

THE COMPUTER MAPPING AND ASSESSMENT OF BOREHOLE AND SAMPLING DATA FOR STABLE MINERALS, PARTICULARLY AS APPLIED IN COAL MINING

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SYNOPSIS

The paper describes a method of producing iso line plans using a mathematical technique for interpolating between boreholes, and its computer application. The iso line plans are used for assessing the value parameters of an ore body. The computer programme also produces means and standard deviations to supplement these iso line plans.

The mathematical techniques tested for the production of a satisfactory fit of the data were the polynomial, Fourier and moving average technique.

The iso line plans are used for assessing the quality of the ore and other salient features such as depth to the floor of the ore body at different points, with a view to deciding how best the ore may be mined.

The plans are plotted by the computer to any scale required.

INTRODUCTION

To assess the economic exploitability of an ore body many tedious and repetitive calculations must be carried out. Observations of borehole or development samples have to be processed yielding measures of ore reserve and quality in each sampled area, and also measures of the worth of the ore body as a whole and of variation in quality from section to section of the ore body.

The observations made on the ore (which we shall term 'value parameters') may embrace a variety of characteristics of the ore. Obvious value parameters are those directly related to the economics of mining, viz. assay values, ore body thickness and depth below surface. Other value parameters however may be almost as important. In the case of coal for example a purchasing contract may specify limits of acceptability on other value parameters such as calorific value, ash content or the percentage of volatile matter. In this case, a detailed study of these value parameters will also be crucial. This detailed study must include an assessment, not only of the body as a whole, but also the regional variation in value of the parameter. An obvious and satisfactory way of displaying this variation is by drawing iso lines of constant value parameter on a map of the ore body and supplementing this with the means and standard deviations of the value parameters taken over all boreholes. Such plans, considered together, show up the zones of significance. The relatively stereotyped nature of the operations involved in this process makes it amenable to computer solution. In order to produce iso line plans on the computer we require some mathematical formula for producing a 'valuation surface'—a mathematical

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function for interpolating between boreholes. This paper deals with one such mathematical technique and its computer implementation. Originally it was developed for the valuation of coal fields and for this historical reason examples cited are of colliery data. Both the technique and the implementation are of potentially much wider application.

TECHNIQUES EXAMINED

A number of mathematical models for valuation surfaces are available and four have been considered for computer implementation.

The first is the 'triangle' method of linear interpolation. In this technique which is used frequently by mining engineers for manual surface fitting, the area is broken up into small triangles defined by boreholes. The value at any point within a particular triangle is estimated by linear interpolation of the values at the three boreholes defining the apices of the triangle. There are several objections to this model. One is that the decomposition of the area into triangles is subjective and that hence results are not reproducible. Another (and much more serious) objection from the statistical point of view is the implicit assumption that the value parameters are measured with perfect accuracy and that the cores obtained from the boreholes are representative of the areas nearby. In practice obviously neither assumption is justified thus no attempt was made to implement this technique.

A second approach which suggests itself is the use of a polynomial. The valuation surface is obtained as a polynomial in x and y co-ordinates. The value of this polynomial at any given point gives the predicted value of the value parameter at that point. The coefficients of the polynomial may be found by means of a traditional least squares fit based on the borehole values given or by the more modern minave and minimax fitting techniques. This technique has been implemented and tried in practical cases but results are, on the whole, disappointing. Polynomials of order 6 and less were found to be inadequate for representing data of the complexity quite commonly found, with the result that valuable information was simply 'smoothed out'. Further disconcerting properties of these least squares polynomials are that they are unduly influenced by boreholes near the periphery of the area considered, and that they must, by their very nature, become unstable at the edges.

A third, related approach involves the use of finite Fourier series rather than polynomials. These series have several advantages over the polynomial series. They are considerably more flexible and can give much better representations of complex surfaces. Further, they are better behaved at the edges. On the other hand, they are inclined to show the sharp discontinuities characteristic of Fourier series, and once again are not capable of representing very complex surfaces. On available evidence, the Fourier series, while better than polynomials, are often simply inadequate for accurate assessment of the borehole data.

In Figs. 1 and 2 comparative iso line plans of seam widths produced by a 6th order polynomial, and a 6th order finite Fourier series are shown. The positions and values (seam widths) of the boreholes are shown in Fig. 3 for comparison. The multiple correlation, which is a measure of the accuracy of the valuation surface in representing the data, is considerably higher for the Fourier than the polynomial surfaces, though both techniques suffer from comparison with the moving average technique discussed below. For example, it is clear that both have smoothed out some useful information—a serious shortcoming.

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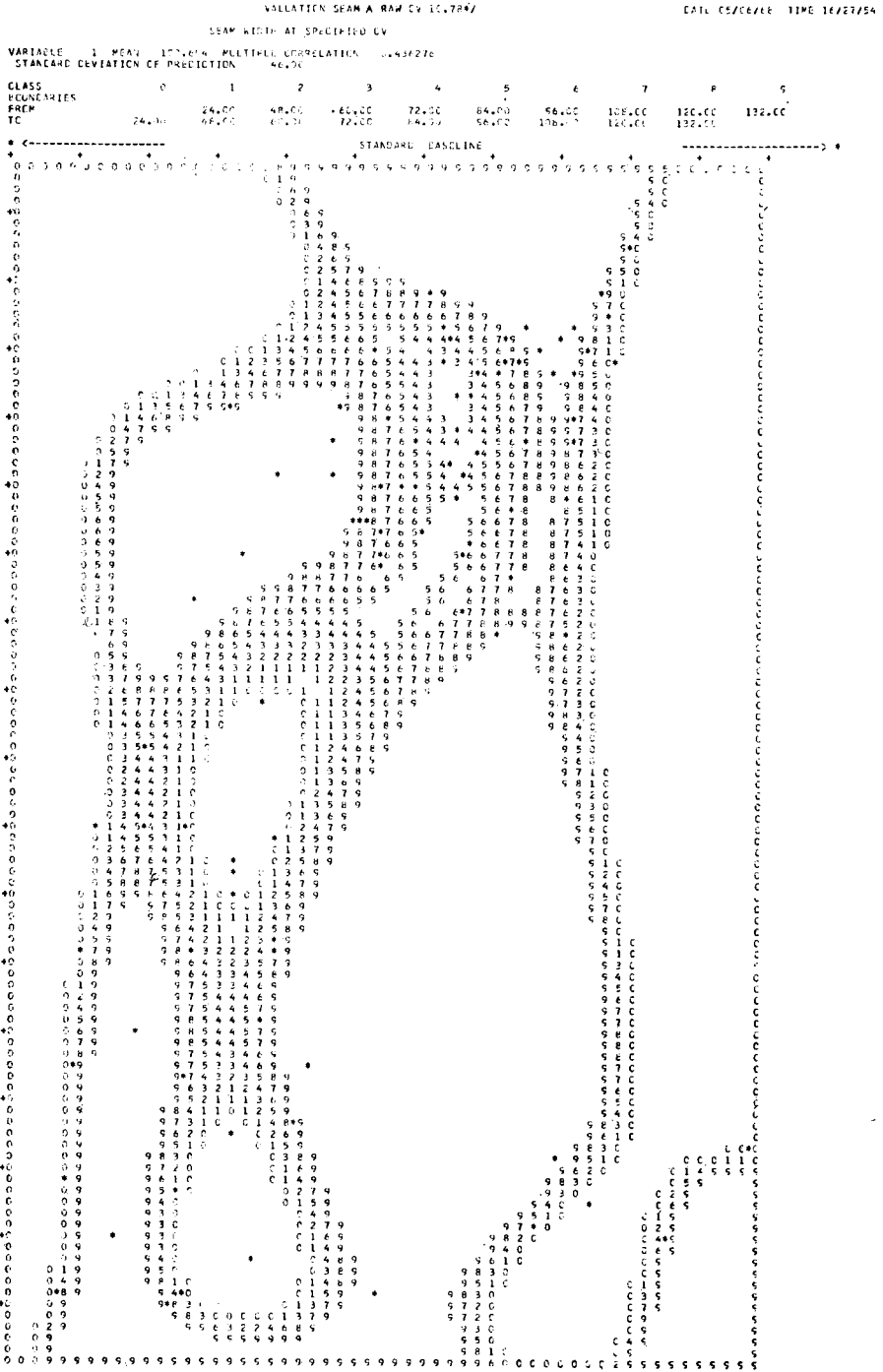


Fig. 1—6th order polynomial fit

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VALLATION: SEAN A RAW CV 10.7847

DATE 05/06/68 TIME 16/32/41

SEAN WIDTH AT SPECIFIED CV

VARIABLE	1 MEAN	100.00%	MULTIPLE	CORRELATION	1.001735																																														
STANDARD DEVIATION OF PREDICTION	40.664																																																		
CLASS	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
BCUNARIES FROM TO	24.000	24.000	40.000	40.000	60.000	60.000	80.000	80.000	100.000	100.000	120.000	120.000	140.000	140.000	160.000	160.000	180.000	180.000	200.000	200.000	220.000	220.000	240.000	240.000	260.000	260.000	280.000	280.000	300.000	300.000	320.000	320.000	340.000	340.000	360.000	360.000	380.000	380.000	400.000	400.000	420.000	420.000	440.000	440.000	460.000	460.000	480.000	480.000	500.000		

Fig. 2—6th order finite Fourier series fit

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/•EVALUATION OF SEAM 1•/

DATE 06/11/68 TIME 23/54/59

MAP OF BOREHOLES WITH SEAM WIDTHS IN INCHES

SCALE ON ALL MAPS: 1: 20000

12" BASELINE

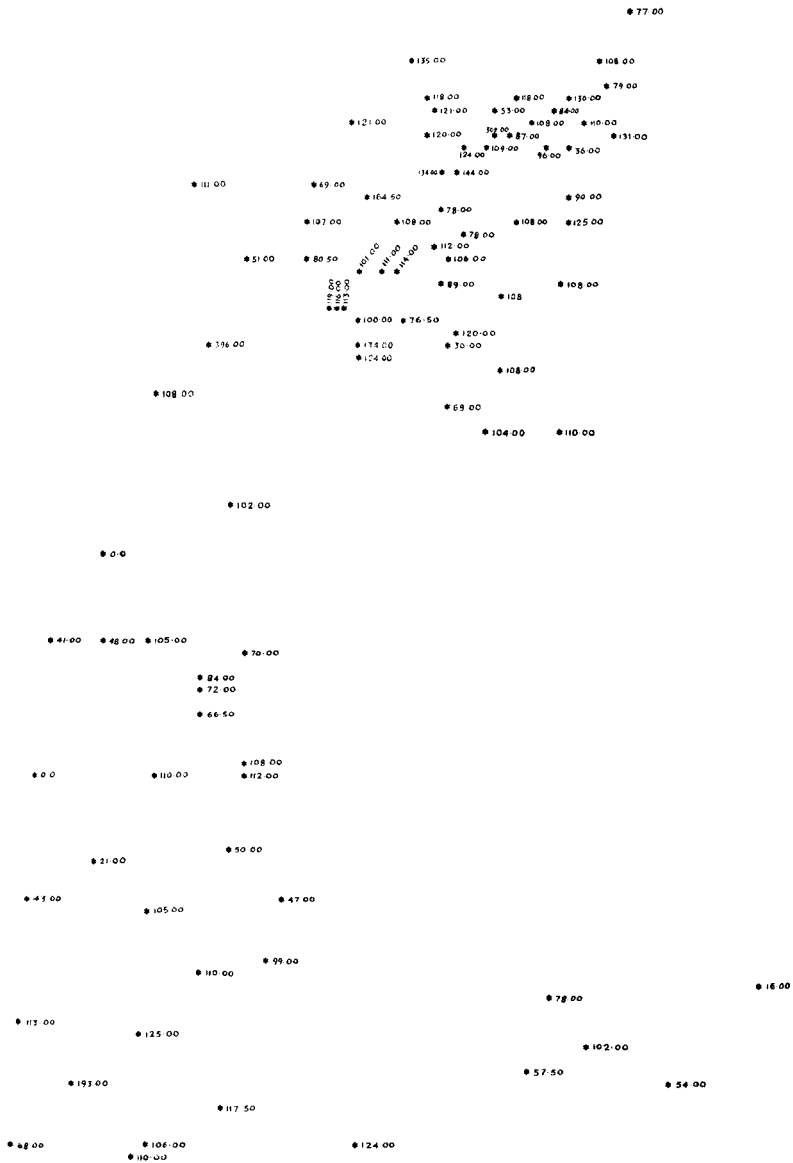


Fig. 3—Position and values of boreholes

The fourth and most satisfactory technique to date is the weighted moving average technique, pioneered in this country by Krige¹. In this technique, the value over any area is estimated by a weighted average of nearby borehole values, the weights decreasing with distance. In terms of this very general definition, the 'triangle' technique is a form of weighted moving average using particularly simple (though subjective) weights. Krige's moving average was designed for the valuation of South African gold mines, where the basic data consist of not borehole, but development and stope samples. The sampling is done at regular intervals over the irregular grid of reef drives and raises, and the stope sampling along stope faces from time to time as these advance. Krige's technique breaks the ore body down into regular squares, and produces estimates of total reserve within each square from data in that and/or the squares in its vicinity. The technique makes efficient use of the regularity in the data, but also relies heavily on that regularity. The intensive sampling on gold mines is motivated by the extreme variability for which gold is notorious. Many other minerals have much more stable properties, and in such cases the regular grid of samples will generally be replaced by a much more haphazard set of borehole readings. In this case, Krige's technique in the form used on the gold mines breaks down.

Accordingly a more general moving average technique is proposed. This moving average does not depend for its success on any regularity in borehole data, and is accordingly well suited to valuation of the more stable minerals.

MOVING AVERAGE TECHNIQUE DEVELOPED

Suppose the observations consist of a set of n measurements r_i of the value parameter at co-ordinates x_i, y_i with $i = 1$ to n . Suppose that we wish to derive an estimate r of the value parameter at some arbitrary point with co-ordinates x, y . The estimate proposed is:

$$r = \frac{\sum_{i=1}^n \frac{r_i}{(ak + d_i^k)}}{\sum_{i=1}^n \frac{1}{(ak + d_i^k)}}$$

where d_i is the distance from x_i, y_i to x, y and a and k are parameters whose physical significance is discussed below.

r is a weighted moving average of all the measurements. The functional form of the weights is particularly simple—the weights assigned to the measurements reducing exponentially as the distance from the borehole to the point predicted increases.

This predictor r does not suffer from any of the drawbacks to the other techniques mentioned above (though of course it has some peculiar to itself). The valuation surface makes allowance for errors of measurement; it is stable at the edges and is not unduly affected by peripheral observations; the surface is smooth but capable of representing data of any complexity; the use of the predictor does not require any systematic placing of observations within the ore field.

This technique has been proposed by Sharapov² and is in use in the United States of America in the special case $a = 0$. The more general case $a > 0$ has not been found in the literature.

As yet no general mathematical formula has been derived which would enable predictions of optimum values for a and k to be made, but values have been suggested

by trial and error. To gain some insight into the physical significance of a and k , let us consider the effect of varying their values.

If $k = 0$, then all points carry the same weight. This yields a perfectly flat valuation surface, the prediction at any point being simply the average of the n measurements. As we increase k , so we increase the rate at which the weighting factor falls off with increasing distance. Thus, if the ore is of an unstable type then the measurements at distant boreholes will provide little information on the value at the point being predicted. In this case, we will use a large value of k to give low weight to such distant boreholes. If, on the other hand, the ore is a stable one, then we will not want the weights to fall off as sharply with increasing distance, and a smaller value of k will be used. Thus k is determined by the large scale stability of the ore.

The value of a on the other hand is determined by the magnitude of errors of measurement and of local variability of the ore. Imagine two boreholes close together. If $a = 0$ then a prediction close to one of them will mirror the actual borehole measurement. If $a > 0$ then this will not occur—any prediction, even at the location of a borehole will not ignore the evidence of other nearby boreholes but will take these borehole measurements into account. Thus a has the effect of a smoothing factor. By making $a > 0$, we allow for the fact that our borehole measurements contain errors by always bringing into use the information from other nearby boreholes. Conceptually the use of the factor a is rather similar to moving all boreholes a fixed distance away from the point being predicted.

If all measurements are highly accurate, and all boreholes are closely representative of the immediately contiguous area, then we require very little smoothing of the data, and should use a value of a close to zero. If there are larger errors or if there is a large measure of highly localized variability in the ore, then rather more smoothing is required, and a larger value of a should be used.

So much for the nature of a and k . The choice of actual value for either parameter is a difficult problem. It is hoped that mathematical geological deposition models will enable optimum values for a and k to be found, but in the meantime these models are not at hand and the problem must be tackled heuristically. This analysis has been partially completed for coal data. On the computer, part of the problem is easily solved by practical considerations. Only k values of 0, 2, 4, 6 . . . are economically feasible—any other values of k would be prodigal of computer time. Initial work suggested the use of $k = 2$, but recently there have been indications that a value $k = 4$ is, in many respects, better, and experimentation on k is continuing.

A tentative value for a is 200 ft. This choice has been made after a study of the plans produced using different values of a . Fig. 4, shows an iso line plan produced by the moving average technique using $k = 2$ and $a = 200$ ft. The data are those used to illustrate the polynomial and finite Fourier techniques. If a is decreased, then a vast amount of spurious fine detail appears—if a is increased then even broad features of the plan are smoothed out.

Some theoretical analysis and experience in using the technique have led to the following comments and conclusions:

- (a) The technique works best when the boreholes are drilled on a regular grid—this is to be expected of almost any interpolation scheme.

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Fig. 4—Moving average fit with K = 2 and A = 200 ft

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- (b) In a 'data void', i.e. an area which has no boreholes near it, the prediction settles down to the average of the value parameter over all boreholes. This is a reasonable prediction in the absence of any information.
- (c) Discontinuities such as dykes or faults, which can ruin a polynomial or Fourier fit, do not have an undue effect on the moving average. Near such a disturbance, some symptomatic crowding of iso lines may occur, but away from the discontinuity the predictor is unaffected.
- (d) The valuation surfaces produced may be (and have been) used to pinpoint areas of interest at which further observations on the ore body should be made, and also to assist in the selection of shaft sites and mining layout.

IMPLEMENTATION

As mentioned earlier, this technique was developed for use in a suite of computer programmes for the assessment of ore fields. It has been embodied in a FORTRAN IV programme written for use on the University of the Witwatersrand IBM System 360 Model 50 computer. The programme is a flexible one, and can handle up to 10 distinct value parameters on any one run. It provides the option of using the moving average technique or a polynomial or finite Fourier series of any order up to 6. (This upper limit can readily be extended.) The programme performs many functions—some of them optional. Fairly comprehensive checks are made on the input data and data errors found are listed out on the printer. Fig. 5 shows an actual error listing. Each type of error is assigned a severity—for example, a borehole whose co-ordinates lie outside the prespecified range raises an error severity of 2. The user specifies the error severity that he is prepared to tolerate, and if an error of higher severity is encountered, the programme completes the error checking, then terminates.

VALLATION SEAM A RAW CV 1C.78*/

INPUT DATA ERROR LISTING

```

4W80  24815. 24086.  3 P.CC
RANGE CHECKS ON VARIABLES  1

4W99  2350C. 16099.  356.CC
RANGE CHECKS ON VARIABLES  1

4WS1C  20402. 11631.  0.C
RANGE CHECKS ON VARIABLES  1

4WS11  16910. 796C.  0.C
RANGE CHECKS ON VARIABLES  1

4WS17  90646. 8825.  126.CC
COORDINATES OUT OF RANGE. BOREHOLE DELETED

4WS18  14661. 8082.  21.CC
RANGE CHECKS ON VARIABLES  1

4WS4C  8465. 20063.  16.CC
RANGE CHECKS ON VARIABLES  1

4WS52  10251. 5191.  193.CC
RANGE CHECKS ON VARIABLES  1

4I  24865. 20741.  164.5C
RANGE CHECKS ON VARIABLES  1

HIGHEST ERROR SEVERITY  2
SEVERITY EXIT PARAMETER  2
NO OF ADMISSIBLE BOREHOLES  91
NO OF ERRORS  9

```

Fig. 5—Error listing

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Certain basic statistics of the value parameters are computed—maxima, minima, means, standard deviations and intercorrelations and these are listed out as in Fig. 6.

Other information available on request includes an analysis of the degree of association between the actual borehole data and the valuation surface. Fig. 7 shows part of a listing of actual observations, the values predicted by the fitted surface, and the errors of prediction. Fig. 8 shows a prediction error histogram. This is invaluable to the statistician studying the properties of the valuation surface. Also available is some other information of a more specialist nature for use in developing mathematical geological models.

The runs are relatively inexpensive. On the University installation, there is an overhead (almost constant) of about R5 for reading in and checking the data. Thereafter the cost of the plans is:

- (a) 6th Order polynomial: R3 per square foot of plan.
- (b) 6th Order Fourier series: R4.50 per square foot of plan.
- (c) Moving average: R4 per square foot per 100 boreholes.

These figures are approximate, and are in any event marginally dependent on a number of other factors.

DATE 20/06/68 TIME 14/22/25									
VALUATION SEAM B.									
SEAM WIDTH	RAW MEANS	RAW YIELD	RAW C.V.	RAW ASH	WASHED YIELD	WASHED C.V.	WASHED ASH	WASHED VOLATILES	RAW C.V. COAL ONLY
103.256		92.150	11.303	25.115	71.308	13.451	13.654	12.355	12.154
STANDARD DEVIATIONS									
28.919		8.370	0.971	4.505	10.782	0.281	1.587	2.029	0.745
MAXIMA									
149.000	100.000	13.100	37.300	90.600	14.100	17.000	14.800	14.100	
MINIMA									
39.000	70.700	8.800	15.600	35.900	12.800	10.000	5.300	10.300	
CORRELATION MATRIX									
1.00000	0.36043	0.44805	-0.46424	0.35395	-0.15078	0.12525	0.15645	0.14337	
0.36043	1.00000	0.61451	-0.55846	0.15021	0.17867	0.07205	-0.23723		
0.44805	0.61451	1.00000	-0.59455	0.07600	-0.00343	-0.02510	0.04561	0.33175	
-0.46424	-0.55846	-0.59455	1.00000	-0.71232	-0.01605	0.04412	-0.02808	-0.40545	
0.35395	0.15021	0.07600	-0.71232	1.00000	-0.05458	0.26025	0.37184	0.58273	
-0.15078	-0.22144	-0.00343	-0.01605	-0.05458	1.00000	-0.08130	0.01254	0.28422	
0.12525	0.17867	-0.02510	0.04412	0.26025	-0.08130	1.00000	0.33611	-0.24657	
0.15645	0.07205	0.04561	-0.02808	0.37184	0.01254	0.33611	1.00000	-0.07853	
0.14337	-0.23723	0.33175	-0.40545	0.58273	0.28422	-0.24657	-0.07853	1.00000	
RMS DEVIATION OF PREDICTING									
22.748	7.707	0.825	4.051	9.982	0.253	1.381	1.862	0.668	

Fig. 6—Basic statistics of observations

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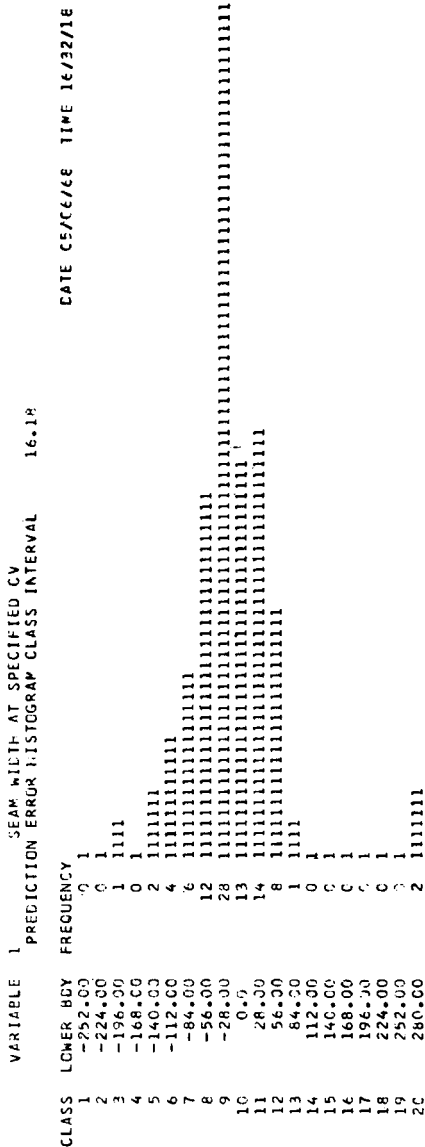


Fig. 8—Prediction error histogram

To date the suite contains two other programmes. The first, also written in FORTRAN IV is used for automatic selection of a mining width in cases where several strata are found in the ore body. All possible widths are studied, and the most satisfactory according to a given criterion is selected. The other programme, written in PL/1 is specific to coal fields. It is used to investigate the association between

common value parameters of coal. A second PL/1 programme under development will be used for comprehensive data checking and for the prediction of missing information according to regression techniques. Another planned extension is the shifting of the iso line planning from the line printer to a digital incremental plotter. Other facilities and programmes may be added as the need for them arises.

APPLICATION OF THE PROGRAMME TO THE COAL MINING SITUATION

There could be four main reasons for opening a coal mine in South Africa today. These are—to supply a power station, to supply an oil from coal plant, to supply the commercial market and to supply special metallurgical coals. All four of these markets may have different quality requirements. Any previous prospecting in a particular coal field may not have been undertaken specifically for any one of these quality requirements. Under the circumstances, certain value parameters may be missing from the previous borehole information. If the opportunity should arise for opening of a captive colliery to supply a power station for example, the mining engineer may be more interested in the raw coal than washed coal values. Some of these may be missing from these previous borehole analyses, especially if certain areas of the coal seam in question contain high quality coal. Using regression techniques and the information that is given for the previous boreholes, many of these values can be reasonably well estimated. Some of the values which show high correlation and therefore lend themselves to prediction are calorific value from ash percentage, washed ash percentage from raw ash percentage and percentage yield from raw ash percentage. Table I shows a typical borehole analysis of a coal seam. It will be noticed that while all the washed coal values are recorded only the raw ash percentage is given. In some cases, the raw coal values may have been entirely left out or if the coal is of poor quality the washed coal values may have been excluded.

To produce the required plans and information the borehole data shown in Table I are punched on to cards. These are fed into the first PL/1 programme which produces the regression curves mentioned above. From these regression curves, missing information is then predicted. This is currently done by hand, but when the second PL/1 programme is complete, it will take over this function. The borehole cards are then fed into the seam selection programme.

If so requested, this programme will attempt to select an optimum mining width; otherwise it will regard the entire seam fed in as the mining width. If manual selection is desired, this may be achieved by removing the data cards corresponding to bands in the seam which it is feasible to exclude from a mining point of view. Exclusion of these bands could have the object of improving quality or mining conditions.

For the selected mining width the programme calculates the average of each value parameter at the boreholes. A summary is printed showing these borehole values, and is transferred to punched cards. Finally, these summary cards are fed into the surface fitting programme for the calculation of the required iso value lines.

The print outs produced by the computer, can be used as they are. However, it has been found that transferring the computer information to plans showing known important geological or surface features is more satisfactory. Fig. 9 shows an iso value plan prepared in this fashion. Any other desired set of iso line plans may of course be plotted and so far the technique has been used to plot 'iso-ash', 'iso calorific value', 'iso volatile' and 'iso width' lines as well as floor contours, middling contours and depth below surface contour lines.

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TABLE I
TYPICAL BOREHOLE ANALYSIS

Thickness in.	Analysis of raw coal (air-dry basis)						Size of coal washed	S.G.	% Float yield	Analysis of float (air-dry basis)						
	Cal. val. lb/lb	% H ₂ O	% ash	% Vol. mat.	% Fix. carb.	Sw. No.				% Total sulphur	Cal. Val. lb/lb	% H ₂ O	% ash	% Vol. mat.	% Fix. carb.	Sw. No.
Excluded	SEAM: Main															
22½			22.0					1.58	83.7	11.63	1.6	20.2	32.8	45.4	1	1.62
24			35.2					1.58	32.8	11.37	2.0	20.4	27.5	50.1	½	1.80
17			13.6					1.58	93.3	13.14	2.1	11.9	35.0	51.0	3	1.75
										%P. in coal A ₁ 0.007 B ₁ 0.008 C ₁ 0.008						

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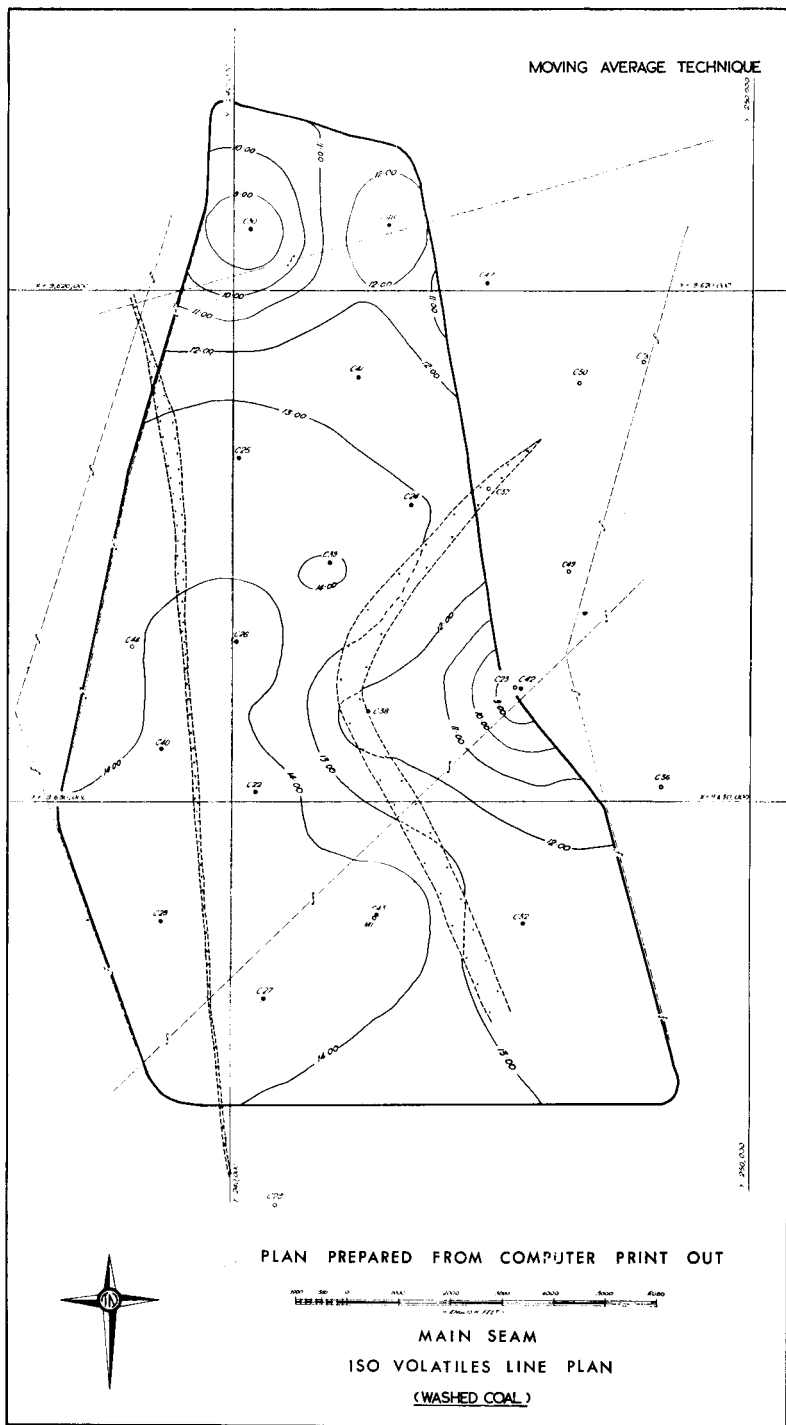


Fig. 9

CONCLUSIONS

A technique has been developed for fitting a reasonable valuation surface to irregular borehole or development data, and has been embodied successfully in a computer programme. This programme has been used effectively in the valuation of coal fields, and it has been found that the technique, apart from fulfilling the function for which it was designed, has cast some light on the problem of siting shafts and further boreholes. Both the technique and the implementation are under further development.

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2. SHARAPOV, I. 'Contribution to the theory of the consistency of ore deposits.' Transactions of the Scientific Coal Institute Perm. Fasc. 4., 1962.

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