The application of the electrical resistance analogue to mining operations

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Discussion:

M. D. G. Salamon* (Fellow): The theoretical principles behind the techniques of calculation used by the authors was formulated and published during the period 1962–65. These principles, together with their practical implications, were summarized in a review paper in 1966.[1] Now, in February, 1970, we have this long and comprehensive paper discussing the practical application of the theory. I think this is good progress. The large number of practical examples treated in the paper clearly demonstrates not only the skill and knowledge of the authors, but also the versatility of the techniques of analysis.

I feel that the techniques described by the authors, and their improved versions, will revolutionize mine planning. I suggest that no shaft system or stoping layout should be planned in future without an adequate rock mechanics analysis. It seems that it will be progressively more difficult to white wash mistakes in rock mechanics and attribute their costly outcome to normal mining risk.

We should not, however, become complacent. There is still a lot to learn and do in the future. I do not want to discuss tonight the question of further research but instead, I will raise two other problems which may hinder the future progress of practical rock mechanics.

To carry out work similar to that described in the paper, mine managements require men who are capable of doing the work and who have the most efficient techniques of analysis at their disposal.

As a research engineer I have been associated with the development of techniques of analysis. I can assure you that these have undergone an amazingly rapid development during the last six years. We have seen the change from the use of the electrolytic tank analogue and manual integration to the method described by the authors, that is, to the employment of the automatic network analogue and computer integration. But this is not the end. In a recent paper Prof R. P. Plewman[2] and his co-authors described a completely digital technique. Here the process of calculating on- and off-reef quantities is integrated into one computer run.

But these sophisticated methods of computation are useless without men to make intelligent use of them. During the last few years a new breed of mining engineers has appeared on the scene — the Rock Mechanics Engineer. I feel it is timely to call the attention of the senior members of this Institute to the fact that this new breed will succeed in carrying out its duties in a valuable manner only if we manage to attract to its ranks young engineers of the right quality. The requirements are high. The top rock mechanics engineers have to be good mine planners and they have to understand the essence of sophisticated theories. Without these attributes they will not succeed in the long term and the industry will not gain the benefit of available knowledge. To attract this quality of engineers the industry will have to ensure that rock mechanics is accepted by all concerned as a career which could lead to the top echelons of our profession.

Finally, I would like to make one or two remarks concerning Part I of the paper.

In Sections 3.1 and 3.2 the authors describe methods of determining the normal stress on the reef plane and the convergence distribution in the excavations. The description as given applies only to a horizontal reef. When the reef is inclined the situation is more complex since a shear component of stress on the intact reef and a ride component in the excavations must then also be determined. These can be calculated by methods similar to that described in the paper.[3]

The practical application of the method of calculating the released energy (Section 3.4) is in the comparison of various stoping layouts to establish an order of preference in terms of decreasing rates of energy release. To carry out these comparisons effectively, the calculations must be carried out in a manner by which the values of energy per unit area obtained are comparable.

I would like to note in this respect that the method of calculation given in the paper is valid without reservation only if there is no elastic contact between the hanging and foot walls in the mined-out area. If there is contact then the calculated energy values are correct only if the mined out area is increased in small steps during the analysis.

My second point in this regard is concerned with the practical method of calculation as described in Sections 3.4 (ii)-(iii). Firstly, I would like to suggest that the energy calculation should be carried out, as far as possible, by using always the same scale on the analogue. Secondly, the reading of current on a pin should be followed by the removal of that pin to obtain the corresponding reading of potential. The product of these two values will be proportional to the energy released during the mining of the small area corresponding to that pin. The method suggested by the authors tends to mask possible danger points in the layouts, since they obtain an average value for a time period, say, for six months.

REFERENCES

1. COOK, N. G. W., HOEK, E., PRETORIUS, J. P. G., ORTLEPP, W. D. and SALAMON, M. D. G. 'Rock mechanics applied to the study of rockbursts.' *J. S. Afr. Inst. Min. Metal*, 66, May 1966, 435.

Chamber of Mines Laboratories.

2. Plewman, R. P., Deist, F. H. and Ortlepp, W. D. 'The development and application of a digital computer method for the solution of strata control problems.' *J. S. Afr. Inst. Min. Metal.*' 70, Sept. 1969, 33.

3. Salamon, M. D. G. 'Elastic analysis of displacement and stress induced by the mining of seam or reef deposits, Part IV.' *J. S. Afr. Inst. Min. Metal.*, 65, Dec. 1964, 319.

R. S. Pearson* (Fellow): The authors are to be congratulated on an excellent paper and on the advance they have made in putting the science of rock mechanics on a sound practical basis. The results produced being in a form which can be readily used by mining engineers.

Prior to October, 1969, for the mines of the Anglo American Corporation in the O.F.S. and Klerksdorp Goldfields, a digital computer was used for the calculation of stresses and displacements from the analogue data. Since then the analogue has been supplemented by digital computer programmes designed to perform routine periodic rock mechanics investigations for these mines. These programmes have been described in a paper read before the Institute by Plewman *et al.*[1]

They have been adapted for use on the IBM 360/40 digital computer situated at Welkom, which has a smaller capacity than the machine for which they were originally written. This version requires a 64K store for the programmes and, in this particular installation, a further 16K for the D.O.S. Supervisor. The excellent 'turn around' facilities at this computer centre and the repetative nature of several aspects to the rock mechanics procedures, together with the storage feature of the digital programme, has reduced the overall dependance on the electrical resistance analogue and eased the load on it.

An indication as to how this technique is being increasingly used is given by the machine running time for these programmes over three months, viz. 18 hours in a total of 28 hours on rock mechanics applications in October, 1969, 22 out of 33 in November and 39 out of 45 in December.

As a matter of interest, the savings effected in the reduction of bunton replacement in one shaft, alone, have more than offset the total cost of the above machine hours.

REFERENCE

1. Plewman, R. P., Deist, F. H., and Ortlepp, W. D. 'The development of a digital computer method for the solution of strata control problems.' *J. S. Afr. Inst. Min. Metal.*, 70, Sept. 1969, 33.

C. L. de Jongh (Visitor): The paper presented here this afternoon, is conclusive proof of the value of the Electric Resistance Analogue. The analogue belonging to the Gold Fields mines was built by Gold Fields Laboratories and is a modified and improved version similar to the new analogue of the Mining Research Laboratories. Although the Gold Fields Analogue was only taken into full operation during April of 1969, it has already proved itself to be an outstanding tool in the treatment of remnant mining and off-reef calculations.

The following example is given as an illustration of its capabilities:—

The Rock Mechanics Department was asked by the management of Libanon mine to investigate the feasibility of mining the payable blocks in their No. 2 Shaft pillar area, while retaining, if possible, the full use of the shaft. The problem was ideally suited to the analogue because of the scattered mining patterns. Calculations obtained with the analogue and the digital computer of Gold Fields in Johannesburg proved the feasibility of mining the desired blocks, viz. the cross-hatched areas within the circular shaft pillar in Fig. 1.

Fig. 1—Mining in No. 2 Shaft pillar at Libanon G.M. Co. Ltd.

The major concern was the possible shortening of the shaft. The predicted shortening between two stations 500 ft apart was calculated as 0.113 ft, while the measured shortening was only 0.06 ft. The predicted shortening between two stations 1 000 ft apart was 0.079 ft and the measured shortening 0.09 ft.

Considering the fact that more than 6 000 square fathoms could be mined out of this pillar with confidence, and with stoping operations extending to within 120 ft of the shaft, a maximum error of less than 1 in. in predicted shortening of the shaft must be regarded as eminently satisfactory.

T. J. Kotze (Graduate): The authors deserve to be congratulated on a thorough and extensive account of what can be achieved practically today in the field of rock mechanics. It should be emphasized that all of this would not have been possible, if it was not for the development of the analogue computer. Without this design tool, the influence of rock mechanics would even today still have been confined to the laboratory and the lecture hall.

The rock mechanics work being done on the mines of Union Corporation Limited, covers most of the spheres indicated by the paper. In particular a great deal of importance has been placed on the determination of critical field stresses for tunnels in different rock strata. With the information obtained future instability

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can now be predicted with confidence. In order to pinpoint the severity of haulage failure accurately, all failure observed is classified into four stages as follows:—

First stage = initial flaking of the sidewalls.

Second stage = formation of slabs without any collapse.

Third stage = slabs collapsing into the excavation.

Fourth stage = total collapse.

In the Evander area the immediate hangingwall quartzite above the reef (H/W2) is much weaker than the immediate footwall quartzite (MK3) and Table I shows that there is a marked difference between the critical field stresses for excavations in the two strata.

TABLE I

CRITICAL FIELD STRESSES FOR EXCAVATIONS

Severity of failure	Critical field stresses for excavations in H/W2 quartzite	Critical field stresses for excavations in MK3 quartzite
Stage 1	41 MN/m^2	41- 55 MN/m^2
Stage 2	41-62 MN/m^2	55- 76 MN/m^2
Stage 3	62 MN/m^2	76-110 MN/m^2
Stage 4	not yet determined	110 MN/m^2

Table I also shows that the critical field stresses are roughly of the same order as those determined in the O.F.S. and Klerksdorp Goldfields.

Based on the criteria used in the table it is now easy to determine the stability of on- and off-reef excavations and to propose methods of protecting them from collapse, i.e. what size reef pillar should be left to protect certain excavations, can an unpay pillar or loss of ground be left over an off-reef excavation without subjecting that excavation to a critical stress, and so on. The degree of stress to which such an excavation is likely to be subjected can be determined in advance and recommendations made as to how best to support the excavation depending on the type of failure likely to occur. In some cases rockbolting and wire mesh support is sufficient. In others waste cuts over the excavations are required. In extreme cases, alternative tunnels may be required so that the initial excavation can be abandoned should there be the possibility of collapse.

To illustrate the use made of the principles described above, two typical examples are quoted below;

(i) Fig. 1 shows a crosscut on Kinross mine which was being developed over mined out ground. A portion of this crosscut was, however, in solid ground and was due to be understoped in the near future. An analysis showed that a total collapse of that portion of the crosscut could be expected during the undermining operation. As it was essential that the development of the crosscut was not interfered with, it was recommended that a by-pass crosscut be developed over mined out ground. This recommendation was carried out and the by-pass proved to be invaluable when a few months later the original crosscut did collapse as predicted.

(ii) Fig. 2 shows a footwall haulage that was giving trouble at Kinross mine. The problem was complicated by the presence of some unpay ground above the troublesome portion. Management was afraid that the build-up of stress when all pay ground was mined out would be excessive and an alternative route underneath mined out ground was contemplated. This problem was analysed on the analogue computer and the results showed that mining of the pay ground should only increase the field stress by a further 6.9 MN/m^2. The haulage, which, by all accounts, was expected to collapse, should not therefore deteriorate much more. Recommendations were made to the mine management that the existing haulage be supported by rockbolting and wire mesh rather than go to the expense of driving a second tunnel.

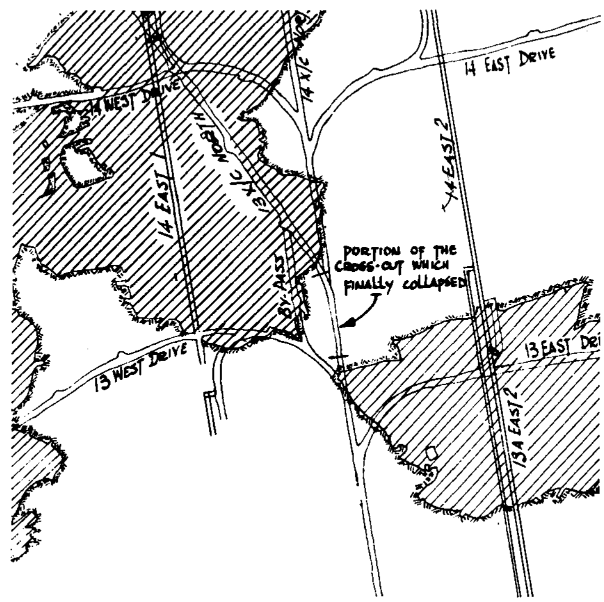

Fig. 1—Cross-cut over mined ground at Kinross Mine

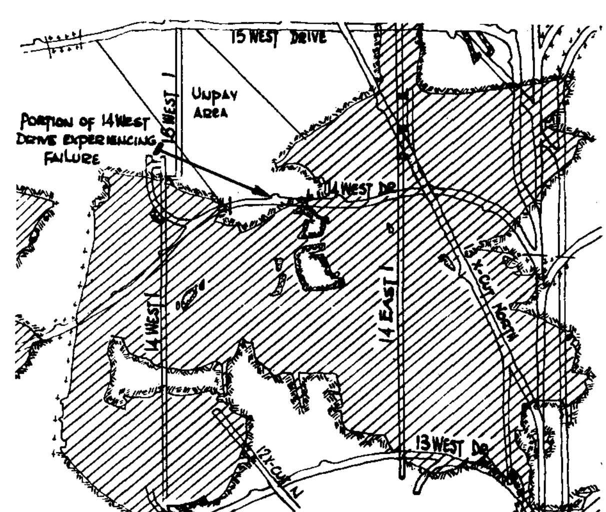

Fig. 2—Failure of footwall haulage at Kinross mine

W. D. Ortlepp (Member): In presenting their paper on the practical application of the electrical resistance analogue, the authors have performed a valuable service and they are to be especially commended for the clear and interesting manner with which they assembled their very comprehensive material.

It is valuable mainly because it finally reveals to mine management the kind of practical benefits which have been promised during the past six years.

In a way it is the natural sequel to the series of papers, commencing in 1964, which described the development of techniques for performing elastic analyses of rock-mass behaviour.

These papers may often have seemed unnecessarily theoretical and remote to the practical mining man impatient for usable solutions. The wide variety of problems, which the authors have shown can now be treated adequately, will surely re-assure him that the results were worth waiting for.

The theoreticians who were responsible for the earlier papers will find it gratifying that their endeavours have proved so useful although some may feel a little uneasy about the simplifications implicit in the authors' treatment of some of the problems.

However, it should be emphasized that the simplifications have, for the most part, been empirically validated. This process of comparing many known instances of damage with the conditions that prevailed at the time—the 'case-history' approach—is now possible because of the facility with which the resistance analogue and digital computer can handle the complicated geometry of actual underground conditions. The benefits to be derived from this approach have been well illustrated by the authors but its importance in the design philosophy can not be over-emphasized. Undoubtedly the use of 'case-history studies' for fixing empirical criteria will continue to be necessary for a long time. Ideally, such criteria should be established for each mining field if not for each individual mine.

The fact that the 8 000 lb/in.^{2} criteria appears to be equally applicable to both O.F.S. and Klerksdorp areas does not mean it is applicable for any tunnel in any gold mine. It is probably as much dependent on the shape of the tunnel as on the immediate geology.

As an illustration of this point the following cases from E.R.P.M. may be of interest.

Main haulages:

The location of two main haulages, each originally cut to about 10 ft × 11 ft with arched roof, is shown in Fig. 1. 58 haulage is in hangingwall quartzites about 500 ft above the reef, and 68 haulage is about 260 ft in the footwall. The maximum principal stresses were computed from an electrolytic analogue analysis, for several points along each haulage. The appearance of the two tunnels at points where the stresses were 14 000 lb/in.^{2} and 20 000 lb/in.^{2} respectively is shown in Figs. 2 and 3.

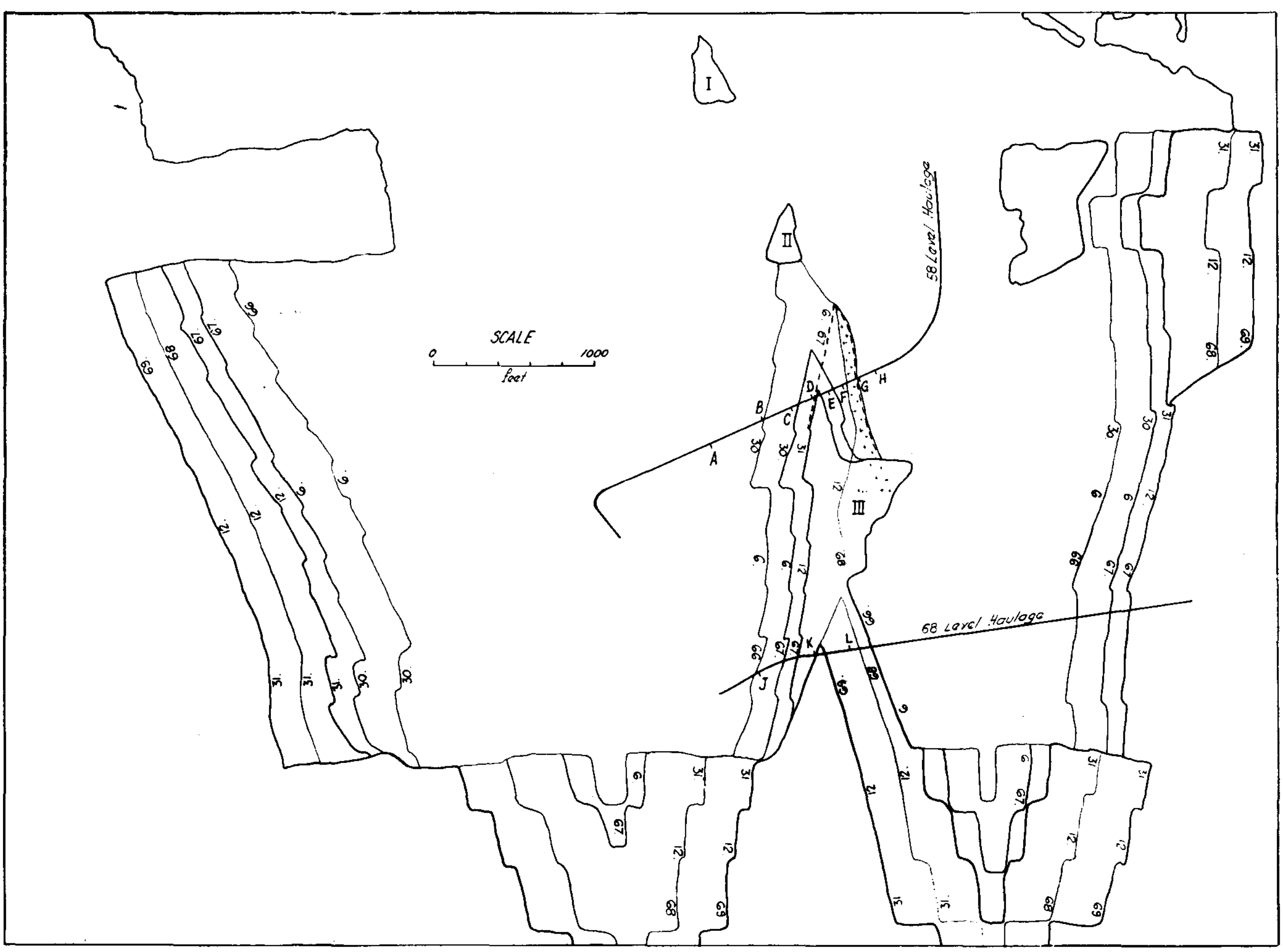

Fig. 1

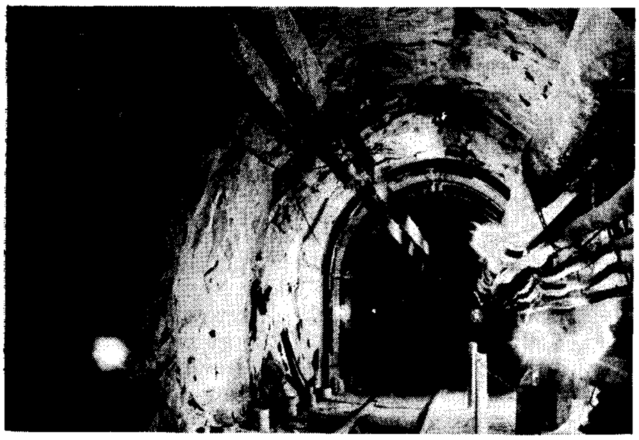

Fig. 2—E.R.P.M. 58 level haulage, in hangingwall quartzites at a depth of 7 960 ft below surface. Theoretical maximum field stress was 14 000 lb/in^2 at C (in Fig. 1)

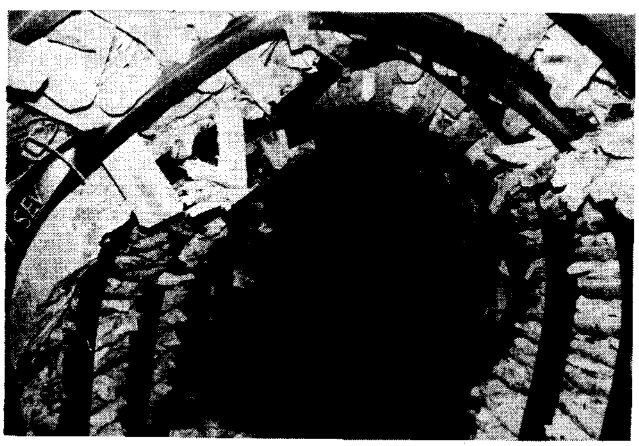

Fig. 3—E.R.P.M. 68 level haulage in argillaceous footwall quartzite at a depth of 9 410 ft below surface. Theoretical maximum field stress was 20 000 lb/in.^{2} at K (in Fig. 1)

With only light steel arches, which show no signs of any distress, the 58 level haulage, in hanging-wall quartzites, has continued to operate without any delay whatso ever. The 68 level haulage which is in very argillaceous quartzite, has long been abandoned.

Reef raise:

An 800 ft long raise of 14 ft $\times$ 5 ft cross-section has recently been completed uneventfully through ground which, according to the electrical analogue, is subjected to total stresses in excess of 60 000 lb/in^2. This raise was designed to be meshed and bolted with 3½ ft and 5 ft yielding bolts. Since the raise is temporarily inaccessible it is not known what standard of bolting was achieved or how it has subsequently behaved.

Unfortunately no empiric criterion for tunnel damage has been established for E.R.P.M., mainly because of the scarcity of reliable case-histories, which is due largely to the very small number of pre-developed tunnels.

No attempt has been made as yet to find a criterion for that type of damage of which there is, regrettably, no scarcity of occurrences on E.R.P.M., Limited, viz. rock-bursts. Here it is much more difficult to obtain reliable records, for various reasons. There is a very wide spectrum of damage intensity possible which can only be subjectively assessed, therefore the threshold at the lower end, which necessarily contains the largest number of incidents, is subject to a variable and indeterminate bias. In an endeavour to reduce this as far as possible, a simplified rock-burst report form, which requires virtually no measurements or laborious written description by the Mine Overseer concerned, has been prepared on E.R.P.M.

Another more serious reason for pessimism lies in the inherently intermittent, perhaps even random, nature of rock-bursts. Quite clearly the rate of energy build up revealed by the analogue for any particular abutment does not fluctuate rapidly but can remain nearly constant for long periods. Yet within such a period there may be long spells without rock-bursts interspersed with short spells of numerous events in quick succession.

In this connection, it appears that the authors always set one pin equal to 50 ft when measuring energy release rates on the analogue. I would like to know whether the effect of scale on the value of energy release rate has been investigated. It appears to me that the effect might be considerable and that it would be advisable for everyone involved in this type of analysis, to adopt a standard scale.

These examples are not intended in any way to cast doubt on the usefulness of the authors' conclusions and achievements which, I repeat, are considerable and of great value to the industry. The intention is merely to show that there are many challenging problems still facing us and that there is perhaps a limit to which the very simplified design philosophy can be extended.

In conclusion, I would like once more to offer my congratulations to the authors on a very fine paper and my hope that they will meet with as much success in the future.

I would like to thank Mr K. E. Steele, General Manager of E.R.P.M., Limited, for permission to publish this contribution.

H. Wagner* (Visitor): In this comprehensive paper the application of the electrical resistance analogue to several practical mining operations has been clearly demonstrated. In this contribution the application of the electrical resistance analogue to a study of idealized shaft pillar problems involving annular and other adjacent pillars is treated.

ANNULAR PILLARS

Mine shafts are one of the most important and expensive elements of a deep-level mine. In order to protect mine shafts and the facilities in their vicinity from stoping operations, large areas of usually payable ground next to the shaft must be left unmined for most of the life of the mine. With increasing depth of the mine the area of these shaft pillars has to be increased in order to maintain the stress concentrations around the structures in the shaft pillar at reasonable values. As a consequence of this, shaft pillars have become a more and more serious problem with increasing mining depths. No obvious alternative to increasing the area of a shaft pillar with depth is apparent.

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Practical experience has shown that the extraction of even small remnant pillars in the vicinity of a shaft pillar may have a substantially deleterious effect on the stability of the shaft pillar. This practical experience indicates that such remnant pillars have more effect on the stability of a shaft pillar than may have been expected.

It is the purpose of this contribution to analyze the effect of pillars adjacent to a shaft pillar, and to show the effectiveness of the subdivision of a shaft pillar into a centre pillar and an annular pillar in reducing the stresses on the centre shaft pillar and those structures within it.

The effect of annular pillars on the stress concentration of a centre pillar

To study the effect of an annular pillar on the stress concentration in a centre pillar some idealized experiments have been conducted on the electrical resistance analogue at the Mining Research Laboratory. For these experiments the idealized mining configuration of a circular centre pillar surrounded by an annular pillar was chosen (Fig. 1), in order to study the effect of the area and of the situation of the annular pillar on the stress in the centre pillar. The total area of the centre pillar and the annular pillar, $A_T = A_{CP} + A_{AP}$, was kept constant, while the ratio between the area of the annular and the central pillars, $R = \frac{A_{AP}}{A_{CP}}$, was varied over the range of $\frac{1}{3}$ to 2. According to the simple geometrical relation $A_{AP} = 2\pi r_{AP} W$, the radius of the annular pillar r_{AP} was varied by changing its width, W, for each ratio, R.

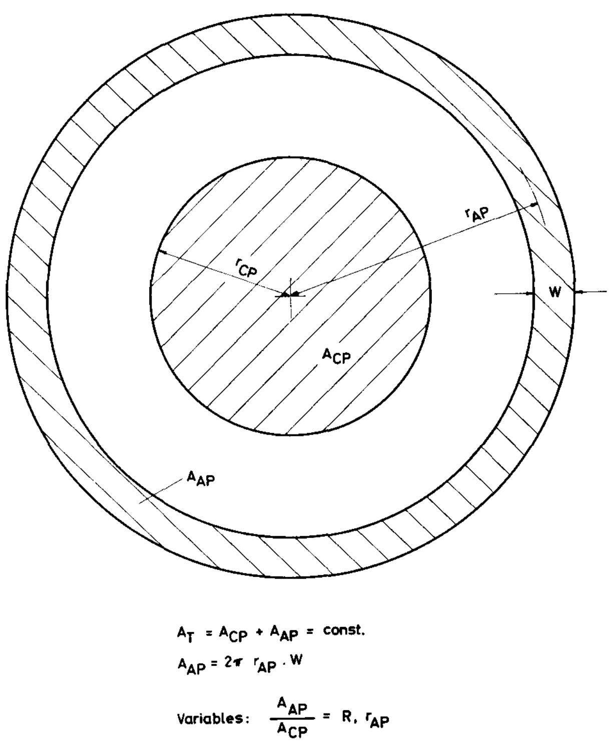


Fig. 1—Idealized mining configuration of a circular central pillar surrounded by an annular pillar

Discussion of the results

The purpose of dividing a shaft pillar into central and annular portions is to reduce the total area of reef required to protect the installations around a shaft. Stresses in the central portion of a divided pillar system can be reduced to values sufficiently small as not to damage these installations at the cost of generating correspondingly higher stresses in the annular portion of the pillar. The annular portion can either be kept free of installations or portions of it can be removed by under- or over-stoping to protect any haulages which may have to pass near it.

The stress concentrations at the centre pillar have been plotted for the different configurations. In order to generalize these results these have been normalized by dividing by the corresponding concentrations for an individual central pillar of the same area as the total area of the combined central and annular pillar system. This allows a direct comparison to be made of the change in stress concentrations for the individual pillar and the sub-divided pillar. Both the average stress and the maximum stress concentrations at the edge of the centre pillar have been plotted separately. The radius of the annular pillar has also been normalized by dividing it by the radius of an individual shaft pillar of the same total area as that of the sub-divided system.

The effect of an annular pillar on the stress concentration at the central pillar for different pillar configurations is shown in Fig. 2 and 3. These show that annular pillars situated near the centre pillar reduce the average stress at the central pillar by between 20 and 30 per cent; however, this effect decreases with increasing distance of the annular pillar from the edge of the central pillar, and annular pillars situated at distances greater than one radius of the centre pillar are ineffective in reducing average stress at the centre pillar. These graphs also indicate that the area of the annular pillar is rather unimportant compared with the distance of the annular pillar from the central pillar.

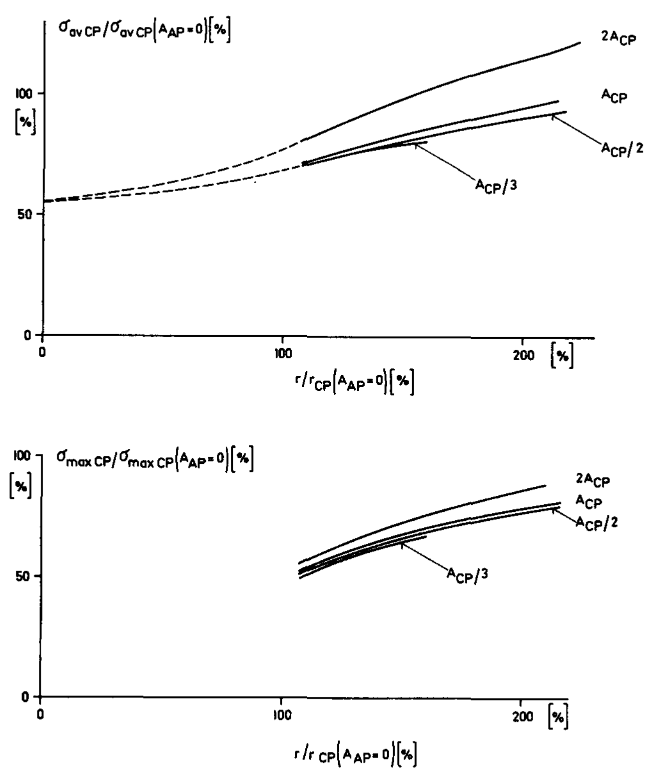


Fig. 2—The effect of the radius of the annular pillar on the stress concentration at the central pillar for different pillar configurations

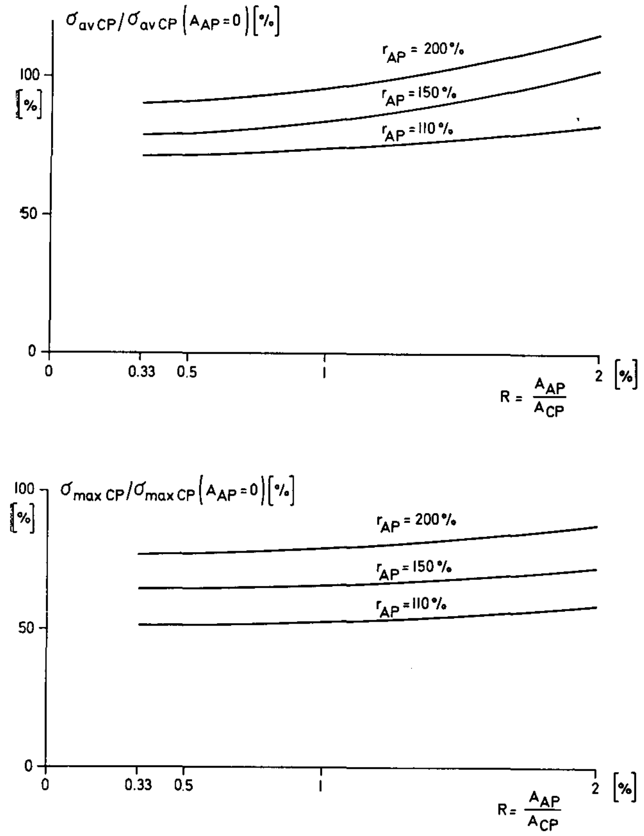


Fig. 3—The effect of the area of the annular pillar on the stress concentration at the central pillar for different pillar configurations

When the effect of an annular pillar on the stress concentration at the edge of the central pillar is considered, the differences between an individual shaft pillar and a subdivided shaft pillar are accentuated further. Annular pillars in the near vicinity of a central pillar reduce the maximum stresses at the edge of the central pillar by 40 to 50 per cent. This effect also decreases with increasing distance from the centre pillar, but annular pillars situated at a distance equal to the radius of the centre pillar are still effective.

The above discussion has indicated that the distance between the central pillar and the annular pillar is the most significant variable in such a system. In order to study the effect of the area of the annular pillar on the stress concentration at the central pillar Fig. 3 has been plotted.

This shows the relation between the area of the annular pillar, A_{AP}, and the stress concentration at the central pillar for different radii of the annular pillar. This graph shows clearly that the stress concentration at the centre pillar is affected little by the area of the annular pillar, as long as the annular pillar is sufficiently strong to carry the stresses imposed upon it. This result is equally valid for the average and the maximum stress concentrations at the central pillar.

The above analysis, which applies to circular shaft pillars in deep-level mining, can be summarized as follows:

(i) The sub-division of an individual circular shaft pillar into a central and an annular pillar is an effective means for reducing the average and maximum stresses at the central shaft pillar.

(ii) The distance between the central pillar and the annular pillar is the most important parameter in respect of reduction of the stress concentrations at the central pillar. The de-stressing effect of an annular pillar decreases with increasing distance.

(iii) The stress level at the central pillar is not influenced by the area of the annular pillar as long as this pillar is sufficiently strong to carry the stresses imposed on it.

Conclusions

The application of the principle of sub-dividing an individual shaft pillar into a central and an annular pillar leads to some important practical conclusions, *viz.* that annular pillars provide a means for reducing the total area of the shaft pillar. To show this for different pillar configurations Fig. 4 has been plotted. This shows the reduction in the total pillar area for different combinations of a central pillar and an annular pillar compared with that of an individual pillar, based on the assumption that the average stress concentrations at the individual shaft pillar and the central pillar remain the same.

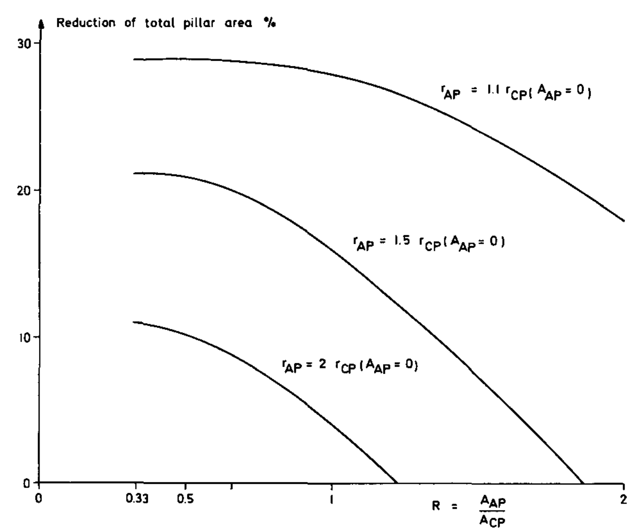


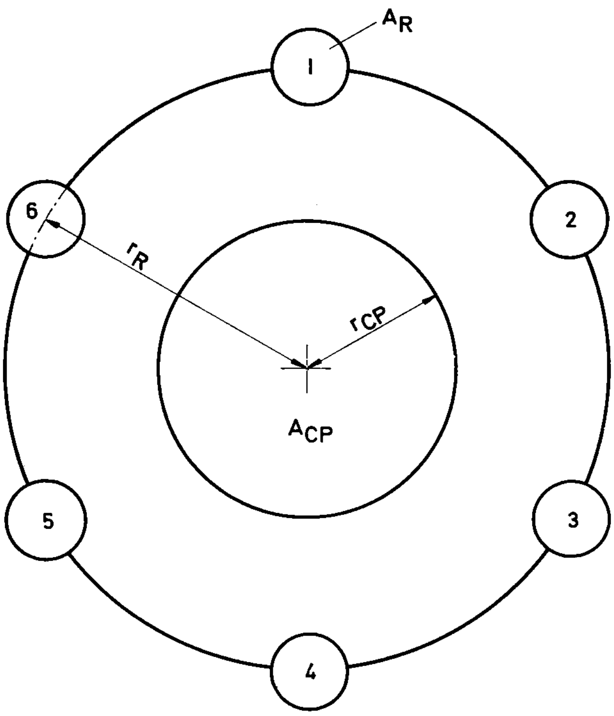
Fig. 4—The reduction in the total pillar area for different pillar configurations (based on the assumption of a constant average stress at the central pillar)

The sub-division of a shaft pillar into a central and an annular portion results in correspondingly higher stresses in the annular portion which must be so placed as not to affect installations in the vicinity of the shaft, or must be partially over- or under-stoped where installations, such as haulages, must pass close to it.

ADJACENT REMNANT PILLARS

Mining remnant pillars in the surrounding of shaft pillars is a fairly common problem in deep-level hard-rock mining. Nevertheless knowledge about the effect on the stability of shaft pillars of mining remnant pillars is still based on a number of adverse experiences. The development of the electrical resistance analogue and computer programmes now offers the means for predicting the effect of mining operations on the stability of shaft pillars, as has been demonstrated by the authors.

The idealized situation of a shaft pillar surrounded by six pillars of equal size has been investigated on the electrical resistance analogue to study the effect on the stress concentration at the shaft pillar of mining remnant pillars. In this investigation the ratio of the area of all six remnant pillars and the shaft pillar, as well as the distance between the remnant pillars and the shaft pillar, have been varied in a similar way to that already described for complete annular pillars (Fig. 5).



$$A_T = A_{CP} + 6A_R = \text{const.}$$

Variables: $R = \frac{6A_R}{A_{CP}}$, r_R

Number of remnants and mining sequence

Fig. 5—Idealized mining configuration for studying the effect of mining remnants on the shaft pillar

Six remnant pillars were chosen because this number offers a wide variety of different combinations of pillar extraction. It allows a detailed study to be made of the effects of an asymmetric mining of remnant pillars on the stress distribution at the shaft pillar.

When studying the effect on the stability of shaft pillars of mining remnant pillars, it is necessary to consider both the average stress and the stress distribution at the shaft pillar separately. While the average stress at the shaft pillar gives some general indications about the change in stress, it does not allow the position of dangerous stress concentrations resulting from asymmetric mining of remnant pillars to be located. In order to study this, curves for stress concentration along the axis of symmetry have been plotted for different pillar configurations.

The effect of mining remnant pillars on the average stress at the shaft pillar

Fig. 6 shows the increase in the average stress at a shaft pillar with the increase in number of extracted remnant pillars. The dotted line in this diagram indicates the theoretical increase in the average stress at an individual shaft pillar the area of which is reduced by the same amount as that of the extracted remnant pillars. The different stress concentrations for the extraction of remnants 2, 3 and 4 have been obtained for different sequences of mining these remnants (as indicated on Fig. 6).

From the results presented in this diagram the following observations can be made:

(i) The extraction of remnant pillars has little effect on the average stress at the shaft pillar, provided more than 30 per cent of the remnant pillars remain unmined. The extraction of the last remnant pillars increases the average stress at the shaft pillar considerably. This result agrees with practical experience, which indicates that the extraction of the last remnant pillars has the most adverse effect on the stability of a shaft pillar.

(ii) The sequence of mining remnant pillars has little effect on the average stress on the shaft pillar.

The effect of mining remnant pillars on the stress distribution at the shaft pillar

Though the above discussion has shown that the sequence of mining remnant pillars has little effect on the average stress concentration at the shaft pillar, the stress distribution at the latter might well be influenced by the sequence of mining operations. In order to show the effect of the sequence of operations on the stress distribution at the shaft pillar the relative change of the stress distribution along the axis of symmetry has been plotted in Fig. 7. This shows the relative change of the stress concentration along the axis of symmetry, X-Y, for two different sequences of extracting three remnant pillars. One gives the relative increase in stress for the situation of three remnant pillars mined next to each other, while the other curve has been obtained when only each second remnant pillar is mined.

The results of this study can be summarized as follows:

(i) The sequence of mining remnant pillars has a considerable effect on the stress distribution in a shaft pillar.

(ii) A concentrated mining of neighbouring remnant pillars leads to a high stress concentration at the edge of the shaft pillar next to the extracted pillars, while the opposite side of the shaft pillar is virtually unaffected. The increase in stress at the edge of the shaft pillar next to the extracted remnant pillar is about three times the increase of the average stress.

(iii) A distributed mining of remnant pillars reduces the high stress concentrations at the edge of the shaft pillar and results in a more uniform increase of the stress concentration.

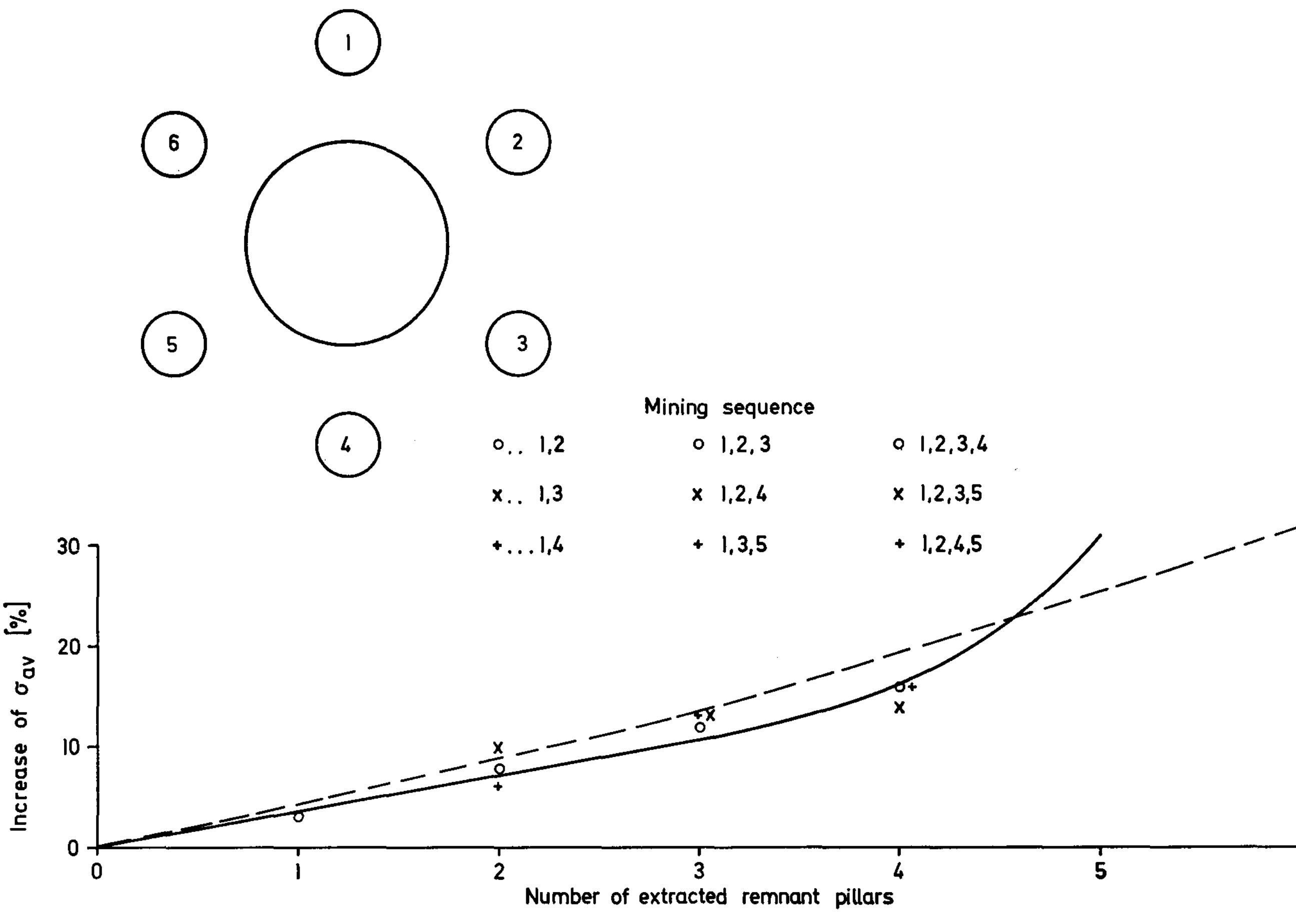


Fig. 6—The increase of the average stress concentration at the shaft pillar with the number of extracted remnant pillars

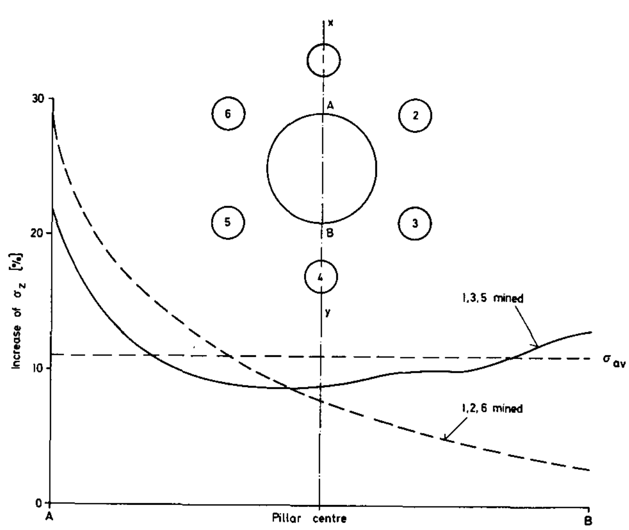


Fig. 7—The effect of the sequence of mining remnant pillars on the stress distribution at the shaft pillar

Conclusions

Based on these results some general conclusions can be drawn about the effect on the stability of shaft pillars of mining remnant pillars.

1. The extraction of remnant pillars has little effect on the stability of a shaft pillar, provided fewer than three-quarters of the remnant pillars are extracted.
2. A distributed extraction of remnant pillars results in a more uniform increase in the stresses at the shaft pillar and avoids the development of high local stress concentrations.
3. Concentrated mining of adjacent remnant pillars results in high stress concentrations at the edge of the pillar next to these pillars. The opposite side of the pillar is virtually unaffected by this mining operation.

Authors' Reply

The authors agree with Mr Ortlepp that the assumptions made when solving some of the problems confronting us tend to be oversimplified. However, we wish to stress that this approach is often the only feasible method with the facilities available. It is often necessary to find the best method of analysing a particular problem without necessarily describing the end conditions exactly. The 'case history' approach has helped us to establish how much we can rely on the results derived from such assumptions.

The 8 000 lb/in.^{2} criterion for haulage damage is not applicable to all areas in the Klerksdorp and Orange Free State gold fields. The criteria in the Klerksdorp area, for example, vary between 7 000 lb/in^2 and 9 000 lb/in.^{2} There are, however, some strata for which no criterion has been established and which clearly show no sign of damage at 14 000 lb/in.^{2} This figure seems to correspond very appropriately with the figure for 58 level haulage at E.R.P.M. The figure of 7 000 lb/in.^{2} stated above relates to a condition in argillaceous quartzites at which sidewall support becomes necessary. It does not describe the stress level at complete collapse. This figure can be raised considerably by the installation of adequate sidewall support as is shown in the paper. In some stratigraphic horizons in the O.F.S., square shaped haulages slab at field stresses of 6 000 lb/in.^{2} The effect of haulage shape and blasting technique on the stability of tunnels is currently under investigation.

Mr Ortlepp is correct in his assumption that a standard scale is used when measuring energy release rates, for the reason that it avoids a scaling factor and it is convenient for current stope face advances.

Mr Kotze's contribution clearly shows again the apparent close relationship between the magnitude of stress levels and the damage which occurs in off reef tunnels. Results from a recent project on the West Rand show that moderate sidewall damage can be expected in a haulage situated in gritty quartzitic rock at 8 000 lb/in.^{2}

Dr M. D. G. Salamon's appeal on behalf of the Rock Mechanics engineer is obviously of considerable interest to us. It has often come to our notice that experience in rock behaviour gained by managers has been lost to the industry when these managers have either been promoted or retired. The Rock Mechanics engineer will ensure that this experience is not lost. Because he acts in an advisory capacity for a number of mines his experience is not limited to a single source. The Rock Mechanics engineers of the various groups often consult one another on a particular problem and in this way increase their fund of knowledge.

The authors feel that the Rock Mechanics engineers now operating in the gold mining industry, have a sufficient background of experience which has been gathered over the past few years from which they can now draw to solve problems at a fraction of the original time and cost. Is the mining industry to lose this also?

The application of the elastic theory to the extraction of shaft pillars, as described by Mr C. L. de Jongh, has yet to be applied extensively. The theory can be beneficial for the siting of expansion joints in guides and pipes, immediately after sinking and immediately before removal of the shaft pillar. In designing the support during the extraction of the shaft pillar the theory can be invaluable. However, a check by means of measurement should be made where ever possible to ascertain the accuracy of the theoretical displacements calculated. Too little is known at present about the *in situ* elastic properties of rock to place complete reliance on theoretical results based solely on specimen tests in a laboratory. Both the authors are closely associated with several shaft pillar extraction programmes currently in progress.

Dr H. Wagner's contribution provides some interesting information on the effect of annular pillars on shaft pillar stresses. It may be of interest to mention that one of the authors has been involved in the design of a shaft pillar system and mining layout for a mine which is intended to work to depths greater than 7 600 ft where the mining heights will be of the order of seven ft. Because of the gold distribution in the reef and the faulted nature of ground in this lease area, it is unlikely that a longwall mining system can be planned.

Extensive studies have been made on the delineation of rib pillars for the long term stability of the workings of this mine. In the process of calculating the protection pillar for the shaft system it was discovered that a 2 000 ft diameter pillar would be inadequate to keep field stresses below the critical levels in some important excavations; however, if rib pillars are left *in situ* in a systematic manner, the shaft pillar size can be reduced by almost 50 per cent in area and still maintain the field stresses at an acceptable level.

The optimization of the spacing and dimensions of such rib pillars has been evaluated using the mining simulation programme referred to by Mr Pearson.

There must be several parallels to the work described by Dr Wagner in practice where sudden deterioration in conditions has occurred in a pillar for no apparent reason. Close investigation would show that somewhere within a couple of thousand feet of the pillar edge a remnant pillar has been stoped out. This action would have increased the span between abutments and considerably increased the stresses within abutments. This effect has often been noted on haulages below a solid block which forms one abutment of a stoping area.