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Pilot and laboratory test procedures for the design of semi-autogenous grinding circuits treating platinum ores

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Mintek has been actively involved for many years in research aiming to improve piloting, plant survey techniques, laboratory test procedures and reliable models for the design of autogenous and semi-autogenous tumbling mills.

Semi-autogenous piloting facilities and procedures used at Mintek are reviewed in this paper. A novel laboratory test developed by Mintek to design semi-autogenous and autogenous milling circuits based on fundamental principles rather than relying on empirical relationships is also presented. Results obtained from the novel test are compared to piloting results using a UG2 ore.

The steady-state and dynamic behaviour of semi-autogenous grinding (SAG) mills are arguably best described using a population model that caters for the breakage kinetics and the material transport of rock and pulp inside the mill. Breakage kinetics are usually expressed in terms of a specific breakage rate function and a separate primary breakage distribution function. Material transport usually invokes the concept of a specific discharge rate function. Unfortunately, the breakage rate and breakage distribution functions for SAG mills are very difficult to estimate uniquely from plant data alone. Simplifying assumptions must be made to overcome this problem, which leads to many unresolved dilemmas with regard to whether or not the derived functions are accurate, precise, and reflect reality. It is shown in this paper that the dilemmas associated with standard population balance models (models that utilize separate breakage rate and breakage distribution functions) can be overcome by expressing breakage behaviour inside a SAG mill in terms of a single function. This function is an energy-based cumulative breakage rate function defined as the fractional rate at which particles and rocks coarser than a given size break to below that size per unit specific energy input. A model for SAG milling is formulated, which invokes the concept of a cumulative breakage rate function. Comparisons are made between the mathematical structure of this "simple model" and standard population balance models.

It is concluded that the novel laboratory test developed by Mintek which has been filed for a patent is accurate and reliable. The simplified approach to model SAG milling may turn out to be the quickest way to fully describe and quantify the performance of these very complex mills.

Introduction

The attractive feature of semi-autogenous grinding mills is that they can be fed directly with coarse ROM (run of mine) ore with 80 per cent passing sizes often in excess of 150 mm. This obviates the need for complex multi-stage crushing and screening plants prior to milling, although a primary crusher and grizzly is often needed to constrain the feed to a top size that can easily be handled and transported to the mill. Large rocks in the ROM ore serve as grinding media that can self-break and also break ore in intermediate and fine sizes. For some ores, effective grinding can occur without the need for steel grinding balls (sometimes referred to as AG or autogenous grinding). However, balls are often added to prevent the build-up of so called critical sizes that do not easily self-break and cannot be efficiently broken down by the largest rocks in the mill. The bulk volume of the balls added is typically a few per cent of the internal mill volume (hence the term SAG or semi-autogenous grinding), but can be as high as thirty per cent. SAG mills with very high ball loads are not uncommon in some South African precious metal operations and are

usually referred to as ROM ball mills.

Although semi-autogenous grinding appears to be a very simple way of achieving large size reduction ratios in a single unit, the design, control, and optimization of grinding circuits incorporating SAG mills are not trivial tasks. One approach is to conduct tests at pilot scale using ROM ore with the same size distribution and composition as the ore likely to be encountered at production scale. Based on piloting tests conducted at Mintek over three decades, it has been found that data generated from a SAG mill with an internal diameter of 1.7 m can provide a good indication of the performance of production mills (Loveday, 1978). Other workers hold similar views (Mosher and Bigg, 2002).

For pilot mills with internal diameters close to 1.7 m, the net specific energy remains essentially invariant to scale-up for a given circuit configuration and grind size. However, there are limitations with regard to the size of the largest rocks that can be handled (250 mm for the Mintek mill). Moreover, some corrections must be made to allow for differences in the grate geometry and aspect ratio of the test pilot mill and the production mill. Obviously, even at pilot

scale it is a costly exercise to explore all grate/liner geometries, operating conditions, and circuit configurations to establish optimal design parameters. Accordingly, there is considerable incentive to develop mathematical models that can be used to accurately extrapolate and interpolate pilot and production data using computer simulation techniques.

Consideration is first given to the use of 'standard' population balance models (models that incorporate separate specific breakage rate and breakage distribution functions) currently available for quantifying the behaviour of SAG mills. It is shown that whilst these models have many attractive features, the functions used to model breakage kinetics and material transport can be very difficult to quantify and give rise to many unresolved dilemmas. It is argued that these dilemmas can be resolved by using functions to model breakage kinetics and material transport that can be measured directly from pilot SAG mills without having to make any *a priori* assumptions about their mathematical structure. Consideration is then given to the formulation of a simple model for SAG milling based on these directly measurable functions.

Although piloting tests are the most reliable method for the design of AG/SAG milling circuits, the cost associated with the test is very high and sometimes the amount of ore required for the test, which is around 90 t, is not available, depending on the stage of the project. Over the past years, several bench-scale laboratory test procedures have been used to design AG/SAG mills. A comprehensive review of bench-scale testwork available was conducted at the 2006 Autogenous and Semiautogenous grinding conference in Canada (McKen, A. and Williams, S., 2006). Recently (Hinde, A.L. and Kalala, J. T., 2007), Mintek has developed a bench-scale test procedures based on fundamental principles rather than relying on empirical relationships. Results obtained from the novel test are compared to piloting results for a developed UG2 platinum ore.

Mintek SAG piloting facility

Mintek comminution piloting facility includes a SAG mill, crushers, a Koppern High Pressure Grinding Roll (HPGR) unit having a diameter of 1 m, a Polysius unit having a diameter of 0.250 m, secondary ball mills, a stirred media detractor (SMD), a Deswik mill, a Derrick screen and hydrocyclones.

The SAG mill has an inside diameter of 1.7 m and an effective length of 0.5 m. The mill operates at 75 per cent

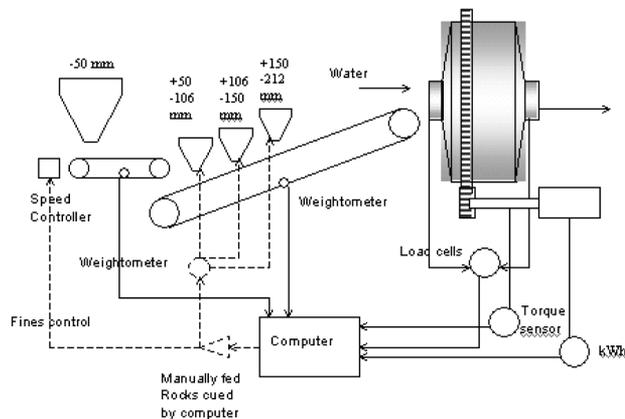


Figure 1. Mintek SAG piloting facility

of critical speed. It is equipped with six peripheral ports (each 75 mm by 175 mm). Each peripheral port can accommodate a range of grate discharge and pebble port geometry.

To control the feed rate to the mill, Mintek piloting system is equipped with the sophisticated StarCS software. The StarCS software provides an automated control of the feed particle size distribution to the mill. The system used at Mintek involves the manual addition of individual rocks in the size classes coarser than 106 mm on the main conveyor to the mill at timed intervals. The inclined conveyor is connected to a weightometer from which the total solids feed rate is obtained. Known masses of rocks in the size classes of typically $-106 + 50$ mm, $-150 + 106$ mm and $-212 + 150$ mm are loaded into buckets. The contents of the buckets are placed at timed intervals on a slow-moving conveyor which discharges onto the main conveyor. At Mintek, this system is computerized to cue the operators (by switching on lights) when to load the rocks and which size class must be added (Figure 1).

The mill power draw is recorded by measuring the torque using a strain gauge on the pinion shaft and by monitoring the motor energy consumption (using a kilowatt-hour meter). The net power is given by the difference between the gross power and the power to overcome drive losses (no-load power).

The mill power and grinding efficiency is affected by the slurry hold-up. The slurry hold-up can be changed during a campaign by increasing or decreasing the total grate discharge opening area. Figure 2 shows an example of a discharge grate used in the pilot mill and Figure 3 shows modifications which can be performed to increase or decrease the total open area.

The hold-up mass of the charge is continuously monitored via four load cells mounted on the mill and drive system. The load cells determine the variation of the total mass of the load to an accuracy of about 2 kg.

Standard SAG models

Basic concepts

The state of the ore charge inside a SAG mill can be characterized by the mass and particle size distribution of each rock-type. Usually, the particle size distribution is expressed as the discrete mass fractions retained on sieves

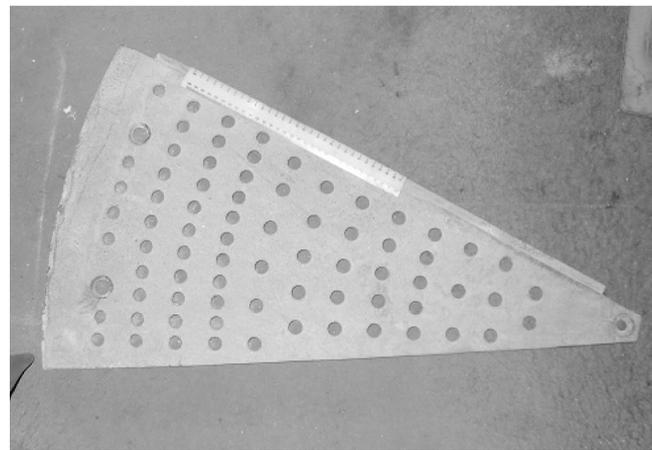


Figure 2. Example of grate discharge used in the pilot mill

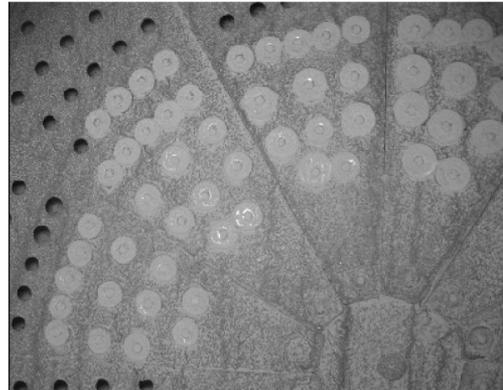
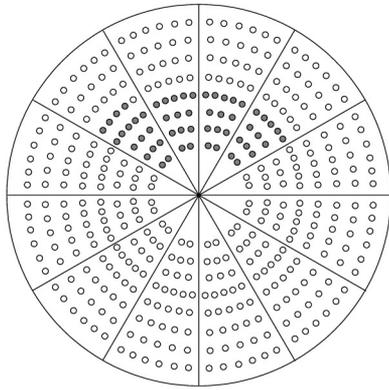


Figure 3. Modifications of grate discharge total opening area

with mesh sizes conforming to a root-two geometric progression. Breakage phenomena causing changes in the state of the ore charge are usually expressed in terms of a specific breakage rate function and a primary breakage distribution function.

The specific breakage rate function, $S_{i,k}$ [h]⁻¹ is defined as the fractional rate at which particles of a given rock type (indexed by the subscript k) break out of a given size class (indexed by the subscript i) per unit time. It is usually desirable to transform the specific breakage function rate function from the time domain to an energy-based domain. This is motivated by the fact that grind size is more closely related to specific energy input, rather than residence time. According to the Herbst Fuerstenau transformation (Herbst and Fuerstenau, 1980), the energy-based breakage rate function, $S_{i,k}^E$ [kWh/t]⁻¹ can be defined by:

$$S_{i,k} = \frac{P}{M} S_{i,k}^E \quad [1]$$

where

P is the mill net power, [kW]

D is the internal diameter of the mill, [m]

L is the effective mill length, [m]

M is the ore holdup mass, [t]

The utility of using an energy-based specific breakage rate function is that it is insensitive to scale-up, whereas a time-based specific breakage rate function increases with mill diameter. This should be self-evident from Equation [1] since:

$$\frac{P}{M} \propto \frac{D^{2.5} L}{D^2 L} \propto D^{0.5}$$

In other words, a time-based breakage function should be proportional to the square root of mill diameter.

The primary breakage distribution function is defined as the size distribution of the progeny fragments breaking out of a given size class before they have had a chance to be re-broken. In other words, the breakage distribution function, $b_{i,j,k}$ for rock type k is defined as the mass fraction of material breaking out of size class j that appears in size class i . The breakage distribution function can also be expressed in cumulative form. The cumulative breakage distribution function, $B_{i,j,k}$ is the mass fraction of progeny particles originating in size class j that are smaller than x [mm]. Note that x_i is the upper size limit of particles in size class i and the subscript i runs from a value of 1 for the

largest particles to a value n for particles in the smallest size class with sizes ranging from zero to x_n . If the mill residence time distribution can be modelled as a simple fully mixed reactor, the following population balance equation can be used to keep track of the holdup mass of particles of given rock type k and size class i inside the mill:

accumulation = flow in–flow out–consumption + generation

$$\frac{dw_{i,k}}{dt} = f_{i,k} - p_{i,k} - S_{i,k}^E \frac{P}{M} w_{i,k} + \sum_{\substack{j=1 \\ i>j}}^{i-1} b_{i,j,k} S_{j,k}^E \frac{P}{M} w_{j,k} \quad [2]$$

where

$w_{i,k}$ is the holdup mass of size class i and rock type k particles, [t]

$f_{i,k}$ is the mill inlet flow rate of size class i and rock type k particles, [t/h]

$p_{i,k}$ is the discharge flow rate of size class i and rock type k particles, [t/h]

t is time, [h]

This formulation of the standard SAG model follows the one available in the MinOcad dynamic simulation package (Herbst and Pate, 2001). In this version of the model, the discharge flowrate is expressed as the product of the holdups and the so called specific discharge rate function, $g_{i,k}$ [h]⁻¹:

$$p_{i,k} = g_{i,k} w_{i,k} \quad [3]$$

After substituting this value for $p_{i,k}$, Equation [2] (for all values of i and k) takes the standard form of a state variable description of a lumped dynamical system (Athans, Dertouzos, Spann, and Mason, 1974). The dependent variables or states are given by the holdups, $w_{i,k}$ inside the mill. The independent variable is the time, t . Measurable outputs, which include discharge flows, the net power, and the ore mass holdup are all functions of the state variables. The resulting set of differential equations can be solved numerically to simulate both the dynamic and steady state behaviour of the mill.

In principle, it should be possible to measure the specific breakage rate and breakage distribution functions directly from pilot-scale SAG batch tests using radioactive tracer techniques to ‘tag-and-track’ parent and progeny fragments

in the mill. However, the technological challenges of doing this are immense and have yet to be addressed. To get around this problem, it is usually assumed that the breakage rate and breakage distribution functions can be expressed in terms of simple analytical equations with a minimal number of parameters. In principle, these parameters can be back-calculated from operating data by statistical methods involving the minimization of weighted sums of squares (Austin, Klimpel, and Luckie, 1984).

Unfortunately, there appears to be very little consensus amongst researchers with regard to the best choice for the mathematical structure of the specific breakage rate and breakage distribution functions. This is understandable if one recognizes that in a SAG mill (unlike a conventional ball mill) there is no clear distinction between the particles doing the grinding (pebbles, rocks, and balls), and the particles being ground. It has been argued (Sepúlveda, 2001) that the effectiveness of the balls as grinding media should be proportional to the power drawn from them. Similarly, one would expect the effectiveness of the pebbles and rocks as grinding media to be proportional to the power they draw. It follows that the formulation of a rigorous population balance model with a clearly-defined specific breakage rate and breakage distribution functions becomes very complex. Suffice to say, some schools of thought prefer not to factor out a specific power component when formulating a population balance model (Austin, Menacho, and Percy, 1987).

There are also differences of opinion with regard to the definitions of breakage and distribution functions. Most researchers treat breakage as a continuous process involving a statistical average of a very large number of breakage events with particles breaking into and out of a given size class. In this case the element, $b_{i,i,k}$ of the breakage distribution function has a value of zero by definition. On the other hand, some researchers view breakage as a sequence of discrete events and make allowance for particles breaking in a given size class to produce progeny fragments that appear in the same size class. In this case the element, $b_{i,i,k}$ can then take on positive non-zero values (Lynch, 1977).

Consideration will now be given to some of the options for 'fixing' the breakage distribution and breakage rate functions in the standard model.

Fixing the primary breakage distribution function

It is generally conceded that the main modes of breakage in SAG mills are abrasion, impact fracture, and attrition.

It has been argued that abrasion grinding is the dominant mode of breakage in AG/SAG mills (Loveday, 2004), which can be defined as the process whereby ore pieces are rubbed against each other and gradually get worn down until they are small enough to be caught between the larger pieces.

Impact fracture has two causes, either the blow of the protruding parts of the mill liner against the ore pieces in the charge or the impact caused by flying balls or pebbles when hitting the charge or the shell liner. The first cause operates at both cascading and cataracting speeds, the second only at cataracting speeds.

Attrition grinding is the usual term for comminution carried out by the action of the grinding media as balls or pebbles on the interstitial fine material.

It should be evident that the breakage distribution function should be structured to cater for all the modes of breakage described above.

One approach to the dilemma of getting an estimate of the breakage distribution function is to postulate a plausible breakage distribution function from laboratory tests using rocks or particles in narrow size ranges (Napier Munn, Morrell, Morrison, and Kojovic, 1996). These tests include simple batch tumbling tests (Austin, Menacho, and Percy, 1987) to provide an estimate of the abrasion component of the breakage distribution function and dropweight tests (Pau and Maré, 1988) on single rocks to get the impact fracture component. It turns out that the size distribution of the products of breakage for both abrasion and impact can be correlated in terms of either a truncated Rosin Rammler distribution or a weighted sum of two truncated Rosin Rammler distributions (Austin, 1993). The truncated Rosin Rammler distribution is given by the equation:

$$P_i^T = \frac{1 - \exp\left(-a\left(\frac{x_i}{d_{\max}}\right)^m\right)}{1 - \exp(-a)}; x_i \leq d_{\max} \quad [4]$$

$$P_i^T = 1; x_i > d_{\max}$$

P_i^T is the mass fraction less than size x_i and d_{\max} is the truncation or maximum particle size. The parameters a and m determine the shape of the size distribution plot. The slope of the size distribution on a log-log scale at fine sizes is determined by m . The parameter a can take on either positive or negative values. Negative values for a can be used to approximate impact size distributions at low energy inputs and positive values can be used to simulate high energy impacts. In the limit as $a \rightarrow 0$, Equation [4] becomes a Schuhmann distribution, which plots as a straight line on a log-log scale.

Figure 4 shows a 'fabricated' estimate of an energy-dependent cumulative breakage distribution function for a 1.7 m diameter pilot SAG mill based on laboratory dropweight and tumbling tests. Many simplifying assumptions were made in the formulation of this function. These included: allowance for size-dependent effects on the distribution of specific energy inputs inside the mill, normalization of the impact breakage distribution with respect to top size for a given specific energy input, and the weighting of the impact and abrasion components to provide the overall breakage distribution function. Given, also, that the tests used to generate this breakage distribution function did not cater for attrition modes of breakage, there is considerable doubt whether the estimated function reflects reality and one would have got the same result from an idealized 'tag and track' test using a continuously fed mill and a complete spectrum of ROM sizes. However, if the breakage distribution function is fixed, only the parameters of the specific breakage rate function need to be back-calculated from subsequent tests conducted on the pilot or production mill.

Fixing the specific breakage rate function

In common with the breakage distribution function, it is usual to assume that the specific breakage rates can be expressed in terms of an analytical equation. Such an equation (Austin, Menacho, and Percy, 1987) is given by:

$$S_{i,k}^E = \frac{\kappa_{1,k} \hat{x}_i^{\alpha_{1,k}}}{1 + \left(\frac{\hat{x}_i}{\mu_k}\right)^{\lambda_k}} + \kappa_{2,k} \hat{x}_i^{\alpha_{2,k}} \quad [5]$$

where $\kappa_{1,k}$, $\kappa_{2,k}$, $\alpha_{1,k}$, $\alpha_{2,k}$, μ_k , and λ_k are parameters that must be estimated by back-calculation. Particle size is

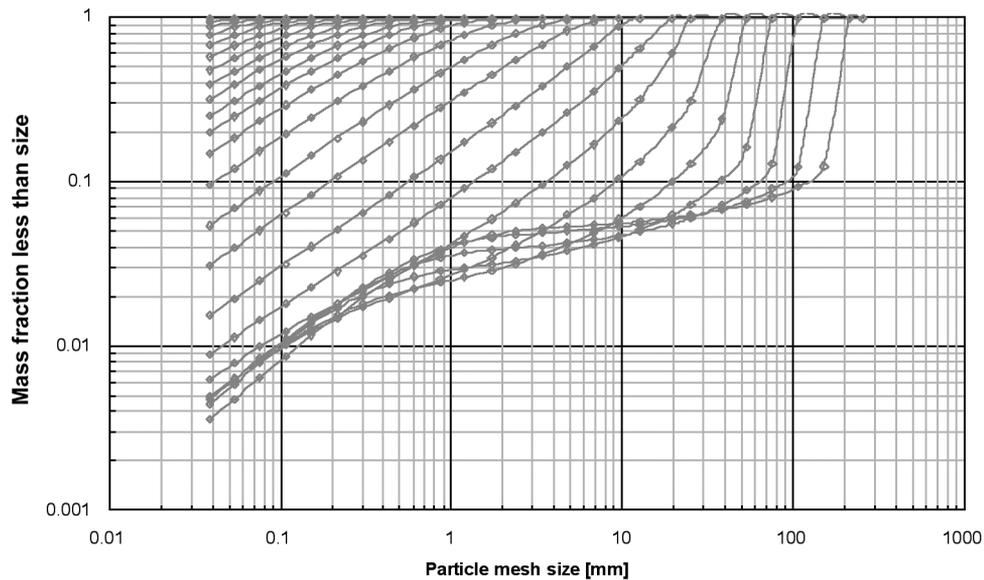


Figure 4. Cumulative breakage distribution function for standard SAG model

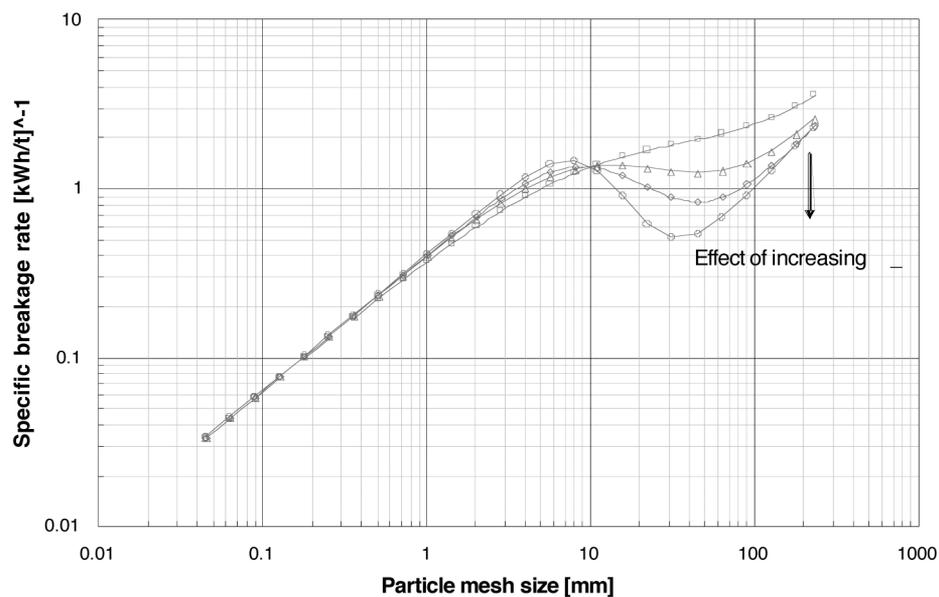


Figure 5. Specific breakage rate function for a pilot SAG mill

usually expressed in terms of the geometric mean size for a given size class, \hat{x}_i [mm]. The first term on the right-hand-side attempts to account for both attrition breakage at fine sizes and abrasion breakage at intermediate sizes. The second term accounts for the effects of self-fracture, which becomes dominant at coarse rock sizes (typically above 100 mm). Figure 5 shows what this function looks like when plotted on a log-log scale and the effect of varying one of the parameters (λ_k). Needless to say, the estimation of all these parameters by back-calculation with a reasonable level of precision and statistical confidence is a challenging task for a single set of conditions, let alone for a wide range of operating variables and mill liner/grate geometries.

The specific discharge rate function

For SAG mills with high aspect ratios ($D/L > 2$), it is generally acknowledged that the mill charge is fully mixed. In this case, the specific discharge rate function can be defined as the ratio of the discharge flowrate of a given

rock type and size interval to the hold-up of the same rock type and size interval inside the mill. This function can be uniquely estimated from measurements of size distributions and mass flows of the feed and discharge streams together with the size distribution and hold-up mass of the mill charge. The size distribution and composition of the mill charge can easily be measured at pilot scale, but this can become a formidable task at production scale. Even at pilot scale, the estimation of the specific discharge rates can be sensitive to small measurement errors. This is especially true for the case where the size distribution of the mill contents is bimodal and close to being gap graded, which is often the case. In the 'gap' region, the specific discharge function is expressed as the ratio of two numbers close to zero, which can give rise to a large variance for the estimated specific discharge rates.

Because of the practical difficulties of measuring specific discharge rates in production mills, it is common to assume the specific discharge rate can be expressed as an analytical

function. For a grate with pebble ports, it is believed that the specific discharge function takes the form (Napier-Munn, Morrell, Morrison, and Kojovic, 1996) given by:

$$\begin{aligned}
 g_{i,k} &= d_{\max} \quad ; \hat{x}_i \leq x_m \\
 g_{i,k} &= d_{\max} (1 + (1 - f_p) \frac{(\ln(\hat{x}_i) - \ln(x_m))}{(\ln(x_m) - \ln(x_g))}) \quad ; x_m < \hat{x}_i \leq x_g \\
 g_{i,k} &= d_{\max} (f_p \frac{(\ln(x_p) - \ln(\hat{x}_i))}{(\ln(x_p) - \ln(x_g))}) \quad ; x_g < \hat{x}_i \leq x_p \\
 g_{i,k} &= 0 \quad ; \hat{x}_i > x_p
 \end{aligned} \quad [6]$$

In this equation (shown graphically in Figure 6), it is assumed that the specific discharge rates for particles up to size x_m (about 1 mm) are constant with a value d_{\max} [h^{-1}]. Above this size the specific discharge rate decreases monotonically with increasing particle with a change in slope at the mesh size, of the grate apertures, x_g [mm], and has a value of zero at the mesh size of the pebble ports, x_p [mm]. The parameter f_p relates to the relative open areas of the grate apertures and pebble ports. Back-calculated estimates of the specific discharge function based on this equation have been reported in the literature (Mainza, Powell, and Morrison, R D, 2006).

A simplified model for SAG milling

Concept of a cumulative breakage rate function

The concept of using a single function to describe breakage kinetics for conventional grinding mills (ball, rod, and pebble mills) is not new (Hinde, and King, 1978), (Finch and Ramirez-Castro, 1981), and (Austin, Klimpel, and Luckie, 1984). The application of the concept of a cumulative breakage rate function to the steady state modelling of a SAG mill was described in 1993 (Austin, Sutherland, and Gottlieb, 1992). The dynamic modelling of an open circuit mill SAG mill using this concept was described in 1993 (Amestica, Gonzalez, Barria, Magne, Menacho, and Castro, 1993) and again in 1996 (Amestica, Gonzalez, Menacho, and Barria, 1996).

The specific cumulative breakage rate function, $K_{i,k}$ [h^{-1}] is defined as the fractional rate at which particles of a given rock type, k above a given size, x_i in the mill break to below that size per unit time. In common with the approach adopted for the standard model (Equation [2]), a transformation relationship is used to define an energy based cumulative breakage rate function,

$$K_{i,k}^E \text{ [kWh/t]}^{-1} \quad (K_{i,k}^E = (M/f) K_{i,k})$$

Consider the fractional rates at which particles above sizes x_i and x_{i+1} break to below these sizes, as indicated in Figure 7. Let the total or cumulative masses above these sizes inside the mill be W_i and W_{i+1} , respectively. If the mill is operated in a simple batch mode the rate of change of mass for particles in size class i and rock type k is given by:

$$\frac{dw_{i,k}}{dt} = W_{i,k} \frac{P}{M} K_{i,k}^E - W_{i+1,k} \frac{P}{M} K_{i+1,k}^E \quad [7]$$

It follows that a single function can be used to uniquely quantify breakage kinetics in a SAG mill and to establish a mass balance for material within a given size class.

Formulation of the cumulative rates population balance model

Consider a population balance for a continuously-fed mill for particles in size class 1 (whose upper mesh size is equal to or greater than the largest particle in the feed) and rock type k . If there are no particles coarser than size x_1 the specific breakage rate of particles coarser than this size, $K_{1,k}^E$ is undefined. It follows that the population balance equation with respect to time for size class 1 is given by:

$$\text{accumulation} = \text{in} - \text{out} - \text{consumption}$$

$$\frac{d(W_{2,k})}{dt} = F_{2,k} - P_{2,k} - W_{2,k} \frac{P}{M} K_{2,k}^E \quad [8]$$

where $F_{2,k}$ and $P_{2,k}$ are the cumulative mass flows of material coarser than size x_2 in the feed and product streams, respectively. For the first size class, Equation [8] is identical to:

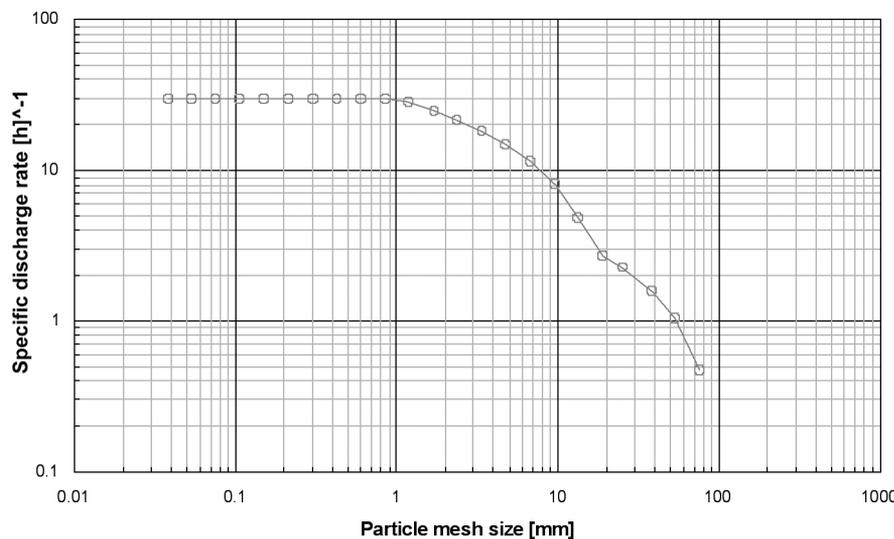


Figure 6. Specific discharge rate function for SAG mill with pebble ports

accumulation = in – out – consumption

$$\frac{d(w_{1,k})}{dt} = f_{1,k} - g_{1,k} w_{1,k} - w_{1,k} \frac{P}{M} K_{2,k}^E \quad [9]$$

The first term on the right-hand side of Equation [9] gives the flowrate of size class 1 and rock type k into the mill. The second term expresses the discharge flow rate in terms of the value of the specific discharge function and mass holdup for size class 1 and rock type k . The third term in Equation [9] assumes that breakage rates are proportional to the product of the specific power input and the mass of size class 1 material inside the mill. When expressed in this form, the energy-based specific breakage rate function, $K_{i,k}^E$ [kWh/t]⁻¹ is assumed to be insensitive to scale-up. It should be noted that the power draw of the mill is strongly dependent on the hold-up of rock and pebbles inside the mill, which makes it necessary to solve this equation by numerical methods.

The accumulation or rate of change of the hold-up of material coarser than x_i ($i = 2, 3, \dots, n-1$) is given by:

$$\frac{d(W_{i,k})}{dt} = F_{i,k} - P_{i,k} - W_{i,k} \frac{P}{M} K_{i,k}^E \quad [10]$$

where the first term on the right-hand-side of this equation is the inlet flowrate of rock type k coarser than size x_i and the second term is the discharge flowrate of material coarser than size x_i . The third term is the rate at which material coarser than size x_i breaks to below this size. A similar equation can be applied to material coarser than size x_{i+1} :

$$\frac{d(W_{i+1,k})}{dt} = F_{i+1,k} - P_{i+1,k} - W_{i+1,k} \frac{P}{M} K_{i+1,k}^E \quad [11]$$

Subtracting Equation [10] from Equation [11] gives the accumulation of material within size class i :

accumulation = in – out + (generation – consumption)

$$\frac{dw_{i,k}}{dt} = f_{i,k} - g_{i,k} w_{i,k} + \left\{ W_{i,k} \frac{P}{M} K_{i,k}^E - (w_{i,k} + W_{i,k}) \frac{P}{M} K_{i+1,k}^E \right\} \quad [12]$$

The mass balance for particles in size class n (the sink interval with particle sizes between zero and x_n) is given by:

$$\frac{d(w_{n,k})}{dt} = f_{n,k} - g_{n,k} w_{n,k} + W_{n,k} \frac{P}{M} K_{n,k}^E \quad [13]$$

The water balance is given by:

$$\text{accumulation} = \text{in} - \text{out} + \frac{dw_0}{dt} = f_0 - g_0 w_0 \quad [14]$$

The above equations can be solved numerically to simulate both the dynamic and steady state behaviour of the mill for any circuit configuration.

The total mass of ore inside the mill is obviously a function of the hold-ups of particles of all rock types and all sizes. The power is a function of the hold-ups of water, ore, and grinding media. A number of model equations are available in the literature for predicting the power draw, such as the one developed for SAG mills given in Equation [15] (Austin, 1990):

$$P = 10.6 D^{2.5} L (1 - 1.03J) \left[(1 - \varepsilon_c) \left(\frac{\hat{\rho}_r}{w_c} \right) J + 0.6 J_B (\rho_B - \frac{\hat{\rho}_r}{w_c}) \right] (\phi_c) \left(1 - \frac{0.1}{2^{9-10\phi_c}} \right) \quad [15]$$

where J and J_B are the static fractional volumetric filling of the mill for the total charge and for the balls, respectively. The effective porosity of the charge is given by ε_c and w_c is the weight of the ore expressed as a fraction of the total mass of ore and water in the mill. The density of the balls and average density of the ore [t/m³] are given by ρ_B and $\hat{\rho}_r$, respectively, and ϕ_c is the mill speed expressed as a fraction of the critical speed.

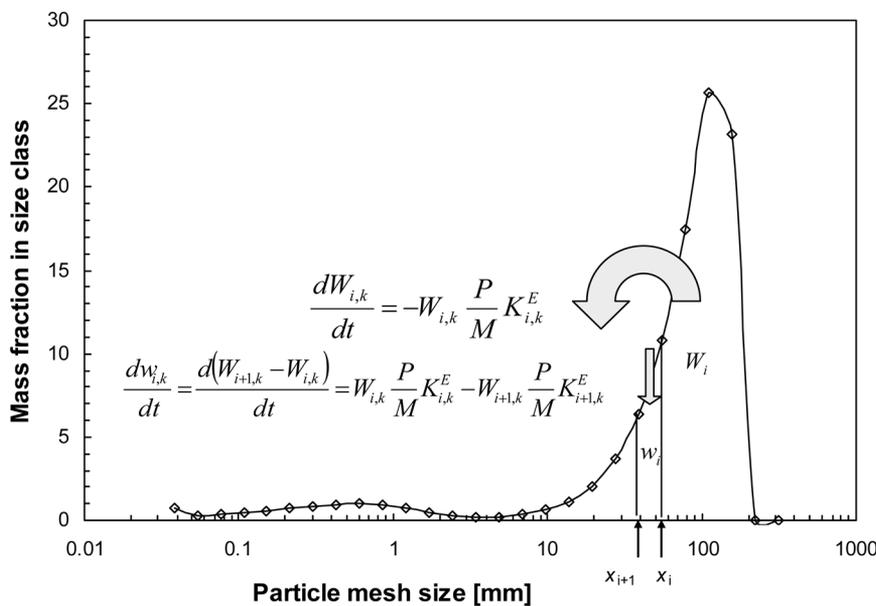


Figure 7. Concept of a cumulative breakage rate function

The Mintek version of the cumulative rates model described above caters for the effects of different aspect ratios (ρ_i) when scaling up pilot data by treating the production mill as a number of fully-mixed reactors in series with back mixing between adjacent reactors.

Measuring the cumulative breakage rate function

It can be shown from the population balance equations for the solids given above (Equations [9], [12], and [13]) that the cumulative specific breakage rate function can be obtained directly from plant data. By definition, there are no particles coarser than x_1 , $K_{1,k}^E$ so is undefined. The specific breakage rates for particles coarser than size x_2 (particles in size class 1 only) can be obtained at steady state by equating the derivative in Equation [9] to zero and rearranging terms:

$$K_{2,k}^E = \frac{f_{1,k} - p_{1,k}}{w_{1,k} \frac{P}{M}} \quad [16]$$

After equating the derivatives to zero in Equation [12], values for the cumulative breakage rate function can be calculated recursively for $i = 2, 3, \dots, (n-1)$:

$$K_{i+1,k}^E = \frac{f_{i,k} - p_{i,k} + W_{i,k} \left(\frac{P}{M}\right) K_{i,k}^E}{(w_{i,k} + W_{i,k}) \frac{P}{M}} \quad [17]$$

From Equation [13] (for $i = n$):

$$K_{n,k}^E = -\frac{f_{n,k} - p_{n,k}}{W_{n,k} \frac{P}{M}} \quad [18]$$

In a pilot mill, all the terms on the right-hand-side of the above equations are directly measurable.

Bench scale SAG testwork

Bench-scale SAG testwork followed by computer simulation studies are becoming a common feature for designing milling circuits at prefeasibility studies stage. Bench-scale comminution testwork used to design AG/SAG mills include Bond ball work index (BBWI), Bond rod work index (BRWI), JKTech drop weight and abrasion tests, SAG Mill comminution test (SMC), SAG power index (SPI), Advanced media competency test (AMCT), McPherson autogenous test and SAG design test. Table I shows a summary of bench-scale testwork equipment used, requirements in terms of mass of sample, top size used and outputs.

These bench-scale testwork have been extensively

discussed at the 2006 Autogenous and Semitogenous conference (Starkey, J. *et al*, 2006; Morrel, S., 2006; McKen, A. and Williams, S., 2006). Assumptions are made to derive parameters like in the JK drop weight test where initially it was assumed that the drop weight test product size distribution for rock of different size obtained at a specific energy input can be normalized. Figure 8 illustrates that the product size distribution cannot be normalized for a UG2 ore.

Mintek SAG testwork

Mintek has been actively involved for many years in research in order to improve laboratory test procedures for the design and optimization of SAG mills. Motivated by the need:

- to reduce the amount of sample used for piloting tests while not compromising on the quality of data generated
- to reduce assumption made in analysing the data generated
- to design a test not relying on a database to interpret the results

Mintek has designed a test to take into account the above-mentioned criteria.

Mintek SAG testwork is conducted in a 0.6 m diameter mill using feed with a top size of 75 mm. The design of the mill was performed to have the same flexibility as the pilot mill. The variable speed drive mill is equipped with grate discharge which can be modified as illustrated in Figure 9.

The mill can run as a fully autogenous or semi-autogenous with top ball size of 80 mm. The mill is equipped with instrumentation for monitoring shaft torque and mill speed, which is linked to a data-logging computer. The net power can therefore be calculated.

Dry looked cycle tests are conducted in dry conditions as locked-cycle until steady state conditions are achieved. Different configurations can be used. The mill discharge can be screened and oversizes sent back to the mill.

At the end of a test, the mill content is weighted, analysed for build-up of a critical size or rock type and sized. The mill discharge is also weighted and sized. The mill product can be used for Bond ball work index testwork.

The sample required to conduct the Mintek SAG test is approximately 300 kg with a top size of 75 mm to explore different conditions.

Comparison between piloting data and Mintek SAG test data on a UG2 ore

Piloting testwork were conducted using a UG2 ore in a configuration shown in Figure 10. The mill was loaded at 30% filling with a ball charge of 6%. The mill discharge

Table I
SAG bench-scale testwork

Testwork	Equipment	Mass of sample (kg)	Top size (mm)	Output
JKTech drop weight test	Drop weight tester	80	63	A, b, ta
SAG Mill Comminution test (SMC)	Drop weight test	20	31.5	DWI, Axb
MinnovEX SAG Power Index (SPI)	Mill having a diameter of 0.3 m	10	19	Specific energy required (kWh/t)
Advanced Media Competency Test (AMCT)	Mill having a diameter	300	165	Pebble competency
MacPherson Autogenous	Mill having a diameter of 0.46 m	175	32	Specific energy required (kWh/t) and MacPherson autogenous work index (AWI)
SAG design test	Mill having a diameter of 0.5 m	20	32	Specific energy consumption

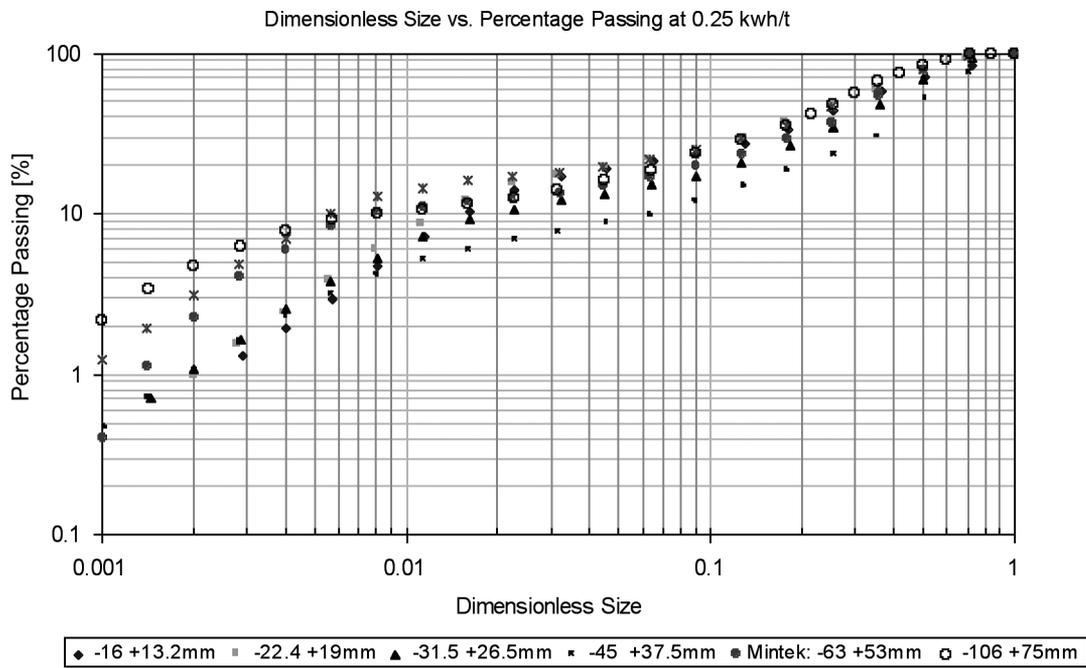


Figure 8. Normalized drop weight size distribution

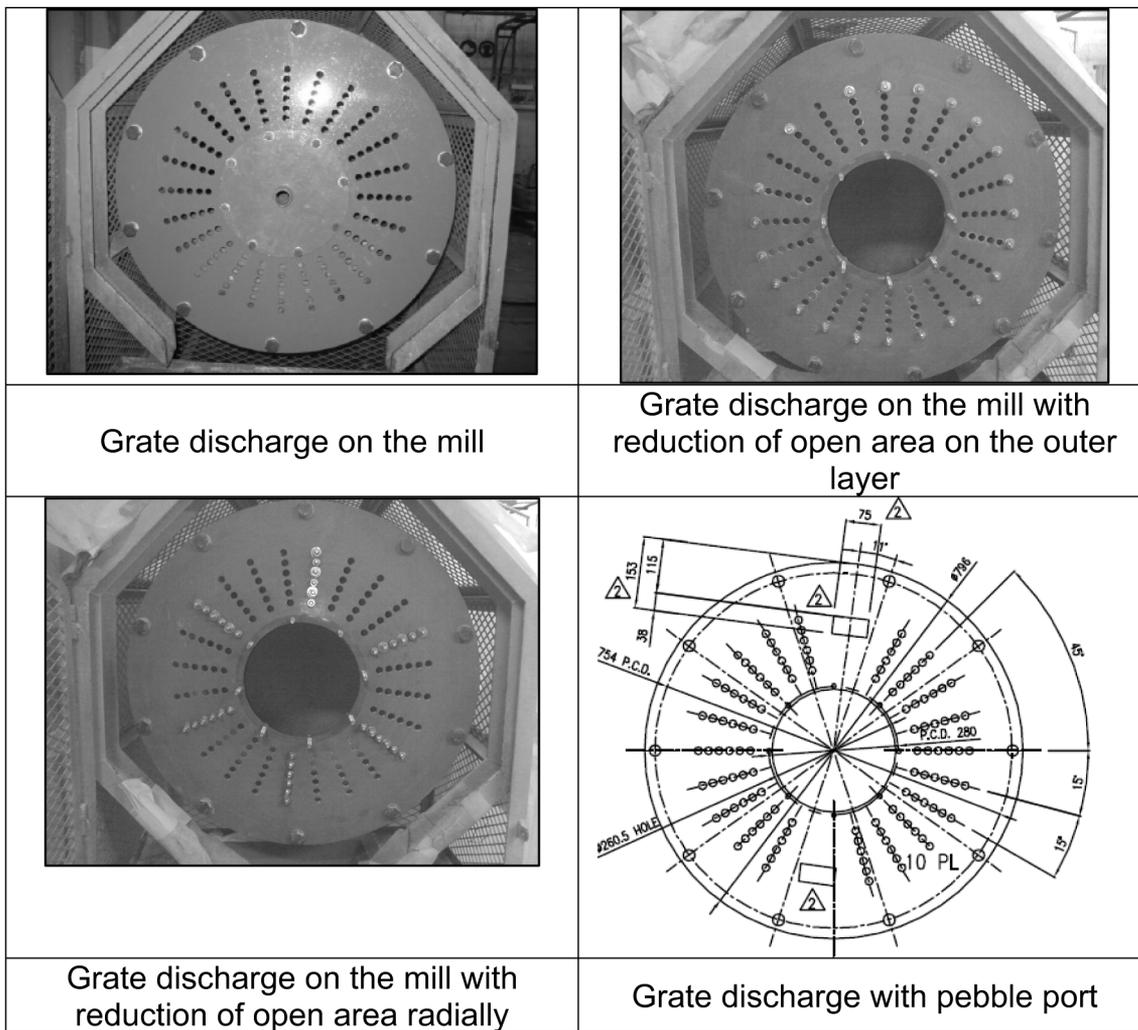


Figure 9. Example of grate discharge used on the Mintek laboratory SAG mill

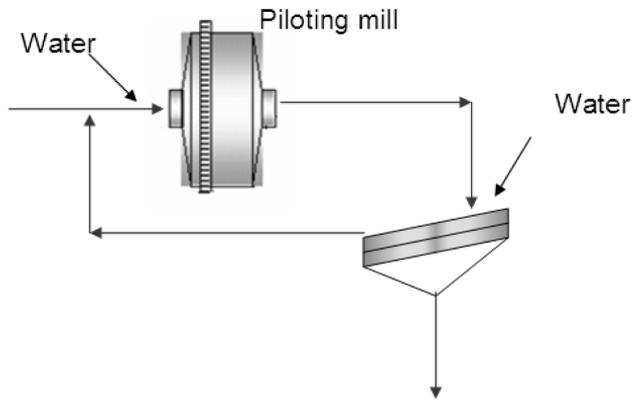


Figure 10. Piloting circuit used for the grinding of UG2 ore

was screened at 1.7 mm. The oversize was recirculated to the mill and the undersize was the final product. The feed rate to the mill was 1.67 t/h to maintain a constant charge in the mill of 30% filling.

After 8 hours, when the mill was running at steady-state, samples were taken for mass balance. The mill content was also sized after a crash stop.

Mintek laboratory SAG testwork was conducted under the same conditions as the piloting testwork. Locked-cycle tests were conducted using the same configuration. Batch grinding were conducted for 5 minutes. The discharge of the mill was screened at 1.7 mm and the oversize recirculated to the mill. Fresh feed was added to the mill according to the feed size distribution to maintain a constant charge in the mill. The process was repeated until steady-state condition was achieved in 11 cycles.

A comparisons in terms of specific energy consumed is shown in Table II. It can be seen that the laboratory Mintek SAG test predict within an accuracy of 5%.

The comparison between the specific discharge rate and the cumulative specific breakage rate are illustrated

Table II
Comparison between specific energy used

	Net power (kW)	Feed rate (t/h)	Specific energy (kWh/t)
Pilot test	8.7	1.67	5.20
Mintek SAG test	0.432	0.087	4.96

respectively in Figures 11 and 12.

The large differences in the specific discharge rate observed below 1 mm is due to the loss in the dust since the test is conducted in dry conditions.

The test facility is currently being modified to reduce dust losses to acceptable levels.

Conclusions

Mintek is equipped with comminution piloting facilities which can be used for the design and optimization of AG/SAG mills.

The breakage processes occurring in SAG mills are complex and any proposed simulation model is going to be an approximation to the actual behaviour of the mills. The concept and application of specific breakage rate and primary breakage distribution functions have been very useful in describing the effects of ball load, ball size and other variables on the performance of conventional tumbling ball mills. It is therefore natural to extrapolate this information to SAG mills. This paper has endeavoured to highlight some of the unresolved issues associated with population balance models for SAG mills that require separate breakage rate and breakage distribution functions to quantify breakage phenomenon. The major dilemma with this approach to SAG modelling is that these functions cannot easily be measured and there is much doubt as to whether or not the values obtained are reproducible and accurately reflect reality. Even if they could be measured directly, it still remains a formidable task to establish how

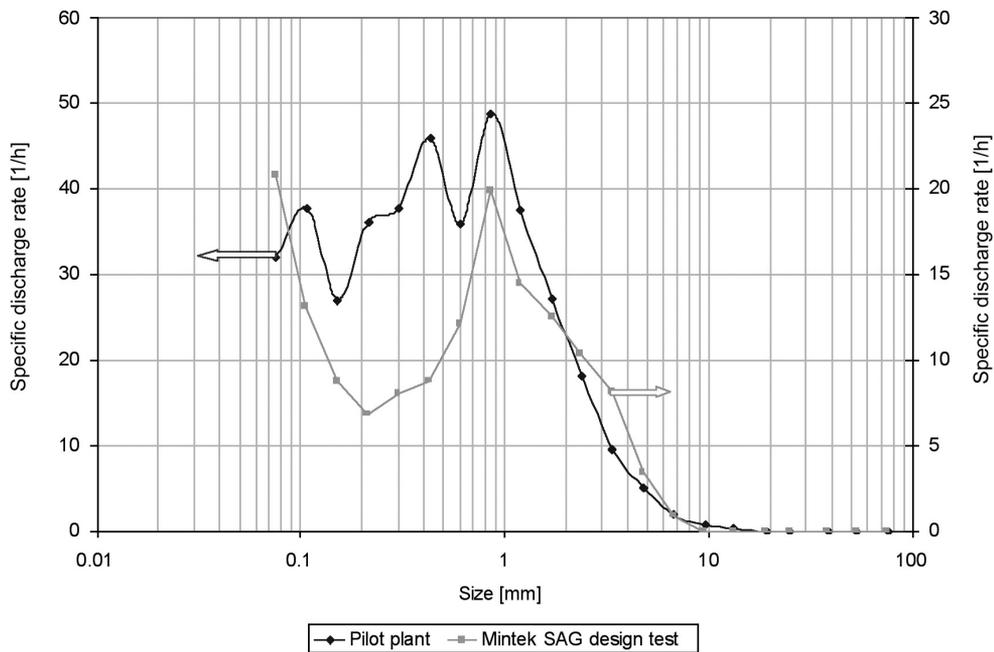


Figure 11. Comparison between the specific discharge rate

these functions change with operating conditions inside the mill and the geometry of liners and grates.

To address the problems associated with the standard approach to modelling SAG mills, a much simpler model structure is proposed which requires fewer assumptions, and where functions describing breakage kinetics and material flow through the discharge grate can be measured directly. However, the 'simple model' does require the development of a set of empirical relations to cover the effect of all variables on the $K_{i,k}^E$ and $g_{i,k}$ functions characterizing breakage behaviour and material transport in and around the mill. Nonetheless, it is believed that this is a very valid approach and may turn out to be the quickest way to fully describe and quantify the performance of these complex mills.

Comminution bench scale laboratory testwork followed by computer simulations are becoming a growing practice to design AG/SAG circuits since piloting tests require more than 80 tons of samples and the cost is very high. This has prompted Mintek to develop a laboratory test procedure that obviates the need for databases by providing estimates of breakage and discharge rate functions that can be used directly to predict the performance of production circuits.

The test developed is conducted in a variable speed and grate discharge laboratory grindmill having a diameter of 0.6 m equipped with torque and speed measurement which are linked to a computer logging system. The test is performed as locked cycle in dry conditions. Data generated were compared to piloting data generated. A good agreement has been found between the pilot data and the Mintek SAG test. More tests are currently being conducted to validate the Mintek SAG test which has been filed for a patent.

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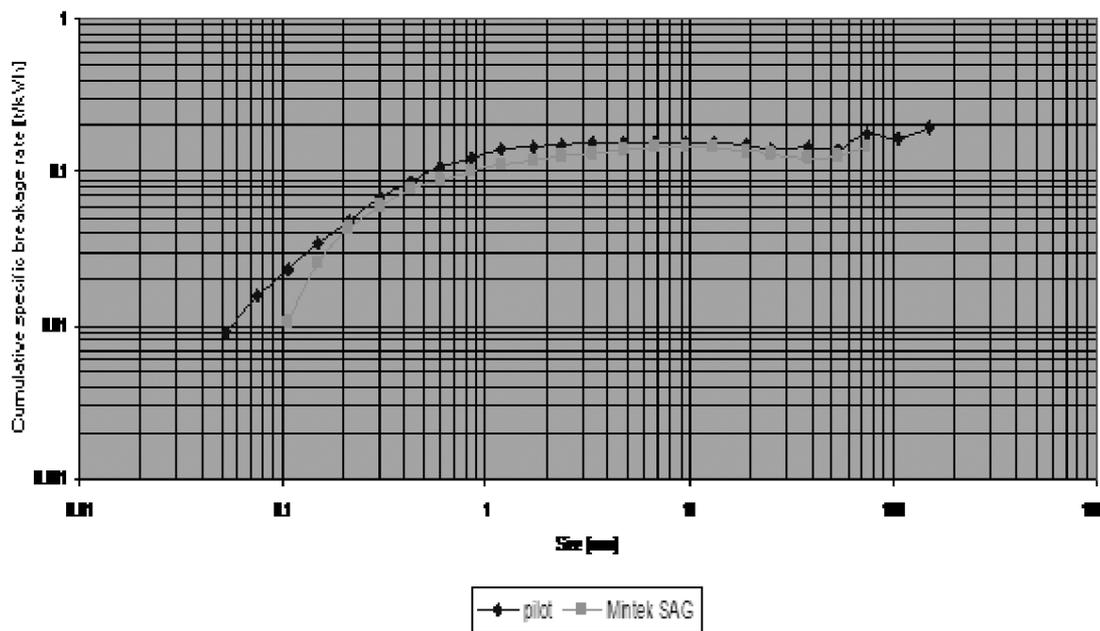


Figure 12. Comparison between the cumulative specific breakage rate

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