Embrace heterogeneity – create value with separation: Elaborations on single particle analyses for calibration and validation test work and sensor-based heterogeneity testing

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Value created by mineral beneficiation processes relies on physical separation (including flotation) of particles containing different grades of metals or minerals of economic interest. The value created is a direct consequence of inducing and exploiting heterogeneity at a certain particle size range. Heterogeneity is a function of the properties of the mineral reserve and the comminution and fractionation steps involved. In the context of sensor-based particle ore sorting, these are the upstream unit processes: mining, crushing, and screening. The heterogeneity of a certain process stream represents the fundamental underlying condition, which is exploited with physical separation. Understanding the heterogeneity characteristics is necessary to design appropriate sampling for subsequent process test work. In the context of sensor-based particle ore sorting, heterogeneity analysis is often done in conjunction with single-particle test work. Single-particle test work is a process in which single fragments of a lot are subjected to sensor characterisation (i.e. imaging) and then assayed individually for calibration and validation activities with a sensor-based particle ore sorting (POS) system. This contribution describes the experiences of sampling of relevant amounts of single fragments and single particle assaying for heterogeneity testing, as well as single particle imaging for calibration and validation with the aim of developing POS processes. In this paper we further demonstrate the method of sensor-based heterogeneity testing to determine the sampling constants K and alpha for a cassiterite application, where the pay-element feature, i.e., tin content, can be well calibrated and validated with the sensor signal of the POS unit. Sensor-based heterogeneity testing becomes a new option for determining material heterogeneity and as a further input for designing optimised process sampling strategies.

INTRODUCTION

Mining has been an integral pillar of global economies for many centuries, providing essential raw materials for society and various industries and contributing significantly to economic development. As we stand on the precipice of an energy transition, the role of mining takes on renewed significance, presenting both challenges and opportunities that resonate across economic, environmental, and social dimensions.

The mining industry is experiencing a technological renaissance, driven by innovations such as advanced sensor technologies automation and artificial intelligence. These advancements, while first and foremost aimed at increasing profitability, also significantly enhance operational efficiency and contribute to minimising environmental impacts. Furthermore, the application of these technologies allows for a more precise and targeted approach to mineral extraction, optimising resource utilisation and reducing waste.

In this context, sensor-based ore sorting stands as a promising innovation that has garnered significant attention in recent years. Sensor-based ore sorting has found application in three different forms, which are named according to the separated material unit: sensor-based batch, bulk and particle ore-sorting (POS). In POS the mining industry benefits from technology developments in the recycling industry, which are being transferred into the mining environment. Recognising this fact helps in understanding the POS equipment and procedures available in mining.

The state of play of sensor-based sorting in recycling is characterised by a transformative shift towards advanced technologies that enhance the efficiency and sustainability of waste processing. Sensor-based sorting systems, utilising technologies such as near-infrared spectroscopy, X-ray transmission (XRT), and optical sensors, have become integral components in modern recycling facilities. These systems enable automated identification (i.e. classification) and separation of various materials, such as plastics, metals, and paper, based on their unique sensor response characteristics. Real-time data acquisition and real-time analysis capabilities of these approaches allow for precise material sorting, significantly reducing contamination and increasing the overall quality of recycled materials, with the challenging aim of producing qualities comparable to virgin material at comparable costs.

The driving force behind the surge in interest in sensor-based ore sorting lies in its ability to improve resource utilisation by reducing the economic cut-off grade, reducing environmental impact, and enhancing overall economic viability. The functional principle of the equipment is introduced in the next chapter.

SENSOR-BASED PARTICLE ORE SORTING

POS is a sophisticated mineral processing technique, usually applied in particle size ranges between 10 mm and 100 mm. In some segments of mineral production, such as diamonds and various industrial minerals, the technology is used for producing (close to) final products. In other segments, it is applied relatively early in the production flow sheet with the aim of separating sub-economic waste from concentrator feeds.

One of the key advancements in sensor-based ore sorting is the integration of adapted high-speed imaging technologies, including colour cameras, XRT, near-infrared spectroscopy, and laser systems. These sensor technologies provide new separation criteria, allowing for subsequent physical separation of single particles into two fractions.

For understanding the characteristics of the process, it is important to not only focus on the equipment or sensor-based particle ore process itself, but also on the supporting functions which together build what we call a sensor-based particle ore 'process island' – as a stereotypical example. Figure 1 shows a simplified process flowsheet for a POS process island. A feed stream, typically a run-of-mine ore, is crushed in two stages. The crushed ore is then screened with vibratory screening equipment to produce the particle size ranges applicable to sensor-based POS.

Readers may find more detailed process flow diagrams of sorting islands in different mineral applications in the literature; for example, Rutledge *et al,*. (2020) describe a POS process island for flint removal from phosphate in three parallel particle size ranges. Robben *et al.,* (2020) describe four XRT sorting machines in parallel size ranges installed in a POS plant for treating marginal tin ore at Mina San Rafael in Peru. In Austria, two XRT units, operating in parallel size ranges are applied to remove low grade waste from run-of-mine tungsten ore (Robben, 2014). An example of a single containerised XRT machine for upgrading of gold ore at the Kensington gold mine in Alaska is operated in one particle size range (Robben, *et a*l., 2017).

The principal process involves several interrelated sub-processes, each contributing to the overall efficiency and effectiveness of the sorting operation, introduced in the following paragraphs.

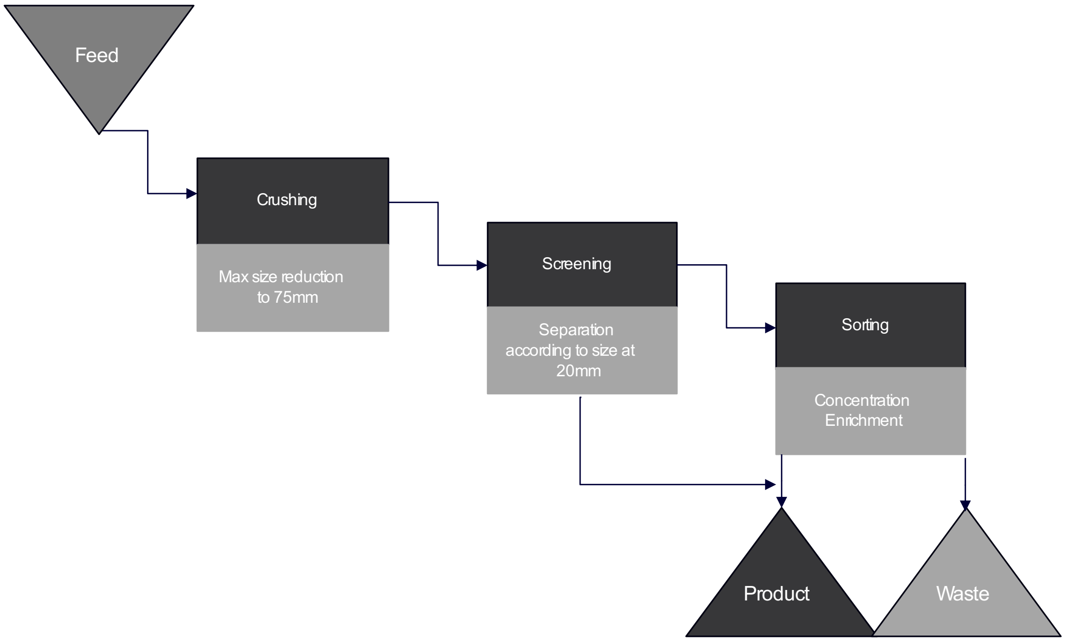

Figure 1. Simplified stereotype process flow sheet of a sensor-based particle ore sorting process island.

Material conditioning – crushing and screening
The process begins with crushing the mined material with primary crushing (and secondary where needed) to produce suitable particle sizes for the applied equipment. Note that crushing is a process that impacts material heterogeneity. By applying different crushing technologies and appropriate parameterisation and process flow configuration, the resulting heterogeneity can be actively influenced. After crushing, screening is necessary to remove undersized (and potentially oversized) particles from the POS equipment feed. This is achieved using vibrating screening equipment.

This process design is a simplified example. Process details and particle size ranges are adaptable depending on the specific project and vary with feed design capacity, type of equipment, ore type, economic drivers etc. Figure 2 shows a functional schematic of an XRT-based POS machine inside view.

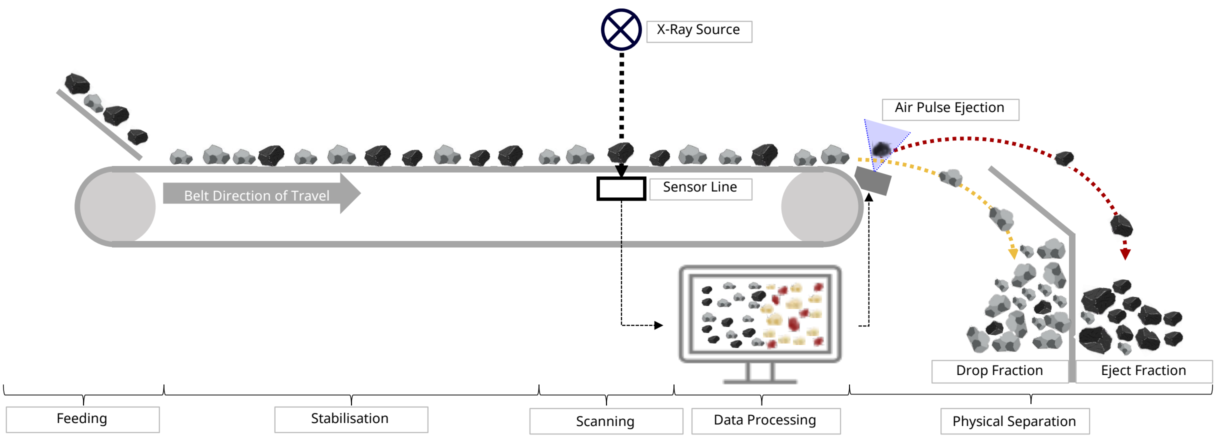

Figure 2. Functional schematic of an XRT-based particle ore sorter (side view).

Material presentation
The aim of the material presentation sub-process is to present single particles to the sensor arrangement. This is achieved by accelerating the stream particles and spreading them out on a larger working width than in the previous conveying mode, which results in the particles being better dispositioned apart

from neighbouring particles. The focus in this process is on achieving a single particle layer with the densest surface cover possible, without particles touching each other and enough distance between each other to allow for a selective ejection of each single particle, i.e., area occupancy. Area occupancy selection is also subject to the specific project and is, next to the particle size and ore density, the factor influencing the design capacity of a POS unit. The first step in the acceleration and spreading of the mass stream is usually actuated by a vibrating feeder, followed by a fast conveyor belt or a chute. These two mechanical presentation setups – the chute and the belt – define a principal discrimination between belt-type and chute-type sensor-based sorters. (Kleine, *et al*., 2011)

Detection – imaging data acquisition

Sensor-based ore sorting relies on a variety of sensor technologies, often XRT, colour cameras or near-infrared spectroscopy, which are arranged as *imaging systems* to detect both location and properties of particles with speeds around 3 m/s (depending on supplier and machine-type). Each sensor type is selected based on the specific mineral characteristics to be identified and used for sorting. XRT, for example, can penetrate materials to reveal their internal composition, while near-infrared-spectroscopy identifies minerals based on their unique spectral signatures in the near-infrared range of the electromagnetic spectrum, but acquired mainly from a very small surficial depth only. This violation of the fundamental sampling principle of surface-based detection, and the adherence of the same by XRT, is one of the reasons why XRT has found wide acceptance across the industry.

Data processing and classification

As the material stream moves along the conveyor, the sensors collect data on its composition. This data is then analysed in real-time using highly dedicated software algorithms, which interpret the sensor signals to identify and classify particles individually into two classes (*drop* and *eject*). For this, each pixel is classified, and object-based image analysis interprets the classified image. These analytical results drive decision-making processes that determine the appropriate action (no action: *drop*; action: *divert or eject*) for each individual material unit (particle).

Physical separation

Once identified and classified, the valuable minerals are selectively separated from the main material flow (*eject* or *divert*). This can be based on physical mechanisms, such as air jets that redirect particles into different chutes. Less commonly used in mining are mechanical flaps diverting the ejected particles. In bulk-ore sorting (BOS) mechanical flaps or diverter gates are diverting an entire mass flow classified as waste by process analytical technology. Due to the scalability of BOS at relatively low capital investment, it can be applied to projects that mine tens of millions of tons per year (Reple, *et al*., 2018).

Overview of effects in sensor-based particle ore sorting process islands

This paragraph describes the different effects of the processes interacting with the material heterogeneity. The original heterogeneity is given by the nature of the mineral deposit targeted for mining. Once ore is broken out of the ground, man-made processes start interacting with its *in-situ* heterogeneity. As they are controlled by humans, the design criteria can be adapted to achieve both the effectiveness and the efficiency needed for technically sound and economically viable processes.

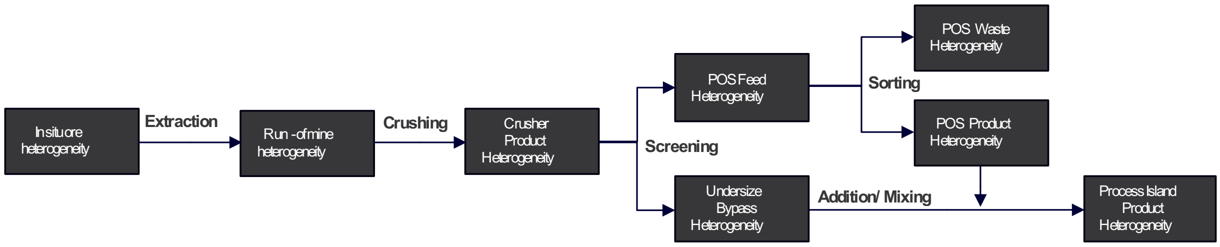

Figure 3. Overview of processes and resulting transformations of material heterogeneity.

Crushing

The processes of size reduction once the material has been mined is summarised here as the term *crushing*. Crushing serves the purpose of reducing the size for enabling continuous belt-based transportation. But more importantly in the context of sensor-based particle ore sorting, crushing serves an integral role in producing a maximum suitable particle size range optimised for the POS equipment, as well as for optimising the heterogeneity in terms of liberation which can then be exploited by the sorting process. For a waste-rejection case aiming for high recovery, the optimisation would aim at maximising the amount of low-grade material below the cut-off grade. Whereas for a concentration case, the goal would be to maximise the amount of material above the product specification. Both can be achieved by designing preferential breakage and choosing the optimum sizing.

Screening

Screening is executed using vibrating screens. The goal of this process is to separate suitable particle sizes from the crusher product stream. Suitable particle sizes in the context of POS are usually in the range of 10 to 100 mm. A maximum to minimum particle size ratio of 3:1 (maximum 5:1) is required. Elaborations on selecting the right particle size range for the process island are subject to many parameters such as the intended sorting equipment and plant capacity as well as economic drivers. This specific topic is not discussed in this paper.

Here we are focusing on the effects on heterogeneity. Due to preferential breakage of minerals and lithologies, the composition differs between the created particle size fractions. For determining the result of a POS process island it is therefore important to understand the characteristics of each size fraction in terms of its mass proportion, distribution of elemental composition as well as distribution of mineralogical compositions.

Sorting

Sensor based particle ore sorting is a physical separation process. Concentration is achieved by separating particles from each other according to a defined separation criterion. The physical integrity of the particles remains intact. This means that the heterogeneity of the feed fraction, described by the grade frequency distribution, is the underlying precondition for the possible separation. In a production environment, it is impossible to obtain a perfect separation, reflecting the real heterogeneity of the particles in a given lot. Nevertheless, reducing particle size can improve separation and reduce the intrinsic heterogeneity. Sorting therefore has the goal to optimally exploit the given heterogeneity. And at the same time, it produces two streams with individual heterogeneity characteristics.

Mixing

A consequence of the process and mere addition of the heterogeneities is mixing of the fine particle size, which bypasses the POS, with one of the sorted fractions. In most cases this is the undersize fraction being combined with the sorted product. This is because fines, especially in metal ore applications and with some few exceptions, usually carry more grade and are worth recovering by further processing.

Overview of effects in sensor-based particle ore sorting processes

The effectiveness of the POS process is a factor between 0%-100%, which is multiplied by the theoretically achievable recovery, derived from the liberation function at a fixed yield. The factor describes the reduced effectiveness of physical separation processes in relation to that theoretically achievable, which is given by the heterogeneity. The process effectiveness factors have been introduced in detail for all defined sub-processes of POS (Robben, 2014). Multiplied with the heterogeneity, the result is the total process effectiveness (TPE).

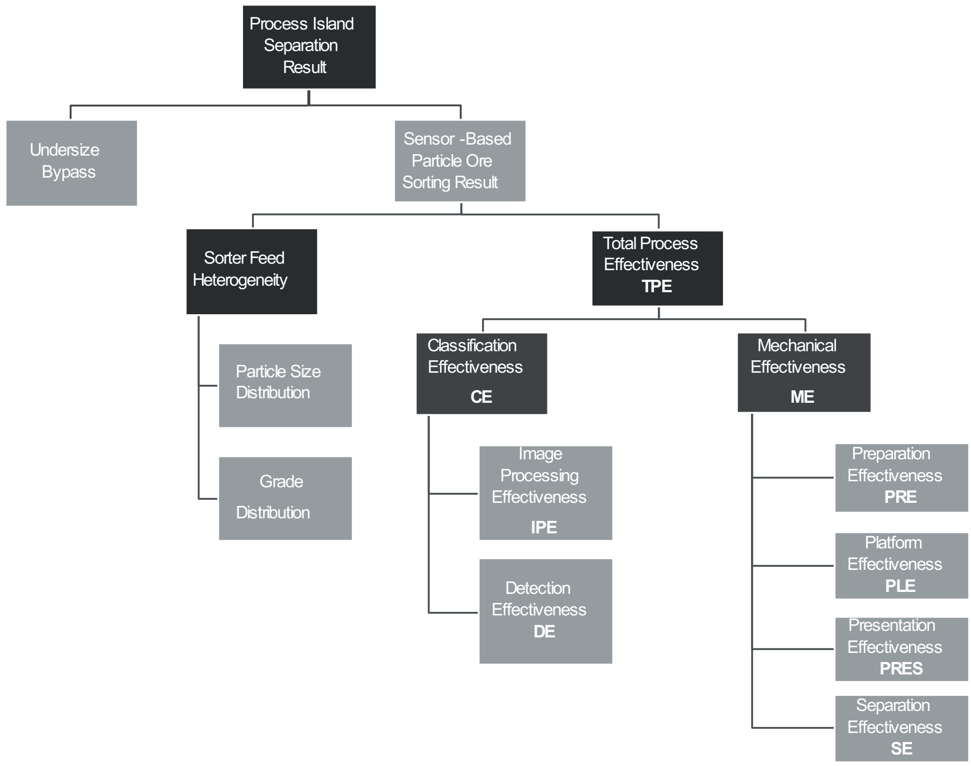

Figure 4. Overview of process effectiveness contributions.

Figure 4 shows an overview of the process effectiveness contributions. They are grouped into two main groups, which are mechanical effectiveness contributions (ME) and classification effectiveness contributions (CE). Please note that terminology has been slightly adapted from the ones originally introduced for clarity. The strong differentiation between ME and CE contributions results from a process where the separation criterion is decoupled from the separation force. Hence it is very useful for understanding and optimising the TPE to differentiate between those factors.

CE is the determining factor, whereas ME is the execution. Both factors combined produce the separation result and both have the power to disable the TPE needed for financial feasibility. While CE is the dominating factor for TPE as it delivers the separation criterion, ME still deserves more attention than it is often given by professionals and operating crews.

The POS separation results in the combination of the material heterogeneity, which must meet a process with a certain TPE to be able to exploit this heterogeneity to an advantage. In most *test work* configurations and process optimisation, these effects are not decoupled from each other. Single particle test work is the only methodology which enables this, and which gives full clarity of the proportions from the individual sorter feed heterogeneity ME and CE contributions. As the feed heterogeneity is the underlying precondition, we will first discuss heterogeneity characterising, followed by elaborating on single particle test work for ME determination and optimisation.

Feature Definition

A feature (i.e., an analytical feature) refers to a distinctive aspect, quality, or characteristic of an individual particle. It is required that a feature stands out or is notable. For the development of the application of POS, for heterogeneity analysis, as well as for the calibration and validation activities before and after the installation of the equipment, it is therefore important to include the most important features that influence the economic value generation and the signal recorded by the equipment. The key for sample selection and test work design is to define and to apply suitable and appropriate features. These can be divided into two main groups.

i) *Pay-element features*, which are the economic elements of interest. These characteristics are identified and prioritised as they determine the value generated by the intended, or applied process. Pay-element features are not necessarily detectable, such as in sensor-based particle sorting for gold ores, where proxies are used (Robben, *et al*., 2017).

ii) Elements or minerals that, if present, have a significant influence on measurement are called *technical features*. Other technical features could include elements or minerals which are of importance for process performance further downstream.

In the context of sensor-based ore sorting, features are usually quantitative characteristics and include rational numbers as grades (i.e., elemental or mineralogical concentrations). Two omnipresent and important categorical features often used to define the cut-off grade are ore and waste. Another important categorical feature is lithology.

Categorical features are a good start for characterising an ore in question. In addition to defining the key pay-element features and technical features, it is also important to define the concentration ranges of those that are expected during operation. A good example of feature definition and how it is applied in the context of POS has been published (Robben & Dumont, 2017).

Heterogeneity characterisation
As the process or sensor-based ore sorting is applied on a specific particle size range, *fractionation* (a newly established sampling unit operation, Esbensen *et al*., (2024)) by screening (i.e., granulochemical characterisation or grade-by-size deportment) is an integral part of heterogeneity characterisation.

Table 1 shows an example of a fractionated tin ore. Tin, which in this case occurs in the form of cassiterite, accumulates in the finer size fractions due to the brittle nature of this mineral. When considering sorting the plus 14 mm size fractions, the screen already upgrades the feed from 1.8% to 2.1%. Screening is a relatively inexpensive exercise and also produces a limited number of fractions subjected to crushing/size reduction, representative mass reduction and assaying.

Table I. Fractionation, granulochemical analysis of an exemplary tin ore, secondary crusher product

Size fraction	***Mass Proportion* [%]**	***Sn* [%]**
< 14 mm	24	2.1
14-22 mm	22	1.9
22-32 mm	24	1.3
32-72 mm	30	1.8
Total	100	1.8

Heterogeneity characterisation for determining grade frequency distributions is costly and seldom exercised, especially in coarse particle sizes +10 mm, while mineral liberation analysis is a method enabling such analysis for particles with a maximum size of 1-2 mm. Methodologies for coarser particles are currently being developed, and it is expected that such characterisations will become more accessible and cost-attractive. One new approach using single particle scanning with a POS machine is introduced here.

The grade frequency distributions shown in Fig. 5 describe the heterogeneity characteristics of another exemplary cassiterite ore. These were derived using XRT-sorting equipment. For this ore, a strong correlation between tin grade (Sn%) and the ratio of the high-density pixel class versus total projection area (XRT-index) was established with an R^2 of 0.9996 (Moslemiyekan, *et al*., 2022). Note that use is made of a logarithmic scale for the Y-axis. This grade frequency distribution is based on a measurement of 9999 particles of a secondary crusher product, which was screened into three size fractions.

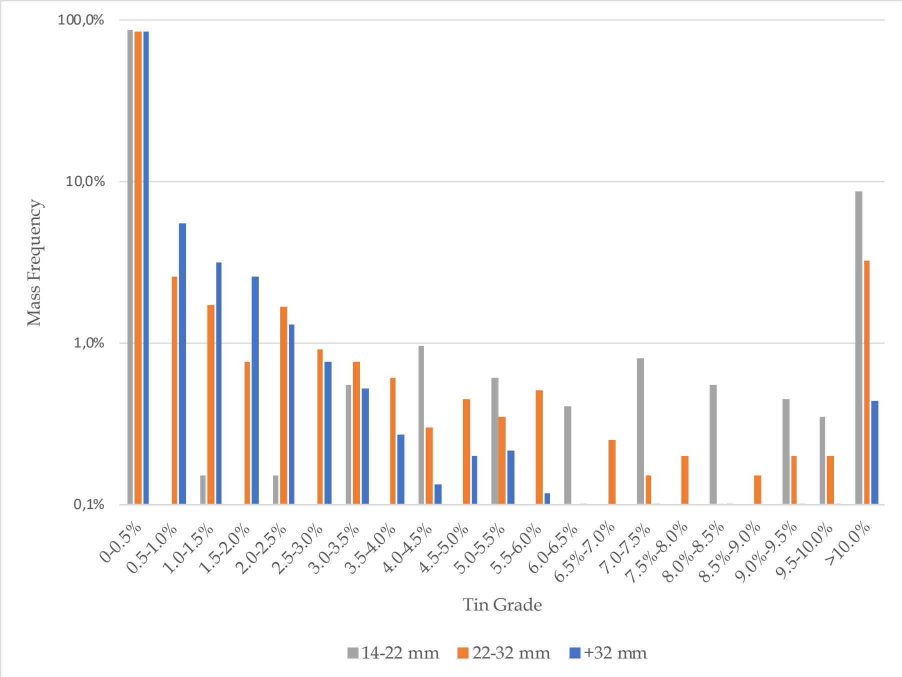

Figure 5. Grade-frequency distribution analysis in fractionated / screened fractions, modified after Moslemiyekan, et al., *(2022).*

The data originates from the production test work executed during the project feasibility stage. For this test work, a 1 mt development waste stockpile at the San Rafael mine, called Cancha 35, was sampled, stratified randomly to a composite sample of 10 t which was sent for separation test work. Note that the three grade frequency distributions are normalised for each size fraction and not offset for the mass they represent in the POS feed stream.

The heterogeneity analysis reveals that in the 14 to 22 mm fraction we have the lowest amount of medium grades between 0.5% Sn and 5% Sn. and high grade (>10% Sn). We therefore have liberation behaviour which is also visible due to the fact that we have about 85% of particles in the two smaller size fractions which do not contain any tin (<0.5% Sn) in comparison to 60% of the mass in the +32 mm fraction. The heterogeneity analysis therefore shows that we can reject significant amounts of sub-economic waste and it also tells us something about the liberation characteristics which may direct crushing activities to reduce the particle size of the +22 mm size fraction.

The project's relevant heterogeneity as grade frequency distribution is transferred, exemplary for the particle size fraction 22-32 mm, into an adapted Mayer II diagram, which was originally introduced for describing liberation as a function (Mayer, 1950). This diagram shows the underlying liberation function enabling physical separation at 100% effectiveness for the 0.1% tin in the rejected waste fraction. The ore heterogeneity would allow rejection of 85% waste (i.e., 15% product yield), upgrading the feed from 0.6% to about 3.1% tin in the product under (unrealistic) ideal conditions.

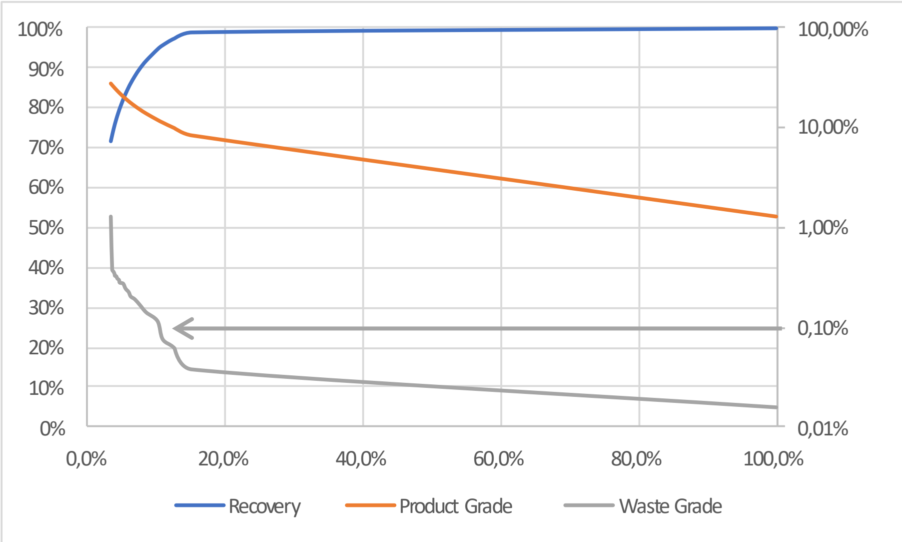

Figure 6. Heterogeneity-based liberation function for the 22-32 mm fraction of secondary crusher product, expressed as a modified Mayer-II diagram.

SINGLE PARTICLE TEST WORK

In project development for sensor-based particle ore sorting, a variety of laboratory tests are typically used to evaluate and develop the optimal process. Test procedures used in project development and process optimisation for POS are:

- Single Particle Test;
- Lithotype Test;
- Variability Test;
- Cascade Test;
- Bench-Scale Test; and
- Performance/Production Test.

Each test offers analysis of different process parameters with varying accuracy, according to the type of ore and the project development stage. These tests are crucial for understanding the heterogeneity characteristics of the ore and determining the most effective sorting strategies. These two aspects are very nearly always analysed together. Effects contributing to the TPE are not well differentiated and analysed separately and this impression is a common source of misinterpretation of test work results.

The only test work that decouples the heterogeneity characteristics of the ore from the effects/ effectiveness of the sorting process, is the so-called a single particle test (SPT) - the most detailed procedure for analysing detection efficiency (DE) on particle level. It provides full information for optimising DE. Fundamental calibration and validation activities are possible by correlating and comparing the sensor imaging data and data interpretation to the true assayed elemental and mineralogical composition. The SPT test is especially suited for projects where the image recorded for one particle does not correlate directly to the pay element(s). The test only gains its full significance if carried out in conjunction with the heterogeneity characterisation of the ore, because this will allow a prediction of the sorting result to be derived (see Figure 4). Furthermore, SPT is especially suited for complex ores where the calibration of sensor image to the grade of a specific particle is multivariate. It is also suitable for green-field projects, where access to sample is limited to (half) drill cores and limited sample availability is balanced with more detailed test work.

Aim

The goal of the SPT is to assess in detail the interaction of the detection system and single particles, to optimise calibration activities and to validate them. The parametrisation of a sensor-based ore sorter's software is based on many variables which each offer considerable degrees of freedom. Costs for SPT are significantly higher than those for other procedures, primarily due to costs associated with scanning *each individual particle*, individual sample preparation and assaying. Therefore, they are less commonly used, but when invoked, it is with one of the following aims:

- Optimisation of DE using calibration and validation on particle scale;
- Evaluation and comparison of different sensors and sensor response to each pay-element feature; correlation of technical features to pay-element features;
- Analysis of elemental and mineralogical composition and associations at the scale of particles tested (usually 20-60 mm);
- Determination of technical potential, suitable project development and testing strategies.

Preparation

As mentioned, definition of appropriate features is key to successful single particle test work. In the case of cassiterite (a particularly simple case) there is only a single pay-element feature, tin grade. The tin grade directly translates into the economic value of each particle. But simultaneously cassiterite is also the relevant technical feature (i.e,. separation criterion), as the high atomic density of tin in the mineral translates into well-discriminating absorption of X-rays while transmitting the particles. As described by Moslemiyekan, a strong correlation between X-ray index and tin grades was established for the San Rafael tin ore (Moslemiyekan, *et al.*, 2022). This means that in the educational example discussed in this paper we have one relevant feature space which is the tin grade.

Sample Acquisition

Particles should be in the particle size of the future equipment feed. A good indication is particle sizing between 20 and 60 mm. The sample selection is based on the heterogeneity characterisation, which delivers the relevant feature spaces. In this sense, samples are selected in order to cover the(se) feature space(s) in *equal amounts*. Samples are specifically <u>not</u> taken to be Theory of Sampling (TOS) compliant.

Instead, particles are selected to <u>cover</u> the relevant feature space(s). The point made here is that an equiprobabilistic sample would deliver a high number for some species in a feature space in the case of it occurring over-proportionately often in the ore. However, for calibration and validation within SPT, we want to spend equal efforts for all relevant species occurring in the ore body and in the future POS feed.

In the example of cassiterite ore, Figure 5 shows that there are more than 80% particles with <u>no tin occurrence</u>, and medium tin grades rarely occur. A campaign to obtain single particles for single particle test work would try to representatively recover particles covering the feature space of tin grade. Whereas barren particles and high-grade particles represent the highest economic impact (negatively and positively respectively), it may be wise to oversample medium grade particles close to the intended separation cut grade, as these may be the most difficult to classify by the detection and data processing system.

For the ore characterised in Figure 5 it is suggested to include 30 to 40 particles in 15 tin grade intervals. Each interval to be 0.1% tin wide. Since the grade of individual particles is unknown when subjecting them to single particle test work, selection of particles in those intervals is a difficult endeavour. A geometallurgical approach is usually taken with geologists selecting hand samples from characterised core (or the mining face) where the context is known, and further analysis has been done.

Please note here that the cassiterite ore is chosen to be an illustrative example. The reader may have discovered that we have a logical short-cut when using the tools for heterogeneity characterisation which then outputs the features which we intend to calibrate for the intended test work. However, this may be an interesting strategy for optimisation of already calibrated and operating equipment.

Test Procedure
Particle registration is the first step. A unique ID is assigned to each of the particles, which are then photographed under controlled conditions, weighed, and packed individually in sealed bags.

Image acquisition is the second step of the experimental work. Particles are fed to the sensor imaging system. This may be with industrial-scale POS equipment, or in specifically designed setups which either move the detection system or the samples. Ideally, image acquisition is conducted at similar speed and geometry compared to the setup in the industrial environments.

After imaging, the particles are split into two sample sets, one for the calibration exercise and the other for the validation. The particles are then destroyed for assaying (crushing, milling, representative mass reduction, elemental and mineralogical assays). Only in coal and iron ore (and other ferrous metals), is a cost-effective strategy of swim-sink-analysis applied for determining particle characteristics. The obtained assay results for the calibration particle set can now be used for parameterisation of the image processing and classification software. In principle, only a binary classification into ore and waste particles is a necessary enabling separation. However, creating an *index* that correlates with the pay-element features enables adjustment and control of the critical cut-grade of the separation. The final stage of the procedure is testing the classification with the validation particles.

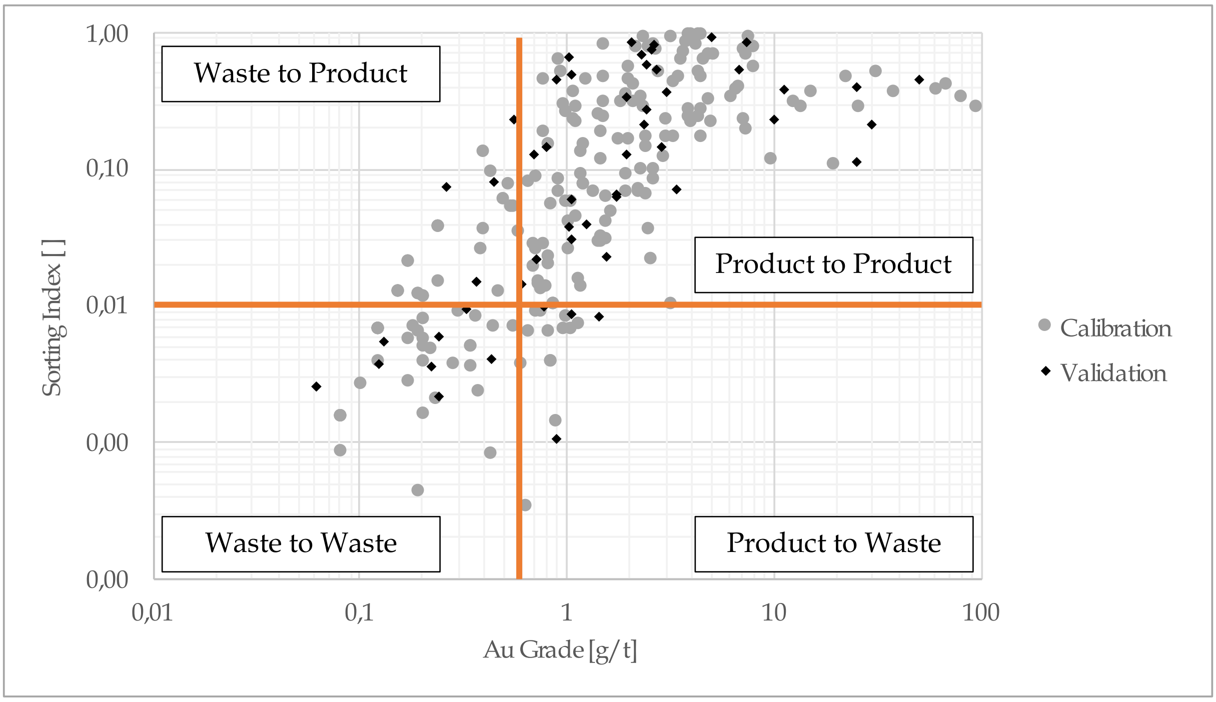

Figure 6. Result of single particle test for a gold ore; assayed gold grade over sorting index.

Figure 7 shows a gold ore example. POS for gold ores is challenging as the technical features have varying correlation to the pay element feature of gold grade. We see a correlation between sorting index and gold grade. However, after calibration, the goal is to validate and evaluate if the application of the sorting index as separation criterion enables a feasible separation.

At a cut-grade of 0.5 g/t Au and a sorting index setting of 0.01, 94.0% of product particles are classified as product (positive-positive allocation; product to product). 72% of waste particles are classified as waste (negative-negative allocation; waste to waste), which does not look like good classification effectiveness. However, this example shows the limits of using classification for evaluation of a physical separation process in mineral processes and emphasises the need for appropriate calibration and validation (while in recycling this is perfectly fine as we are dealing with lots consisting of particles with categorical features). We note that high-grade particles are reliably classified as product. With decreasing gold grade, we have a decreasing probability of classification as waste. Positive to note here is that the high-grade ore between 20 and 100 g/t is reliably recovered. This example has been installed in industry because the grade frequency distribution combined with the TPE results in a favourable separation result.

SAMPLING FOR PROCESS CONTROL AND OPTIMISATION

For POS process islands, there are principally three material streams which are needed to be assayed for process control and operational optimisation. These are the sorter drop fraction, the sorter eject fraction and the undersize stream. The feed can be calculated as the addition of those three. Each of the material streams inherits its own heterogeneity characteristics and fluctuations (variability) that need to be understood for designing appropriate sampling protocols.

The most critical fraction is the depleted sorting fraction (usually waste), which has the lowest grade and the coarsest particle sizing. This is further aggravated by the fact that POS is a physical separation process, where the separation force is decoupled from the separation criterion. This means that (given a high CE medium) and high-grade particles have a similar chance of being misplaced. We therefore have a chance of high-grade particles being seldom misplaced and therefore a 'nuggety' case for sampling. This is in contrary to a – simply speaking – normal distribution created by other physical separation processes such as dense-media-separation.

Introduction to Intrinsic Heterogeneity

When in project development for POS we may also use the analysis for what Chieregati *et al*., call the optimal approach "sampling for sampling" or "sampling calibration" (Chieregati, *et al*., 2023). This is an approach undertaken early in project development between advanced exploration and pre-feasibility study, with the work undertaken either internally or by an expert. In this context, the data generated for SPT can also be used for assessing the material heterogeneity, as we intend to use sampling for process control and process optimisation. Heterogeneity tests are critical for optimising sampling protocols to contribute to a variogram with a low nugget effect (<50%) so that estimation variances are minimised (Chieregati, *et al*., 2023; Francois-Bongarcon, 2004; Dominy, 2014). To achieve low nugget variances, larger primary sample masses may be required, and subsequent sample splitting must be optimised with regards to the FSE (Chieregati, *et al*., 2023).

When operating in the context of POS, we are looking at relatively coarse particle sizes, where extraction of TOS-compliant primary samples is critical. Delimitation and extraction errors need to be reduced before turning to the understanding of the Fundamental Sampling Error (FSE) as the irreducible minimum that can be achieved even after homogenisation of a sample, as it is the only sampling error that can be estimated before performing sampling (Petersen, *et al*., 2005). The FSE depends on the constitutional heterogeneity, which reflects the internal composition between individual fragments of sampled ores, and relates to sample weight, mineral fragment size, and shape, liberation of the analytical phase of interest (pay-element feature), pay-element grade, pay-element mineral phase density and gangue density (Minnitt, *et al*., 2007).

Experimental approaches available are the 50-fragment method (Gy, 1988), the heterogeneity test (HT), the sampling tree experiment (STE) or the duplicate series/sample analysis, and the segregation free analysis (SFA) (Pitard, 2019; Magri, 2007; Minnit, *et al*., 2007; Minnitt, *et al*., 2011; Dominy & Xie, 2016). Debates have addressed the limitations of some approaches and the true nature of variability measured in heterogeneity experiments (Pitard, 2011). All the tests are based on assaying individual or *groups of particles*, which is a cost-effective approach. However, aren't some of those tests determining distributional heterogeneity as they are based on assaying groups of fragments?

We want to introduce a new sensor-based approach here, using the data output from a POS machine as it delivers single-particle analyses, like Gy's 50-fragment method, which comes close to determining the so-called constitutional or intrinsic heterogeneity (CH or IH). The new method presented in this section may add a source of input with a low entry barrier to an integrated approach tackling (especially heterogeneous) ores (Chieregati, *et al.*, 2023).

We return to our illustrative cassiterite example case, to determine the sampling constants, K and α, in the modified formula for estimating the FSE, based on the estimation of the intrinsic heterogeneity of the lot (IH_L). The determination of K and α allows the calibration of Gy's formula to our specific example ore following the work done by Chieregati *et a*l., (2023). The estimation of IH_L allows the calculation of the FSE and subsequently the optimisation of sampling protocols. Equation 1 shows Gy's original formula (Gy, 1982):

$$s_{FSE}^2 = \left(\frac{1}{M_S} - \frac{1}{M_L}\right) fgcld^3 \qquad [1]$$

where s_{FSE}^2 is the relative variance of the FSE, M_S [g] is the mass of the sample, M_L [g] is the mass of the lot, d [cm] is the nominal top-size (d_{95}) of the particles (called fragments) in the lot, and the other variables are dimensionless: f [-] is the shape factor, g [-] is the granulometric factor, c [-] the mineralogical factor, l [-] the liberation factor. François-Bongarcon proposed a modified equation based on Gy's original formula as follows (Francois-Bongarcon, 1998):

$$s_{FSE}^2 = \left(\frac{1}{M_S} - \frac{1}{M_L}\right) fgcld_l^b d^a \qquad [2]$$

Here d_l [cm] is the liberation diameter and b is (3-α) where α is determined experimentally. IH_L comprises of the components of the FSE equation at a given d value:

$$IH_L = fgcld_1^b d^a = Kd^a \qquad [3]$$

IH_L can therefore be substituted into Gy's formula:

$$s_{FSE}^2 = \left(\frac{1}{M_S} - \frac{1}{M_L}\right) IH_L = \left(\frac{1}{M_S} - \frac{1}{M_L}\right) Kd^a \qquad [4]$$

The source data for determining K and α is single particle (object) data output from the marginal cassiterite ore, which is separated using POS at the San Rafael mine. The feed is fractionated into three size classes: 14-22 mm, 22-32 mm and 32-75 mm. We have the single particle scans of particles in the three size fractions as characterised in Table I and Figure 5. The data displayed in this grade-frequency-distribution is used for the sensor-based heterogeneity test. The X-ray attenuation image of each particle is translated into a tin grade (Moslemiyekan, *et al.*, 2022). For this, a number of pixels of the planar X-ray image projection of each particle is transferred into a square mesh sizing, which again is converted into particle weight M_q [g]. With each the derived particle grade a_q, the corresponding particle mass M_q, and the total mass of particles M_Q, the weighted average grade of the lot is calculated using formula 6 with the number of particles in the object list n.

$$a_Q = \frac{1}{M_Q} \sum_{q=1}^{n} a_q \, M_q \qquad [5]$$

Table II shows the number of particles recorded per fraction as well as the average calculated tin grade. Please note that although we are virtually sampling single particles, a measurement uncertainty is inherited in the system of detection and calibration of x-ray attenuation to tin content.

The estimated constant factor of intrinsic heterogeneity EST IH_L [g] is then calculated for each of the three size fractions using the simplified method described by Gy (1988) and Pitard (1993) according to Equation 6.

$$EST\ IH_L = g \sum_{q=1}^{n} \frac{(a_q - a_Q)^2}{{a_Q}^2} \frac{{M_q}^2}{M_Q} \qquad [6]$$

The dimensionless granulometric factor g [] is the mass proportion of the respective size fraction in the secondary crusher stream feeding the POS sorters. For the derivation of the sampling constants K and α, the nominal particle size d [cm] for each particle size is calculated using Equation 7 and the upper screen cut d_{95} and lower screen cut d_5 of the size fraction:

$$d = \sqrt[3]{\frac{d_5^3 + d_{95}^3}{2}} \qquad [6]$$

To check the validity of the results, an additional size fraction is evaluated using the data of a different single particle analysis with X-ray tomography. For a detailed description of the study, please refer to Robben *et al.*, (Robben, *et al.*, 2022). The results are summarised in Table II:

Table II. Estimation of intrinsic heterogeneity derived from XCT and XRT data for San Rafael ore

Measurement principle	*Min. particle size d_5 [mm]*	*Max. particle size d_{95} [mm]*	*Nominal particle size d [cm]*	*Mass proportion M/g [%]*	*No. of particles in scan n []*	*Tin Grade a_Q [%]*	*EST IH_L*
XCT	2	4	0.02	5	4947	6.2	0.02
XRT	14	22	2.29	22	2000	1.9	37
XRT	22	32	3.36	24	2000	1.3	378
XRT	32	72	7.30	30	6000	1.1	8712

The result is plotted in a bi-log graph with the nominal fragment diameter d on the x-axis and IH_L on the y-axis. This allows for the estimation of K and α using the regression line equation, where the constant of proportionality in the power function is K and the exponent of the power function is α. In order to compare the two measurement principles, K and α are derived for the XRT measurements only, and then for the XRT measurements combined with the XCT measurement. The bi-log graphs for those two analyses are displayed in the Figure 7. It can be seen that the XRT only value of K (1.0006) is very close to the XRT+XCT K value of 1.1199. Also, α is closely related with 4.6177 for the XRT only measurement and 4.5399 of the XRT+XCT combination.

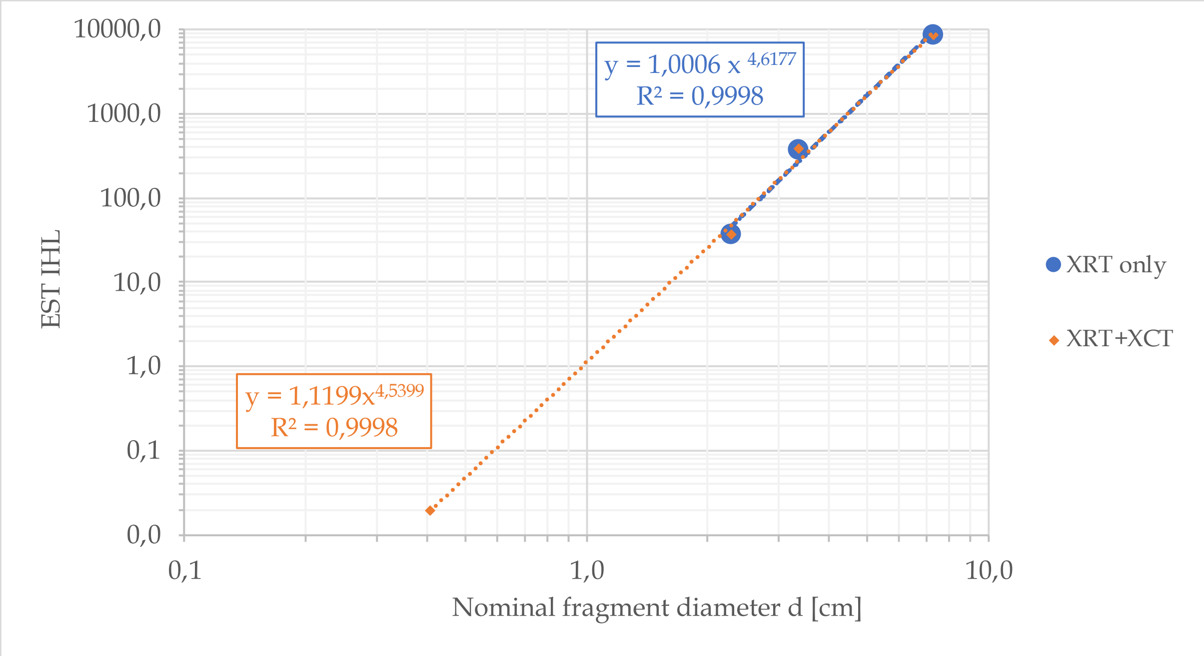

Figure 7. Sensor-based heterogeneity test results for San Rafael cassiterite ore.

The estimation of the intrinsic heterogeneity of the lot IH_L is important for understanding the properties of the ore. However, for supervising and optimising POS process islands, sampling needs to be conducted on the output fractions. Using and adapting the process modelling results of Moslemiyekan, *et al.*, (2022), the grade-frequency-distributions of the product and waste stream of the three size fractions are derived and the IH_L is calculated for each of those six streams. Table III and Figure 8 show the results.

Table III. Estimation of inherent heterogeneity derived for the simulated sorted fractions

				Feed		***Product***		***Waste***	
Nominal particle size d [mm]	***Mass proportion M/g [%]***	***Mass yield Y [%]***	***Recovery R [%]***	***EST IH_L***	***Tin Grade a_Q [%]***	***EST IH_L***	***Tin Grade a_Q [%]***	***EST IH_L***	***Tin Grade a_Q [%]***
2.3	22	25	91,9	42	1.9	6	7.0	75	0.10
3.4	24	50	96,9	378	1.3	1452	2.5	488	0.08
7.3	30	56	94,0	8712	1.1	2375	1.8	3634	0.24

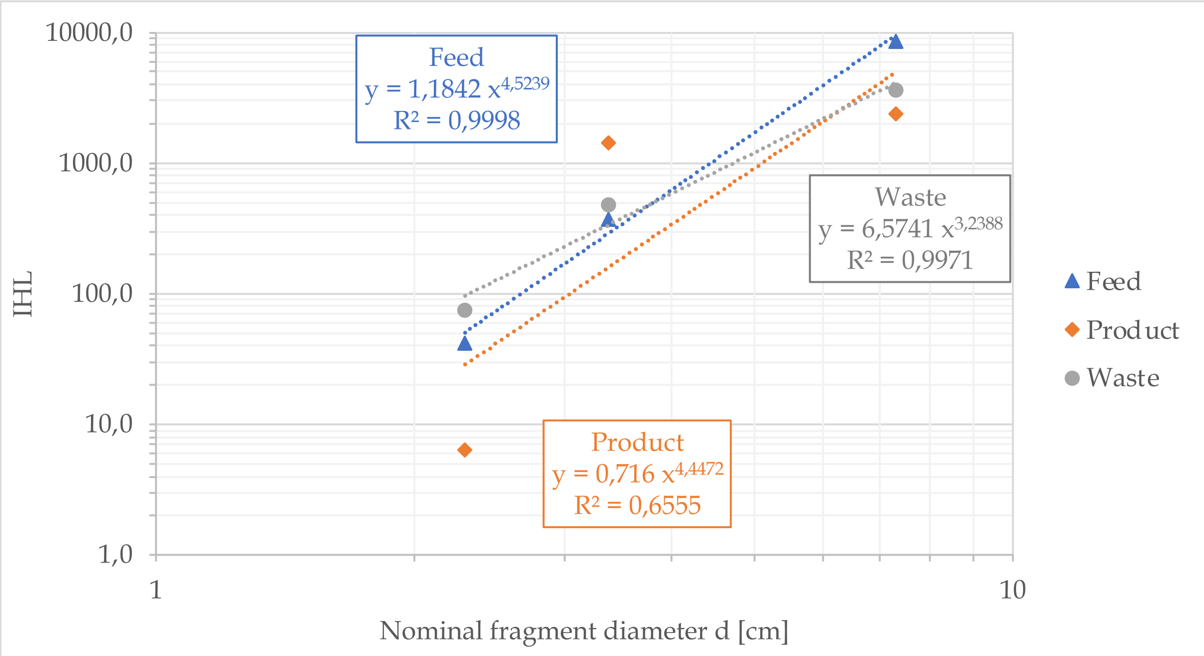

Figure 8. Sensor-based heterogeneity test results for San Rafael cassiterite ore.

CONCLUSIONS

New technologies are opening up possibilities to determine heterogeneity of particulate lots. Understanding material heterogeneity is necessary to evaluate a potential application of all sensor-based ore sorting processes, especially for POS. The grade-frequency-distribution, described for the pay-element feature(s), describes the heterogeneity which can be exploited with a physical separation process. At the same time, efforts taken in project development and POS plant auditing and optimisation deliver data, which has a broader relevance, and which can be used for sampling strategy design and optimisation. Thus, it is enables equipment setup and optimisation during operation when sampling protocols deliver reliable information.

Although the sensor-based heterogeneity test differs essentially from the ones proposed by Gy (1988), Pitard (1993), François-Bongarçon (1988), and Minnitt *et al*,. (2011), since the whole lot is assayed through the analysis of each individual fragment in the lot, we have shown that it is possible – with a known correlation of technical features of the ore and pay-element features – to derive K and α from the sensor-based single particle volume or image scans. This source of data can therefore also be utilised for optimising and recalibrating minimum sample masses and sampling protocols. During project development, it can assist in designing sampling and test work strategies. During operation, it can be applied to permanently review sampling strategies and to potentially adjust them for cost-effective process control. This is valid, as the comparison between IH_L derivation in particle sizes between 14-75 mm and 2-4 mm has shown.

For process engineers, understanding the grade-frequency distributions delivers transparent and detailed information about the ore. For POS as well as for downstream processes, this information can be used to optimally exploit heterogeneity, or in the words of Mayer, “the liberation function of the particulate lot” (Mayer, 1950).

Spending the efforts on SPT enables optimum calibration and a true validation of the POS equipment and allows for maximising TPE and the process island separation result or, in other words, the creation of economic value for the operation. At the same time, it delivers the input for designing and optimising sampling strategies.

One question may arise: why do we need sampling for sorting at all, when the machine is able to determine IH_L as demonstrated in this paper? The answer is that we have multiple effects on TPE that are not captured by the detection system. Furthermore, we need an independent instance to check on a system where calibration or ore properties may drift and the total analytical error becomes significant and indistinguishable from FSE. A good example of a sampling station for a POS installation has been described by Robben *et al*., (2014).

OUTLOOK

Debates about heterogeneity tests are many and range from the sample selection method (by randomly collecting individual fragments or by splitting the lot using riffle splitters or rotary dividers) to the formulation for estimating the intrinsic heterogeneity of the lot and, consequently, the fundamental sampling error. Different approaches have been proposed based on the first "50-fragment method" proposed by Gy (1988) and his original formula to calculate the relative variance of the FSE. The validity of K and α calibration is a matter on which a unanimous conclusion has not yet been reached.

However, regardless of which approach/formulation is correct, this article demonstrates that sensor-based heterogeneity testing is technically viable and can be integrated with process development and lot characterisation activities. We intend to further develop sensor-based heterogeneity testing to reduce the measurement inherent instrument variability by calibrating and validating the analytic grade (pay-element features) optimally to the imaging data. Furthermore, comparative work on traditional heterogeneity testing shall deliver further understanding of the similarities and differences of the approaches.

The sensor-based particle-based heterogeneity testing for many ores delivers grade-frequency-distributions which do not follow statistical distributions. We expect that TOS can further evolve by testing and understanding particle-based constitutional or intrinsic heterogeneity.

NOMENCLATURE

POS	Sensor-based particle-ore sorting
XRT	X-ray transmission
XCT	X-ray computed tomography
ME	Mechanical Effectiveness Contributions
CE	Classification Effectiveness Contributions
TPE	Total Process Effectiveness
IPE	Image Processing Effectiveness
DE	Detection Effectiveness
PRE	Preparation Effectiveness
PLE	Platform Effectiveness
PRES	Presentation Effectiveness
SE	Separation Effectiveness
SPT	Single Particle Test
TOS	Theory of Sampling
FSE	Fundamental Sampling Error
HT	Heterogeneity Test
STE	Sampling Tree Experiment
DSA	Duplicate Series Analysis
IH_L	Intrinsic Heterogeneity of a lot
s_{FSE}^2	relative variance of the FSE
M_s	mass of the sample [gr]
M_L	mass of the lot [gr]
g	shape factor [-]

d nominal top size of the lot, d_{95} [cm]
c mineralogical factor [-]
l liberation factor [-]
a_Q weighted average grade
a_q particle grade
d nominal particle size
R recovery [%]
Y product yield [%]

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