Valuation of recoverable resources by kriging, direct conditioning or simulation.

D. G. Krige
Private Consultant, South Africa,

W. Assibey-Bonsu
Gold Fields Limited, South Africa

ABSTRACT: For the estimation of recoverable resources for mining projects when the data are limited, available techniques yield useful global estimates but all suffer from the problem of being unable to provide acceptable individual block estimates on a selective mining scale. For the purpose of an objective and practical comparison of available techniques an extensive data base for a 3D ore body was simulated. The data are on a close grid (13824 point values) to provide a basis for 'actual' block grades and for 'actual' grade, production and financial 'life of mine' profiles for feasibility studies. The position at the stage when only limited data are available, was covered by selecting a set of 216 widely spaced data points from the complete simulation. This data set was then used for block valuations based on ordinary kriging with an adequate search routine, direct conditioning applied to each block to remove the effects of smoothing and seven repeated simulations on a close grid. The block valuations for each technique provided comparative global grade, tonnage and relative profit estimates; also for twelve consecutive production periods and for the indicated grade and financial profiles for life of mine and feasibility studies. The results are analysed in detail in order to rate the relative efficiencies and uncertainties of the techniques.

1. INTRODUCTION

1.1. General

The estimation of recoverable reserves when mining selectively to a cut-off has received much attention in recent years. The two main scenarios are:

i) Realistic global tonnage-grade estimates for long term life-of-mine production and feasibility analyses based on the selection of blocks on a scale dictated by the planned Selective Mining Units (SMU blocks).

Where only exploration data are available, but permit of a reasonable estimate of the spatial structure(s) of the grade data, global tonnage-grade estimates can be produced via various techniques without the specific valuation of individual SMU blocks. This aspect applies in the case of an exploration project or to virgin sections of existing mines where the data are inadequate for SMU block by block valuations, but have to feature in long term planning.

ii) Estimates of the detailed pattern of the grade distribution, as far as possible, on a (SMU) block by block basis, for use in the mining plan. The objective is to provide a guide to the mine planners for short and medium planning

iii) of the selection of blocks in the two main categories above and below the cut-off respectively and for meaningful feasibility studies.

1.2. Short term planning

For short term planning under scenario ii), ore reserve blocks have to be valued on the available data in the final form on a basis, which ensures the maximum reduction in uncertainty, i.e. with the minimum error variance. This is impossible without, at the same time, eliminating all conditional biases. This, in turn, can only best be ensured by a proper kriging estimate. The use of all relevant data is also essential, e.g. closely spaced blast hole data in an open cast mine as well as an adequate search routine (Krige 1999a).

Originally such estimates on ordinary or simple kriging were called Best Linear Unbiased Estimates (BLUE) with 'Unbiased' covering the elimination of conditional and any other biases.

Any estimates at this stage, which are based on an inadequate search routine will still incorporate conditional biases and be unacceptable; they should not be called 'kriging', and should not be passed by a 'competent person' for the publication of reserves in terms of any of the codes at present being formulated or updated world wide (Krige 1999a).

1.3. Medium term estimates, direct conditioning and simulations

For medium term planning under scenario ii) the relevant SMU blocks will have to be valued on the detailed final data as used for the short term planning, but with this data to be extrapolated, and/or on widely spaced exploration data as referred to under scenario i) above.

Kriged SMU block valuations at the stage when the data are either to be extrapolated beyond the short term blocks or are still widely spaced, will be conditionally unbiased if based on a proper search routine. They will have the lowest level of uncertainty, but will, unavoidably, be *smoothed*; i.e. they will have a smaller dispersion variance than that of the final SMU distribution. These smoothed estimates will generally tend to show a large tonnage and a lower average grade above a cut-off, than those which can be expected at the final SMU stage. Various techniques are available to correct for this feature such as, e.g. uniform conditioning, indirect or direct (Assibey-Bonsu and Krige 1999). The technique of direct distributions as referred to in that paper, is now called 'Direct Conditioning' by the authors.

Direct Conditioning can provide, for each 'smoothed' block or set of blocks, the average percentage of ore and the corresponding grade, which can be expected to be selected for mining when the more closely spaced data become available. Note that such techniques can be applied only to block estimates which are conditionally unbiased. These estimates can then be used in the life of mine programme for feasibility studies. However, it does not provide, and no technique can ever fully provide, at this early stage, for a practical detailed definition of the in situ SMU blocks which will eventually be selected or discarded.

Repeated conditional simulations were suggested by various authors at the International Symposium on Simulation in Perth (Oct. 1999), as providing an alternative practical measure for the estimation of the overall grade patterns and of the uncertainties inherent in doing early mine planning on the basis of any selected individual simulation ('median' or otherwise).

A conditional simulation requires basically only a realistic model of the spatial structure of the data and the co-ordinates and grades for the data available for the conditioning of the simulation. However, such a simulation provides only one of an infinite number of equally possible versions of the grade pattern.

A series of simulations could, provide a useful indication of the global grade pattern and of the overall uncertainty in this grade pattern, provided the available data are adequate to provide acceptable estimates of the global grade and spatial structure.

However, when any subdivisions of the simulation estimates for the ore body are used to predict the pattern of results over time as well as the uncertainties over short mining periods, e.g. the individual year(s) of production, the data for each subdivision or year will be more limited and the outcome could be misleading (Krige 1999a, Krige and Assibey-Bonsu 1999b and 2000).

This paper presents a practical and detailed comparison of the indicated techniques mainly for medium planning based on a closely spaced 3D database. The closely spaced 3D database will provide the criteria for these comparisons via 'actual' follow-up grades for the SMU blocks as valued by the various techniques.

2. DATA BASE FOR THE ANALYSES

2.1. General

The Turning Band method was used to simulate a 3D lognormal ore body with 13824 point values spaced 5m apart, with a mean grade of 1, a variance of 1.7 and an anisotropical spatial structure with the nugget at 0.7, ranges of 19 to 35m, and the net sill at 1.0. From these values a set of 216 (6x6x6) values was drawn to represent a widely spaced grid of exploration data. The model for the spatial structure was accepted at that used for the comprehensive simulation. Therefore, this analysis does not cover the uncertainties inherent in estimating the relevant parameters from the limited exploration data. This aspect was covered by Dowd et al (2000).

The complete grid of point values was used to provide kriged grades, i.e. 'actual' follow-up values, for a set of 1728 SMU blocks (measuring 10x10x10m).

2.2. Techniques and ore body sections

The set of 216 values was also used to provide corresponding block estimates as follows:

Technique 1: Conditionally unbiased kriged estimates.

Technique 2: These estimates were adjusted to eliminate the smoothing effect, i.e. for Direct Conditioning estimates.

Technique 3: Simulations of 7 grids of 110592 point values each, with a search routine of up to 56 values. The average of 64 point values in each block were averaged to provide a set of *7* simulation estimates for these blocks.

The sets of 1728 block estimates and the corresponding 'actual' values, were subdivided into

12 sections representing, say, 12 individual years of production for a 12 year total mine life. Each section thus had 144 blocks each estimated by these 3 techniques and with their ‘actual’ grades also known. On application of a range of cut-offs, the ‘actual’ percentages of blocks above each cutoff, their ‘actual’ pay grades and contents are calculated, as well as the corresponding sets of estimates. The estimates for the 144 blocks in each section are referred to as section estimates.

Confidence levels for the estimates were arrived at as follows:

‘Actuals Limits’: Deviations of estimates from actuals for all 3 techniques.

Observed: Deviations of simulation estimates from their pooled mean (technique 3).

Kriged estimates (technique 1): via kriging variances.

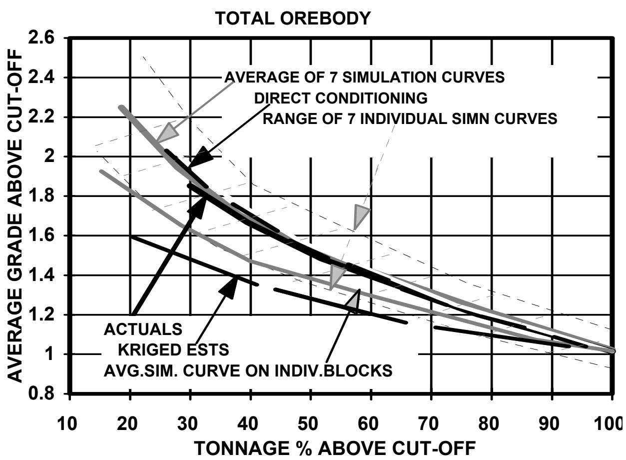


Figure 1 showing the ‘actual’ tonnage grade curve for the whole ore body as well as the corresponding estimates.

3. RESULTS OF THE ANALYSES

3.1. Global estimates

The results of the analyses are summarised in Figures 1 to 4 and in Table 1 for the ‘actuals’ and the corresponding estimates for techniques 1 to 3.

Technique 3 was further subdivided into:

Technique 3a: Individual simulations

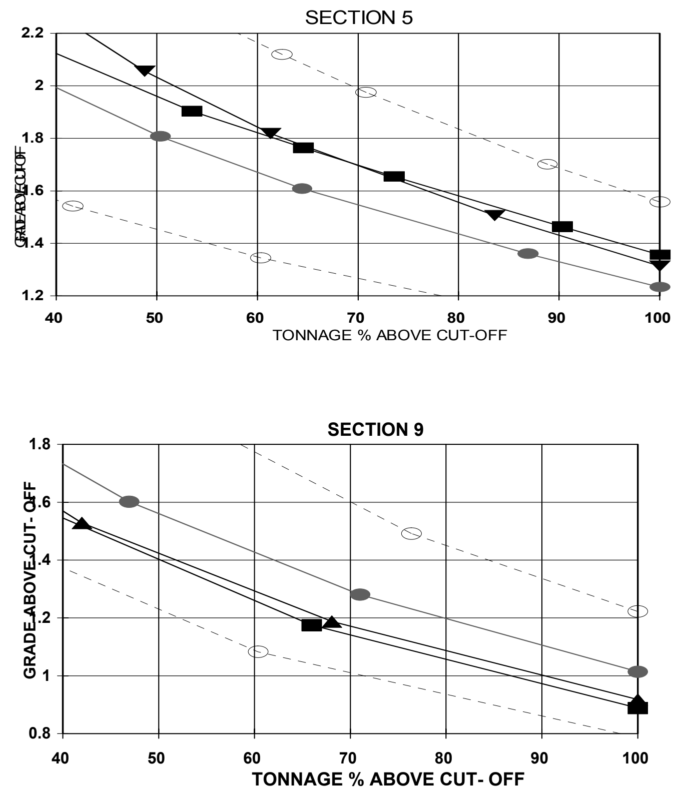
Technique 3b: Averages of 7 simulation estimates for each block

Technique 3c: Estimates based on section averages of tonnages and contents above cut-off for the 7 simulations.

Figure 1 shows the global tonnage grade curves for the whole ore body. The direct conditioning (technique 2) and the simulation averages (technique 3c) agree very closely with the ‘actuals’ and are both acceptable. Simulation technique 3b is smoothed like the kriged estimate, i.e. unacceptable. The lower and upper limits for individual simulations, shown in broken lines, have a very wide spread. Also these estimates show a much wider misclassification of blocks actually above and below cut-off (Krige and Assibey-Bonsu 1999b and 2000).

3.2. Section estimates

Figures 2a and 2b show 2 examples of section tonnage grade curves (sections 5 and 9), as for Figure 1, but excluding the unacceptable kriged and simulation estimates (techniques 1 and 3b).



Figures 2a and 2b showing the tonnage grade curves for sections 5 and 9 as follows:

Actual: line with square markers

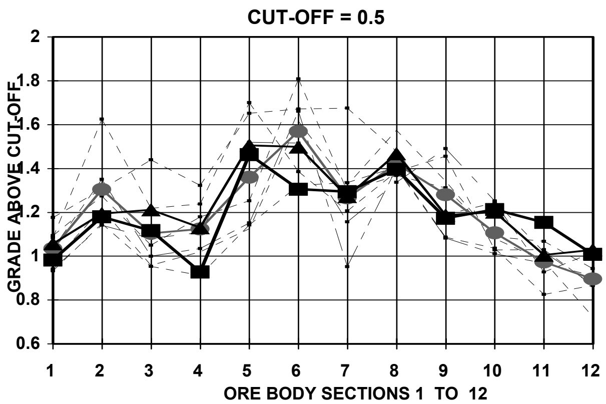
Direct conditioning (2): line with triangle markers

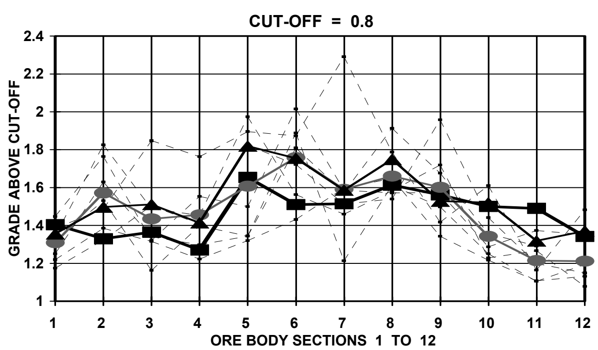
Simulations: Average of 7 curves (3c): gray line with filled in circles

Individual extreme curves (3a): thin lines with open circles.

The spread of individual simulations is much wider than for the ore body as a whole and indicates that it is dangerous to use individual simulations for planning or feasibility studies. In 7 of the 12 sections the direct conditioning estimates (2) are closer to the ‘actuals’ than the average simulation curves (3c), in 2 sections these estimates are very similar and in the remaining 3 sections the simulation estimates are somewhat better than direct conditioning. On balance, therefore, the latter is clearly the best choice.

Figures 3a and 3b show the section by section estimates for grades above two cut-offs of 0.5 and 0.8 respectively. The patterns for other cut-offs, i.e. 0.0, 1.0, 1.2 and 1.5 are very similar. These patterns again confirm that individual simulations can give misleading results; also, that the direct conditioning technique (2) is preferred to repeated simulations (3).





Figures 3a and 3b showing the 'actual' grades above cut-offs of 0.5 and 0.8 for all 12 sections of the ore body as well as the corresponding estimates as follows:
'Actual': Heavy lines with rectangles.
Direct conditioning (2): Lines with triangles
Simulations (3c): Average of 7 curves: Grey +circles
(3a): Individual simulations: Thin lines

3.3. Relative profit estimates

Figures 4a and 4b show the relative profit results for all sections for cut-offs of 0.5 and 0.8 where

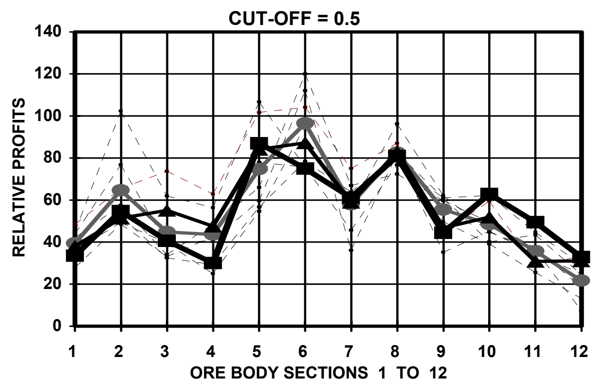
Relative profits = % tons(grade - cut-off).

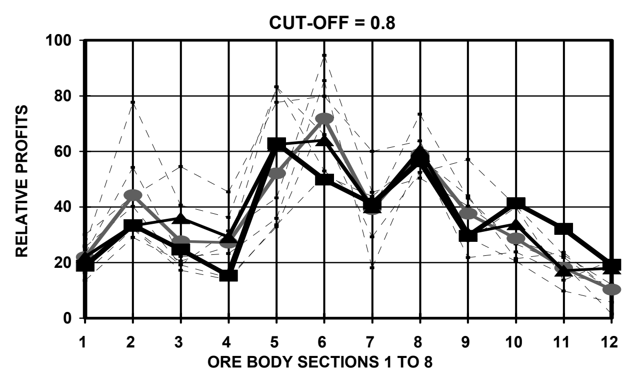
These provide relative measures of the patterns of the cash flows over the life of the project for the estimates vs. the 'actuals'. Again the spread of individual simulations (3a) is very wide (thin lines) and direct conditioning (2) shows better results than the average of 7 repeated simulations (3c).

3.4. Individual block selections

Direct conditioning provides some guide for individual blocks. Simulation averages 3a also meet this requirement but are conditionally biased, and technique 3c does not provide individual block estimates. Thus direct conditioning is more useful for preliminary mine planning.

As far as any actual selection of blocks has to be made to a limited extent at the stage before the final data for SMU's become available, the kriged blocks which form the base of the direct conditioning estimates, can also be used with an upward adjusted cut-off level to minimise the misclassification of blocks (Krige and Assibey-Bonsu 1999a).





Figures 4a and 4b showing the 'actual' relative profits for cut-offs of 0.5 and 0.8 for all 12 sections of the ore body as well as the corresponding estimates on the same basis as for Figures 3a and 3b.

3.5. Standard errors of estimates

A further useful criterion for comparing the efficiencies of different estimation techniques, is that of the expected and actual standard errors (SE) of the section estimates. Table 1 shows the actual standard errors % (SE %) of the estimates, as well as the expected levels for cut-offs, 0.0 to 1.0. For individual simulations (3a), the expected levels of uncertainty are also misleadingly low compared to the ‘actuals’. This is due to the fact that these simulation estimates are conditionally biased and the increase in the standard error is not reflected in the expected levels (for more details see also Krige and Assibey-Bonsu, 2000).

The direct conditioning estimates (2) provide significantly lower standard errors. From the kriging variances available from the relevant kriged estimates (1) for sections, the corresponding average section SE % for cut-off of 0.0, can be calculated and agrees well with the ‘actual’ SE % from the follow-up results (1). The confidence levels for direct conditioning estimates can, therefore, be approached via the kriging variances.

4. CONCLUSIONS

i) For the ore body as a whole with 216 available data for the estimates, both direct conditioning and the global average tonnage grade curve(3c) from repeated simulations are very close to the ‘actual’. However, only the former provides some useful information on individual blocks.

ii) For section estimates with much more limited data direct conditioning on the whole shows tonnage grade curves closer to those. of averages of repeated simulations as well as lower standard errors for the estimates.

iii) Averaging of repeated simulations for each block leads to a smoothed tonnage grade curve approaching that of kriging and is not recommended for the whole orebody or for sections.

iv) Individual simulations have a very wide spread for the whole ore body, much more so for individual sections, and thus also for detailed planning and feasibility studies; also they show more misclassifications of block grades (Krige 1999a). Also, the spread of individual simulations for sections significantly undervalues the actual uncertainty levels as shown by ‘actual’ follow up grades.

v) Although the uncertainty levels for grades above cut-off for sections remain reasonably constant as the cut-off is increased, those for relative profits rise sharply.

TABLE 1: Average standard errors % (SE %) of estimates for individual sections

			Actual SE % of estimates			
			Cut-off			
			0	**0.5**	**0.8**	**1**
Estimation Technique		**No.**	**Grades above cut-off**			
Direct Conditioning		**2**	**13**	**10**	**10**	**11**
Simulations:	**Section averages 1/7**	**3c**	**13**	**12**	**12**	**12**
	do. with indiv. block avgs.	**3b**	**13**	**14**	**16**	**16**
Individual Sims:	**Actuals: S.E.%s**	**3a**	**20**	**18**	**18**	**18**
	Estimated: S.E.%s	*3a*	*14*	*12*	*13*	*12*
Proper Kriged Estimates- Actual S.E.%		**1**	**13**	**18**	**20**	**19**
Estimated from kriging variances		*1*	*14*			
			Relative profits			
Direct Conditioning		**2**	**13**	**25**	**36**	**44**
Simulations:	**Section averages 1/7**	**3c**	**13**	**26**	**39**	**49**
	do. after indiv. blk. avgs.	*3c*	*13*	*25*	*37*	*47*
Individual Sims:	**Actuals: S.E.%s**	**3a**	**20**	**38**	**56**	**71**
	Observed: S.E.%s	*3a*	*14*	*26*	*37*	*45*
Proper Kriged Estimates S.E.%		**1**	**13**	**26**	**41**	**54**

Notes on Techniques for Table 1:

1. Ordinary Kriged estimates, adequate search routine -- smoothed, but forms basis for 2.

1. SE%s derived from kriging variances associated with the kriged estimates no. 1.

2. Direct Conditioning applied to individual SMU blocks – best technique with lowest SE%.

3a. Deviations of individual simulations from the corresponding actuals – wide spread.

3a. Deviations from the mean of each set of 7 simulations – SE% badly underestimated.

3b. Average of 7 simulations for each block pooled for each section – poor, conditionally biased.

3c. Average of tonnage-grade simulations 1/7 for each section, smoothed similar to technique 1.

5. REFERENCES

Assibey-Bonsu W. and Krige D.G., (1999). Use of direct and indirect distributions of selective mining units for estimation of recoverable resource/reserves for new mining projects. *APCOM'99 Symposium, Colorado School of Mines, Golden, Oct. 1999*.

Dowd P.A. and Pardo-Iquzquiza E.(2000). Incorporating model uncertainty in geostatistical simulation, *6th International Geostatistics Congress, South Africa, April 2000.*

Krige D.G.(1999a). Conditional biases and uncertainty of estimation in geostatistics. *Proc. APCOM'99 Symposium, Colorado School of Mines, Oct. 1999.*

Krige D.G. and Assibey-Bonsu W.(1999b). Practical Problems in the estimation of recoverable reserves when using simulation or block kriging techniques. *Conditional Simulation Conference, Perth Australia, October, 1999.*

Krige D.G. and Assibey-Bonsu W.(2000). Limitations in accepting repeated simulations to measure the uncertainty in recoverable resource estimates particularly for sections of an ore deposit. *6th International Geostatistics Congress, South Africa, April, 2000.*