Note on a Volumetric Technique for the Determination of Stress in Rock

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SYNOPSIS

A technique is described for measuring the change in volume in a borehole drilled in a rock mass subjected to stress and strain. It is shown that the volumetric strain or change in volume provides a practical estimate of the the stress in the rock.

INTRODUCTION

Existing instruments for measuring stresses in rock are designed to measure the stress at specific points in the rock. Work done with the doorstopper type of stress-measuring instrument developed by Leeman[1] has shown that there may be considerable fluctuations in stress from point to point in coal pillars, and that large numbers of such stress readings are required before a reliable picture can be formed of the general stress distribution and total load on the pillar.

The technique described in this paper measures the change in volume of boreholes in rock, which represents the total effect of changes in stress on all the individual points on the wall of the borehole. The method is simple and straightforward and can be used by ordinary mining personnel for routine measurements of stress in rock.

THE RELATIONSHIP BETWEEN CHANGE IN VOLUME OF A BOREHOLE AND CHANGE IN STRESS AROUND THE BOREHOLE

Consider a horizontal borehole of diameter D and length Z in rock which is loaded simultaneously by three stresses Px, Py and Pz applied to remote boundaries as in Fig. 1.

Equations based on the theory of elasticity and relating the change in vertical and horizontal diameters and length of the borehole with the stresses Px, Py and Pz have been presented by Kirsch[2], Kuun[3], Fenner[4], Everling[5] and Hoek[6].

Hoek has shown that:

$$u = \frac{PyD}{E}\{(1+k)-\nu L+2(1-k)(1-\nu^2)\} \quad \ldots\ldots.(1)$$

$$v = \frac{PyD}{E}\{(1+k)-\nu L-2(1-k)(1-\nu^2)\} \quad \ldots\ldots\ldots(2)$$

$$w = \frac{PyZ}{E}\{L-\nu(1+k)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where u = change in vertical diameter of the borehole.
v = change in horizontal diameter of the borehole.
w = change in length of the borehole.
D = original diameter of the borehole.
Z = original length of the borehole.
E = Young's Modulus for the rock.
ν = Poisson's Ratio for the rock.
Py = Vertical stress on the borehole.

$$k = \frac{\text{Horizontal stress perpendicular to the borehole}}{\text{Vertical stress}} = \frac{Px}{Py}$$

$$L = \frac{\text{Horizontal stress parallel to the borehole}}{\text{Vertical stress}} = \frac{Pz}{Py}$$

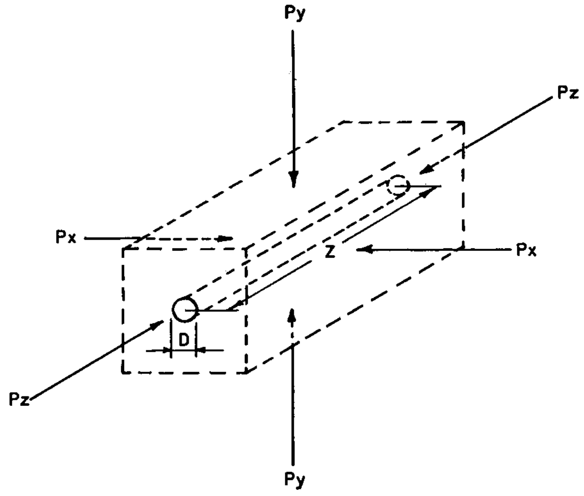

Fig. 1

With the aid of these equations it is theoretically possible to determine the change in volume of a borehole under altered stress conditions.

A change in the stressfield surrounding a hole will deform the hole, so that it will no longer be circular in section, but as the changes u, v and w are small compared to the original diameter and length of the borehole, it is a sufficiently accurate approximation to say that the change in volume, Vc, due to the change in stress is:

$$Vc = \frac{\pi D^2 Z}{4}\left\{\frac{u}{D}+\frac{v}{D}+\frac{w}{Z}\right\} \quad \ldots\ldots\ldots\ldots\ldots(4a)$$

and substituting the values of u, v and w in equations (1), (2) and (3):

$$Vc = \frac{\pi D^2 ZPy}{4E}\{k(2-\nu)+L(1-2\nu)-\nu+2\} \ldots.(4b)$$

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and since $k = \frac{Px}{Py}$ and $L = \frac{Pz}{Py}$

$$Vc = \frac{\pi D^2 Z}{4E} \{Py\,(2 - \nu) + Px\,(2 - \nu) + Pz\,(1 - 2\nu)\} \quad \ldots .(4c)$$

It is apparent that the vertical stress Py and the horizontal stress Px at right angles to the hole, have an equal effect on the change in volume of the hole, whereas the horizontal stress Pz parallel to the hole has a lesser effect.

Equation (4c) contains three unknowns, Px, Py and Pz; these three unknowns cannot be determined individually by measuring the volume change unless two of them are small in comparison with the remaining one, or a close estimate of their ratios is available.

It may be noted that equation (4c) can be written in the following form:

$$Vc = \frac{\pi D^2 Z}{4E} \{(2 - \nu)\,(Px + Py) + (1 - 2\nu)\,Pz\} \ldots (4d)$$

which form reveals that the sum $Px + Py$ can be measured accurately in every instance where the Pz component is small. This is always the case in a hole drilled perpendicular to the surface of an excavation, if the measurement is carried out in close vicinity of the rock face.

Furthermore, Fenner[4], has shown that in stable strata, unaffected by geological disturbances and away from excavations, the horizontal stresses Px and Pz are equal and dependent only on the vertical stress Py due to gravity and Poisson's Ratio ν for the rock, i.e.

$$\frac{Px}{Py} = \frac{Pz}{Py} = k = L = \frac{\nu}{1 - \nu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Poisson's ratio for most rocks is between 0·12 and 0·33, hence the ratio of the horizontal to vertical stress calculated from equation (5) should lie between 0·14 and 0·5. Tectonic forces of geological origin may however cause greater horizontal stresses, and values of k and L greater than 1·0 have been obtained from stress measurements in some rock formations. It is also evident that horizontal stresses greater than the vertical stress may be present in the footwall and hangingwall of excavations in rock.

In South Africa, stress measurements made in rock away from excavations have shown that the ratios k and L seldom exceed $\frac{\nu}{1 - \nu}$. This indicates that horizontal stresses of geological origin are absent. It should also be remembered that stress measurements are often required to be made in pillars surrounded by excavations, which will effectively reduce the ratios k and L to $\frac{\nu}{1 - \nu}$ and less, even though they may have been greater than $\frac{\nu}{1 - \nu}$ before the excavations were made.

It may therefore be accepted that a fairly accurate estimate of the vertical stress Py in pillars may be obtained by using the ratio $k = L = \frac{\nu}{1 - \nu}$ in equation 4(b).

The validity of this supposition was tested and confirmed by experiments described later in this paper.

DEVELOPMENT OF A MEASURING TECHNIQUE

The method that immediately suggested itself was the drilling of a borehole on a slight down grade, connecting the mouth of the hole with a graduated burette and filling the hole with water, so that changes in water level could be noted on the burette. The holes are however seldom watertight, and several methods of sealing the walls by means of epoxy resins, polyesters, etc., proved unsuccessful.

The next method tried was to insert a very thin and soft latex tube into the hole, connect it to the burette and fill it with water. During 1966, 21 boreholes were drilled and equipped in this manner at Tweefontein Colliery. Very interesting fluctuations in volumes were obtained; these followed roughly the calculated theoretical trends, but the tubes registered a continuous slow drop in water level, which invalidated the results. It was found that the holes had not been completely filled due to the water pressure in the tubes being insufficient to expand them tightly up against the walls of the holes.

Oversized tubes were tried, but pockets of air trapped between the walls of the holes and the latex tubes made the results unreliable.

During 1967 numerous different materials and procedures were tested at Klippoortjie and Delmas Collieries. The General Mining and Finance Corporation Limited was sufficiently interested to make a mining graduate available to assist in the task. Fig. 2 illustrates the technique now in use.

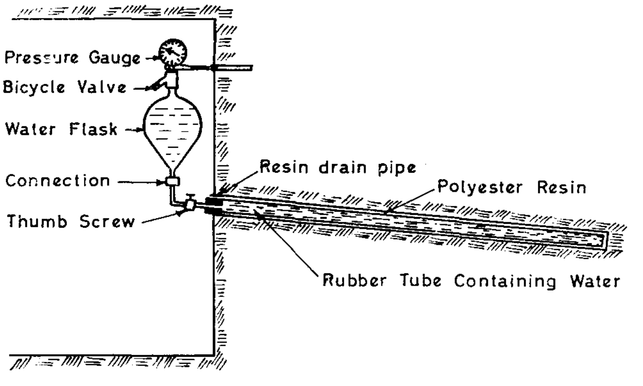

Fig. 2

A measured quantity of polyester resin (polylite 8120) mixed with the necessary curing agents is poured down the hole, and a thin walled rubber tube (the diameter of which is $\frac{1}{8}$ in. less than that of the hole) filled with water, is pushed into the resin by means of a thin steel rod. A stopper, which is inserted in the rubber tube, fits into the mouth of the hole with sufficient clearance to allow insertion of a thin pipe 2 in. long, through which excess resin drains off. The stopper is secured to the rock-face with plaster of Paris; as soon as this has hardened, more water is pumped into the tube by means of the arrangement shown. Most of the remaining resin is thus forced out of the hole leaving only a thin layer.

After about 40 minutes the layer of resin has set into a soft and flexible solid and the drainpipe is pulled out. The small hole left by the pipe is sealed with plaster of Paris, the rubber tube is closed-off by means of a thumb-screw and the waterflask is removed.

All holes are equipped in this manner and left standing for 24 hours. By then the resin has cured completely and temperatures are stable. A $\frac{3}{8}$ in. graduated glass tube with a T piece fitted with a bicycle valve and a pressure gauge on the end, is now filled with water and connected to the rubber tube protruding from the hole. The thumbscrew is released after the required pressure has been applied by means of a bicycle pump. The water level is arranged to stand near the top of the glass tube, and any change in volume can be read off.

APPLICATIONS OF THE TECHNIQUE

A. Experiments to test the reliability of the technique:

1. Tests in the laboratory:

A block of Waterberg Sandstone was cut by diamond saw to dimensions 36 in. $\times$ 6 in. $\times$ 9 in., and a hole 29 in. long with diameter $1\cdot5$ in. was drilled along its long axis on a downgrade of 5°.

The sandstone block was placed between the platens of a 200 tons press and a load of 150 tons was applied. The area under load was 36 in. $\times$ 6 in. less the area of the hole, which was $43\cdot5$ in.^{2}. This equals $172\cdot5$ in.^{2} resulting in a stress of 1,740 lb/in.^{2}. While under this stress, the hole was instrumented as described before.

The water in the glass tube was placed under a pressure of 11 lb/in.^{2}. The load applied by the press was reduced in steps of 25 tons to zero and again increased to 150 tons in steps of 25 tons. The change in water level in the glass tube was noted.

Fig. 3 shows the relationship between the change in volume of the hole and the load. Also shown on the graph in dotted lines is the relationship as calculated from equation (4b) using $k = L = \frac{\nu}{1 - \nu}$. The graph shows a slight deviation from straight line relationship and also a small difference between the values for decreasing and increasing load.

These can be attributed to experimental inaccuracies. The agreement between the measured change in volume and the calculated change is considered satisfactory.

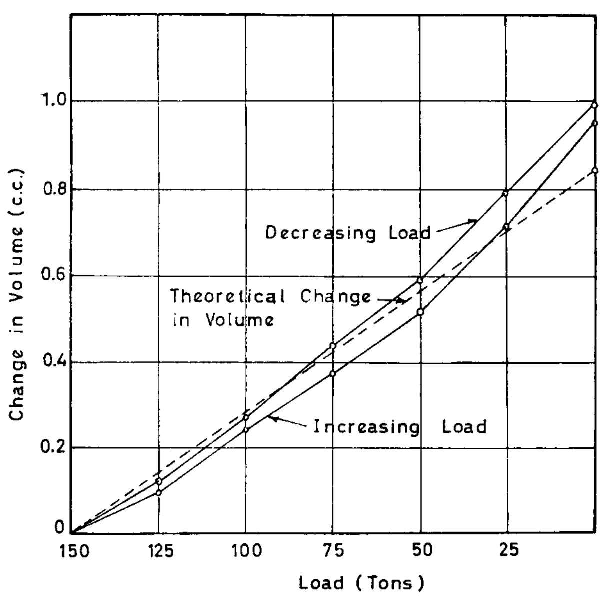

Fig. 3

2. In situ tests at Wolwekrans Colliery:

Three boreholes were drilled into each of two cubes of coal, which were being loaded to destruction by officers of the South African Council for Scientific and Industrial Research. The boreholes 6 ft long by 1 in. dia were drilled as shown in Fig. 4 and equipped with tubes in resin as described above. The cubes of coal, each with faces of 2 metres, were loaded by a system of hydraulic jacks placed between the hangingwall and the top face of the cube. The change in volume of the holes was then measured by the change in water level on burettes connected to the tubes inside the holes.

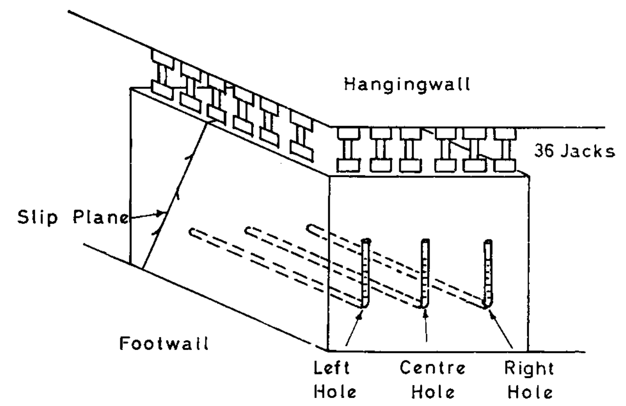

Fig. 4

The load applied to the pillar was registered by a pressure gauge in the pump system of the hydraulic jacks. It was increased in steps of 250 tons up to 1,250 tons, reduced in steps back to zero, and then taken up to fracture load. This was approximately 2,000 tons (equal to 640 lb/in.^{2}. The changes in volume recorded by the instruments are depicted graphically in Fig. 5 where they are compared with the changes in volume derived from equation (4b), again assuming $k = L = \frac{\nu}{1 - \nu}$.

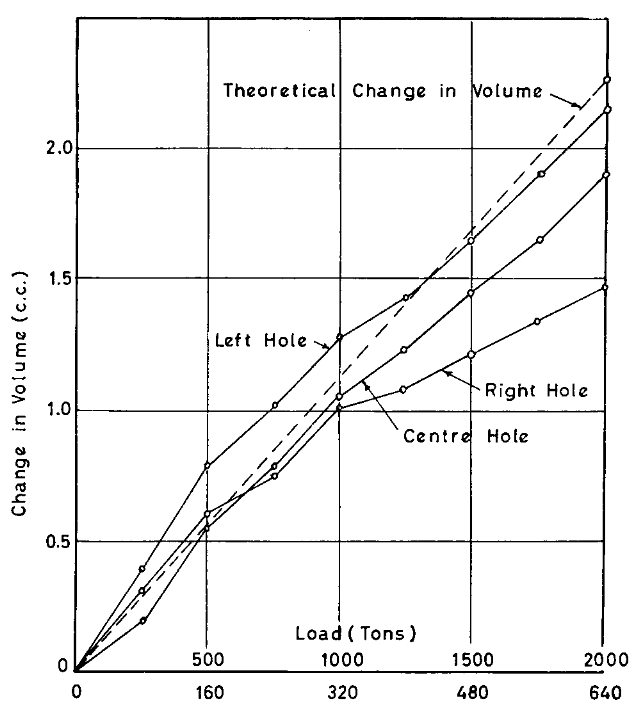

Fig. 5—Relationship between changes in volume of boreholes in a cube of coal, and load on the cube

The graph shows good agreement between the actual changes in volume registered by the instruments and the calculated values. The left and right holes were drilled symmetrically in the cube of coal and should have shown a similar change in volume. The difference indicates uneven distribution of the load on the cube, probably caused by the presence of a slip plane in the coal (see Fig. 4). This is probably also the explanation for the change in volume of the centre hole being less than that of the left hole.

B. Measuring stresses in coal pillars at Blinkpan Colliery

The measurements were done in Bethal 13 South Panel of Blinkpan Colliery during the period 15th to 18th July, 1968. Fig. 6 indicates the position of 15 instrumented boreholes, and also shows surface contours over the panel and roof elevations in the bords.

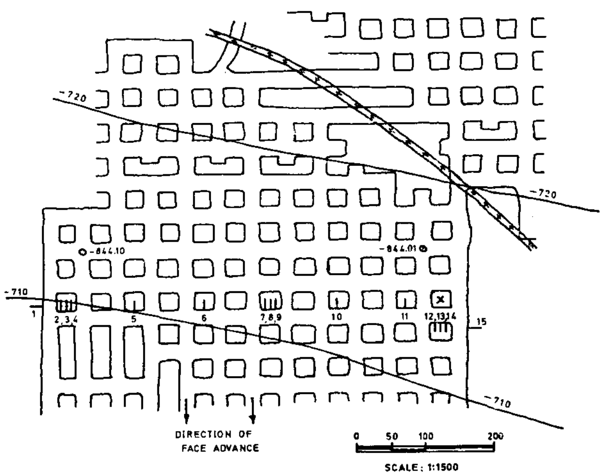

Fig. 6—Bethal 23 South Panel, Blinkpan Colliery

No. 4 seam is mined in this panel. There are 12 pillars each measuring 32 ft square between the two boundaries of unmined coal. Bords are 23 ft wide and the pillar height is 8 ft. The 15 holes drilled were all 5 ft 6 in. long and of $1\frac{1}{2}$ in. dia. Rockbolts inserted in the pillar marked 'X' would have interfered with slot cutting (destressing of holes), and this necessitated the drilling of holes Nos. 12 to 14 in the opposite pillar and No. 15 in the boundary in line with Nos. 12 to 14. The holes were drilled midway between roof and floor at a downgrade of 5 degrees, cleaned out and instrumented. A box was fitted over each instrument to protect it from debris made by the coal cutter in the stress release operations which were carried out on the following day.

A Universal coal cutter was used to destress the coal around the boreholes. Slots were cut above and below and to the left and right of the holes at a distance of 24 in. and to a depth of 7 ft.

The water level in the glass tube was noted before cutting commenced and again after each slot was completed. Fig. 7 shows a front view of the instrument after completion of the four slots.

Unfortunately the available coal cutter was an unwieldy machine and difficulty was often experienced in withdrawing the jib from the vertical slots. This resulted in damage to some of the instruments, before all four slots had been cut. Later the vertical slots were cut at a slight angle to the direction of the holes.

Slotcutting operations were commenced with a pressure of 15 lb/in.^{2} on the water in the tube. As the water level dropped in the tube with increasing volume of the hole, the pressure registered by the gauge dropped proportionately. More accurate results could probably have been obtained by bringing the pressure back to 15 lb/in.^{2} before taking each reading, but it was considered that the pressure drop could make only a slight and negligible difference.

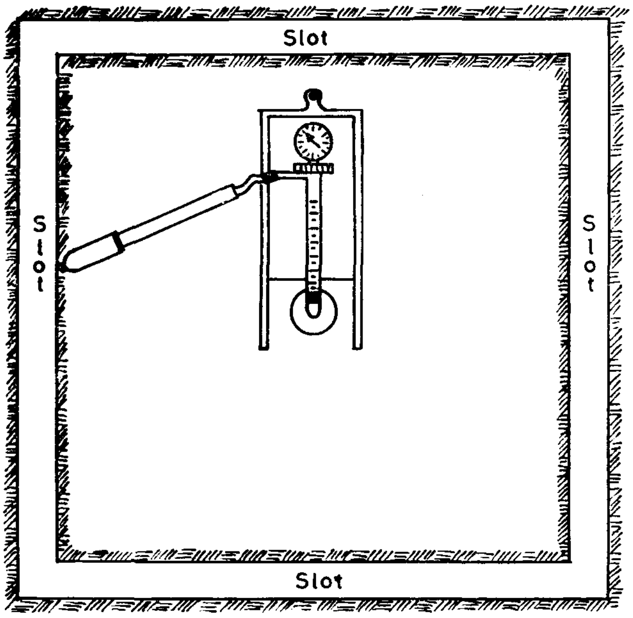

Fig. 7

Note:

Holes Nos. 1 and 15 were drilled into the boundaries of the panel. The side holes Nos. 2, 4, 7, 9, 12 and 14 were drilled 24 in. from the pillar sides, and only three slots were required to destress them completely. The remaining holes were all drilled at the centres of pillar faces.

The water level dropped further after each successive slot was cut, but each following slot had a lesser effect. In fact, in the case of holes Nos. 3, 6, and 13 the water level had actually risen slightly in the burette after the last (top) slice had been removed.

A Young's Modulus $E = 0{\cdot}55 \times 10^6$ lb/in.^{2} and Poisson's Ratio $\nu = 0{\cdot}13$ were again used to convert change in volume to stress.

The value of k and L in equation 4(b) was taken as being equal to $\frac{\nu}{1-\nu}$.

Using these values the factor obtained to convert change in volume to stress was 1 cc equals 127 lb/in.^{2}.

The results obtained are given in Table I. The holes were $5{\cdot}5$ ft long, and the values given in the last column thus represent the stress at a distance of $2{\cdot}75$ ft into the coal.

According to Salamon[8] the average stress on a coal pillar is not easily predicted and usually depends on many factors. Assuming that the whole weight of the overburden is carried by the pillars, and that the weight increases by $1{\cdot}1$ lb/in.^{2} for every foot of depth, he presents the formula $p = 1{\cdot}1H\left(\frac{w+B}{w}\right)^2$ for square pillars, H being the depth below surface, w the width of the pillars, B the width of bords, all in feet, and p is the maximum possible load on the pillars in lb/in.^{2}.

TABLE I

Hole No.	Water level before slot cutting	Water level after cutting				Total change in volume cc	Calculated stress in hole (lb/in.²) 1 cc = 127 lb/in.²
		Bottom slot	Left slot	Right slot	Top slot		
1	1·22	2·60	3·68	4·60	4·72	3·50	445
2	1·28	3·71	Not done	4·00	Inst. damaged	2·72	345
3	1·70	2·59	4·01	4·28	4·21	2·51	319
4	2·30	4·00	Hole damaged			1·70	216
5	0·78	2·43	3·20	4·00	Not done	3·22	409
6	1·18	2·11	2·51	2·86	2·81	1·63	207
7	1·10	3·19	Not done	3·72	4·15	3·05	388
8	1·15	2·32	2·99	4·02	4·45	3·30	419
9	2·96	6·67	Inst. damaged			3·71	471
10	0·88	2·48	2·91	3·58	4·41	3·53	448
11	0·89	2·87	3·26	3·61	3·90	3·01	382
12	1·17	3·80	5·68	Not done	5·52	4·35	553
13	1·85	3·85	4·75	5·36	5·20	3·35	425
14	1·18	4·28	Not done	4·30	4·30	3·12	396
15	1·12	2·81	3·08	3·29	3·75	2·63	334
						Average:	384

At Blinkpan the depth H of the pillars below surface is 135 ft, the pillars are 32 ft square and the bords 23 ft wide. Substituting these values in Salamon's formula, we obtain $p = 438$ lb/in.². Theoretical work done by Salamon and Orovecz[9] shows that the load on a coal pillar is not distributed uniformly, but decreases from the sides towards the centre at a ratio which may be as much as 10:6. The average value $p = 438$ lb/in.² could therefore be interpreted to mean a probable 540 lb/in.² on the side, 330 lb/in.² at the centre of the pillar, and 504 lb/in.² at a distance 2·75 ft from the face of the pillar.

It will be noted that a stress greater than this was obtained at only one hole (No. 12). On all the other holes the measured stress was appreciably lower, which confirms the view that the full weight of the overburden is not carried by the pillars inside a panel surrounded by barrier pillars.

CONCLUSIONS

The results obtained hold promise that the volumetric technique may develop into a practical, simple and inexpensive stress measuring tool.

It is apparent that the change in volume in a borehole measures the combined effect of vertical and horizontal stresses, but the true value of the vertical stress can be calculated from the change in volume only if the horizontal stresses are accurately known. It has however, been illustrated that in practice a near estimate of the vertical stress in pillars is obtained by taking the ratio of horizontal stress to vertical stress as $\frac{\nu}{1-\nu}$. An improvement in the technique, entailing the actual measurement of this ratio, is at present being developed.

The measurements at Blinkpan Colliery show that the actual stress on various coal pillars in a panel may be rather different from values calculated from theory. This should be taken into account in bord and pillar design.

ACKNOWLEDGEMENTS

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