Optimising the production schedule for transition mines

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Current technologies have made the transition from surface mining to underground mining feasible and economically viable. The transition is challenging, especially for deposits that require exploitation with both methods. The existing research addresses a more complex transition mine planning problem, which integrates production scheduling with transition depth optimisation to maximise net present value (NPV). However, the research does not consider some realistic operational features in the problem setting. Our paper aims to develop an integrated mixed-integer linear programming (MILP) model to determine the fleet size and production schedule, which maximises the NPV as a function of the production scheduling and transition depth. We also investigate the extent to which operational features of transition mines affect the optimal production schedule and the maximum NPV. Our model includes new constraints to consider development cost, fleet size, stockpile, and the crown pillar. A case study is used to validate the model, with a comparative sensitivity analysis to obtain operational insights.

Keywords: Transition mines, underground mining, open pit, production scheduling, MILP, optimisation

RELEVANT WORK

The formulation of surface and underground production scheduling problems as linear programming (LP) models is quite common. However, all research in the literature presents a different perspective on operational features and parameters that impact the NPV (Opoku and Musingwini 2013, MacNiel and Dimitrakopoulos 2018). Such features include stockpile options (Bley *et al.*, 2012, Moreno *et al.*, 2017), blending constraint (Bley *et al.*, 2012), ventilation constraint (Brickley, 2015) and geotechnical constraint (Nezhadshahmohammad *et al*., 2018).

Current LP formulations contain a foundation of operational constraints, including the block and activity precedence constraint (first developed by Bienstock and Zuckerberg, 2010), mining and plant resource constraint, grade constraint, slope constraint, and reserve constraint. It can be argued that all the operational parameters and restrictions mentioned above imposed in optimising a mine's production schedule based on the NPV are essential for obtaining a realistic NPV. However, solving the production scheduling optimisation with millions of variables and constraints often results in unfeasible solutions with high computational costs. This is especially true for production schedule optimisation. The magnitude of the transition problem has evoked the level of simplification in previous work (Chowdu *et al.,* 2021). Although these operational restrictions are essential, there is a need to determine how relevant and sensitive the NPV is to them to prioritise their incorporation in the model. We investigate the extent to which operational features of transition mines accounted for by researchers in decision models affect the optimal production schedule obtained.

The selection of operational parameters and restrictions are highly dependent on the characteristics and needs of the mine. Therefore, a sensitivity analysis cannot be conducted for all mining constraints and parameters. The case study used is based on the transition from an open pit to a block caving operation. The operational features evaluated for the transition mine problem were generated from the literature survey and meetings with experts from several mines in Chile. We present the first work to evaluate the impact of operational features on the NPV, specifically for transition mines. Our work uses a mixed-integer linear programming (MILP) model which incorporates fleet size optimisation and the area of influence of the drawpoints as polygons instead of blocks, and the partial extraction of draw columns.

MILP Formulation – Base Case

Open Pit

Sets and Indices

$b \in B$	*Set of blocks to be mined*
$t \in T$	*Set of periods in which blocks will be extracted*
$b' \in B_b$	*Set of blocks b' that precede block b vertically and/or horizontally*
m	*Index indicating mining*
p	*Index indicating processing*

Parameters

d	*Discount rate (%)*
$\underline{q}_t$	*Lower limit of grade target in period t*
$\overline{q}_t$	*Upper limit of grade target in period t*
$\underline{R}_{mt}$	*Minimum available mining resources in period t (tons)*
$\overline{R}_{mt}$	*Maximum available mining resources in period t (tons)*
$\underline{R}_{pt}$	*Minimum available plant resources in period t (tons)*
$\overline{R}_{pt}$	*Maximum available plant resources in period t (tons)*
p_t	*Price of ore in period t*
t_b	*Tonnage of material (ore + waste) in block b*
o_b	*Tonnage of ore in block b*
$\overline{r}_b$	*Percentage recovery for block b*
q_b	*Percentage of ore in block b (grade quality)*
c_{mbt}	*Cost per ton of mining block b*
c_{pbt}	*Cost per ton of processing block b*

Decision Variables:

x_{bt}	*Takes on a value of 1 if block b is mined in period t, 0 otherwise*

$$\max\left[\sum_b \sum_t \left\{\frac{\left(p_t * \bar{r}_b * q_b - c_{pbt}\right) * o_b - c_{mbt} * t_b}{(1+d)^t}\right\} * x_{bt}\right] \quad [1]$$

$$\underline{R}_{mt} \le \sum_{b\in B} t_b * x_{bt} \le \overline{R}_{mt} \quad \forall t \in T \quad [2]$$

$$\underline{R}_{pt} \le \sum_{b\in B} o_b * x_{bt} \le \overline{R}_{pt} \quad \forall t \in T \quad [3]$$

$$\sum_{t=1}^{\tau+1} x_{b',t} \ge \sum_{t=1}^{\tau} x_{bt} \quad \forall\, b \in B, \tau \in T \quad [4]$$

$$\underline{q}_t * \sum_{b=1}^{B} o_b \le \sum_{b=1}^{B} o_b * q_b * x_{bt} \le \overline{q}_t * \sum_{b=1}^{B} o_b \quad \forall t \in T \quad [5]$$

$$\sum_{t=1}^{T} x_{bt} \le 1 \quad \forall b \in B \quad [6]$$

$$x_{bt} \in \{0,1\} \quad \forall b \in B, t \in T \quad [7]$$

Block Caving

Sets and Indices:

T	*Set of periods*
D	*Set of drawpoints*
$\check{S}^d$	*Set of drawpoints that precede drawpoint* $d \in D$

Parameters:

d	*Discount rate (%)*
$\hat{r}_{dt}$	*Value of draw column associated with drawpoint d. Summed value of blocks in d*
N_t	*Maximum number of drawpoints being extracted in period t.*
$\overline{N_{nt}}$	*Upper limit of the number of new drawpoints allowed in period t*
$\underline{N_{nt}}$	*Lower limit of the number of new drawpoints allowed in period t*
T_d	*Total tonnage of material in draw column above drawpoint d*
Q_d	*Total tonnage of ore in draw column above drawpoint d*
$\overline{q}_{dt}$	*Upper limit of grade target of drawpoint d in period t*
$\underline{q}_{dt}$	*Lower limit of grade target of drawpoint d in period t*
q_d	*Average grade of drawpoint d*
$\overline{M}_{mt}$	*Upper limit of mining capacity in period t*
$\underline{M}_{mt}$	*Lower limit of mining capacity in period t*
$\overline{M}_{pt}$	*Upper limit of plant capacity in period t*
$\underline{M}_{pt}$	*Lower limit of plant capacity in period t*
$\overline{R}_{dt}$	*Maximum possible draw rate of drawpoint d in period t.*
$\underline{R}_{dt}$	*Minimum possible draw rate of drawpoint d in period t.*
A_d	*Maximum number of periods a drawpoint d can be extracted, assuming continuous extraction*

Decision Variables

x_{dt}	*Takes on a value of 1 if the extraction of drawpoint d is initiated in period t, and 0 otherwise*
y_{dt}	*Continuous variable, representing the portion of drawpoint to be extracted in period t.*
z_{dt}	*Takes on a value of 1 if drawpoint d is extracted in period t, and 0 otherwise*

$$\max\left[\sum_d\sum_t\left\{\frac{(p_t * \bar{r}_d * q_d - c_{pdt}) * Q_d - c_{mdt} * T_d}{(1+d)^t}\right\} * y_{dt}\right] \quad [8]$$

$$\underline{M}_{mt} \le \sum_{d\in D} T_d * y_{dt} \le \overline{M}_{mt} \qquad \forall t \in T \quad [9]$$

$$\underline{M}_{pt} \le \sum_{d\in D} Q_d * y_{dt} \le \overline{M}_{pt} \qquad \forall t \in T \quad [10]$$

$$\sum_{t=1}^{\tau+1} x_{d',t} \ge \sum_{t=1}^{\tau} x_{dt} \quad \forall\, d \in D, \tau \in \mathrm{T}, \mathrm{d}' \in \check{S}^d \quad [11]$$

$$\underline{q}_{dt} * \sum_{d\in D} Q_d * y_{dt} \le \sum_{d\in D} Q_d * q_d * Y_{dt} \le \overline{q}_{dt} * \sum_{d\in D} Q_d * y_{dt} \quad \forall t \in T \quad [12]$$

$$\sum_{t\in T} x_{dt} = 1 \qquad \forall d \in D \quad [13]$$

$$\textstyle\sum_{\tau=0}^{t} x_{dt} \ge z_{dt}, \qquad \forall d \in D, t \in T \quad [14]$$

$$\textstyle\sum_{t\in T} z_{dt} \le A_d \qquad \forall d \in D \quad [15]$$

$$A_d(z_{d,t} - z_{d,t+1}) - \textstyle\sum_{\tau=0}^{t} z_{d\tau} \le 0 \qquad \forall d \in D, t = \{0,1,\dots,|T|-1\} \quad [16]$$

$$\underline{\mathrm{N}_{\mathrm{nt}}} \le \textstyle\sum_{d\in D} x_{dt} \le \overline{\mathrm{N}_{\mathrm{nt}}} \qquad \forall t \in T \quad [17]$$

$$\textstyle\sum_{d\in D} x_{dt} \le \underline{\mathrm{N}_{\mathrm{n1}}} \qquad \forall t \in T \quad [18]$$

[19]

$\sum_{d \in D} z_{dt} \leq N_t \qquad \forall t \in T$

$$y_{dt} \leq z_{dt} \qquad \forall d \in D, t \in T \qquad [20]$$

$$\underline{R}_{dt} z_{dt} \leq Q_d y_{dt} \qquad \forall d \in D, t \in T \qquad [21]$$

$$\sum_{t=1}^{T} y_{dt} \leq 1, \qquad \forall d \in D \qquad [22]$$

$$\sum_{t=1}^{\tau+1} z_{d',t} \geq \sum_{t=1}^{\tau} z_{dt} \quad \forall d \in D, \tau \in T, d' \in \check{S}^d \qquad [23]$$

$$x_{dt}, z_{dt} \in \{0,1\} \qquad \forall d \in D, t \in T \qquad [24]$$

Experimental analysis

Fleet size

This analysis aims to evaluate the impact of fleet size on the NPV. The open pit model was modified as follows.

Parameters

$cicle_b$	*Truck cycle per block*
$cost_{truck}$	*Cost per truck*

New Decision Variables:

cn_t	*number of trucks needed in each period*
cc_t	*number of trucks that must be purchased in each period*

The objective function indicated in Equation [25] accounts for the cost of purchasing new trucks.

$$\max\left(\left[\sum_b \sum_t \left\{\frac{(p_t * \bar{r}_b * q_b - c_{pbt}) * o_b - c_{mbt} * t_b}{(1+d)^t}\right\} * x_{bt}\right] - \sum_t \frac{cc_t * cost_{truck}}{(1+d)^t}\right) \qquad [25]$$

Subject to:

$$cn_0 = cc_0 \quad \forall t \in T \qquad [26]$$

$$\sum_{\tau=1}^{t+1} cc_\tau \geq cn_t \quad \forall t \in T \qquad [27]$$

$$\sum_{b=1}^{B} x_{bt} * cicle_b \leq cn_t \quad \forall t \in T \qquad [28]$$

Material Destinations

Four decision variables are used in this formulation, considering three possible destinations associated with block movements - dump, plant or stockpile, and stockpile to plant. The model is reformulated as follows:

Sets and Indices

$t \in T_{stock}$	*Set of periods in which blocks can be sent to plant from the stockpile*
$t \in T_{delta}$	*Set of periods in which blocks are sent to plant from stockpile after the mine extraction period*

Parameter

re	*Material rehandling cost*

Decision Variables:

B_{bt}	*Takes on a value of 1 if block b is mined in period t and sent to dump, 0 otherwise*
S_{bt}	*Takes on a value of 1 if block b is mined in period t and sent to stockpile, 0 otherwise*
P_{bt}	*Takes on a value of 1 if block b is mined in period t and sent to plant, 0 otherwise*
SP_{bt}	*Takes on a value of 1 if block b is sent from the stockpile to plant in period t, 0 otherwise*

$$\max\left[\sum_{b}\sum_{t\in T}\left\{\frac{\left(p_t*\bar{r}_b*q_b-c_{pbt}\right)*o_b*P_{bt}-c_{mbt}*t_b*(B_{bt}+S_{bt}+P_{bt})}{(1+d)^t}\right\}\right.$$
$$\left.-\sum_{b}\sum_{\tau\in T_{Stock}}\left\{\frac{\left(p_\tau*\bar{r}_b*q_b-c_{pb\tau}\right)*o_b*SP_{b\tau}-re*t_b*SP_{b\tau}}{(1+d)^\tau}\right\}\right] \quad [29]$$

$$\underline{R}_{mt} \le \sum_{b\in B} t_b * (B_{bt}+S_{bt}+P_{bt}) \le \overline{R}_{mt} \quad \forall t \in T \quad [30]$$

$$\underline{R}_{pt} \le \sum_{b\in B} o_b * (SP_{bt}+P_{bt}) \le \overline{R}_{pt} \quad \forall t \in T \quad [31]$$

$$\underline{R}_{pt} \le \sum_{b\in B} o_b * (SP_{bt}) \le \overline{R}_{pt} \quad \forall t \in T_{delta} \quad [32]$$

$$\sum_{t=1}^{\tau+1}(B_{b't}+S_{b't}+P_{b't}) \ge \sum_{t=1}^{\tau}(B_{bt}+S_{bt}+P_{bt}) \quad \forall b \in B, \tau \in T \quad [33]$$

$$\underline{q}_t * \sum_{b=1}^{B} o_b\,(SP_{bt}+P_{bt}) \le \sum_{b=1}^{B} o_b * q_b * (SP_{bt}+P_{bt}) \le \overline{q}_t * \sum_{b=1}^{B} o_b\,(SP_{bt}+P_{bt}) \quad \forall t \in T \quad [34]$$

$$\sum_{t=1}^{T} P_{bt} \le 1 \quad \forall b \in B \quad [35]$$

$$\sum_{t=1}^{T} B_{bt} \le 1 \quad \forall b \in B \quad [36]$$

$$\sum_{t=1}^{T} S_{bt} \le 1 \quad \forall b \in B \quad [37]$$

$$\sum_{t=1}^{T_{stock}} SP_{bt} \le 1 \quad \forall b \in B \quad [38]$$

$$\sum_{t=1}^{T}(B_{bt}+S_{bt}+P_{bt}) \le 1 \quad \forall b \in B \quad [39]$$

$$\sum_{b=1}^{B} SP_{b0} \le 1 \quad [40]$$

$$\sum_{t=0}^{t+1} SP_{bt} \ge \sum_{t=0}^{t+1} SP_{b,t+1} \quad \forall b \in B, t \in T \quad [41]$$

$$\sum_{t=0}^{t\in T} SP_{bt} = \sum_{t=0}^{t\in T_{stock}} SP_{b,t+1} \quad \forall b \in B \quad [42]$$

$$B_{bt} \in \{0,1\} \quad \forall b \in B, t \in T \quad [43]$$

$$S_{bt} \in \{0,1\} \quad \forall b \in B, t \in T \quad [44]$$

$$P_{bt} \in \{0,1\} \quad \forall b \in B, t \in T \quad [45]$$

$$SP_{bt} \in \{0,1\} \quad \forall b \in B, t \in T_{stock} \quad [46]$$

Development cost

In this scenario, we consider the impact of integrating the development costs into the block caving formulation. The model accounts for the development cost per metre and the metres of advance per period. The objective function is reformulated as shown in Equation [47]

Parameters

d_c	*Development cost per metre*
adv_t	*Development per metre advance per period*
T_{mt}	*Total distance of a tunnel from the transport level to the surface*

$$\max\left(\left[\sum_{d}\sum_{t}\left\{\frac{\left(p_t * \bar{r}_d * q_d - c_{pdt}\right) * Q_d - c_{mdt} * T_d}{(1+d)^t}\right\} * y_{dt}\right] - d_c * adv_t * T_{mt}\right) \qquad [47]$$

Mine footprint
In the base scenario, rectangular arrays are used to optimise the extraction sequence on a block level. In this scenario, we consider the area of influence by each drawpoint which is defined by a hexagonal area. The optimal footprint is selected such that the overall value of the operation is maximised. This scenario is computationally expensive considering the direction of extraction and the combination of drawpoints in each direction. The MILP in this scenario remains unchanged, but the input for the production scheduling block caving model is changed.

Integrated Model
The integrated model is a combination of the open pit and block caving model. A crown pillar constraint is incorporated into the model, as shown in Equation [48].

$$x_{bt} * \left(cz_b - b_{centroid} - U_{level} - col_{height} + CP\right) \geq 0 \quad \forall\, b\, \in B, \forall\, t\, \in\, T \qquad [48]$$

where:

d	*Set of drawpoints*
cz_b	*The centroid coordinates on the z-axis of a block*
$b_{centroid}$	*The minimum distance between the centre of a block to any face of the block*
U_{level}	*The undercut level*
col_{height}	*The column height related to the drawpoints*
CP	*Crown Pillar height*

In Equation [49] variable x_{bt} is replaced by the block, stockpile, and processing plant variables B_{bt}, S_{bt}, and P_{bt}, respectively.

$$(B_{bt} + S_{bt} + P_{bt}) * \left(cz_b - b_{centroid} - U_{level} - col_{height} + CP\right) \geq 0 \quad \forall\, b\, \in B, \forall\, t\, \in\, T \qquad [49]$$

Case Study
The technical and economic parameters used in the model are presented in Tables 1 and 2. The values are determined based on market conditions such as copper price and average cost values used by Chilean mines at the time of this research. The height of the copper (Cu) deposit is approximately 600 metres with a cut-off grade of 0.3%Cu. The block model consists of 22,490 blocks with dimensions of 10m x10m x10 m.

Table I. Technical and economic parameters

Parameter	Value	Unit
Cu Price	$3,791.912	USD/ton Cu
Crown pillar height	30	m
Discount rate	10	%
Average surface mining cost	$10.15	USD/ton
Surface processing cost	$7.5	USD/ton Cu
Underground mining cost	$10	USD/ton
Underground processing cost	$7	USD/ton Cu
Recovery	70	%

Table II. Fleet size optimisation parameters

Item	Value
Truck cost	5 MUSD
Nominal capacity	15 ton
Real capacity	8.4 ton

Results and discussion

Fleet size

Tables 3 and 4 show the sensitivity analysis results and the base case scenario. From the base case results, the number of trucks needed varies in each period. However, no additional trucks are purchased after Period 1. Compared to our reformulated model, 13% fewer trucks are needed over the evaluated scheduling horizon. However, the incorporation of fleet size reduces the productivity (29.7%), plant capacity (29.7%), and NPV (28.9%) in each period.

Table III. Open pit base case

Item	Period 1	Period 2	Period 3	Total
Mine capacity [Mton]	47.79	41.04	43.2	132.03
Plant capacity [Mton]	47.79	41.04	43.2	132.03
NPV [MUSD]				294.97
Time [s]				130
Trucks needed	19	17	19	19
Trucks purchased	19	0	0	19

Table IV.Fleet size results

Item	Period 1	Period 2	Period 3	Total
Mine capacity [Mton]	33.75	29.43	29.7	92.88
Plant capacity [Mton]	33.75	29.43	29.7	92.88
NPV [MUSD]				209.61
Time [s]				200
Trucks needed	14	14	14	14
Trucks purchased	14	0	0	14

Destinations

Although three scheduling periods are used in the destination scenario, two additional periods are added to send the remaining materials from the stockpile to the plant (Table V). The NPV is significantly higher (10.5 %) than the base case scenario (Table III) based on the results.

Table V. Fleet size results

Item	Period 1	Period 2	Period 3	Period 4	Period 5	Total
Dump [Mton]	31.65	34.95	38.16	0	0	104.76
Plant [Mton]	19.3	19.1	19.2	0	0	57.6
Stockpile [Mton]		11.12	7.41	0	0	18.53
Stockpile to plant [Mton]	0	0	0	18.53	0	18.53
NPV [MUSD]						325.97
Time [s]						23,670

Development cost

The base case and the reformulated model results are shown in Tables VI and VII, respectively. The differences in NPV (0.15% for the maximum NPV highlighted) are not significant; however, this cannot be generalised to all mines or mining methods.

Table VI. Underground base case scenario results

Height	NPV [MUSD]	Mton
100-190	629.7	67.5
150-240	3427.8	132.3
200-290	4146.1	132.3
250-340	4442.9	132.3
300-390	4658.3	132.3
350-440	4520.6	132.3
400-490	4220.0	132.3
450-540	4182.8	132.3
500-590	3371.3	132.3

Table VII. Underground development cost scenario results

Height	NPV [MUSD]	Mton
100-190	618.9	67.5
150-240	3417.9	132.3
200-290	4137.1	132.3
250-340	4434.8	132.3
300-390	4651.1	132.3
350-440	4514.3	132.3
400-490	4214.6	132.3
450-540	4178.3	132.3
500-590	3367.7	132.3

Mine footprint

Based on the results in TableVIII, by incorporating footprint optimisation in the production scheduling optimisation, the maximum NPV (highlighted) is significantly increased (4.8%).

Table VIII. Underground mine footprint scenario results

Height	NPV [MUSD]	Mton
100-190	686.4	58.7
150-240	3633.5	112.5
200-290	4560.7	116.4
250-340	4665.1	113.8
300-390	4891.2	116.4
350-440	4882.2	113.8
400-490	4599.8	121.7
450-540	4433.8	128.3
500-590	3539.9	121.7

CONCLUSIONS

This research aims to evaluate the extent to which operational features incorporated by researchers in the production schedule optimisation of transition mines affect the NPV of the operation. We developed an experimental analysis that evaluates the sensitivity of the NPV to the development cost, truck fleet size, mining footprint, and ore destination. An Intel Xeon Silver 4210 CPU @2.20 GHz computer with 10 cores and 32 GB RAM and a Gurobi® Optimiser wass used to solve the MILP. Based on the result, we conclude that the factors evaluated significantly impact the production schedule generated and, consequently, the NPV. However, the level of sensitivity of the NPV to these factors is a function of the orebody characteristics and the mining methods employed. Future work includes evaluating more levels of the experimental factor and evaluating a more realistic approach to incorporating ore dilution in the underground model.

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