A Note on the Stress Concentrations at the bottom of a Flat-Ended Borehole

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SYPOPSIS

The 'doorstopper' technique for determining the state of stress in rock requires that the stress concentration factors at the flat end of a borehole be known. Published values of these factors are widely variant. An axisymmetric finite element analysis of this problem has been made, yielding elastic stress concentration factors for different values of Poisson's ratio.

INTRODUCTION

Of several methods for determining the state of stress in rock, the 'doorstopper' technique is perhaps the most popular. In this method a strain gauge rosette is cemented to the bottom of a flat-ended borehole, and the strain relief brought about by overcoring this with an annulus of the same outside diameter as the borehole is measured, Leeman[3], (1964). The state of stress can be calculated from such measurements in at least three boreholes of different orientations, if the elastic constants of the rock and the stress concentrations at the bottom of the flat-ended borehole are known.

No analytical solution is available for the stress distribution around a flat-ended borehole. Several investigators have attempted to determine the stress concentration factors experimentally, for example, Galle and Wilhoit[2] (1962), Leeman[3] (1964), Bonnechere and Fairhurst[1] (1968), Pallister[4] (1969), and Van Heerden[6] (1969), but the numerical values of their results vary by almost fifty per cent, with one exception. The point on which there is complete agreement is that a normal stress perpendicular to the axis of the borehole generates no stress across the flat end in a direction at right angles to it. This allows an important analytical simplification to be made, namely, that the stress concentration factors for normal stresses parallel and perpendicular to the axis of the borehole can be found from the solution for an axisymmetric stress distribution.

This note presents the stress concentration factors as determined by axisymmetric finite element analysis (Wilson[7], 1965). Because the stresses in three-dimensional elasticity problems depend on Poisson's ratio (Timoshenko and Goodier[5], 1951), the calculations were made for values of Poisson's ratio from 0 to 0·4. It should be noted that these results, determined for uniform applied stresses, are applicable only where negligible stress gradients existed in a small volume of rock around the borehole prior to its being drilled.

METHOD OF SOLUTION

The basis for axisymmetric finite element analysis is imagining a continuous body of revolution to be comprised of a finite number of axisymmetric 'ring elements', which are connected at 'nodal circles' (Fig. 1). The relationship between the nodal circle forces and displacements, or the stiffness, of each ring element is determined by the geometry and stress-strain behaviour of the element. By systematic addition of ring element stiffness a system of equilibrium equations relating the resultant nodal circle forces and the unknown nodal circle displacements is developed for the entire finite element network. Solution of these equations gives the solution of the problem because the ring element stresses are determined uniquely by the ring element stiffnesses and the nodal circle displacements (Wilson[7], 1965).

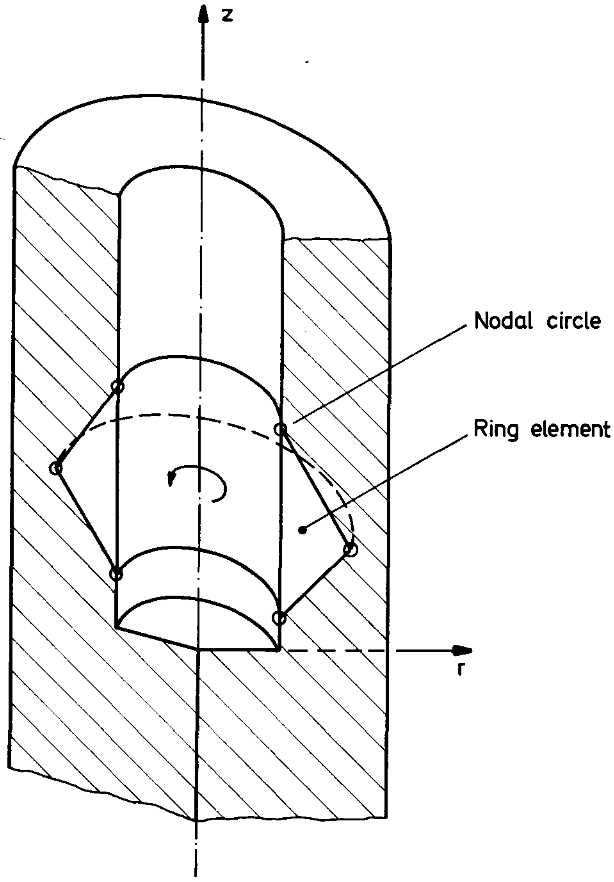

Fig. 1—Diagram of an axisymmetric triangular ring element

The finite element model used to solve this problem is depicted in Fig. 2. A semi-finite flat-ended borehole was represented by a cylindrical cavity, with a length to diameter ratio of 10:1, in a cylindrical body. The finite-element mesh included 433 triangular ring elements and 244 nodal circles, with 8 ring elements and 9 nodal circles across one half the end of the cavity.

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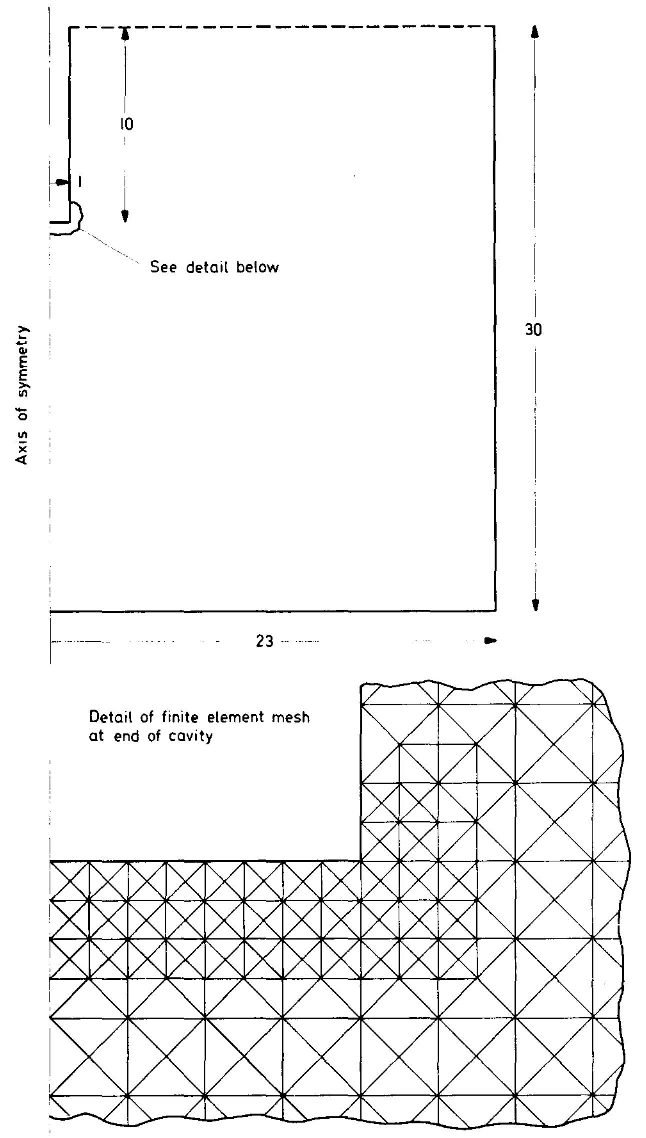

Fig. 2—Diametral section through a cylindrical cavity showing the finite element approximation to a flat-ended borehole in an infinite medium

The stress concentrations at the ends of the cavity, due to stresses applied parallel and perpendicular to the axis of symmetry, were determined for Poisson's ratios of 0, 0·1, 0·2, 0·3 and 0·4. In all cases the force-displacement equations were solved by iteration until the sum of the residuals was less than 10^{-4}.

RESULTS

For each value of Poisson's ratio the stress concentration at the centre of the end of the cavity was found by evaluating the stress in each element at its centroid and extrapolating to the surface to find the stress concentration, as illustrated in Fig. 3. These factors, as a function of Poisson's ratio, for stresses parallel and perpendicular to the axis of symmetry are presented in Table I. They are almost uniform from the centre to a radius of about 0·4 R, where R is the radius of the cavity.

The finite element solution is compared with published values for the stress concentration factors in Fig. 4 and 5.

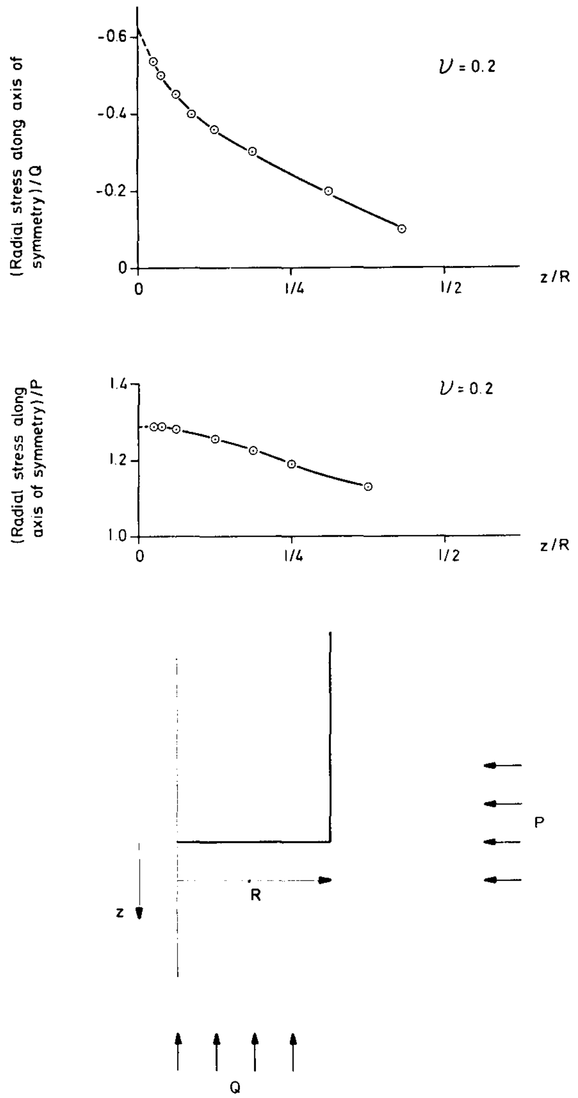

Fig. 3—Example of the determination of stress concentration factors by extrapolztion of element stresses to the surface

TABLE I

AXIAL AND RADIAL STRESS CONCENTRATION FACTORS FOR VARIOUS VALUES OF POISSON'S RATIO

ν	F_A	F_R
0	−0·455	1·350
0·1	−0·540	1·325
0·2	−0·620	1·287
0·3	−0·700	1·210
0·4	−0·780	1·110

EXAMPLE

If the stresses remote from the borehole are σ_x, σ_y, and σ_z, with the axis of the borehole in the z-direction (Fig. 6), then the stresses at the flat end of the borehole are

$$\sigma_x' = F_R\sigma_x + F_A\sigma_z$$
$$\sigma_y' = F_R\sigma_y + F_A\sigma_z.$$

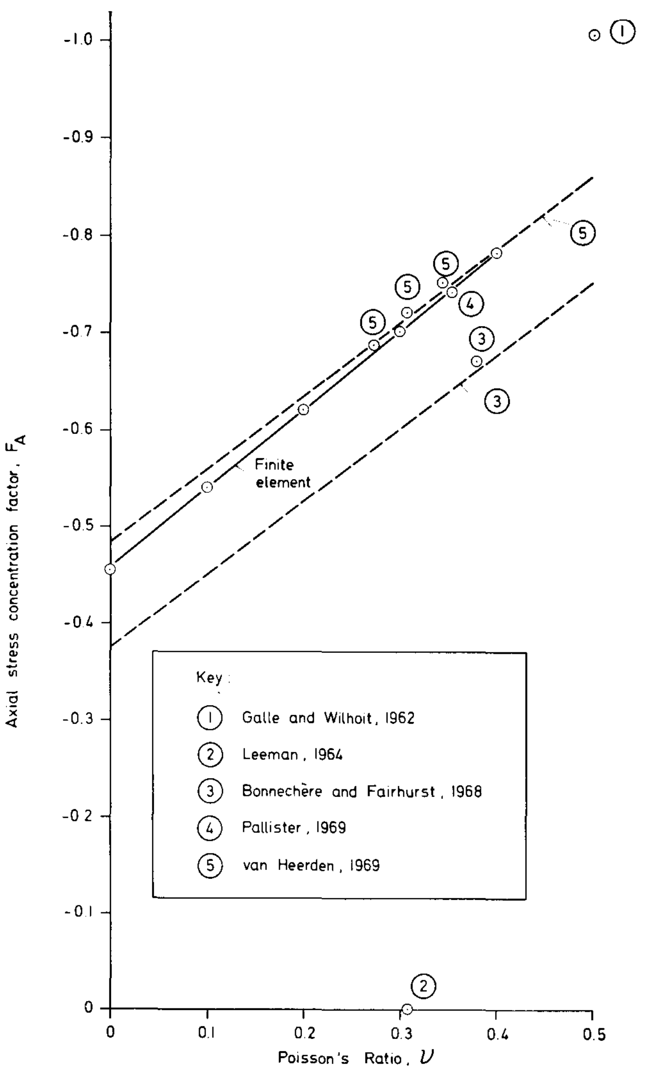

Fig. 4—Comparison of finite element solution with experimentally determined axial stress concentration factors

If Poisson's ratio is 0·2 then, from Table I,

$$\sigma_x' \doteq 1\cdot 29\,\sigma_x - 0\cdot 62\,\sigma_z$$
$$\sigma_y' \doteq 1\cdot 29\,\sigma_y - 0\cdot 62\,\sigma_z.$$

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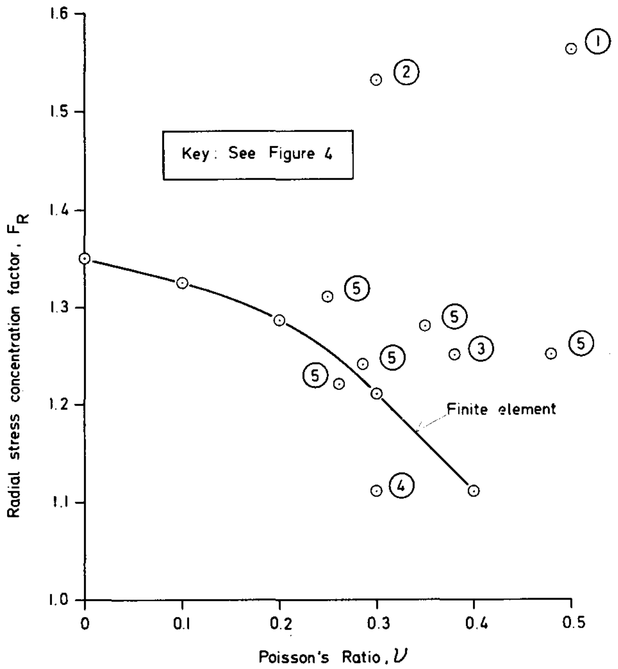
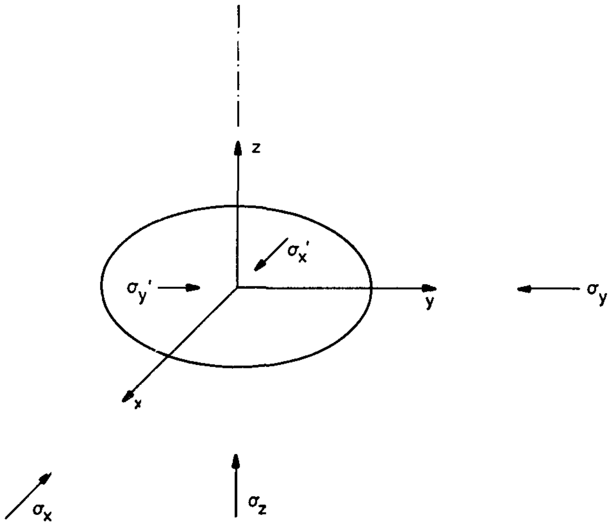


Fig. 5—Comparison of finite element solution with experimentally determined radial stress concentration factors



$$\sigma_x' = F_R\,\sigma_x + F_A\,\sigma_z$$
$$\sigma_y' = F_R\,\sigma_y + F_A\,\sigma_z$$

Fig. 6—Example of the application of the stress concentration factors