THE 'DOORSTOPPER' AND TRIAXIAL ROCK STRESS MEASURING INSTRUMENTS DEVELOPED BY THE C.S.I.R.

By E. R. Leeman* (Visitor)

INTRODUCTION

The first references in the literature to the so-called C.S.I.R. 'doorstopper' strain cell were made in 19641, 2, 3, 4. These were the climax of nearly two years of experimental and developmental work with the instruments and the operating techniques involved.

Since that time numerous field measurements have been made with 'doorstoppers' and they have been developed to a stage where they are in commercial production and are being supplied to users throughout the world. No detailed description of the latest 'doorstopper' equipment has, however, appeared in print and this paper is written to bring the literature up to date in this respect.

The triaxial strain cell was developed three years ago5, 6, 7 and was first described at the First Congress of the International Society for Rock Mechanics in Lisbon in 19668. Since that date laboratory tests and—particularly important—field tests9, 9 have demonstrated the validity and practicality of the instruments and techniques. These instruments have also been developed to a stage where they have been released for commercial production. In this form they are very different from those described two years ago and a full description of them at this stage seems to be justified.

THE INFORMATION SUPPLIED BY 'DOORSTOPPER' AND TRIAXIAL STRAIN CELLS

Both the 'doorstopper' and triaxial strain cells were designed specifically to determine the absolute stress in rock using an overcoring stress relieving technique. They were not designed to measure changes in stress. They may, however, be used for the latter purpose provided the glue which is used to stick the strain gauges to the rock is known to possess sufficiently stable strain-time characteristics as not to affect the accuracy of the results during the time the measurements are made.

The 'doorstopper' was designed primarily to measure the major principal stress in situations where its direction and those of the two other principal stresses were either known or could be assumed. For example, at great depth in undisturbed virgin ground, it is reasonable to assume that the maximum principal stress is vertical and that the other two lie in a horizontal plane. The borehole in which the 'doorstopper' is installed would thus be drilled in the horizontal plane parallel to one of the minor principal stresses, the direction of which would either have to be assumed or determined as described later. The stresses measured would be the vertical major principal stress and the principal stress acting in a horizontal plane normal to the borehole axis.

It is possible, as will be shown later, to obtain the complete state of stress using 'doorstoppers'. In this event, it is necessary to make measurements in three boreholes drilled in any three known directions to each other and relative to a system of three orthogonal axes.

The triaxial strain cell was designed to determine the complete state of stress in a single borehole drilled in any direction in any stress field. At first glance it would appear to supersede the 'doorstopper'. This is not necessarily true since there are

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situations, such as in coal and weak, soft rock where it might be impossible to obtain the relatively long lengths of core necessary (20 in. long and 34 in. outside and 14 in. inside diameter) to effect a satisfactory stress relief cycle when using the triaxial cell. In using the 'doorstopper' a length of as little as 2 in. of 18 in. diameter core is required for successful overcoring.

It has also been demonstrated in coal that unreliable stress results are given when the stresses are obtained by multiplying the measured strain relief readings obtained by doorstoppers by the elastic constants of the coal. If, however, an assimulator (described later) is used to obtain the stresses very reliable stress results are obtained with doorstoppers. At present no assimulator has been evolved by which the stresses can be obtained directly from triaxial strain cell strain readings. They still have to be calculated using the elastic constants of the rock. Thus, in such materials as some coals it would seem unwise to use the triaxial cell and preferable to use the doorstopper.

It is also possible that the triaxial strain cell could be used initially to obtain the directions of the minor principal stresses after which doorstoppers could be used in a borehole drilled in a direction parallel to one of these stresses. The two instruments may therefore be considered as complementary to one another.

It must furthermore be emphasized that both instruments depend upon linear stress-strain behaviour of the rock during stress relieving and a knowledge of the relationship between stress and strain on unloading. In this connection the discovery that even fractured rock on unloading and reloading exhibits a reasonably linear stress strain behaviour, the width of the unloading/reloading stress strain 'hysteresis' being relatively small, gives hope that the stress measuring technique may even be used in fractured rocks for stress determinations. This behaviour no doubt accounts for the successful use of the assimulator in conjunction with doorstoppers for stress determinations in some coals which could not be described as elastic materials in the classical sense because of the presence of cracks, cleats, etc.

THE DOORSTOPPER STRAIN CELL

It should perhaps first be explained that the name 'doorstopper' was given to these strain cells because of their resemblance in colour, shape and size in the early days to the red rubber cylindrical blocks used as doorstops in most homes. The name therefore possesses no scientific connotation whatever!

*The principle underlying the 'doorstopper' method of determining the stress in rock*

A borehole is drilled into the rock to the depth at which it is desired to measure the stress. Strain gauges are glued on the flattened end of the borehole. The depth of the borehole is then extended using the coring crown used to drill the borehole to the original depth. This is in effect an overcoring or trepanning operation which relieves the stresses present on the flattened end of the borehole and results in changes of strain which are measured by means of the strain gauges. The strain readings are multiplied by the elastic constants of the rock and the stresses which were present on the end of the borehole before overcoring are obtained. If the relationship between the stresses present on the flattened end of a borehole and those in the surrounding rock is known, the stress in the surrounding rock (which it is desired to measure) may be calculated.

*The relationship between the stresses on the flattened end of a borehole and those in the surrounding rock*

The stress at a point in the rock can be represented by the system of stresses \( \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz} \) and \( \tau_{zx} \) illustrated in Fig. 1(a). If a borehole be drilled as shown in
Fig. 1(b) in the Z direction of the co-ordinate system used to specify the directions of the six stress components the stresses on the flattened end of the borehole will be as shown in Fig. 1(c). The stresses therefore which will be measured by strain gauges glued on the flattened end of the borehole will be \( \sigma_X, \sigma_Y \) and \( \tau_{XY} \). To obtain the stresses \( \sigma_X, \sigma_Y \), etc. in the surrounding rock from the stresses measured on the end of the borehole the relationship between them must be known.

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![Diagram](image-url)

Fig. 1—The stresses on the flattened end of a borehole
No theoretical relationship has yet been derived. Laboratory measurements, however, on loaded prisms and cylinders of steel, rock and araldite by Bonnechere and Van Herden using electrical resistance strain gauges and photoelasticity to determine the stress distributions on the flat end of boreholes drilled into the specimens have thrown light on the subject. The object of their investigation was to determine the effect of the stress components, $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{yz}$ and $\tau_{zx}$ in the rock upon the stress components $\sigma'_x$, $\sigma'_y$ and $\sigma'_{xy}$ acting upon the end of the borehole.

They wrote $\sigma'_x$, $\sigma'_y$ and $\tau'_{xy}$ as follows:

$$
\begin{align*}
\sigma'_x &= a \sigma_x + b \sigma_y + c \sigma_z \\
\sigma'_y &= a \sigma_y + b \sigma_x + c \sigma_z \\
\tau'_{XY} &= d \tau_{XY}
\end{align*}
$$

and found the following values, for $a$, $b$, $c$ and $d$

<table>
<thead>
<tr>
<th></th>
<th>Bonnechere</th>
<th>Van Heerden</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>$b$</td>
<td>0</td>
<td>-0.064</td>
</tr>
<tr>
<td>$c$</td>
<td>-0.75 $(0.5 + \nu)$</td>
<td>-0.75 $(0.645 + \nu)$</td>
</tr>
<tr>
<td>$d$</td>
<td>1.25</td>
<td>—</td>
</tr>
</tbody>
</table>

Other investigators have obtained other values, particularly for $a$ but it is believed that the above values are the most reliable available. The author is inclined to accept the value of $c$ obtained by Van Herden since it is the result of several measurements.

It should be mentioned that the value of $a = 1.53$ obtained by, amongst others, the author, resulted from using cubes instead of prisms in the laboratory tests. The necessary conditions of uniform stress cannot be and were not achieved in the cubes and caused the higher erroneous value of $a$ to be obtained.

The value of $d$ gives the effect of $\tau_{XY}$ upon strain readings from strain gauges glued on the end of the borehole. Since $\tau_{XY}$ has the same effect as two normal stresses, of equal magnitude and opposite sign, acting at right angles to each other and at 45 degrees to $\tau_{XY}$, the value of $d = 1.25$, the same as $a$ would seem to follow.

The shear components $\tau_{yz}$ and $\tau_{zx}$ would appear to have no effect on the stresses in the $XY$ plane. Since they have the effect of only rotating this plane around the $X$ and $Y$ axes respectively strain gauges glued on it should not be influenced by them.

The value of $b = -0.064$ is small and may be neglected. Hence we can write

$$
\begin{align*}
\sigma'_x &= 1.25 \sigma_x - 0.75 (0.645 + \nu) \sigma_z \\
\sigma'_y &= 1.25 \sigma_y - 0.75 (0.645 + \nu) \sigma_z \\
\tau'_{XY} &= 1.25 \tau_{XY}
\end{align*}
$$

The distribution of $\frac{\sigma'_x}{\sigma_x}$ and $\frac{\sigma'_y}{\sigma_y}$ over the end surface of the borehole, obtained photoelastically, is shown in Fig. 1(d). As will be seen from this figure $\frac{\sigma'_x}{\sigma_x}$ and $\frac{\sigma'_y}{\sigma_y}$ are constant over approximately the middle third of the end of the borehole having the values of $a = 1.25$ and $b = 0$ (approx.). Strain measurements should therefore be made in this area.
The determination of the stresses from doorstopper strain readings.

(a) Measurements in one borehole only

Since only three strain measurements are made with a doorstopper during any single overcoring operation it is not possible to obtain any more information than the magnitudes and directions of the principal stresses acting on the end of the borehole before overcoring (and thence of course the stresses in the surrounding rock as indicated previously).

Thus if measurements are to be made in only one borehole it should be drilled parallel to one of the minor principal stresses. The end of the borehole then becomes the principal plane in which the major and the other minor principal stress act.

If as shown in Fig. 1(b) the borehole is drilled in a direction parallel to the \( \sigma_x \) principal stress, the stresses \( \sigma'_x, \sigma'_y \) and \( \tau'_{XY} \) (see Fig. 1(c)) will be measured by a doorstopper glued on the end of the borehole.

If the configuration of the gauges in the rosette is as shown in Fig. 1(e), and they are glued on the end of the borehole so that the \( A \) and \( B \) directions are parallel to the \( X \) and \( Y \) directions respectively, then the normal stresses \( \sigma'_A = \sigma'_X \) and \( \sigma'_B = \sigma'_Y \) and \( \tau'_{AB} = \tau'_{XY} \) are given by (see Appendix)

\[
\sigma'_A = \sigma'_X = \frac{E}{2} \left( \frac{e_A + e_B}{1 - \nu} + \frac{e_A - e_B}{1 + \nu} \right) 
\]

\[
\sigma'_B = \sigma'_Y = \frac{E}{2} \left( \frac{e_A + e_B}{1 - \nu} - \frac{e_A - e_B}{1 + \nu} \right) 
\]

\[
\tau'_{AB} = \tau'_{XY} = \frac{E}{2} \left( \frac{2e_C - (e_A + e_B)}{1 + \nu} \right) 
\]

where \( E \) and \( \nu \) are the Young's modulus and Poisson's ratio of the rock and \( e_A, e_B \) and \( e_C \) are the strains measured in the \( A, B \) and \( C \) directions respectively.

If \( \tau'_{XY} = 0 \), then \( \sigma'_X \) and \( \sigma'_Y \) are the principal stresses and the directions of the gauges \( A \) and \( B \) are parallel with the principal directions.

If \( \tau'_{XY} \neq 0 \), then \( \sigma'_X \) and \( \sigma'_Y \) are not the principal stresses which are then given by

\[
\sigma'_{1,2} = \frac{E}{2} \left\{ 1 - \nu \right\} \left( e_A + e_B \right) \pm \left( 1 + \nu \right) \sqrt{\left( 2e_C - (e_A + e_B) \right)^2 + (e_A - e_B)^2} 
\]

and their directions by

\[
\tan \phi'_{1,2} = \frac{2(e_{1,2} - e_A)}{2e_C - (e_A + e_B)} 
\]

where \( \phi' \) is measured anti-clockwise from the \( OA \) direction as shown in Fig. 1(e).

The stresses in the surrounding rock \( \sigma_X, \sigma_Y \) and \( \tau_{XY} \) are then given by substitution in equations (1), (2) and (3).

(b) Measurements in three boreholes

As will be seen from equations (4), (5) and (6) the normal components of stress \( \sigma'_A \) and \( \sigma'_B \) and shear stress component \( \tau'_{AB} \) on the end of any borehole are easily obtained from the doorstopper strain readings \( e_A, e_B \) and \( e_C \).

If measurements are made in three boreholes inclined at any angle to one another, it is necessary to resolve the components \( \sigma'_A, \sigma'_B \) and \( \tau'_{AB} \) measured in each borehole in three arbitrarily chosen orthogonal directions in order to bring them into relationship with each other.
The stress at any point in a body can be defined by six independent stress components $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{yz}$ and $\tau_{zx}$ with respect to any arbitrarily chosen coordinate system represented by three axes $OX$, $OY$ and $OZ$ as shown in Fig. 2.

![Diagram showing generalized coordinate system](image)

The normal component of stress $\sigma$ acting on any plane $PQR$ has the direction cosines $l$, $m$ and $n$ and is given by

$$\sigma = l^2 \sigma_x + m^2 \sigma_y + n^2 \sigma_z + 2lm \tau_{xy} + 2mn \tau_{yz} + 2nl \tau_{zx} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots 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The components $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{yz}$, $\tau_{zx}$ in the surrounding rock will be found by substitution in equations (1), (2) and (3) as follows:

\[
\begin{align*}
\sigma'_A(1) &= \sigma'_x(1) = a_1 \sigma_x + b_1 \sigma_z \\
\sigma'_A(2) &= \sigma'_x(2) = a_2 \sigma_x + b_2 \sigma_y \\
\sigma'_B(3) &= \sigma'_y(3) = a_3 \sigma_y + b_3 \sigma_x \\
\sigma'_A(3) &= \sigma'_z(3) = a_4 \sigma_z + b_4 \sigma_x \\
\tau'_{AB}(1) &= \tau'_{xy}(1) = a_5 \tau_{xy} \\
\tau'_{AB}(2) &= \tau'_{yz}(2) = a_6 \tau_{yz} \\
\tau'_{AB}(3) &= \tau'_{zx}(3) = a_7 \tau_{zx}
\end{align*}
\]

and $\tau'_{AB}(1) = \tau'_{xy}(1) = a_5 \tau_{xy}$  \hspace{1cm} (17)

$\tau'_{AB}(2) = \tau'_{yz}(2) = a_6 \tau_{yz}$  \hspace{1cm} (18)

$\tau'_{AB}(3) = \tau'_{zx}(3) = a_7 \tau_{zx}$  \hspace{1cm} (19)

*Only three of these six equations are required to obtain $\sigma_x$, $\sigma_y$ and $\sigma_z$, e.g. $\sigma'_x(1)$, $\sigma'_y(1)$ and $\sigma'_z(1)$. If, however, all six are measured, a useful check on the results is obtained.
If measurements are made in three coplanar boreholes (a configuration usually more suited to underground circumstances) a similar procedure to the above is followed.

Thus, if, as shown in Fig. 4 the three boreholes 1, 2 and 3 are drilled parallel to and at angles of $\delta_2$ and $\delta_3$ respectively to the OZ axis, the direction cosines of each stress component and their components in the three directions are as shown in the table in Fig. 4.

![Diagram](image)

<table>
<thead>
<tr>
<th>STRESS COMPONENT</th>
<th>DIRECTION COSINES</th>
<th>SUBSTITUTING THE DIRECTION COSINES IN EQUATIONS (14) AND (15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{III}$</td>
<td>$l$ $m$ $n$</td>
<td>$\sigma_{III} = \sigma_{III}'$</td>
</tr>
<tr>
<td>$\sigma_{III}'$</td>
<td>1 0 0</td>
<td></td>
</tr>
<tr>
<td>$\tau_{ABC}$</td>
<td>1 $\delta_2$ $\delta_2$</td>
<td>$\tau_{ABC} = \tau_{ABC}'$</td>
</tr>
<tr>
<td>$\tau_{ABC}'$</td>
<td>0 1 0</td>
<td></td>
</tr>
<tr>
<td>$\tau_{ABC''}$</td>
<td>$\cos\delta_2$ 0 0</td>
<td>$\tau_{ABC''} = \tau_{ABC''}'$</td>
</tr>
<tr>
<td>$\tau_{ABC'''}$</td>
<td>$\cos\delta_2$ $\cos\delta_3$ $\sin\theta$</td>
<td>$\tau_{ABC'''} = \tau_{ABC'''}'$</td>
</tr>
</tbody>
</table>

Fig. 4—Normal stress components in three boreholes drilled in the xz plane at $\delta_1$, $\delta_2$, and $\delta_3$ to the OZ ordinate.

The components $\sigma_X$, $\sigma_Y$, $\sigma_Z$, $\tau_{XY}$, $\tau_{YZ}$ and $\tau_{ZX}$ in the surrounding rock will be found by substitution in equations (1), (2) and (3) as before (see corresponding equations for three orthogonal boreholes—(11) to (19)).

Similar procedures are followed for boreholes drilled at any angles to one another.
The three principal stresses in the rock \( \sigma_1 = \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the three roots of the well-known equation:

\[
\sigma_i^3 - \sigma_i^2 (\sigma_x + \sigma_y + \sigma_z) + \sigma_i (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2) - (\sigma_x \sigma_y \sigma_z - \sigma_x \tau_{yz}^2 - \sigma_y \tau_{zx}^2 - \sigma_z \tau_{xy}^2 + 2\sigma_x \tau_{yz} \tau_{zx}) = 0 .
\]

This can be written in the forms

\[
\sigma_i^3 + B\sigma_i^2 + C\sigma_i + D = 0 \quad \text{...........................................(21)}
\]

and

\[
W_i^3 + W_i + \beta = 0 \quad \text{...........................................(22)}
\]

where

\[
\sigma_i = W_i - \frac{B}{3} \\
\alpha = \frac{1}{3} (3C - B^2) \\
\beta = \frac{1}{27} \left( 2B^2 - 9BC + 27D \right)
\]

Equation (22) has real roots \( W_1, W_2 \) and \( W_3 \) if

\[
\frac{\beta^2}{4} + \frac{\alpha^3}{27} < 0
\]

and have the values

\[
W_1 = 2 \times \sqrt{-\frac{\alpha}{3}} \quad \cos \left( \frac{\phi}{3} \right)
\]

\[
W_2 = 2 \times \sqrt{-\frac{\alpha}{3}} \quad \cos \left( \frac{\phi}{3} + 120^\circ \right)
\]

\[
W_3 = 2 \times \sqrt{-\frac{\alpha}{3}} \quad \cos \left( \frac{\phi}{3} + 240^\circ \right)
\]

where \( \phi = \cos^{-1} \left\{ \frac{-\beta/2}{\sqrt{-\frac{\alpha^3}{27}}} \right\} \)

Thus it is possible to determine \( W_1, W_2 \) and \( W_3 \) and hence \( \sigma_1, \sigma_2 \) and \( \sigma_3 \).

The direction cosines \( l_1, m_1, n_1 \) of each principal stress \( \sigma_i = \sigma_1, \sigma_2, \sigma_3 \) are found by substitution in equation (9) written as

\[
\sigma_i = l_i^2 \sigma_x + m_i^2 \sigma_y + n_i^2 \sigma_z + 2l_i m_i \tau_{xy}, 2m_i n_i \tau_{yz} + 2n_i l_i \tau_{zx}
\]

The overcoring technique used with doorstoppers

The overcoring technique used is represented diagrammatically in Fig. 5.

A standard \( BX \) diamond coring crown is used to drill the borehole (approximately 2\( \frac{1}{2} \) in. diameter) to the depth in the rock at which it is desired to determine the stress as shown in Fig. 5(a). The end of the borehole is ground flat and smooth with specially prepared square faced and flat faced diamond impregnated bits, photographs of which are shown in Fig. 6.
The 'doorstopper' and triaxial rock stress measuring instruments developed by the C.S.I.R.—E. R. Leeman

(a) BX borehole drilled to the required depth and end flattened and polished with diamond tools.

(b) Strain cell bonded onto end of borehole and strain readings recorded.

(c) Borehole extended with BX diamond coring crown thereby stress relieving the core.

(d) BX core, with strain cell attached, removed and strain readings taken.

Fig. 5—The overcoring technique using a doorstopper.
The direction in which the borehole is drilled should be parallel to one of the minor principal stresses as indicated in a previous paragraph if measurements in only one borehole are to be made. If measurements are being made in three boreholes to determine the complete state of stress then the directions in which the boreholes are drilled is governed by the considerations outlined in the previous paragraph.

Having cleaned and dried the end of the borehole, a doorstopper is glued on to it and strain gauge readings taken when the glue has hardened. The installing tool is then removed leaving the doorstopper stuck on the end of the hole as shown in Fig. 5(b).

Using the same BX coring crown as was used to drill the borehole, the length of the borehole is extended as shown in Fig. 5(c)—thereby relieving the stresses on the end of the borehole. The core is broken off the end of the hole as shown in Fig. 5(d) with the doorstopper still attached to it and the strain relieved readings taken from the doorstopper strain gauges.

The stresses present on the end of the borehole before overcoring may then be calculated from the change in strain readings as indicated in the previous paragraph.

**Description of the doorstopper strain cell equipment**

As will be seen from the diagrammatic sketch of one of the strain cells ('doorstopper') in Fig. 7, a rectangular strain gauge rosette (consisting of three strain gauges
The "doorstopper" and triaxial rock stress measuring instruments developed by the CSIR—E. R. Leeman measuring in the 0 degree (horizontal) 45 degree and 90 degree (vertical) directions is moulded into a rubber casting which fills a plastic shell. Four gold plated connector pins are moulded in the plastic shell, as shown in Fig. 7, in such a way that when they are plugged into the inserting tool (described later) they effect electrical contact between the strain gauges and the strain indicating instrument. The connections to the pins are shown in the lower illustration in Fig. 7.

The presence of a keyway in the plug section of the plastic shell, ensures that the "doorstopper" can be plugged into the installing tool with only one possible orientation.

![Diagram of doorstopper](image)
The diameter of the 'doorstopper' was chosen so that it can be used in a standard $BX$ diamond drilled borehole which is approximately 2$\frac{3}{4}$ in. in diameter.

When a 'doorstopper' is installed on the end of a $BX$ borehole, changes in strain on the surface of the rock resulting from the overcoring operation are transmitted to the strain indicating instrument via the strain gauges which are glued on the rock. The rubber and plastic shell serve to protect the strain gauges from damage and from water during the overcoring operation. Normally, sufficient glue would be used to ensure that the base of the plastic shell is also glued to the rock surface.

A photograph of doorstoppers is given in Fig. 8, one showing the strain gauge rosette, the other glued to a short length of core.

![Fig. 8—A doorstopper (left) showing the rosette strain gauge and (right) glued to a short length of core](image)

A cross sectional drawing of the installing tool used to glue a doorstopper on the end of the borehole is given in Fig. 9 and a photograph in Fig. 10. As will be seen in Fig. 9, the 'doorstopper' and compensating gauge plug into a piston unit which slides against a compression spring inside a simple brass tubular unit. An orienting mercury switch is also housed in the piston unit to enable the doorstopper to be accurately oriented. The electric lead cable is connected to the instrument through an eight-channel plug. Ventilation holes, visible also in the photograph in Fig. 10, are machined in the tubular body of the tool to ensure that the temperature of the disc of rock on which the dummy gauge is glued reaches the same temperature as that upon which the measuring 'doorstopper' is attached as quickly as possible.
The 'doorstopper' and triaxial rock stress measuring instruments developed by the C.S.I.R.—E. R. Leeman

Fig. 9—Doorstopper strain cell installing tool

Fig. 10—The doorstopper installing tool showing a doorstopper in the foreground
The compensating dummy gauge consists of a ‘doorstopper’ glued to a \( \frac{1}{4} \) in. length of BX core of the rock in which the stresses are to be measured. In fact, this is prepared initially by gluing a ‘doorstopper’ upon the end of the borehole in which the stresses are to be measured, overcoring and cutting off a \( \frac{1}{4} \) in. length from the core at the end on which the ‘doorstopper’ is glued. This ‘doorstopper’ is very easily plugged into the inner plug of the piston unit and acts as a temperature compensating dummy gauge.

When the installing tool with a ‘doorstopper’ plugged into it is pushed up to the end of the hole by means of specially designed installing rods, and pushed against the end of the borehole, the piston unit is forced into the tubular body of the tool against the compression spring until the spring pressure switch, shown in Fig. 9, closes. A lamp lights up in the control box to which the installing tool is connected and indicates that the desired load of approximately 10 lb is being applied to the ‘doorstopper’. This must be maintained until the glue has hardened. The installing tool can then be removed from the borehole leaving the doorstopper glued to the end of the borehole.

A photograph of the control box is shown in Fig. 11. This houses the electrical lead, the installing tool, spare doorstoppers, the strain indicator and control panel. On the control panel are two lamps, one connected to the mercury orienting switch and the other to the spring pressure switches in the installing tool. A three-way switch is provided to enable readings from each of the three strain gauges in the doorstopper rosette to be made in turn.

**Glues and rock drying agents used**

Any of the commercially available strain gauge cements may be used to glue the doorstoppers to the rock. Araldite strain gauge cement has been found very reliable. Its setting time is, however, rather long. The setting time should on the other hand not be so short that the doorstopper cannot be installed before the glue has wholly or partly set.

In coal, Philips strain gauge cement No. P.R. 9244/04, a quick setting cement, has been successfully used.

The use of a water dispellant has also been proved very beneficial in producing reliable and consistent gluing on of the doorstoppers. In this connection a 10 per cent mixture in pure alcohol of Union Carbide Silane Coupling Agent No. A 1120 has been found to be very effective. This enables doorstoppers, and triaxial cells for that matter to be used in wet rock.

**The doorstopper borehole assimulator**

The determination of the stresses from the doorstopper strain measurements described in a previous paragraph requires a knowledge of the elastic moduli of the rock. Due to the variations in these properties, depending upon the locality and the reliability of the laboratory methods used to determine them, serious errors can be introduced in the calculated stress results.

In order to overcome this difficulty a so-called ‘borehole assimulator’ was designed in which the stress relieved core to which the doorstopper is glued, is loaded in such a way as to restore the stresses which existed in it before the core was overcored. Two radial loads at right angles are applied to the core hydraulically and the oil pressures required to restore the doorstopper strain readings to their values before overcoring are a measure of the original stresses on the end of the borehole.
The "doorstopper" and triaxial rock stress measuring instruments developed by the C.S.I.R.—E. R. Leeman

Fig. 11—The control box/carrying case for the doorstopper equipment
It can be shown\textsuperscript{16, 17} that if a cylindrical core is loaded biaxially under the pressures \( p \) and \( q \), each uniformly distributed over a quarter of its circumference as shown in Fig. 12 the radial, tangential and shear stresses at the centre are given by

\[
\sigma_r = \frac{1}{2} \left[ (p + q) + \frac{4}{\pi} (p - q) \cos 2\theta \right]
\]

\[
\sigma_\theta = \frac{1}{2} \left[ (p + q) - \frac{4}{\pi} (p - q) \cos 2\theta \right]
\]

\[
\tau_{r\theta} = \frac{2}{\pi} (p - q) \sin 2\theta
\]

Fig. 12—A cylindrical core subjected to hydraulic pressures \( p \) and \( q \) over four equal segments
The radial strains for $\theta = 0^\circ$, $45^\circ$ and $90^\circ$ are given by Hooke’s Law for plane stress as follows:

\[ e_{r0} = \frac{1}{2E} \left[ (1 - \nu) (p + q) + \frac{4}{\pi} (1 + \nu) (p - q) \right] \]

\[ e_{r45} = \frac{1 - \nu}{2E} (p + q) \]

\[ e_{r90} = \frac{1}{2E} \left[ (1 - \nu) (p + q) - \frac{4}{\pi} (1 + \nu) (p - q) \right] \]

The components of stress on the end of a borehole in a uniform stress field with principal stresses $P$ and $Q$ are given by

\[ \sigma'_r = \frac{1.25}{2} \left[ (P + Q) + (P - Q) \cos 2\theta \right] \]

\[ \sigma'_\theta = \frac{1.25}{2} \left[ (P + Q) - (P - Q) \right] \cos 2\theta \]

\[ \tau'_r = \frac{1.25}{2} (P - Q) \sin 2\theta \]

where the value of 1.25 is the factor $a$ referred to in paragraph on page 308.

The radial strains for $\theta = 0^\circ$, $45^\circ$ and $90^\circ$ are given by Hooke’s Law for plane stress as follows:

\[ e_{r0}' = \frac{1.25}{2E} \left[ (1 - \nu) (P + Q) + (1 + \nu) (P - Q) \right] \]

\[ e_{r45}' = \frac{1.25}{2E} (P + Q) \]

\[ e_{r90}' = \frac{1.25}{2E} \left[ (1 - \nu) (P + Q) - (1 + \nu) (P - Q) \right] \]

Thus for the state of stress in the core in the assimulator to be identical to that at the end of the borehole:

\[ e_{r0} = e_{r0}' \]

\[ e_{r45} = e_{r45}' \]

\[ e_{r90} = e_{r90}' \]

Solving for $P$ and $Q$ in terms of $p$ and $q$ we obtain

\[ P = \frac{1}{2.50} \left[ \left( 1 + \frac{4}{\pi} \right) p + \left( 1 - \frac{4}{\pi} \right) p \right] \]

\[ Q = \frac{1}{2.50} \left[ \left( 1 - \frac{4}{\pi} \right) p + \left( 1 + \frac{4}{\pi} \right) q \right] \]
Thus by applying the hydraulic pressures $p$ and $q$ on the rock core to restore the strain, the original principal stresses $P$ and $Q$ in the rock can be obtained, without requiring any knowledge of the properties of the rock.

A diagrammatic sketch of an assimulator* is given in Fig. 13 and a photograph in Fig. 14. As can be seen in Fig. 13, the pressures $p$ and $q$ are applied around the circumference of the rock core via the neoprene seals which fit snugly around it. In Fig. 14 the assimulator is shown assembled in a carrying case. Pressure pump handles, pressure gauges and protractor for orienting the core can be clearly seen in the photograph.

The assimulator is used as follows: Having obtained the three strain readings after overcoring, the directions of the principal stresses represented by $\theta$ are determined from equation (8).

The core with the doorstopper glued to it is inserted in the assimulator and oriented at angle $\theta$. The hydraulic pressures $p$ and $q$ to be applied to the core will then act in the same directions, relative to the gauges on the core, as the principal stresses $P$ and $Q$ before overcoring. The pressures $p$ and $q$ required to restore the strain readings to those before overcoring are obtained and substituted in equations (23) and (24) to obtain $P$ and $Q$.

**The accuracy of doorstopper stress measurements**

Van Heerden\(^{15}\) has carried out tests to determine the accuracy of doorstopper stress measurements in coal which could by no means be described as the classical material from the point of view of homogeneity, isotropy and continuity assumed in elastic theory. In these tests large 3 ft $\times$ 3 ft $\times$ 6 ft high coal specimens were cut \textit{in situ} in coal pillars underground and known uniaxial compressive stresses were applied to them by means of hydraulic jacks. The stresses in the pillars were measured by means of doorstoppers installed in them and this enabled a comparison to be made between applied and measured stresses.

The results obtained are included in Table I from which it can be seen that the vertical stresses $\sigma_1$ obtained by using the assimulator agreed remarkably well with the applied stresses. The horizontal stresses $\sigma_2$ were zero and very small stresses were measured.

On the other hand the stresses obtained by calculation using the elastic moduli determined in compression tests on small cylindrical specimens in an underground laboratory (to eliminate influences of temperature and humidity changes) in most instances did not give very good results (see Table I). It must therefore be emphasized that for materials such as coal reliable results cannot be expected, if they are calculated by multiplying the strain readings by the elastic constants obtained on small laboratory specimens, and that the assimulator should be used to obtain the stresses in such materials. It is in fact recommended that the assimulator be used in stress measurements even in hard homogeneous rock in spite of the fact that reliable results using the elastic constants can be expected in such rock.

In solid homogeneous rock, consistent and reliable results are easily attainable. Attention, in this connection is drawn to a recent publication\(^{18}\) in which the results obtained with doorstoppers were compared with those obtained with a borehole deformation gauge. The test showed remarkable agreement in the results and demonstrated that the doorstopper was very much easier to use.

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*For reasons of accuracy two versions of the assimulator have been developed, one for low strength rock such as coal, and one for high strength rock such as quartzite.
# TABLE I

Comparison of results obtained in coal using the assimulator and by calculation based upon elastic constants obtained in the laboratory

<table>
<thead>
<tr>
<th>Relief of strain during overcoring of doorstopper (micro strain)</th>
<th>Results obtained with the assimulator</th>
<th>Assimulator pressures (lb/ft²)</th>
<th>Principal stresses (lb/ft²)</th>
<th>Stresses obtained using elastic constants</th>
<th>Actual vertical stress ( \sigma_z ) applied by jacks on the coal pillar (lb/ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_{90} )</td>
<td>( \epsilon_{45} )</td>
<td>( \epsilon_{0} )</td>
<td>( \epsilon_{90} )</td>
<td>( \epsilon_{45} )</td>
<td>( \epsilon_{0} )</td>
</tr>
<tr>
<td>+485</td>
<td>+55</td>
<td>-225</td>
<td>-485</td>
<td>-155</td>
<td>+225</td>
</tr>
<tr>
<td>+1,045</td>
<td>+360</td>
<td>-495</td>
<td>-1,045</td>
<td>-200</td>
<td>+495</td>
</tr>
<tr>
<td>+625</td>
<td>+210</td>
<td>-260</td>
<td>-625</td>
<td>-240</td>
<td>+260</td>
</tr>
<tr>
<td>+1,130</td>
<td>+100</td>
<td>-295</td>
<td>-1,130</td>
<td>-120</td>
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<tr>
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<td>-140</td>
<td>-805</td>
<td>-195</td>
<td>+140</td>
</tr>
<tr>
<td>+2,570</td>
<td>+415</td>
<td>-380</td>
<td>-2,570</td>
<td>-400</td>
<td>+380</td>
</tr>
<tr>
<td>+570</td>
<td>+305</td>
<td>-80</td>
<td>-570</td>
<td>-250</td>
<td>+130</td>
</tr>
<tr>
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<td>+300</td>
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<td>-465</td>
<td>-280</td>
<td>+120</td>
</tr>
<tr>
<td>+790</td>
<td>+230</td>
<td>-210</td>
<td>-790</td>
<td>-230</td>
<td>+210</td>
</tr>
<tr>
<td>+1,500</td>
<td>+440</td>
<td>-340</td>
<td>-1,500</td>
<td>-400</td>
<td>+340</td>
</tr>
</tbody>
</table>
The 'doorstopper' and triaxial rock stress measuring instruments developed by the C.S.I.R.—E. R. Leeman
Numerous measurements with doorstoppers throughout the world have confirmed the faith placed in this technique and the doorstopper equipment is now being commercially manufactured.

THE C.S.I.R. TRIAXIAL STRAIN CELL

The C.S.I.R. triaxial strain cell was designed to obtain the complete state of stress in rock in a single borehole.

The theoretical basis for the measuring technique

The stress at a point in a body is presented by six components, namely three normal stress components \( \sigma_x, \sigma_y, \sigma_z \) and three shear stress components, namely \( \tau_{xy}, \tau_{yz}, \tau_{zx} \), as illustrated in Fig. 1(a) where the point is 'blown up' to a cube element.

If a circular hole is drilled in an elastic rock mass subjected to a uniformly distributed state of stress given by the stress components defined by Fig. 15 the stress in the vicinity of the hole is changed. With reference to a system of cylindrical co-ordinates \( r-\theta-z \), as defined by Fig. 15, the stress at any point \( r-\theta-z \) in the vicinity of the hole with radius \( a \) drilled parallel to the \( Z \)-axis, is given by six components, namely three normal stress components \( \tau_r, \sigma_\theta, \sigma_z \) and three shear stress components
\[ \begin{align*}
\sigma_r &= \frac{\sigma_x + \sigma_y}{2} \left( 1 - \frac{a^2}{r^2} \right) + \frac{\sigma_x - \sigma_y}{2} \left( 1 + 3 \frac{a^2}{r^4} - 4 \frac{a^2}{r^2} \right) \cos \theta + \\
\tau_{XY} &= \left( 1 + 3 \frac{a^2}{r^4} - 4 \frac{a^2}{r^2} \right) \sin \theta \\
\sigma_0 &= \frac{\sigma_x + \sigma_y}{2} \left( 1 + \frac{a^2}{r^2} \right) - \frac{\sigma_x - \sigma_y}{2} \left( 1 + 3 \frac{a^2}{r^4} \right) \cos \theta - \tau_{XY} \left( 1 + 3 \frac{a^2}{r^4} \right) \sin \theta \\
\sigma_z &= -\sqrt{(2(\sigma_x - \sigma_y) \frac{a^2}{r^2} \cos \theta + 4\tau_{XY} \frac{a^2}{r^2} \sin \theta)} + \sigma_z \\
\tau_{r0} &= \frac{\sigma_x - \sigma_y}{2} \left( 1 - 3 \frac{a^2}{r^4} + 2 \frac{a^2}{r^2} \right) \sin \theta + \tau_{XY} \left( 1 - 3 \frac{a^2}{r^4} + 2 \frac{a^2}{r^2} \right) \cos \theta \\
\tau_{0z} &= \left( -\tau_{ZX} \sin \theta + \tau_{YZ} \cos \theta \right) \left( 1 + \frac{a^2}{r^2} \right) \\
\tau_{rz} &= \left( \tau_{ZX} \cos \theta + \tau_{YZ} \sin \theta \right) \left( 1 - \frac{a^2}{r^2} \right)
\end{align*}\]

\(a, b, c\) as illustrated in Fig. 15. The following six expressions yield the relationship between these components and the components \(\sigma_x\) etc. of the stress in the rock mass before the hole was drilled.

Fig. 15—Stress system around a hole in the rock mass
If \( r = a \) in equations (25) to (30) expressions for the stresses at any point on the wall of the borehole are obtained. It will be noticed that at \( r = a , \sigma _r = 0 , \tau _{r \vartheta } = 0 \) and \( \tau _{r z } = 0 \). This leaves \( \sigma _\vartheta \), \( \sigma _z \) and \( \tau _{\vartheta z } \) which can be measured with a rosette of three gauges having the same configuration as that used in the doorstoppers.

Thus if the \( A \) and \( B \) gauges (see Appendix) are lined up with the \( \vartheta \) and \( Z \) directions, i.e. with the circumferential and axial directions of the borehole, \( \sigma _A = \sigma _\vartheta , \sigma _B = \sigma _Z \) and \( \tau _{AB} = \tau _{\vartheta z} \).

In the triaxial strain cell three rosettes are glued to the walls of a borehole as shown in Fig. 16. Rosette 1 is glued on at \( \theta = \pi \) (on the ‘sidewall’ of the borehole), rosette 2 at \( \theta = \frac{\pi}{2} \) (on the ‘roof’ of the borehole) and rosette 3 at \( \theta = \frac{3\pi}{4} \) (at an intermediate point between rosettes 1 and 2). The gauges in each rosette are aligned as indicated above.

![Diagram of rosettes and borehole](image)

**Table:**

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \sigma _r )</th>
<th>( \sigma _\vartheta )</th>
<th>( \tau _{r \vartheta } )</th>
<th>( \tau _{r z } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \sigma _{r1} = \sigma _{\vartheta 1} )</td>
<td>( \sigma _{\vartheta 2} )</td>
<td>( \tau _{r \vartheta 1} )</td>
<td>( \tau _{r z 1} )</td>
</tr>
<tr>
<td>( \pi / 2 )</td>
<td>( \sigma _{r2} = \sigma _{\vartheta 2} )</td>
<td>( \sigma _{\vartheta 3} )</td>
<td>( \tau _{r \vartheta 2} )</td>
<td>( \tau _{r z 2} )</td>
</tr>
<tr>
<td>( 3\pi / 4 )</td>
<td>( \sigma _{r3} = \sigma _{\vartheta 3} )</td>
<td>( \sigma _{\vartheta 4} )</td>
<td>( \tau _{r \vartheta 3} )</td>
<td>( \tau _{r z 3} )</td>
</tr>
</tbody>
</table>

The six stress components \( \sigma _1 , \sigma _2 , \tau _{1 \vartheta } , \tau _{1 z } , \tau _{2 \vartheta } , \tau _{2 z } \) are found from the equations marked \( \ast \) as follows:

\[
\begin{align*}
\sigma _1 &= \frac{1}{2} \{ \sigma _{11} + \sigma _{22} \} \\
\sigma _2 &= \frac{1}{2} \{ \sigma _{11} + \sigma _{22} \} \\
\tau _{1 \vartheta } &= -\frac{1}{2} \{ \sigma _{12} + \sigma _{21} \} \\
\tau _{1 z } &= \frac{1}{2} \{ \sigma _{12} + \sigma _{21} \} \\
\tau _{2 \vartheta } &= \frac{1}{2} \{ \sigma _{12} + \sigma _{21} \} \\
\tau _{2 z } &= -\frac{1}{2} \{ \sigma _{12} + \sigma _{21} \}
\end{align*}
\]

Fig. 16—The components of stress around the periphery of a borehole measured in the three rosettes of the triaxial strain cell.
The components of stress at the three measuring points \( i = 1, 2, 3 \) (for \( \theta = \pi \)),
\[
\begin{align*}
\frac{\pi}{2^2} \frac{\pi}{4} \sigma_\theta(i) &= \sigma_A(i), \quad \sigma_\phi(i) = \sigma_B(i) \quad \text{and} \quad \tau_{YZ}(i) = \tau_{AB}(i)
\end{align*}
\]
are given in Fig. 16. The values of \( \sigma_A(i), \sigma_B(i) \) and \( \tau_{AB}(i) \) are obtained from the strain gauge readings as before (see Appendix) from
\[
\begin{align*}
\sigma_A(i) &= \frac{E}{2} \left\{ \frac{e_{AI} + e_{BI}}{1 - \nu} + \frac{e_{AI} - e_{BI}}{1 + \nu} \right\} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (31) \\
\sigma_B(i) &= \frac{E}{2} \left\{ \frac{e_{AI} + e_{BI}}{1 - \nu} - \frac{e_{AI} - e_{BI}}{1 + \nu} \right\} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (32) \\
\tau_{AB}(i) &= \frac{E}{2} \left\{ \frac{2e_{CI} - (e_{AI} + e_{BI})}{1 + \nu} \right\} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (33)
\end{align*}
\]
The six stress components in the rock \( \sigma_X, \sigma_Y, \sigma_Z, \tau_{XY}, \tau_{YZ} \) and \( \tau_{ZX} \) can then be obtained from the nine equations in the table in Fig. 16. Obviously, only six equations are required. Those marked with an asterisk in Fig. 16 have been used for this purpose and give the expressions for \( \sigma_X, \sigma_Y, \tau_{XY}, \tau_{ZX} \) and \( \tau_{YZ} \) in terms of the measured \( \sigma_A(i), \sigma_B(i) \) and \( \tau_{AB}(i) \) included at the foot of the table in Fig. 16.

It is in fact unnecessary to use all nine of the gauges in the three rosettes—only six are required. However, the three gauges which measure the strains in the direction of the axis of the borehole (the \( Z \) or \( A \) direction) should give identical readings since \( e_{B(i)} = e_{Z(i)} \) is constant around the circumference of the borehole. All of the results can in fact be obtained from the following strain readings

<table>
<thead>
<tr>
<th>Rosette 1</th>
<th>Rosette 2</th>
<th>Rosette 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction ( Z )</td>
<td>( e_A(i) ) or ( e_A(g) )</td>
<td>( e_A(g) )</td>
</tr>
<tr>
<td>Direction 0</td>
<td>( e_B(i) )</td>
<td>( e_B(g) )</td>
</tr>
<tr>
<td>Direction 45°</td>
<td>( e_C(i) )</td>
<td>( e_C(g) )</td>
</tr>
</tbody>
</table>

Having obtained \( \sigma_X, \sigma_Y, \sigma_Z, \tau_{XY}, \tau_{YZ} \) and \( \tau_{ZX} \) the principal stresses and their directions in the rock can be calculated as indicated on page 311. A computer programme has however been compiled which enables this information to be obtained directly from the strain readings.

The technique used for in situ stress measurements

To apply the above results to determine the absolute stress in rock an overcoring technique is used. A standard \( NXCU \) (\( NX \) casing) borehole (3½ in. diameter) is drilled to the depth at which it is desired to determine the stress—as shown in Fig. 17. The end of this borehole is ground flat with a suitable diamond flattening bit to ensure accurate collaring of the \( EX \) (1¼ in. diameter) borehole which is drilled into the end of and concentric with the \( NXCU \) borehole for a distance of 18 in. as shown in Fig. 17(b). The flat end surface of the \( NXCU \) borehole also ensures the proper plugging of the \( EX \) borehole against ingress of cooling water during the overcoring operation performed later. The strain cell containing the three strain gauge rosettes is then inserted into the \( EX \) portion of the borehole and the rosettes glued in position midway along its length as shown in Fig. 17(c). The strain gauge readings are taken when the glue used to glue the rosettes in position has hardened. The installing tool is removed.
from the borehole, leaving the strain cell in the EX portion of the borehole. The mouth of the EX borehole is plugged to prevent cooling water used during the following overcoreing operation from entering this borehole and damaging the strain cell. The EX portion of the borehole is next overcored using the NXCU coring crown used to drill the NXCU portion of the borehole as shown in Fig. 17(d).

![Diagram](image)

**Fig. 17—The overcoreing technique with the triaxial strain cell**
The cylindrical core containing the rosette is removed from the borehole as shown in Fig. 17(e) and the stress relieved strain readings taken from each gauge. The difference in the strain gauge readings before and after overcoring are used to calculate the required values of $\sigma_0$, $\sigma_z$, $\sigma_{0z}$ and hence $\sigma_X$, $\sigma_Y$, $\sigma_Z$, $\tau_{XY}$, $\tau_{YZ}$, $\tau_{ZX}$ and thereafter the principal stresses and their directions.

Description of the triaxial strain cell equipment

(i) The strain cell and installing tool

A photograph of a triaxial strain cell is shown in Fig. 18. A photograph and a drawing of a strain cell showing its various components are shown in Figs. 19 and 20 respectively. From these it can be seen that it consists of five distinct units, four of which all fit on to the main body unit. The strain gauge rosette unit slides over the main body unit and the projections on each of the three plugs slide in the rosette plug guides. The temperature compensating gauge unit plugs into the two pins in the main body unit. The two end covers also fit over the main body unit and form one composite unit as shown in Fig. 18.

Fig. 18—The C.S.I.R. triaxial strain cell
The 'doorstopper' and triaxial rock stress measuring instruments developed by the C.S.I.R.—E. R. Leeman

Fig. 19—Photograph of triaxial strain cell showing its component parts
The cell is installed in the borehole by means of an installing tool illustrated in Figs. 21 and 22. It plugs into the installing tool as shown in Fig. 21. When this is done the wedge rod seen in Fig. 22 enters the main body of the cell. The wedge rod is connected to a piston which is actuated by air pressure through flexible tubing connected to the air pressure pipe connection. Thus when the piston is forced out under pressure, the wedge rod moves out and forces out the three rosette units into contact with sidewalls of the borehole. Thus if glue was smeared on the rosettes beforehand, pressure is supplied for a sufficiently long length of time for the gauges to be effectively glued to the sidewalls of the borehole.

As will be seen from Fig. 22, a mercury switch is built into the installing tool to ensure that the cell is correctly oriented in the borehole.

Seeing that the strain cell is installed in an EX pilot hole (1\(\frac{1}{4}\) in. diameter) drilled into the end of an NXC7 borehole (3\(\frac{1}{2}\) in. diameter), it is necessary to fit the installing tool into a coupling unit as shown in Fig. 23 whose diameter is just slightly smaller than 3\(\frac{1}{4}\) in. This is, in turn, connected to the installing rods by which the whole assembly is pushed into the borehole. Note the electrical lead and compressed air lines connected to the installing tool in Fig. 23.

It should be noted that on completing a stress relieving cycle the strain cell can be removed from the rock core, dismantled, the used rosette unit removed and replaced by a new one*. The cell may then be used again. Furthermore, the temperature compensating dummy gauge unit* is easily replaced by another one when the cell is to be used in different rocks.

*These units will be available commercially.
Fig. 21—The cell being plugged into the installing tool

Fig. 22—Strain cell installing tool
The 'doorstopper' and triaxial rock stress measuring instruments developed by the C.S.I.R. — E. R. Leeman

(ii) The control box and strain indicator

The control box and strain indicator used are similar to that used with the doorstopper, shown in Fig. 11. The main difference is in the control panel where a nine-way switch (connected to the nine strain gauges in the three rosettes) replaces the three-way switch in the doorstopper control panel.

(iii) Auxiliary equipment

A syringe spray is available for spraying the sidewalls of the EX borehole with cleaning and water repellant liquids before the cell is installed.

An EX borehole plug is available to plug the mouth of the EX borehole after the installing tool has been removed and just before overcoring. This ensures that drill cooling water does not enter the EX borehole and thus damage the strain cell during the overcoring cycle.

The operations involved in using these units are very simple indeed even at the end of boreholes 50 ft or more long.
Laboratory and field measurements with triaxial strain cell

Tests with the triaxial strain cell in steel cubes in the laboratory have confirmed the validity of the method, and have confirmed the decision to release it for manufacture on a commercial basis. This will make generally available equipment for which there have already been world-wide enquiries.

REFERENCES

17. Salamon, M. D. G. Private communication.
APPENDIX

THE USE OF A (0°, 45°, 90°) THREE GAUGE ROSETTE TO DETERMINE THE STRESS COMPONENTS AT A POINT

Suppose the strains at any point 0 on the surface to which the rosette is glued are measured by means of a strain gauge rosette containing three strain gauges $G_A$, $G_B$ and $G_C$ oriented relative to two axes $X$, $Y$ as shown in Fig. A.

By a simple resolution of the strains in Fig. A it can be shown that the relations between the strains $e_A$, $e_B$ and $e_C$ measured by the strain gauge $G_A$, $G_B$ and $G_C$ and the state of strain at 0, are given by:

$$
e_A = e_X \cos^2 \phi_A + e_Y \sin^2 \phi_A + \gamma_{XY} \sin \phi_A \cos \phi_A \quad \text{(A.1)}$$

$$
e_B = e_X \cos^2 \phi_B + e_Y \sin^2 \phi_B + \gamma_{XY} \sin \phi_B \cos \phi_B \quad \text{(A.2)}$$

$$
e_C = e_X \cos \phi_C + e_Y \sin^2 \phi_C + \gamma_{XY} \sin \phi_C \cos \phi_C \quad \text{(A.3)}$$

where $e_X$ and $e_Y$ are the normal strains in the $X$ and $Y$ directions and $\gamma_{XY}$ the tangential strain at 0.

If now the strain gauges are arranged as shown in Fig. B such that $G_A$ measures in the direction $X$, $G_B$ in the direction $Y$ and $G_C$ measures at 45° to $G_A$ and $G_B$ then

$\phi_A = 0, \quad \phi_B = \frac{\pi}{2}, \quad \phi_C = \frac{5\pi}{4}$

and substituting in equations (A.1) — (A.3) above we obtain.

$$e_A = e_X$$

$$e_B = e_Y$$

$$e_C = e_{45°} = \frac{1}{2} \{(e_X + e_Y) + \gamma_{XY}\}$$

$$= \frac{1}{2} \{(e_A + e_B) + \gamma_{AB}\}$$

$\therefore \quad \gamma_{AB} = 2e_C - (e_A + e_B)$

The magnitudes of the principal strains, $e_1$ and $e_2$ may be calculated from the strain gauge readings $e_A$, $e_B$ and $e_C$ given by $G_A$, $G_B$ and $G_C$ by substitution in the following equation:

$$e_{1,2} = \frac{1}{2} \{(e_A + e_B) \pm \sqrt{[(e_A - e_B)^2 + \gamma_{AB}^2]}\}$$

$$= \frac{1}{2} \{(e_A + e_B) \pm \sqrt{[(e_A - e_B)^2 + (2e_C - (e_A + e_B))^2]}\} \quad \text{(A.4)}$$

Their directions may be calculated from the following expressions:

$$\tan 2\phi_p = \frac{\gamma_{AB}}{e_A - e_B} = \frac{2e_C - (e_A + e_B)}{e_A - e_B}$$

Writing $\sqrt{[(e_A - e_B)^2 + (2e_C - (e_A + e_B))^2]} = \sqrt{X}$

$$\sin 2\phi_p = \frac{e_{AB}}{\sqrt{X}} = \frac{2e_C - (e_A + e_B)}{\sqrt{X}} \quad \text{(A.5)}$$

$$\cos 2\phi_p = \frac{e_A - e_B}{\sqrt{X}} \quad \text{(A.6)}$$
The 'doorstopper' and triaxial rock stress measuring instruments developed by the C.S.I.R.—E. R. Leeman

![Diagram A](image1)

![Diagram B](image2)
The 'doorstopper' and triaxial rock stress measuring instruments developed by the C.S.I.R.—E. R. Leeman

\[ \tan \phi \equiv \frac{2(e_1 - e_2)}{2e_c - (e_A + e_B)} \]

Notice, from (A.4) that

\[ e_1 + e_2 = e_A + e_B \]

and \[ e_1 - e_2 = \sqrt{X} \]

(A.7)

and (A.8)

Now if \( \sigma_1 \) and \( \sigma_2 \) are the principal stresses on the surface of the borehole.

\[ \sigma_1 = \frac{E}{1 - \nu} (e_1 + \nu e_2) \]

(A.9)

\[ \sigma_2 = \frac{E}{1 - \nu} (e_2 + \nu e_1) \]

(A.10)

where \( E \) = Young's modulus of the rock

\( \nu \) = Poisson's ratio of the rock

The components of the normal stresses \( \sigma_A \) and \( \sigma_B \) in the \( A \) and \( B \) directions respectively and the tangential stress \( \tau_{AB} \) are given by:

\[ \sigma_A = \frac{1}{2} (\sigma_1 + \sigma_2) + \frac{1}{2} (\sigma_1 - \sigma_2) \cos 2\phi_p \]

(A.11)

\[ \sigma_B = \frac{1}{2} (\sigma_1 + \sigma_2) - \frac{1}{2} (\sigma_1 - \sigma_2) \cos 2\phi_p \]

(A.12)

\[ \tau_{AB} = \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\phi_p \]

(A.13)

In terms of strain, substituting for \( \sigma_1 \) and \( \sigma_2 \) from (A.9) and (A.10) in (A.11) to (A.13)

\[ \sigma_A = \frac{E}{2} \left[ \frac{e_1 + e_2}{1 - \nu} + \frac{e_1 - e_2}{1 + \nu} \cos 2\phi_p \right] \]

(A.14)

\[ \sigma_B = \frac{E}{2} \left[ \frac{e_1 + e_2}{1 - \nu} - \frac{e_1 - e_2}{1 + \nu} \cos 2\phi_p \right] \]

(A.15)

\[ \tau_{AB} = \frac{E}{2} \left[ \frac{e_1 - e_2}{1 + \nu} \sin 2\phi_p \right] \]

(A.16)

and substituting (A.5) to (A.6) in (A.14) to (A.16) we obtain

\[ \sigma_A = \frac{E}{2} \left[ \frac{e_A + e_B}{1 - \nu} + \frac{e_A - e_B}{1 + \nu} \right] \]

\[ \sigma_B = \frac{E}{2} \left[ \frac{e_A + e_B}{1 - \nu} - \frac{e_A - e_B}{1 + \nu} \right] \]

\[ \tau_{AB} = \frac{E}{2} \left[ \frac{2e_c - (e_A + e_B)}{1 + \nu} \right] \]