THE USE OF LINEAR PROGRAMMING IN THE OPTIMAL DESIGN OF FLOTATION CIRCUITS INCORPORATING REGRIND MILLS

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by

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Abstract

Two linear models, the second being a subset of the first, are proposed for the simulation of flotation plants by use of *Linear Programming*. The first linear model produces the circuit structure, as well as the optimal flow rates of the valuable element between any number of flotation banks incorporating any number of recycle mills. An optimal grade for the valuable element in the concentrate is given by the second model. Operating conditions in the flotation banks and recycle mills are included as bounds in these models, permitting their possible application in *expert systems*. The simulated circuit structure, concentrate grade and recoveries closely resemble those of similar industrial flotation plants. The only data required by the simulation models are the feed rates of the species of an element, and their separation factors which are estimated from a multi-parameter flotation model.

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Introduction

Froth flotation is one of the most commonly used unit operations for the separation of valuable minerals from associated gangue impurities. The process is based on the property of some mineral particles to adhere stronger than others to air bubbles formed by an air flow in chemically treated water. Flotation plants usually consist of cells which are grouped into banks and inter-connected in a predefined manner(Lynch *et al.*, 1981). Although considerable research has been published on the chemistry and process engineering of flotation, it is still extremely difficult to design and control such plants.

Fine grinding is normally required to adequately liberate a fine-grained ore before flotation (Johnson, 1987). Over-grinding of the ore could cause lower recoveries of the valuable mineral, increased entrainment of hydrophilic gangue and consequently lower grades. Composite particles containing both valuable and gangue minerals cause problems in flotation. Many of the complexities of flotation circuits are due to attempts to find the most efficient way to treat composite particles so that recovery of the valuable minerals is maintained at a maximum while dilution of the concentrate by gangue is minimised (Lynch *et al.*, 1981). In some flotation plants intermediate tailings and/or concentrate must be reground prior to further processing.

When designing flotation plants, it is attempted to produce an optimal structure as well as optimal operating conditions. Few authors (King, 1976; Sutherland, 1977) have considered the effect of regrind mills in their simulation routines, which are normally based on numerical methods.

Mehrotra and Kapur(1974) derived an optimal flotation structure from a generalized circuit by direct search methods. The simulator of King(1976) consisted of mass balance equations defined for mineral classes in a fixed structure, which were solved iteratively. Niemi *et al.*(1982) included a recycle conditioning tank in the design of flotation networks. Williams and Meloy (1983), and Williams *et al.*(1986), used a technique

called "circuit analysis" to simulate mineral separation processes. Unit operations were assumed to be linear, so that the ensuing linear algebraic equations could be solved without the use of any numerical methods.

Green (1982, 1984) and Green *et al.* (1985) developed mass balance equations in terms of split factors and bounds on the flow rates in flotation banks. The resulting linear models could be solved by use of linear programming. An optimal structure was based on the valuable element, with the flow rates of the remaining mineral species being calculated by an iterative procedure. It is noteworthy that the flow rates of the valuable mineral after this iterative procedure were not the same as those produced by the linear programming procedure. Furthermore, the circuits produced by this method are difficult to apply in practice owing to their complexity.

In a previous paper, Reuter *et al.* (1988) optimised mineral separation circuits by solving two sequential linear models, but based the structure on total flow rates. However, regrind mills were not considered and the concentrate grades were not necessarily optimal.

It is the aim of the present paper to propose a general optimisation routine for flotation plants which incorporate regrind mills. Both the recovery and grade of valuable mineral in the concentrate will be optimised by use of two sequential linear programming models. Realistic solutions are yielded by this model, as will be illustrated by three sample problems.

Basis of Simulation Model

Significant progress has been made in recent years on the mathematical modelling of mills (Klimpel and Austin, 1984; Austin *et al.*, 1987a and b). Nevertheless, more work is needed before these models could be used reliably. It is possible, however, to predict bounds on the size distribution caused by a mill.

Various methods for the measurement of mineral liberation and the modelling thereof have been proposed (King, 1979; Malvik, 1982; Holland-Batt, 1983; Finch and Petruk, 1984). It is possible to quantitatively estimate the mineral liberation for different particle sizes from image analyses on polished ore sections (King, 1979). Hence, bounds could be determined for the change in mineralogy within different particle sizes caused by milling.

Flotation is influenced not only by particle size and liberation, but also interactively by surface properties and conditions in the pulp and froth phases. Consequently, it is extremely difficult to predict flotation behaviour solely from milling and liberation models.

In this paper flotation species s are defined by lumping milling, liberation and flotation response in one batch flotation test, where the recovery R of element k is given by a model such as (Lynch *et al.*, 1981; Ross, 1984)

$$R = \Phi[1 - \{(1 - \Omega)\cdot\exp(-k_f t) + \Omega\cdot\exp(-k_s t)\}] \qquad ---1$$

with $[1-\Phi]$ being the non-floating, $[\Phi\cdot\Omega]$ the slow-floating and $[\Phi(1 - \Omega)]$ the fast-floating species. Such flotation tests may be conducted on material from different positions in the flotation plant, and before and after a regrind mill. The effect of a mill will be simulated here by a matrix which transforms flotation species l into s. Suitable models (Ross, 1988) may be used to convert the values of k_f and k_s to $k^s{}_{kj}$ for plant conditions. The element k may also be a lumped entity such as the total sulphur content.

The separation factor $ym^s{}_{kj}$ of species s at bank j defined by

$$ym^s{}_{kj} = \frac{y^s{}_{kj}}{m^s{}_{kj}} \qquad \text{----2}$$

could be related to the mean retention time τ_j in a cell, the N_j cells in bank j and the rate constants $k^s{}_{kj}$ derived from Eq. 1 (Lynch et al., 1981):

$$ym^s{}_{kj} = = (1 + k^s{}_{kj}\ \tau_j)^{N_j} - 1 \qquad \text{----3}$$

Hence, bounds on $ym^s{}_{kj}$ could be estimated if bounds on $k^s{}_{kj}$ and τ_j are known, or derived from plant experience.

In contrast with most other simulators, this simulator includes the separation characteristics of a mineral species at a particular bank as bounds and not as a model included in the mass balance equations. This is realistic if it is considered that most flotation, milling and liberation models are empirical and suitable for a particular application only. A *knowledge based system* could be used to define and modify such bounds.

The mass balance equations which define the flow rates of flotation species s of elements k and the structural parameters in a flotation plant are non-linear (Reuter *et al.*, 1988), and must be linearised in order to use linear programming methods. This is achieved here by structuring them as two sets of linear equations, the second set being a subset of the first.

An optimal circuit structure and flow rates for the valuable element between flotation banks are produced by the first linear model, which is based on the maximization of the recovery of the valuable element. The second linear model produces the flow rates of the non-valuable elements subject to the flow rates of the valuable element and circuit structure produced by the first linear model. In the second model the grade of the final concentrate is maximized, which is equivalent to the minimization of the recovery of the non-valuable elements. These models are described separately in the next two sections.

Maximization of the Recovery of the Valuable Element h

Linear Programming Model I optimises the flow rates and the path of the valuable element h between the flotation banks and through the mills. This simulation model will be developed here by formulating all possible constraints imposed on the mass balance equations.

Mass balance constraints

The mass balance equations define the flow rates of the valuable element h between the flotation banks and through the mills. Since a particular element is conserved in a mill at steady state, the flow rate through it remains constant. However, the mill could affect the floatability of an element, including h, and hence its separation characteristics in the flotation banks. Therefore, the mill does affect the flow rates of concentrate and tailings of the valuable element h from the flotation banks.

Figure 1 defines the mass flow rates of the valuable element h between two flotation banks i and j. The three steady state mass balance equations for the valuable element h are:

$$\sum_{j,i\neq j} rm^{h}_{ij} + \sum_{j,i\neq j} ry^{h}_{ij} + u_{hi} = m_{hi} + y_{hi} \qquad \text{----4}$$

$$\sum_{i,i\neq j} rm^{h}_{ij} + b_{hj} = m_{hj} \qquad \text{----5}$$

$$\sum_{i,i\neq j} ry^{h}_{ij} + a_{hj} = y_{hj} \qquad \text{----6}$$

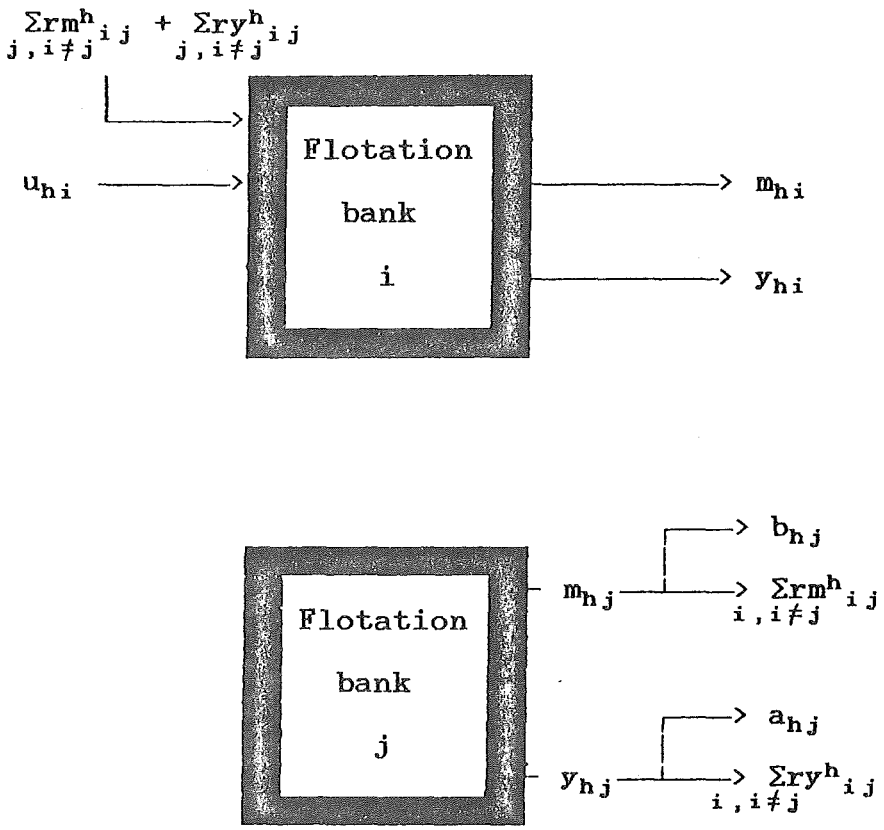


Figure 1 Steady state flow rates of the valuable element h from flotation bank j to i.

External constraints

A number of external constraints should be imposed on the three mass balance equations:

- The production capacity of a flotation bank limits the total flow rate of the valuable element h in its feed. These constraints also determine which banks produce concentrate and which produce tailings. If desired, fresh feed can be supplied to more than one flotation bank (Eq. 7). The bounds on the (multiple) feed rate of the valuable element h, final concentrate flow rate and the final tailings flow rate are defined by the following constraints:

$$u^L_{hj} \leq u_{hj} \leq u^U_{hj} \qquad \text{----7}$$

$$a^L_{hj} \leq a_{hj} \leq a^U_{hj} \qquad \text{----8}$$

$$b^L_{hj} \leq b_{hj} \leq b^U_{hj} \qquad \text{----9}$$

- All the flow rates of the valuable element h from a flotation bank have a lower bound of zero to ensure that no flow rate becomes negative.

- Certain recycle streams ij which are not physically meaningful must be excluded, e.g. recycling of rougher tailings to the cleaner banks (Eqs. 10 and 11):

$$rm^h_{ij} = 0 \qquad \text{----10}$$

$$ry^h_{ij} = 0 \qquad \text{----11}$$

Flotation constraints

The degree of separation of the valuable element h into concentrate and tailings is determined by the efficiency of the flotation process. In this paper the valuable element h is characterized by three flotation species s, i.e. a fast floating,

a slow floating and a non-floating species. Bounds on separation factors $ym^s{}_{hj}$ may be obtained from operational experience, or by fitting a flotation model such as Eq. 1 to batch or plant data:

$$(ym^s{}_{hj})^L \; m^s{}_{hj} \leq y^s{}_{hj} \leq (ym^s{}_{hj})^U \; m^s{}_{hj} \qquad \text{----12}$$

By defining the separation factor $ym^s{}_{hj}$ as bounds, the optimization model becomes well-suited for inclusion in a *knowledge based simulator* that permits the intelligent choice of bounds.

Milling affects the flotation rate constants, and hence the separation factor $ym^s{}_{hj}$, by changing amongst others the degree of mineral liberation or the surface chemistry of the different minerals. This is taken into account here by shifting the bounds on constraint 12. The transformation of species by milling will be considered in the second simulation model.

The separation of the valuable element h in the feed into a concentrate and a tailings stream is a function of the fraction of species s present in the valuable element h in the tailings ($F^s{}_{hj}$), and the separation characteristics of species s:

$$y_{hj} = (\sum_s F^s{}_{hj} \; ym^s{}_{hj}) \; m_{hj} \qquad \text{----13}$$

$$\text{where} \quad F^s{}_{hj} = (m^s{}_{hj})/(\sum_s m^s{}_{hj}) \qquad \text{----14}$$

An upper and a lower bound on Eq. 14, together with Eq. 12, produce Eq. 15:

$$(YM_{hj})^L \; m_{hj} \leq y_{hj} \leq (YM_{hj})^U \; m_{hj} \qquad \text{----15}$$

$$\text{where} \quad (YM_{hj})^L = \sum_s (ym^s{}_{hj})^L (F^s{}_{hj})^L$$

$$(YM_{hj})^U = \sum_s (ym^s{}_{hj})^U (F^s{}_{hj})^U$$

Objective function

The aim of the optimization is to maximize the recovery of valuable element h in the concentrate , subject to the constraints 2, 4 to 11, and 15. This can be stated as:

$$MAXh = \sum_{j} w_{hj} \; a_{hj} \qquad \text{----16}$$

where

MAXh = is the objective function which maximizes the production of valuable element h

w_{hj} = the price weight for the production of a_{hj} (e.g. monetary units/tonne)

j = bank(s) from which concentrate is vented.

Linear Programming Model I

Linear Programming Model I consists of Eqs. 2, 4 to 11, 15 and 16. This model produces the optimal circuit structure, and the optimal flow rates and recovery of the valuable element h.

Maximization of the Concentrate Grade

Linear Programming Model II simulates the flow rates of all species s of elements k in such a way that the concentrate grade is maximized. This model is subject to the optimal flow rates for valuable element h and the circuit structure produced by *Linear Programming Model I*, as well as the separation constraints for all species s and the transformation effects produced by the mill(s).

Figure 2 defines the steady state flow rates of all species s in elements k between banks i and j, and Figure 3 presents the transformation of species in the mill(s). Mass balance equations will be formulated with reference to these diagrams. (Variables denoted by an * are constants produced by *Model I*).

Mass balance equations for species s

The general structure of the mass balance equations for all flotation species s is very similar to that developed for the valuable element h in ***Model I***:

$$\sum_{j,i\neq j} t^*_{ij} m^s_{kj} + \sum_{j,i\neq j} c^*_{ij} y^s_{kj} + \sum_{j,i\neq j} M^{sk}_{ij} + \sum_{j,i\neq j} Y^{sk}_{ij} + u^s_{ki} = m^s_{ki} + y^s_{ki} \qquad \text{----17}$$

$$\sum_{i,i\neq j} t^*_{ij} m^s_{kj} + \sum_{i,i\neq j} e^*_{ij} m^s_{kj} + \beta^*_{Tj} m^s_{kj} = m^s_{kj} \qquad \text{----18}$$

$$\sum_{i,i\neq j} c^*_{ij} y^s_{kj} + \sum_{i,i\neq j} g^*_{ij} y^s_{kj} + \alpha^*_{cj} y^s_{kj} = y^s_{kj} \qquad \text{----19}$$

The optimal flow rates of h produced by ***Model I*** are included as:

$$M^*_{hj} = \sum_s m^s_{hj} \qquad \text{----20}$$

$$Y^*_{hj} = \sum_s y^s_{hj} \qquad \text{----21}$$

$$A^*_{hj} = \sum_s a^s_{hj} \qquad \text{----22}$$

$$B^*_{hj} = \sum_s b^s_{hj} \qquad \text{----23}$$

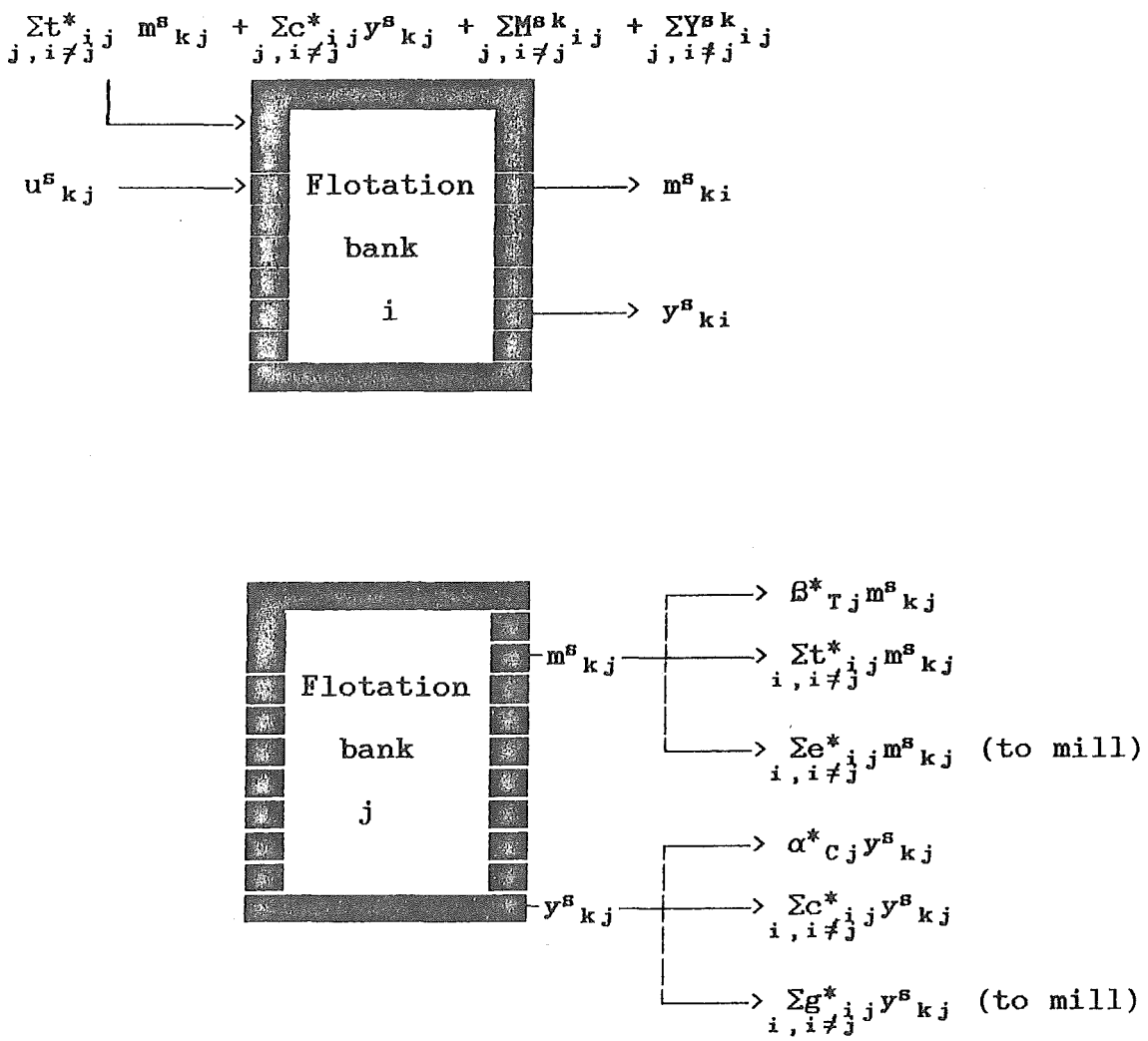


Figure 2 Steady state flow rates of flotation species s of an element k between banks i and j, and through mills in streams g^*_{ij} and e^*_{ij}.

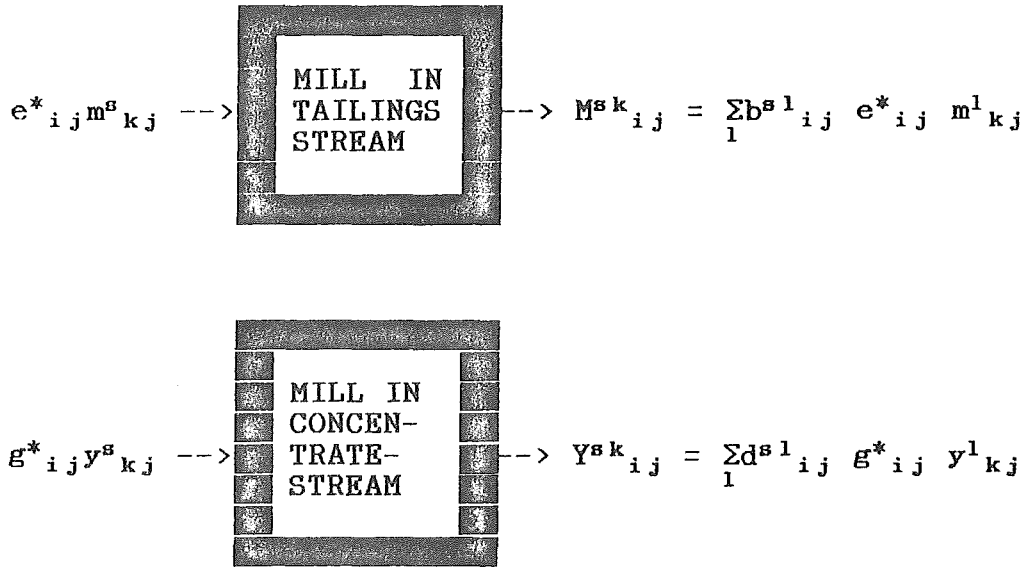


Figure 3 The transformation of flotation species l of element k into flotation species s in the mills situated in streams g^*_{ij} and e^*_{ij}.

External constraints

The bounds on the (multiple) feed rates of the flotation species s are given by:

$$(u^s{}_{kj})^L < u^s{}_{kj} < (u^s{}_{kj})^U \qquad \text{----24}$$

Grade constraints

The grade constraint for element k is defined as:

$$(G^T{}_{kj})^L \leq \frac{\sum_s m^s{}_{kj}}{\sum_k \sum_s m^s{}_{kj}} \leq (G^T{}_{kj})^U \qquad \text{----25}$$

Flotation bank constraints

The separation constraints for the non-valuable elements are similar to those for the valuable elements given in Eq. 12, so that the following general separation constraint may be defined:

$$(ym^s{}_{kj})^L \; m^s{}_{kj} \leq y^s{}_{kj} \leq (ym^s{}_{kj})^U \; m^s{}_{kj} \qquad \text{----26}$$

Mill constraints

In Figure 3 the transformation matrix for the mill is $b^{sl}{}_{kij}$ in stream $e^*{}_{ij}$ and that for the mill in stream $g^*{}_{ij}$ is $d^{sl}{}_{kij}$. Eqs. 27 and 28 define the transformation of flotation species l of element k in a mill into flotation species s:

$$M^{sk}{}_{ij} = \sum_l b^{sl}{}_{kij} \; e^*{}_{ij} \; m^l{}_{kj} \qquad \text{----27}$$

$$Y^{sk}{}_{ij} = \sum_l d^{sl}{}_{kij} \; g^*{}_{ij} \; y^l{}_{kj} \qquad \text{----28}$$

These constraints may be extended into bounded constraints if the tranformation matrices are not known exactly:

$$\sum_{l}(b^{sl}{}_{kij})^{L}\ e^{*}{}_{ij}\ m^{l}{}_{kj} > M^{sk}{}_{ij} > \sum_{l}(b^{sl}{}_{kij})^{U}\ e^{*}{}_{ij}\ m^{l}{}_{kj} \qquad \text{----29}$$

$$\sum_{l}(d^{sl}{}_{kij})^{L}\ g^{*}{}_{ij}\ y^{l}{}_{kj} > Y^{sk}{}_{ij} > \sum_{l}(d^{sl}{}_{kij})^{U}\ g^{*}{}_{ij}\ y^{l}{}_{kj} \qquad \text{----30}$$

Objective function

The objective function of *Linear Programming Model II* is the maximization of the grade of the valuable element h, subject to Eqs. 17 to 30. This is equivalent to the minimization of the recovery of the non-valuable gangue elements.

$$GRADE = \sum_{j}\sum_{s}(a^{s}{}_{hj} - a^{s}{}_{zj}) \qquad \text{----31}$$

where

GRADE	=	objective function which maximizes the grade.
$a^{s}{}_{hj}$	=	valuable species in concentrate.
$a^{s}{}_{zj}$	=	non-valuable species in concentrate.
j	=	bank(s) from which concentrate is vented.

Linear Programming Model II

Linear Programming Model II consists of Eqs. 2, and 17 to 31, and yields the optimal flow rates of the non-valuable elements and water, and the maximum grade of the final concentrate.

Application Algorithm

In order to design an optimal flotation circuit, *Linear Programming Models I and II* should be solved in succession as described below:

(a) Define bounds on the (multiple) feed rate(s) of the valuable element, and bounds on the flow rates of final concentrate and final tailings in Eqs. 7 to 9.

(b) Establish bounds imposed on the recycle streams in Eqs. 10 and 11, i.e. to exclude those recycle streams which are not practically feasible.

(c) Bounds on the separation factors for all species of the valuable element in Eq. 12 must be established from fundamental models, batch flotation data, plant experience or by an appropriate *knowledge based system.*

(d) Bounds on the composition of the valuable element in terms of species s, i.e. the distribution of floatabilities of the valuable element, should be selected for the construction of Eq. 15.

(e) Any linear programming package may now be used to solve *Linear Programming Model I.*

(f) The structural parameters and the optimal flow rates for the valuable element produced by *Linear Programming Model I* may now be used to construct Eqs. 17 to 23.

(g) The feed flow rates of the flotation species s to a bank j should be bounded in Eq. 24.

(h) Production requirements must be established, which could be related to the constraints on the grade of element k in the tailings streams, as defined in Eq. 25.

(i) Constraints on the separation factors for all species s of all elements k in Eq. 26 could be derived from batch flotation data, plant experience or an appropriate *knowledge based system*.

(j) If a mill is included in the flotation circuit, Eqs. 27 to 30 should be formulated. For this purpose a matrix for the transformation of species l to species s must be constructed from milling, liberation and flotation data measured in batch experiments, pilot plants or industrial operations. An appropriate *knowledge based system* may also be used to update these bounds on the transformation matrix.

(k) A linear programming package could be used to solve *Linear Programming Model II*, which yields the flow rates of flotation species s that maximize the grade of the valuable element in the final concentrate.

(l) The solution of *Linear Programming Model II* produces a set of exact separation factors for each species s and a transformation matrix for the mills. Theoretical flotation, liberation and milling models may subsequently be used to specify the operating conditions in the flotation cells and mills which will produce an optimal recovery and grade of the valuable element.

In the sample problems described below a linear programming package FMPS on a UNIVAC 1100 computer was used to solve the models. Less than 5 seconds of central processor time was required in each case.

Problem 1: A 2 Bank Circuit for the Flotation of Pyrite

A simple 2 bank rougher-cleaner circuit for the flotation of pyrite(FeS_2) from a quartzitic ore will be used here to demonstrate the applicability of the simulation model to the design of flotation plants which incorporate recycle mills. The performances of circuits with and without mills will be compared. While the bounds on separation factors are hypothetical, they are similar to those for real ores (Liddell and Dunne, 1984; Hanekom, 1987). Element 1 (Pyrite, FeS_2), element 2 (Gangue) and element 3 (Water) are separated into concentrate and tailings streams.

The bounds on the separation factors for the three species of the three elements are given in Tables 1 and 2 for a circuit that includes a recycle mill between the rougher and cleaner bank. In this example it is assumed that N_j=5 and τ_j=5 (min) for all banks and mineral species respectively. Although these values will differ between banks in practice, identical values are used here for simplicity. All flow rates are given in tonnes/h.

Table 1 Bounds on separation factors (ym^s_{k-j}) for rougher (j=1)

	Element		
Species	k=1	k=2	k=3
s=1	1060-448401	4.83-96	0.06-0.28
s=2	7-156	0.4-2.7	0
s=3	0	0	0

Table 2 Bounds on separation factors (ym^s_{k-j}) for cleaner (j=2)

	Element		
Species	k=1	k=2	k=3
s=1	1330-640972	3-45	0.06-0.28
s=2	1-524	1.1-11.2	0
s=3	0	0	0

Rougher bank

By using the separation factors for the species of the valuable element, bounds may be established for constraint 15. The upper and lower bounds on YM_{h1} ($h=FeS_2$) for the rougher bank are produced as follows:

$$(YM_{h1})^L = \sum_s (ym^s{}_{h1})^L (F^s{}_{h1})^L$$
$$= (1060 \times 0) + (7 \times 0) + (0 \times 0)$$
$$= 0$$

$$(YM_{h1})^U = \sum_s (ym^s{}_{h1})^U (F^s{}_{h1})^U$$
$$= (448401 \times 0.000053) + (156 \times 0.0015) + (0 \times 1)$$
$$= 24$$

The values for $(F^s{}_{hj})^L$ and $(F^s{}_{hj})^U$ could be obtained from experimental batch flotation data, plant data or set by the designer as a design variable.

Hence,

$$0 \le (y_{FeS_2})_1 \le 24(m_{FeS_2})_1$$

Cleaner bank

The same procedure is followed to obtain bounds for the separation of pyrite at the cleaner bank, i.e. $(YM_{h2})^L$ and $(YM_{h2})^U$

$$(YM_{h2})^L = \sum_s (ym^s{}_{h2})^L (F^s{}_{h2})^L$$
$$= (1330 \times 0) + (16.6 \times 0) + (0 \times 0)$$
$$= 0$$

$$(YM_{h2})^U = \sum_s (ym^s{}_{h2})^U (F^s{}_{h2})^U$$
$$= (640972 \times 0.00001) + (524 \times 0.0049) + (0 \times 1)$$
$$= 9$$

Hence,

$$0 \le (y_{FeS_2})_2 \le 9(m_{FeS_2})_2$$

In this example milling increases the mineral liberation and the flotation rates in the cleaner bank. If a mill is not included in the circuit, the flotation rates and hence the separation factor will be lower. The bounds $(YM_{h2})^L$ and $(YM_{h2})^U$ for the cleaner bank in a circuit that does not include a mill are:

$$(YM_{h2})^L = \sum_s (ym^s{}_{h2})^L (F^s{}_{h2})^L$$
$$= (1191 \times 0) + (10.6 \times 0) + (0 \times 0)$$
$$= 0$$

$$(YM_{h2})^U = \sum_s (ym^s{}_{h2})^U (F^s{}_{h2})^U$$
$$= (573824 \times 0.00001) + (243 \times 0.001) + (0 \times 0)$$
$$= 6$$

Hence,

$$0 \leq (y_{FeS_2})_2 \leq 6(m_{FeS_2})_2$$

From the above data the following simulation models can be constructed for a two bank flotation plant:

Linear Programming Model I

Objective function

$$MAXh = 200(a_{FeS_2})_2$$

Mass balance constraints

$$(rm^{FeS_2})_{12} + (ry^{FeS_2})_{12} + (u_{FeS_2})_1 = (m_{FeS_2})_1 + (y_{FeS_2})_1$$

$$(rm^{FeS_2})_{21} + (ry^{FeS_2})_{21} + (u_{FeS_2})_2 = (m_{FeS_2})_2 + (y_{FeS_2})_2$$

$$(rm^{FeS_2})_{12} + (b_{FeS_2})_2 = (m_{FeS_2})_2$$

$$(rm^{FeS_2})_{21} + (b_{FeS_2})_1 = (m_{FeS_2})_1$$

$$(ry^{FeS_2})_{12} + (a_{FeS_2})_2 = (y_{FeS_2})_2$$

$$(ry^{FeS_2})_{21} + (a_{FeS_2})_1 = (y_{FeS_2})_1$$

Feed constraints (fixed)

$$(u_{FeS_2})_1 = 8.4$$

$$(u_{FeS_2})_2 = 0$$

Concentrate constraints

$$(a_{FeS_2})_1 = 0$$

$$0 \leq (a_{FeS_2})_2 \leq 20$$

Tailings constraints

$$0 \leq (b_{FeS_2})_1 \leq 20$$

$$(b_{FeS_2})_2 = 0$$

Recycle constraint

$$(rm^{FeS_2})_{21} = 0$$

Separation constraints for valuable element (mill included)

$$0 \leq (y_{FeS_2})_1 \leq 24(m_{FeS_2})_1$$

$$0 \leq (y_{FeS_2})_2 \leq 9(m_{FeS_2})_2$$

Separation constraints for valuable element (no mill included)

$$0 \leq (y_{FeS_2})_1 \leq 24(m_{FeS_2})_1$$

$$0 \leq (y_{FeS_2})_2 \leq 6(m_{FeS_2})_2$$

Linear Programming Model II

The structural parameters and the flow rates of the valuable element h (FeS_2), which are produced by *Linear Programming Model I*, are shown in Figure 4 and are also included in this model. The structural parameters are:

$t^*_{12}=1;$ $t^*_{21}=0;$

$c^*_{12}=0;$ $c^*_{21}=1;$

$a^*_{C1}=0;$ $a^*_{C2}=1;$

$b^*_{T1}=1;$ $b^*_{T2}=0$

The elements k in this model are FeS_2 (valuable element), Gangue (G) and water (W).

Objective function

$$GRADE = \sum_s (a^s_{FeS_2})_2 - \sum_s a^s_{G\ 2} - \sum_s a^s_{W\ 2}$$

Mass balance constraints for species s

$$t^*_{21}\ m^s_{k1} + Y^{sk}_{21} + u^s_{k2} = m^s_{k2} + y^s_{k2}$$

$$t^*_{12}\ m^s_{k2} + c^*_{12} y^s_{k2} + u^s_{k1} = m^s_{k1} + y^s_{k1}$$

$$t^*_{21} m^s_{k1} + b^*_{T1} m^s_{k1} = m^s_{k1}$$

$$t^*_{12} m^s_{k2} + b^*_{T2} m^s_{k2} = m^s_{k2}$$

$$Y^{sk}_{21} + a^*_{C1} y^s_{k1} = y^s_{k1}$$

$$c^*_{12} y^s_{k2} + a^*_{C2} y^s_{k2} = y^s_{k2}$$

Flow rates of valuable element (FeS_2) produced by Model I

$$8.92 = \sum_s (y^s_{FeS_2})_1 \qquad 8.028 = \sum_s (y^s_{FeS_2})_2$$

$$0.372 = \sum_s (m^s_{FeS_2})_1 \qquad 0.892 = \sum_s (m^s_{FeS_2})_2$$

$$0.372 = \sum_s (b^s_{FeS_2})_1 \qquad 8.028 = \sum_s (a^s_{FeS_2})_2$$

Feed rates for species s

FeS_2

$$(u^1_{FeS_2})_1 = 5.46$$

$$(u^2_{FeS_2})_1 = 2.772$$

$$(u^3_{FeS_2})_1 = 0.168$$

Gangue (G)

$u^1_{G\,1} = 6.124$

$u^2_{G\,1} = 2.624$

$u^3_{G\,1} = 282.852$

Water (W)

$u^1_{W\,1} = 370$

Separation factors of species s

These values are given in Tables 1 and 2.

Grade constraint

$$\frac{\sum_s m^s_{W2}}{\sum_k \sum_s m^s_{k2}} > 0.3$$

This constraint ensures that the fraction of water in the final concentrate is larger than 0.3.

Transformation matrix for gangue material in the mill

The transformation of species within the valuable element has been taken into consideration in the first simulation model. Hence, the transformation matrix of only the gangue element is considered in this model:

$$\begin{array}{c} \text{Species} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0.5 & 0.5 & 1 \end{bmatrix} \end{array}$$

This matrix implies that 0.5 of the fast floating species of the gangue has been transformed to the non-floating species. The same holds for the slow floating fraction.

Final Results

The operating conditions could now be estimated by applying Eq. 3 to the optimal separation factors for the different species of the three elements. Tables 3 and 4 summarize the optimal rate constants, from which operating conditions could be estimated by use of phenomenological models (Ross, 1988). The flow rates of the elements k produced by Model II are given in Figure 4, which shows a practically feasible two bank circuit configuration.

A sulphur grade of 32.2% and a recovery of sulphur of 95.4% in the concentrate are produced by the circuit without a regrind mill. These values could be improved to 41.3% and 95.6% respectively by the inclusion of a mill in the concentrate stream from the rougher to the cleaner. These values are typical of those attained in practice.

Table 3 Flotation rates ($k^s_{k,j}$ min^{-1}) in rougher (j=1)

	Element		
Species	k=1	k=2	k=3
s=1	2.5	0.085	0.002
s=2	0.156	0.014	0
s=3	0	0	0

Table 4 Flotation rates ($k^s_{k,j}$ min^{-1}) in cleaner (j=2)

	Element		
Species	k=1	k=2	k=3
s=1	2.7	0.064	0.01
s=2	0.062	0.032	0
s=3	0	0	0

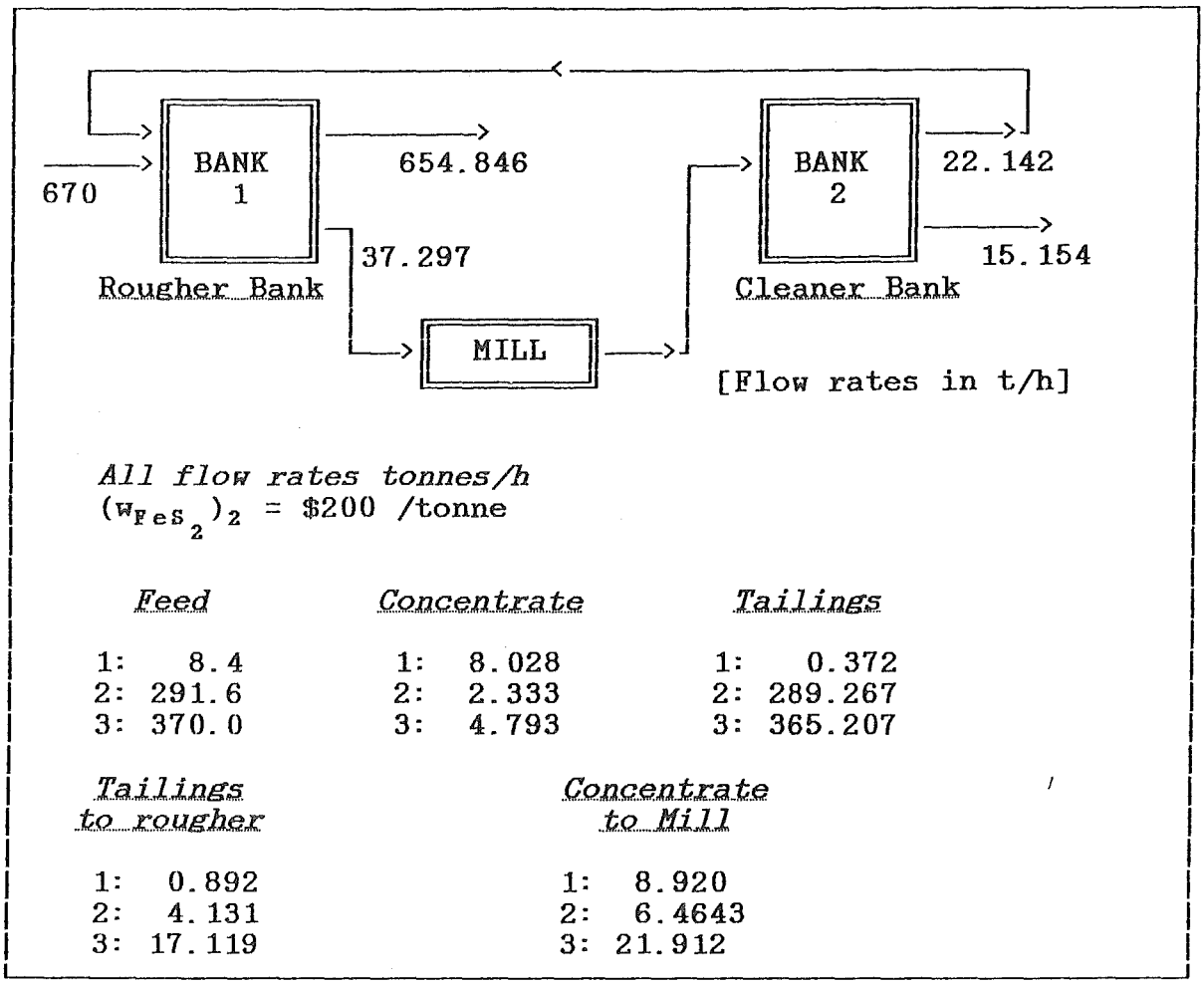


Figure 4. Circuit structure and flow rates of elements for Problem 1.

Problem 2: A 5 Bank Circuit for the Flotation of Cassiterite

The valuable element concentrated in this circuit is cassiterite (SnO_2), with gangue (G) and water (W) constituting the non-valuable elements. Five flotation banks, i.e. a rougher, scavenger, cleaner, recleaner I and recleaner II, will be used here to demonstrate the applicability of the optimization procedure. Again, the effect of a regrind mill will be investigated. Hypothetical bounds on the separation factors are similar to data given by Sutherland (1981) and Frew and Trahar (1982). Whereas the use of Models I and II was explained in more detail in Problem 1, only the more essential information will be summarized here.

The bounds on the separation factors are summarized in Tables 5 and 6. It is assumed here that N_j=8 and τ_j=4 (min) for both the rougher and scavenger banks, whereas N_j=8 and τ_j=2 (min) apply to the cleaner banks. A mill is assumed to be included in the concentrate stream from the rougher bank to the recleanerI bank.

Table 5 Bounds on separation factors (ym^s_{kj}) in both the rougher (j=1) and scavenger (j=2)

	Element		
Species	k=1	k=2	k=3
s=1	38.3-5764800	12.8-200475	0.097-0.85
s=2	1.0-6560	1.7-6560	0
s=3	0	0	0

Table 6 Bounds on separation factors (ym^s_{kj}) in cleaners (j=3 to 5)

	Element		
Species	k=1	k=2	k=3
s=1	4.8-6560	0.33-7.2	0.024-0.19
s=2	0.1-8.2	0.23-2.76	0
s=3	0	0	0

From these data the following constraints could be formulated for the valuable element cassiterite (SnO_2) for each flotation bank:

$$0 \leq (y_{SnO_2})_1 \leq 11.00(m_{SnO_2})_1$$

$$0 \leq (y_{SnO_2})_2 \leq 7.00(m_{SnO_2})_2$$

$$0 \leq (y_{SnO_2})_3 \leq 0.70(m_{SnO_2})_3$$

$$0 \leq (y_{SnO_2})_4 \leq 0.60(m_{SnO_2})_4$$

$$0 \leq (y_{SnO_2})_5 \leq 0.45(m_{SnO_2})_5$$

Without a mill being included in the concentrate stream from the rougher to recleaner I, the separation factor for recleaner I (bank 4) is:

$$0 \leq (y_{SnO_2})_4 \leq 0.35(m_{SnO_2})_4$$

The transformation matrix for the gangue material is:

$$\begin{array}{c} \text{Species} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0.5 & 0.5 & 1 \end{bmatrix} \end{array}$$

The circuit structure and the flow rates of the elements for the case where a mill is included, are depicted in Figure 5. Operating conditions were estimated by the application of Eq. 3, and are summarized in Tables 7 to 11.

The simulation produces an unusual circuit configuration for a flotation circuit comprising of five flotation banks, although there are certain distinct similarities to existing industrial flotation plants. For example, the tailings are recycled to previous banks. The circuit which does not include a mill,

produces an Sn grade of 51.1% and an Sn recovery of 44.0%. When a recycle mill is included in the circuit as depicted in Figure 5, the Sn grade in the final concentrate increases to 67.3% and the Sn recovery to 75.4%.

Table 7 Flotation rates ($k^s_{k,j}$ min^{-1}) in rougher (j=1)

Element			
Species	k=1	k=2	k=3
s=1	0.146	0.9	0.003
s=2	0.091	0.5	0
s=3	0	0	0

Table 8 Flotation rates ($k^s_{k,j}$ min^{-1}) in scavenger (j=2)

Element			
Species	k=1	k=2	k=3
s=1	0.146	0.097	0.017
s=2	0.095	0.033	0
s=3	0	0	0

Table 9 Flotation rates ($k^s_{k,j}$ min^{-1}) in cleaner (j=3)

Element			
Species	k=1	k=2	k=3
s=1	1.0	0.018	0.001
s=2	0.033	0.013	0
s=3	0	0	0

Table 10 Flotation rates (k^s_{kj} min^{-1}) in recleaner I (j=4)

Element			
Species	k=1	k=2	k=3
s=1	0.123	0.018	0.011
s=2	0.028	0.013	0
s=3	0	0	0

Table 11 Flotation rates (k^s_{kj} min^{-1}) in recleaner II (j=5)

Element			
Species	k=1	k=2	k=3
s=1	1.0	0.018	0.011
s=2	0.16	0.013	0
s=3	0	0	0

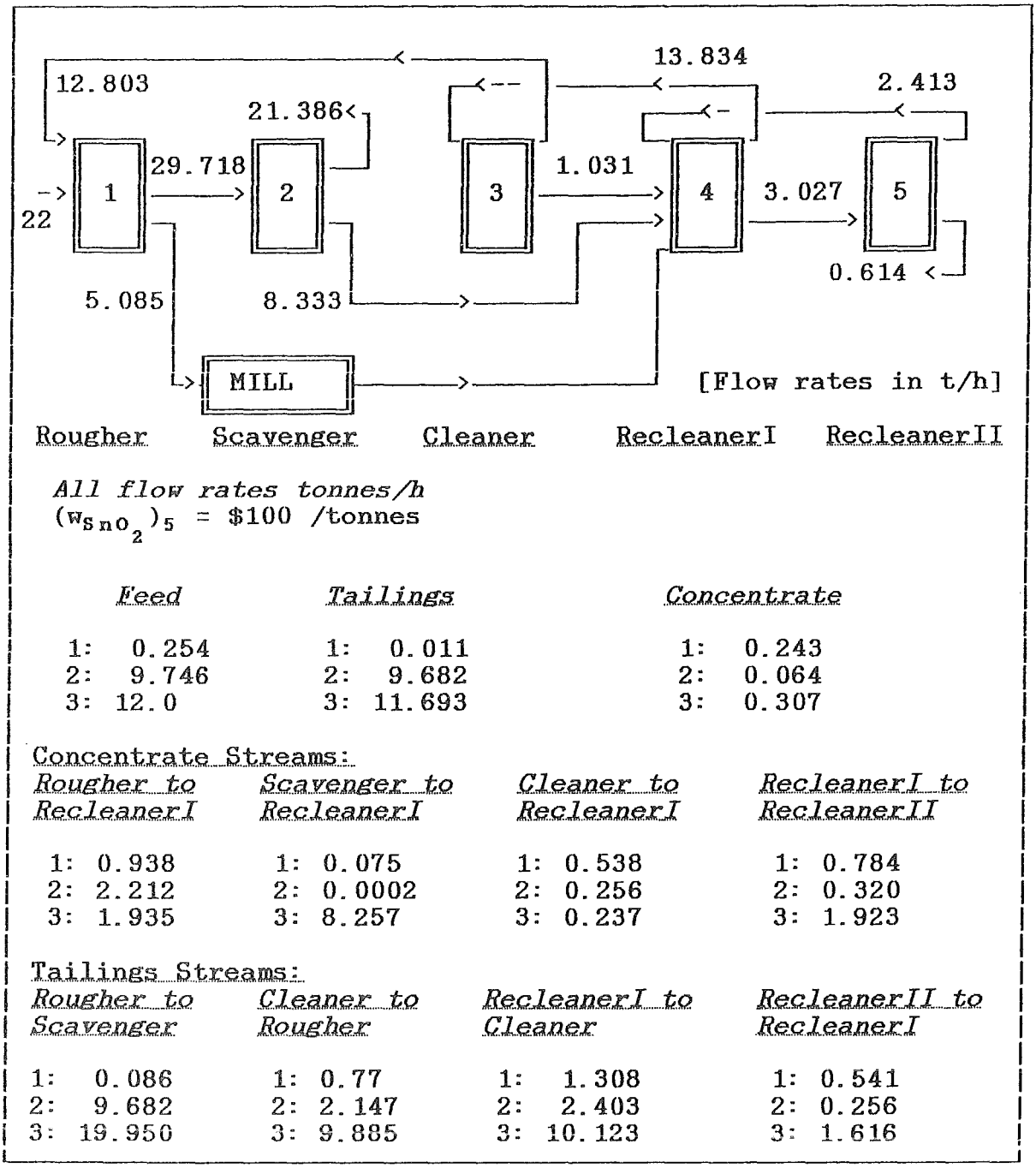


Figure 5. Circuit structure and flow rates of elements for Problem 2.

Problem 3: A 4 Bank Circuit for the Flotation of Galena

As in Problem 2, the models are not presented in detail, and only the more important data are given here. A fixed circuit structure is selected and the influence of two regrind mills on optimal flotation performance will be determined. The final circuit incorporating the mills is shown in Figure 6. Mills are placed in the tailings recycle between the cleaner and rougher, and in the concentrate stream between the rougher and the recleaner.

Bounds on the separation factors for the flotation of galena (PbS) from gangue (G) material were chosen so as to approximate the flotation data given by Forssberg *et al.* (1982) and Sutherland (1977). These bounds are given in Tables 12 to 14. In this problem it was assumed that N_j=8 and τ_j=2 (min) for the rougher and cleaners, and N_j=10 and τ_j=2 (min) for the scavenger.

Table 12 Bounds on separation factors (ym^s_{kj}) in rougher (j=1)

	Element		
Species	k=1	k=2	k=3
s=1	2.84-63	0.46-6.53	0.024-0.087
s=2	0.02-3.0	0.024-0.087	0
s=3	0	0	0

Table 13 Bounds on separation factors (ym^s_{kj}) in scavenger (j=2)

	Element		
Species	k=1	k=2	k=3
s=1	3.3-109	0.48-2.7	0.14-0.48
s=2	0.27-1.59	0.14-0.48	0
s=3	0	0	0

Table 14 Bounds on separation factors ($ym^s_{k,j}$) in cleaner (j=3) and recleaner (j=4)

Element			
Species	k=1	k=2	k=3
s=1	4.1-112.4	1.07-15.8	0.04-0.48
s=2	0.082-2.0	0.082-1.19	0
s=3	0	0	0

By the application of these data the following constraints could be formulated for the valuable element galena (PbS):

$$0 \leq (y_{PbS})_1 \leq 2.1(m_{PbS})_1$$

$$0 \leq (y_{PbS})_2 \leq 0.7(m_{PbS})_2$$

$$0 \leq (y_{PbS})_3 \leq 0.8(m_{PbS})_3$$

$$0 \leq (y_{PbS})_4 \leq 2.1(m_{PbS})_4$$

Without the two mills in the circuit, as shown in Figure 6, the separation factors for the valuable element galena in the rougher and recleaner banks change to:

$$0 \leq (y_{PbS})_1 \leq 0.6(m_{PbS})_1$$

$$0 \leq (y_{PbS})_4 \leq 0.8(m_{PbS})_4$$

The transformation matrix for the gangue material in both mills is taken to be:

$$\begin{array}{c} \text{Species} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0.5 & 0.5 & 1 \end{bmatrix} \end{array}$$

The flow rates of the three elements for this circuit are depicted in Figure 6. The operating conditions were estimated by the application of Eq. 3, with the results being summarized in Tables 15 to 18.

The simulation produces a typical sulphide flotation circuit consisting of four banks. It is clear that the two mills have a marked effect on the performance of this circuit. The recovery of the valuable element Pb in this circuit is 95.6%, while the grade produced is 68.8 %Pb. Without these mills included in the circuit, the grade and recovery drop to 30.2 %Pb and 87.7% respectively.

Table 15 Flotation rates (k^s_{kj} min^{-1}) in rougher (j=1)

	Element		
Species	k=1	k=2	k=3
s=1	0.5	0.033	0.002
s=2	0.091	0.018	0
s=3	0	0	0

Table 16 Flotation rates (k^s_{kj} min^{-1}) in scavenger (j=2)

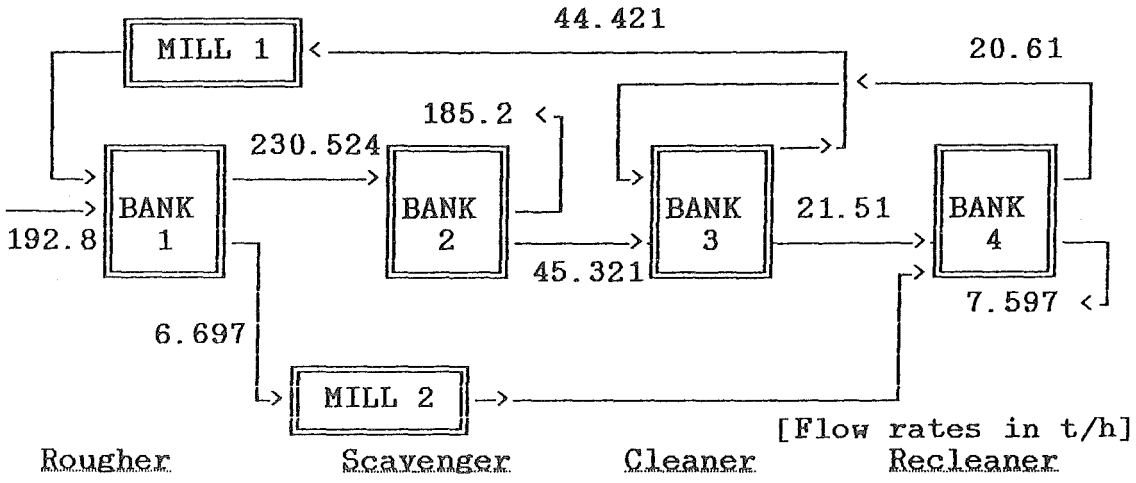
	Element		
Species	k=1	k=2	k=3
s=1	0.1	0.025	0.025
s=2	0.041	0.008	0
s=3	0	0	0

Table 17 Flotation rates (k^s_{kj} min^{-1}) in cleaner (j=3)

	Element		
Species	k=1	k=2	k=3
s=1	0.156	0.064	0.034
s=2	0.051	0.007	0
s=3	0	0	0

Table 18 Flotation rates ($k^s_{k,j}$ min^{-1}) in recleaner (j=4)

Element			
Species	k=1	k=2	k=3
s=1	0.6	0.064	0.02
s=2	0.079	0.007	0
s=3	0	0	0



All flow rates tonnes/h
$(w_{PbS})_4$ = \$100 /tonne

Feed	*Tailings*	*Concentrate*
1: 3.1	1: 0.762	1: 2.338
2: 93.3	2: 92.904	2: 0.396
3: 96.4	3: 91.537	3: 4.863

Concentrate Streams:

Rougher to Recleaner	*Scavenger to Cleaner*	*Cleaner to Recleaner*
1: 2.719	1: 0.534	1: 0.732
2: 0.727	2: 0.849	2: 0.462
3: 3.251	3: 43.938	3: 20.316

Tailings Streams:

Rougher to Scavenger	*Cleaner to Rougher*	*Recleaner to Cleaner*
1: 1.296	1: 0.915	1: 1.113
2: 93.753	2: 1.180	2: 0.792
3: 135.474	3: 42.326	3: 18.705

Figure 6. Circuit structure and flow rates of elements for Problem 3.

Conclusions and Significance

The solutions to the sample problems serve to demonstrate a number of properties of the simulation model, as discussed below:

(a) The circuit structures produced by *Linear Programming Model I* are similar to those encountered in industry. Furthermore, *Linear Programming Model II* produces grades and recoveries which compare favourably with those of similar flotation plants in industry. Hence, the two-step approach for the optimization of a non-linear simulation problem produces meaningful results. It was shown that the effect of regrind mills on the recovery and grade could be simulated realistically. Although not demonstrated by the sample problems, the simulation model could also consider a multiple feed strategy as described by Williams *et al.* (1986).

(b) The simulation model produces solutions within a few seconds, which makes it more time efficient than iterative numerical procedures. This feature makes the model particularly attractive for inclusion in a control algorithm.

(c) Most flotation models yield only approximate predictions of recoveries and grades. Since the separation characteristics are included here as constraints rather than a flotation model, the inaccuracies in any flotation model may be introduced as an upper and a lower bound. This implies that only approximate and not precise separation data are required by the simulation model.

The construction of bounds for the separation characteristics is meaningful, since the operating conditions in a flotation plant may vary greatly and hence, may not be defined accurately by a theoretical or an empirical separation model. Therefore, the bounding of the separation characteristics as is done in this simulation model also embraces the dynamics encountered in a flotation circuit.

(d) The data for the estimation of bounds may be obtained from a data base, which may be updated by the user or by the simulation model in a control loop. The inclusion of operating conditions as bounds makes this model particularly attractive for use in a *knowledge based system*, which could manipulate the available data base to estimate bounds.

(e) The only data required by the simulation models are the separation factors for the fast and slow floating species of a particular element k, and the feed composition of element k in terms of the fast floating, slow floating and non-floating species.

Once a flotation plant has been designed and commissioned, the model presented here may be used to simulate and, if used in a control algorithm, control the operation of the plant. If, for example, there are drastic changes in the ore body and subsequently in the flotation conditions, this model may be used to suggest an alternative circuit and new operating conditions.

Notation

a_{hj} = concentrate recovery of valuable element h from flotation bank j

$a^s{}_{hj}$ = concentrate recovery of species s of valuable element h from flotation bank j

$a^s{}_{zj}$ = concentrate recovery of species s of non-valuable elements z from flotation bank j

$a^L{}_{hj}$ = lower bound on a_{hj}

$a^U{}_{hj}$ = upper bound on a_{hj}

$A^*{}_{hj}$ = concentrate flow of valuable element h from flotation bank j. Optimal value produced by *Linear Programming Model I.*

b_{hj} = tailings recovery of valuable element h from flotation bank j

$b^s{}_{hj}$ = tailings recovery of species s of valuable element h from flotation bank j

$b^L{}_{hj}$ = lower bound on b_{hj}

$b^U{}_{hj}$ = upper bound on b_{hj}

$B^*{}_{hj}$ = tailings flow of valuable element h from flotation bank j. Optimal value produced by *Linear Programming Model I*

$b^{sl}{}_{kij}$ = breakage of species s of element k into species l by use of a mill in tailings recycle ij

$(b^{sl}{}_{kij})^L$ = lower bound on $b^{sl}{}_{kij}$

$(b^{sl}{}_{kij})^U$ = upper bound on $b^{sl}{}_{kij}$

$c^*{}_{ij}$ = fraction of y_{hj} that is recycled from flotation bank j to i. Optimal value produced by *Linear Programming Model I*

$d^{sl}{}_{kij}$ = breakage of species s of element k into species l by use of a mill in the concentrate recycle ij

$(d^{sl}{}_{kij})^L$ = lower bound on $d^{sl}{}_{kij}$

$(d^{sl}{}_{kij})^U$ = upper bound on $d^{sl}{}_{kij}$

$e^*{}_{ij}$ = tailings stream that includes a mill. Optimal value produced by *Linear Programming Model I*

$F^s{}_{hj}$ = $m^s{}_{hj} / \sum_s m^s{}_{hj}$: Fraction of species s in element k in the tailings

$G^T{}_{kj}$ = grade of element k in tailings stream j

$(G^T{}_{kj})^L$ = lower bound on tailings grade

$(G^T{}_{kj})^U$ = upper bound on tailings grade

$g^*{}_{ij}$ = concentrate stream that includes a mill. Optimal value produced by *Linear Programming Model I.*

GRADE = objective function that maximizes the grade.

k_f = rate constant of the fast-floating species

k_s = rate constant of the slow-floating species

$k^s{}_{kj}$ = flotation rate of the species s of element k in flotation bank j

m_{hj} = tailings flow of valuable element h from bank j

$m^s{}_{kj}$ = tailings flow of species s in element k from bank j

$M^*{}_{hj}$ = tailings flow of the valuable element h from flotation bank j. Optimal value produced by *Linear Programming Model I*

$M^{sk}{}_{ij}$ = $\sum_l b^{sl}{}_{kij}\ e^*{}_{ij}\ m^l{}_{kj}$ Flow rate of species s of element k from a mill situated in the tailings recycle stream ij

MAXh = total earnings due to the optimal recovery of valuable element h

N_j = number of cells in flotation bank j

R = recovery in batch flotation

$rm^h{}_{ij}$ = recycle of valuable element h in the tailings from flotation bank j to i

$ry^h{}_{ij}$ = recycle of valuable element h in the concentrate from flotation bank j to i

t = flotation time

$t^*{}_{ij}$ = fraction of m_{hj} that is recycled from flotation bank j to i. Optimal value produced by *Linear Programming Model I*

$Y^*{}_{hj}$ = concentrate flow of valuable element h from flotation bank j. Optimal value produced by *Linear Programming Model I*

y_{hj} = concentrate flow of valuable element h from flotation bank j

$y^s{}_{kj}$ = concentrate flow of species s in element k from flotation bank j

$ym^s{}_{kj}$ = separation factor: $y^s{}_{kj}$ / $m^s{}_{kj}$

$(ym^s{}_{kj})^L$ = lower bound on $ym^s{}_{kj}$

$(ym^s{}_{kj})^U$ = upper bound on $ym^s{}_{kj}$

YM_{hj} = $\sum_s F^s{}_{hj}\ ym^s{}_{hj}$

$(YM_{hj})^L$ = lower bound on $(\sum_s F^s{}_h ym^s{}_{hj})$

$(YM_{hj})^U$ = upper bound on $(\sum_s F^s{}_{hj}\ ym^s{}_{hj})$

$Y^{sk}{}_{ij}$ = $\sum_l d^{sl}{}_{kij}\ g^*{}_{ij}\ y^l{}_{kj}$ Flow rate of species k from a mill situated in concentrate recycle stream ij

u_{hi} = feed of valuable element h to flotation bank i

$u^s{}_{kj}$ = feed of species s of element k to flotation bank j

$u^L{}_{hj}$ = lower bound on u_{hj}

$u^U{}_{hj}$ = upper bound on u_{hj}

$(u^s{}_{kj})^L$ = lower bound on $u^s{}_{kj}$

$(u^s{}_{kj})^U$ = upper bound on $u^s{}_{kj}$

w_{hj} = price weight of valuable element h in the objective function MAXh

Greek letters

$\alpha^*{}_{Cj}$ = fraction of y_{hj} that is concentrate product. Optimal value produced by *Linear Programming Model I.*

$\beta^*{}_{Tj}$ = fraction of m_{hj} that is tailings product. Optimal value produced by *Linear Programming Model I.*

Φ = ultimate recovery at infinite time

Ω = fraction of the ultimately floatable species that is slow floating

τ_j = retention time of all species s in bank j

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