinary kriging of blocks at 2x2(hor.)x6(vert.). The results are summarised in Figure 9 and, as could be expected, show significant improvements in efficiency. The improvements are more noticeable for the shorter variogram range.

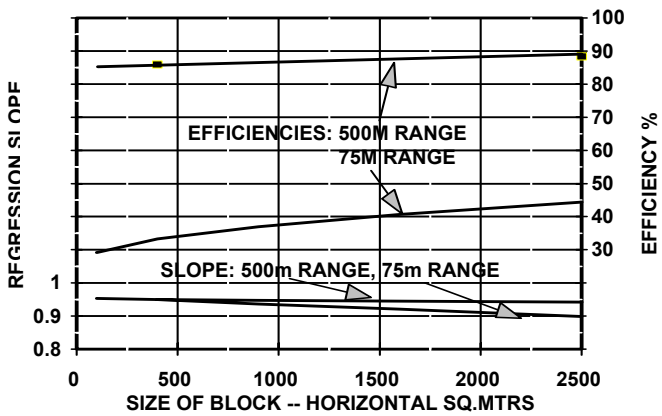


FIGURE 9: Showing the effect on efficiencies and regression slopes of changes in block size.

9.Blocks Valued by Extrapolation of Data

The above analyses all cover the position where the ore block lies inside the data grid and the valuation is therefore effected on an ***interpolation*** basis. However, in many practical situations the block lies outside the data grid, at best adjacent thereto. This is generally the case in the deep South African gold mines where advance development on the ore horizon is limited and most ore blocks have to be valued on the latest sampling data on the relevant stope face and on data further back in the stoped out ground. It is therefore a clear case of ***2D extrapolation.*** A similar situation arises in open cast mines where blast hole grades are used to evaluate the block as drilled and also using the data from neighbouring and nearby blocks. Even where this is done on a co-kriging basis with the addition of widely spaced drill hole data(see Krige and Dunn,1995) the process is still largely one of ***3D extrapolation***

9.1 2D EXTRAPOLATION ON POINT DATA

Figures 10 and 11 show the effects of block size and of distance extrapolated given a data grid of 50x50m and the spatial structures specified. It is clear from Figure 11 that, as the extent of extrapolation increases, the slope and efficiencies for Ordinary Kriging decrease rapidly. The efficiencies for Simple Kriging are significantly better but presupposes

knowledge of the global or local mean grade for an area covering the blocks being valued. Figure 10 highlights the negative effect of a higher nugget effect even when more data is used(10x2 vs.4x2) and also the fact that extrapolation is more reliable for larger block sizes.

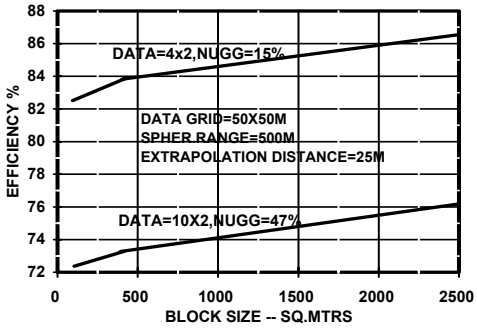


FIGURE 10 : Effect of block size, data and nugget on efficiencies

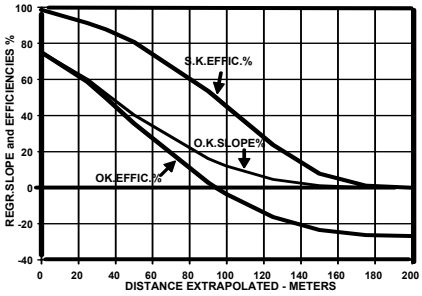


FIGURE 11 : Effect of distance extrapolated on slope and efficiencies -- data grid=50x50m,data used=10x2. range=200m,nugget=40%

9.2 : 2D EXTRAPOLATION ON BLOCKED DATA

Where the data grid is dense(e.g. blast hole grades) and/or highly variable(e.g. gold deposits) a better spread of data is often achieved by regularising the data into data blocks. Figure 12 shows the data patterns required for regression slopes of 95% for data blocks of 20x20m, ore blocks of 20x20 (and one case of 40x40m) and extrapolation distances of 20 and 40m.

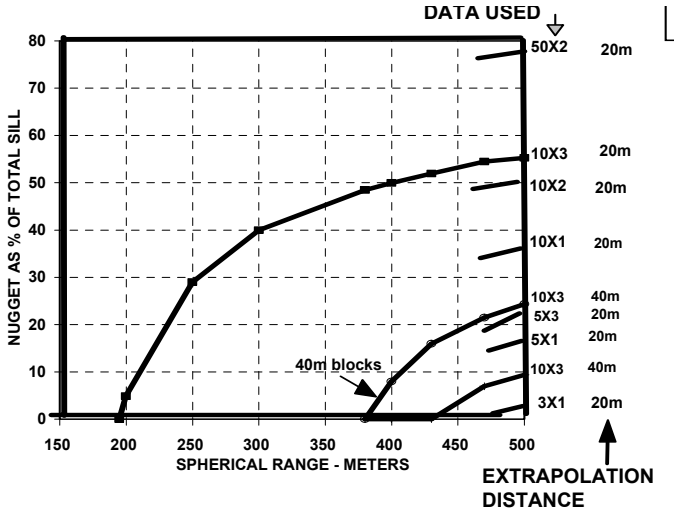


FIGURE 12 : Effects of extrapolation distances and patterns of data blocks used -- 2D case. above curves slope<95% =unacceptable.

5

Figure 12 shows that for 20x20m blocks :

i)extrapolation for 20m requires the 30 or more data blocks to be used, unless the nugget approaches zero and the range 500m.

ii)extrapolation to 40m will not be conditionally unbiased, even with 30 data blocks used, unless the nugget approaches zero and the range 500m.

iii)extrapolation of larger blocks (40x40m) to a distance of 40m imposes less stringent limitations.

Extrapolation ,therefore, requires a much more careful approach than interpolation.

9.3 3D EXTRAPOLATION ON BLOCKED DATA

The 3D case of extrapolation on regularised data (20x20x12.5m) is covered in Figure 13. It shows the combinations of nugget, range, extrapolation distance and data patterns required to result in a regression slope of 95 %.

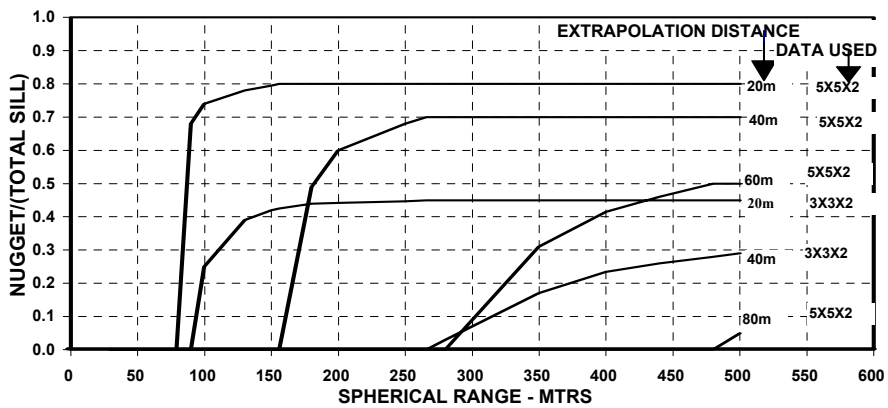


FIGURE 13 : Effects of range, nugget, data patterns(regularised blocks) and extrapolation distances on regression slopes for the 3D case

As in the 2D case above, the requirements of range, nugget and data to be used increase rapidly as the extrapolation distance increases. However, extrapolation up to 60m will still be conditionally unbiased provided the range exceeds 400m, the nugget is less than 40% and the data accessed is at least 5x5x2(vert). Extrapolation to 80m seems undesirable.

9.4: POINT DATA VS. REGULARISED BLOCK DATA -- 3D EXTRAPOLATION

Figure 14 is based on point data available on a 5x5m horizontal pattern and vertically as composites on a bench height basis of 12.5m, with a spherical range of 100m and a nugget of 67%. The data is used either directly to value 20x20x12.5m ore blocks or on a regularised data block basis of 20x20x12.5m. Extrapolation covered a range from zero to 8 benches vertically. The variogram for data blocks was derived from the point data using a nugget effect equivalent to the error variance for blocks valued on 16 internal point values, the net sill set at the block variance and the range as for point values. The data pattern used was 10x10x1 for both points and data blocks.

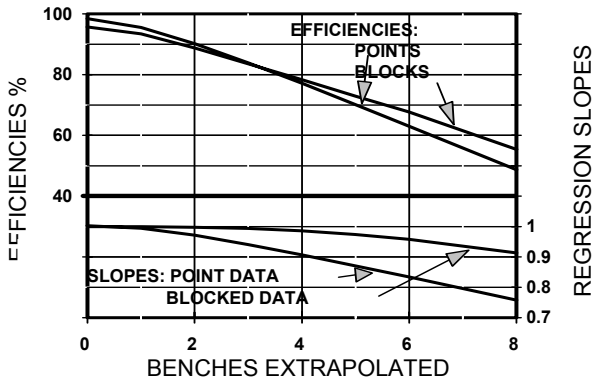


Figure 14 : Comparison of effect of using regularised block data versus point data in 3D extrapolations

The use of regularised data blocks shows a distinct advantage over point values for the regression slopes and to a lesser extent also for efficiencies. This is due to the larger spread of data used in the data block case (200x200m horizontally) compared to the point data case(50x50m). Note that to cover the same data spread as for block data a total of 1600 data points would have to be accessed. As extrapolation extends to 2 or more benches the use of regularised data seems preferable.

10. Conclusions

Individual block valuations form an essential part of ore reserve assessments and of the in situ selection process as ore or waste. Whatever technique is used it is essential that these valuations should be conditionally unbiased.

In this study the range of variations in the factors affecting conditional biases and block valuation efficiencies was, of necessity, limited but have shown that when ore blocks are kriged for ore reserve or whatever other purpose:

follow-up and/or validation analyses should be a routine exercise;

the kriging program should cater for the recording of the expected regression slope and valuation efficiency together with an appropriate plotting facility to identify the specific blocks where problems are indicated;

proper attention should be given to the search routine to ensure the use of a data pattern which will avoid conditional biases; factors such as the total number of data used and the number of composites per drill hole are important. Use special care when extrapolating;

it is totally unacceptable to endeavour to reduce the smoothing effect of kriging at the expense of introducing conditional biases;

where problems are encountered give consideration to larger ore blocks, the use of regularised data blocks and to the need for a closer data grid.

11. Acknowledgements.

The assistance provided by the Ore Evaluation Department of Anglo-American mainly via the relevant computer program is appreciated as well as comments and suggestions from colleagues in that department and in the geostatistical community at large.

12. References

Krige,D.G.(1994) An analysis of some essential basic tenets of geostatistics not always practised in ore valuations, *Computer Applications and Operations Research in the Mineral Industries,1st Regional APCOM, Slovenia*, 15-28.

Krige,D.G.and Dunn,P.G.(1995).Some practical aspects of ore reserve estimation at Chuquicamata copper mine, Chile, *Application of Computers and Operations Research in the Mineral Industries, APCOM XXV, Brisbane, Australia, July 1995*, 125-133.

Krige,D.G.(1996) A basic perspective on the roles of classical statistics, data search routines, conditional biases and information and smoothing effects in block valuations, *Conference on Mining Geostatistics, Kruger National Park, Sept..1994,UNISA Publisdhers.*